One-Neutron Transfer Reactions in Even and Odd Tin Isotopes and a Realistic Nucleon-Nucleon Potential*

R. ALZETTA
Istituto di Fisica Teorica dell' Università, Trieste, Italy

AND

J. SAWICKI
International Centre for Theoretical Physics and Istituto di Fisica Teorica dell' Università, Trieste, Italy

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Low-lying energy levels and wave functions of the even and odd isotopes of tin ($A=116$–121) are calculated with 0-, 2-, and 4- and with 1- and 3-quasiparticle Tamm-Dancoff theories with Tabakin's realistic two-nucleon potential renormalized for core polarization by Gmitro *et al.* All the spectra are in good over-all agreement with experiment. The corresponding spectroscopic factors for the one-neutron stripping and pickup reactions are calculated and found to be in reasonable general agreement with the data of Schneid *et al.* and of Yagi *et al.* No adjustable parameters are involved in the theory (the single-particle input parameters are those of Bando).

I. INTRODUCTION

An important tool for analyzing details of microscopic nuclear wave functions is one- and two-nucleon transfer reactions. Spectroscopic structure factors (of fractional parentage) for one-nucleon transfer reactions (stripping and pickup) give information on states of both the even and the odd nucleus involved and on their most important "parentage" interrelation. The even and odd tin isotopes have recently received a great deal of attention both experimentally and theoretically as typical representatives of the so-called vibrational nuclei. In particular, successful descriptions of the spectra of nearly all the Sn isotopes have been obtained in terms of quasiparticle (qp) Tamm-Dancoff methods.[1–7] The qp configuration mixing must be limited to taking only the five valence neutron subshells explicitly into account. On the other hand, such a description can also be justified for all the low-lying states provided we take into account, through a renormalization of the residual nuclear force, the effects of the excited configurations of at least the most important subshells of the 50-50 core nucleons. The states of the odd isotopes can be understood as superpositions of one- and three-qp excitations[2] out of the qp vacuum $|0\rangle$, while those of the even isotopes are described in terms of the excitation of zero-, two-, and four-qp.[3–7] We call the respective approximations qp Tamm-Dancoff 13 (QTD 13), and qp second Tamm-Dancoff (QSTD). All the basic spurious kets due to the nucleon-number nonconservation in the qp formalism must be projected out before diagonalizing the corresponding secular matrices.

In Refs. 2 and 8 the spectroscopic factors for the one-neutron transfer reactions in Sn were calculated assuming the QTD 13 eigenvectors for the states of the odd Sn isotopes. The ground states of the corresponding even Sn isotopes were taken to be the qp vacuum $|0\rangle$ and simple one-qp [independent-qp-mode (IQM)] states—for the odd isotopes—in describing (d,p) reactions on odd isotopes. The excited final states of the corresponding even isotopes were described in the simple two-qp Tamm-Dancoff (QTD) approximation.

One of us[9] has generalized the method to systematically calculate the same spectroscopic factors with the QTD13 description for the odd Sn isotopes and with the QSTD description for the even Sn isotopes. The numerical calculations of Ref. 9 were, like those of Refs. 2, 3, and 8, limited to a simple, purely phenomenological Gaussian-Wigner two-body residual nuclear force.

It is the aim of the present paper to extend the calculations of Refs. 2, 8, and 9 to a realistic nucleon-nucleon potential renormalized for core polarization.[4] We have chosen to use the nonlocal potential of Tabakin[10] with the second-order core-polarization terms as calculated in Ref. 4 (the variant labelled "*S2*" in Ref. 4); the unperturbed single-particle (sp) energies are the same as those of Ref. 4 taken from Bando.[11] The renormalized Tabakin force of Ref. 4 gives rather satisfactory QTD and QSTD results for the spectra of

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[1] R. Arvieu, Ann. Phys. (Paris) 8, 407 (1963); R. Arvieu *et al.*, Phys. Letters 4, 119 (1963).

[2] T. T. S. Kuo, E. Baranger, and M. Baranger, Nucl. Phys. **79**, 513 (1966).

[3] P. L. Ottaviani, M. Savoia, J. Sawicki, and A. Tomasini, Phys. Rev. **153**, 1138 (1966); P. L. Ottaviani, M. Savoia and J. Sawicki, Phys. Letters **24B**, 353 (1967); A. Rimini, J. Sawicki, and T. Weber, Phys. Rev. **168**, 1401 (1968); P. L. Ottaviani, M. Savoia, and J. Sawicki, Nuovo Cimento **56B**, 149 (1968).

[4] M. Gmitro, J. Hendekovič, and J. Sawicki, Phys. Letters **26B**, 252 (1968); Phys. Rev. **169**, 983 (1968); M. Gmitro and J. Sawicki, Phys. Letters **26B**, 493 (1968).

[5] A. Rimini, J. Sawicki, and T. Weber, Phys. Rev. Letters **20**, 676 (1968).

[6] M. K. Pal, Y. K. Gambhir, and Ram Raj, Phys. Rev. **155**, 1144 (1967); Ram Raj, Y. K. Gambhir, and M. K. Pal, *ibid.* **163**, 1004 (1967).

[7] L. S. Hsu, Nucl. Phys. **A96**, 624 (1967).

[8] E. Baranger and T. T. S. Kuo, Nucl. Phys. **A97**, 289 (1967).

[9] R. Alzetta, Nuovo Cimento (to be published).

[10] F. Tabakin, Ann. Phys. (N. Y.) **30**, 51 (1964).

[11] H. Bando, in Proceedings of the International Conference on Nuclear Structure, Tokyo, 1967 (unpublished); and Progr. Theoret. Phys. (Kyoto) **38**, 1285 (1967).

Sn^{116} and Sn^{120} (except that the 2_1^+ state lies slightly too high). In the present calculations we have used the QSTD eigenvectors of Sn^{116} of Refs. 4 and 12, but the ground-state ($|0_1^+\rangle$) eigenvector is modified (and other $|0_n^+\rangle$ vectors are only slightly modified). In fact, the spurious states $|\Psi_{sp4}\rangle$ of Refs. 3 and 4, at least with our particular choice of the sp basis and of the pairing force, have large vacuum $|0\rangle$ components and thus, when projected out of the 0^+ QSTD secular matrix, cause an appreciable depletion of the $|0\rangle$ component in $|0_1^+\rangle$. This indicates the possibility of a similar situation in the $J^\pi \neq 0^+$ states when the higher-order spurions $|\Psi_{sp6}\rangle$ involving six-qp components are projected out in a six-qp (0, 2, 4, and 6) Tamm-Dancoff calculation. Namely, the projection of such spurions may significantly deplete the two-qp components of the corresponding physical eigenvectors. This question is still open and under investigation. It is also evident that the unprojected Bardeen-Cooper-Schrieffer (BCS) ground state is not a very satisfactory starting point (reference ground state) for microscopic spectroscopy under the circumstances. An exactly particle-number-conserving model with the same dimensions of QSTD becomes highly desirable.[13] In our QSTD case, in the situation described above, one would have a model in which the theory of the $J^\pi \neq 0^+$ states would be somehow divorced from that of the QSTD 0^+ states. On the other hand, it is just the interrelations between the $|0_1^+\rangle$ state and all the other states $J^\pi \neq 0^+$ of both even and odd isotopes which are of main interest for our theory of nuclear reactions. In particular, in our present applications we have the QTD13 eigenvectors for the states of the odd Sn isotopes in which the spurions $|\Psi_{sp5}\rangle$ containing one-, three-, and five-qp components could have appreciable one-qp components, thus being able to deplete such components in a five-qp Tamm-Dancoff theory, rendering them substantially different from our QTD13 eigenvectors. This question also is still open and under investigation. Here again an exactly number-conserving QTD13 theory (of the QTD13 dimensions) would be highly desirable.

Part of the blame for such an uncomfortable situation in a given particular case (nucleus, sp input parameters, nuclear force, etc.) may be due to the lack of the Hartree-Fock-Bogolubov (HFB) self-consistency.

In this situation we have chosen to project out the spurions

$$|\Psi_{sp4}'\rangle = \mathfrak{N}' A_{JM}^\dagger(cc')(\hat{N}-N_0)|0\rangle, \qquad (1)$$

where $A_{JM}^\dagger(cc')$ is a normalized two-qp creation operator of spin J, $\hat{N}$ and N_0 are the nucleon-number operator and its correct eigenvalue, respectively, and $\mathfrak{N}'$ is the normalization constant. These spurions differ from $|\Psi_{sp4}\rangle$ of Refs. 3 and 4 by the absence of the vacuum ($|0\rangle$) components. Such a (somewhat arbitrary) prescription for spurion projection (definition of four-qp spurions) has been implicitly applied in Refs. 6 and 7. This variant of QSTD involves, then, the ansatz that a given particular BCS solution is essentially almost nonspurions and that the only four-qp correlations in $|0_1^+\rangle$ are due to the dynamical effect of the H_{40} term of the qp-transformed interaction Hamiltonian H_{int}. Although this ansatz is, at least in principle, unjustified, it has the advantage of a unique, coherent description of all the 0^+ and other QSTD states and of their relation to the QTD13 states of the odd isotopes. Moreover, the spurious effects of the vacuum $|0\rangle$ itself (such as of the fluctuation of $\hat{N}^2-N_0^2$, etc.) may have only very little influence on spectroscopic factors for nucleon transfer reactions in this variant where the spurions in QTD13 and in QSTD are both treated on the same footing. Clearly, the whole question of whether or not to project out the $|0\rangle$ component of $|\Psi_{sp4}\rangle$ in QSTD exclusively concerns the 0^+ states and, in practice, only the $|0_1^+\rangle$ eigenvector. In the Appendix we compare our spectroscopic factors computed with $\langle 0|\Psi_{sp4}'\rangle = 0$ with those which result when $|\Psi_{sp4}\rangle$ is used ($\langle 0|\Psi_{sp4}\rangle \neq 0$), and we find only small numerical differences between the two variants; agreement with experiment is sometimes even better with $\langle 0|\Psi_{sp4}\rangle \neq 0$.

In the expressions derived in Ref. 9 for the spectroscopic factors there are, in addition to the (0qp-1qp) and (1qp-2qp) terms calculated in Refs. 2 and 8, also the terms (2qp-3qp) and (3qp-4qp).

We compute the QTD13 and QSTD eigenvectors for the renormalized Tabakin force, and then the corresponding spectroscopic factors utilizing the formulas of Ref. 9 for the following stripping and pickup reactions:

$$Sn^{116}(d,p)Sn^{117}, \quad Sn^{118}(p,d)Sn^{117},$$
$$Sn^{118}(d,p)Sn^{119}, \quad Sn^{120}(d,t)Sn^{119},$$
$$Sn^{120}(d,p)Sn^{121}$$

(the first five levels of the final nucleus), and

$$Sn^{117}(d,p)Sn^{118}, \quad Sn^{117}(p,d)Sn^{116},$$
$$Sn^{119}(d,p)Sn^{120}, \quad (0_{1,2,3}^+, 2_{1,2,3}^+).$$

Our results for the spectra of QDT13 and QSTD and for the spectroscopic factors are compared with the recent experimental data of Schneid *et al.*[14] and of Yagi *et al.*[15]

II. EIGENSTATES OF QTD13 AND QSTD

The computation of QTD13 eigenvalues and eigenvectors for the odd tin isotopes follows exactly the method of Kuo *et al.*[2] The QSTD results obtained for the same renormalized Tabakin force of Ref. 4 are

[12] M. Gmitro, A. Rimini, J. Sawicki, and T. Weber, International Center for Theoretical Physics, Trieste, Report No. IC/68/29 (unpublished); and Phys. Rev. **173**, 964 (1968).

[13] For the idea of such a method see M. H. MacFarlane, *1965 Lectures in Theoretical Physics* (University of Colorado Press, Boulder, Colo., 1966), p. 583.

[14] E. J. Schneid, A. Prakash, and B. L. Cohen, Phys. Rev. **156**, 1316 (1967).

[15] K. Yagi *et al.*, Nucl. Phys. **A111**, 129 (1968).

exactly the same as in Ref. 4 except that the 0^+ states are now computed with the spurions $|\Psi_{sp4}'\rangle$ rather than with $|\Psi_{sp4}\rangle$ (no $|0\rangle$ components in the new four-qp spurions of $J^\pi=0^+$).[12]

For the odd Sn isotopes 117, 119, and 121, we give our QTD13 excited energy levels in Table I and Figs. 1–3. The three-qp total (%) weights in the corresponding eigenvectors are given in parentheses. The general over-all agreement with the data of Refs. 14–16 is reasonably good, given the fact that there are no *ad hoc* adjustable parameters in the present calculation. From the comparison with the QTD13 results of Kuo *et al.*[2] given in Figs. 1–3, we see that they are as good as ours in spite of the fact that they use a purely phe-

TABLE I. The lowest-lying QTD13 energy levels (in MeV) calculated for tin isotopes with $A=117$, 119, and 121. The total three-qp weights (in%) of the corresponding eigenvectors are given in parentheses. The corresponding experimental levels of Schneid *et al.*[a] are also given for comparison.

117			119			121		
	QTD 13	Expt.[a]		QTD 13	Expt.[a]		QTD 13	Expt.[a]
$(\frac{1}{2})_1^+$	0.0 (5.6)	0.0	$(\frac{1}{2})_1^+$	0.0 (5.1)	0.0	$(\frac{3}{2})_1^+$	0.0 (3.2)	0.0
$(\frac{11}{2})_1^-$	0.003(2.0)	0.32	$(\frac{3}{2})_1^+$	0.09(3.6)	0.024	$(\frac{11}{2})_1^-$	−0.14(0.9)	0.05
$(\frac{3}{2})_1^+$	0.17 (4.0)	0.16	$(\frac{11}{2})_1^-$	−0.08(1.3)	0.08	$(\frac{1}{2})_1^+$	0.01(4.6)	0.05
$(\frac{7}{2})_1^+$	0.48 (4.7)	0.72	$(\frac{7}{2})_1^+$	0.62(5.8)	0.79	$(\frac{7}{2})_1^+$	0.74(7.2)	0.93
$(\frac{5}{2})_1^+$	0.86 (4.8)	1.03	$(\frac{5}{2})_1^+$	1.02(6.7)	0.93	$(\frac{5}{2})_1^+$	1.11(76.7)	1.12

[a] Reference 14.

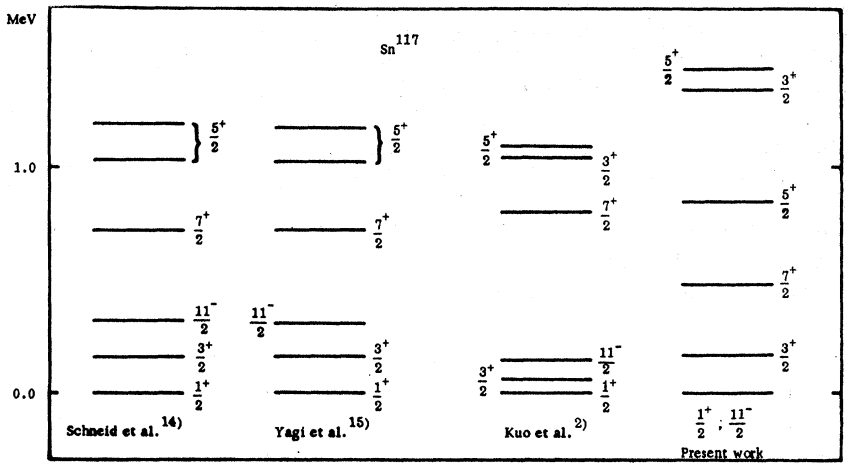

FIG. 1. Comparison of QTD13 energy levels for Sn^{117} with the experimental data of Refs. 14 and 15 and the values obtained by Kuo *et al.* (Ref. 2).

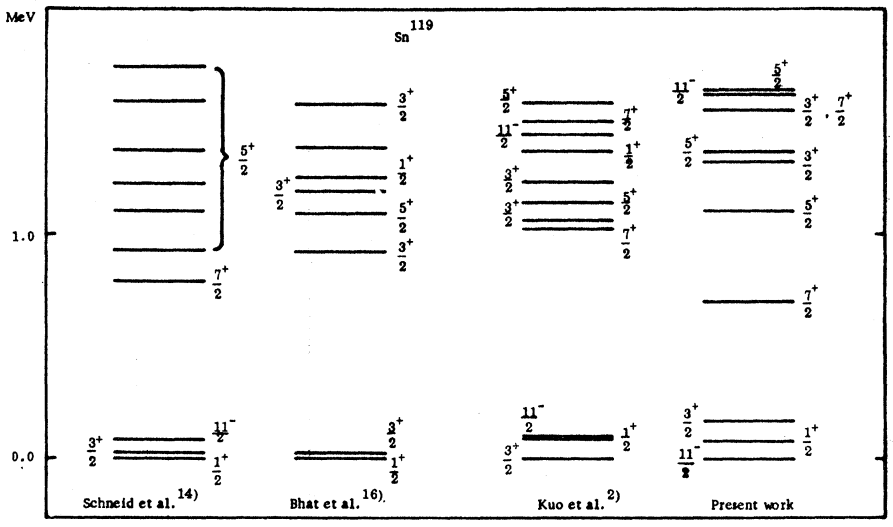

FIG. 2. Comparison of QTD13 energy levels for Sn^{119} with the experimental data of Refs. 14 and 16 and the values obtained by Kuo *et al.* (Ref. 2).

[16] M. R. Bhat *et al.*, Phys. Rev. **166**, 1111 (1968).

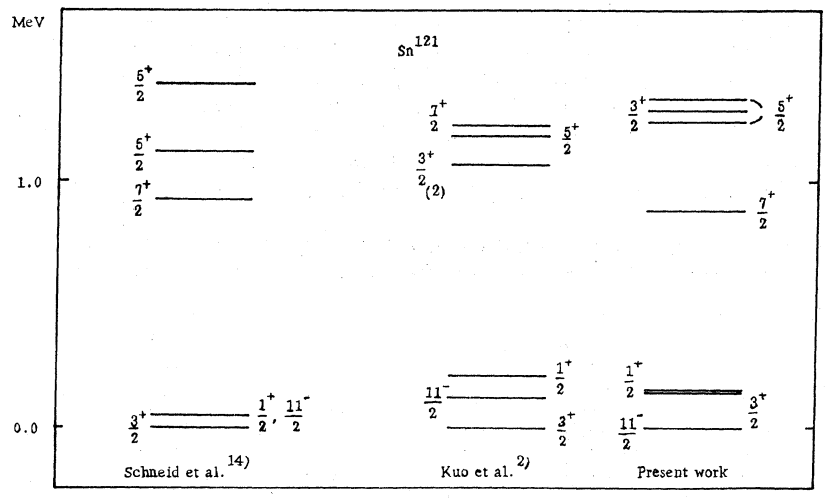

FIG. 3. Comparison of QTD13 energy levels for Sn^{121} with the experimental data of Ref. 14 and with the levels calculated by Kuo *et al.* (Ref. 2).

nomenological nucleon-nucleon interaction with parameters adjusted on odd-even mass differences and other data. The QSTD eigenvalues for the states $0_{1,2,3}^+$ and $2_{1,2,3}^+$ of $Sn^{116,118,120}$ are given in Figs. 4–6 and are compared with the data of Refs. 14 and 15. Again, as in Ref. 4, a generally good agreement with experiment is obtained, the best being for the 120 isotope. The four-qp percentages are generally small, much smaller than those of Ref. 3 based on the Gaussian-Wigner force of Kuo *et al.*[2] One has to exclude the "two-phonon" interpretation of such states where a "phonon" is defined as a low-lying collective QTD mode.

III. SPECTROSCOPIC FACTORS FOR ONE-NEUTRON TRANSFER REACTIONS

The spectroscopic factor for a (d,p) reaction on an even isotope, $S_{\mathcal{J}}{}^{(+)}(0_1{}^+,\mathcal{J}_n{}^\pi)$, is defined as

$$S_{\mathcal{J}}{}^{(+)}(0_1{}^+,\mathcal{J}_n{}^\pi)=|\langle\mathcal{J}_n{}^\pi(\mathrm{A}+1)\|c_{\mathcal{J}}{}^\dagger\|0_1{}^+(A)\rangle|^2, \quad (2)$$

where $c_\alpha{}^\dagger$ creates a neutron in the sp state α, $|0_1{}^+(A)\rangle$ is the QSTD ground state of the target (A) nucleus, and $|\mathcal{J}_n{}^\pi(A+1)\rangle$ is the final QTD13 state of the odd isotope.

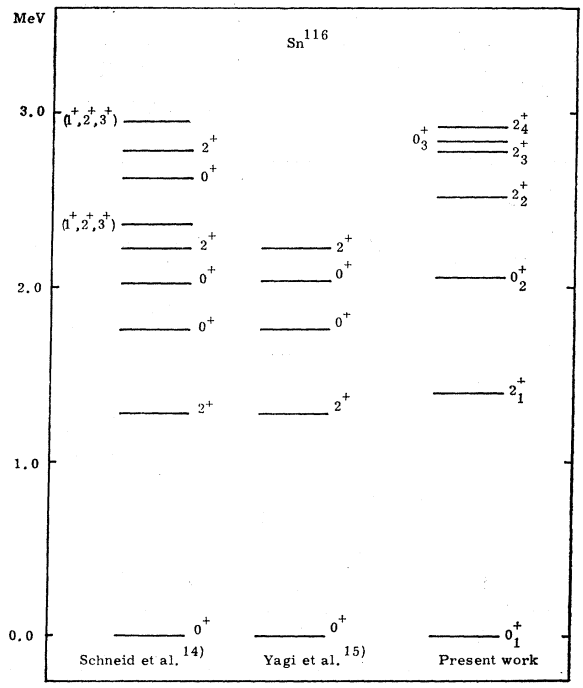

FIG. 4. Comparison of the QSTD energy levels 0^+ and 2^+ of Sn^{116} with the experimental data of Refs. 14 and 15.

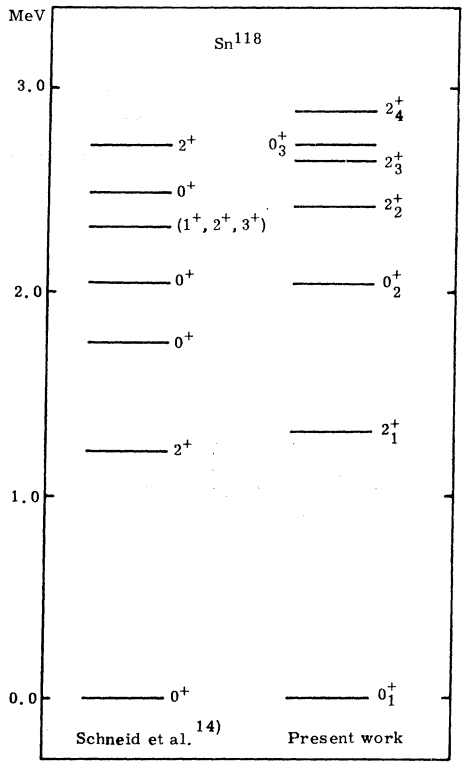

FIG. 5. Comparison of the QSTD energy levels 0^+ and 2^+ of Sn^{118} and the experimental levels of Ref. 14.

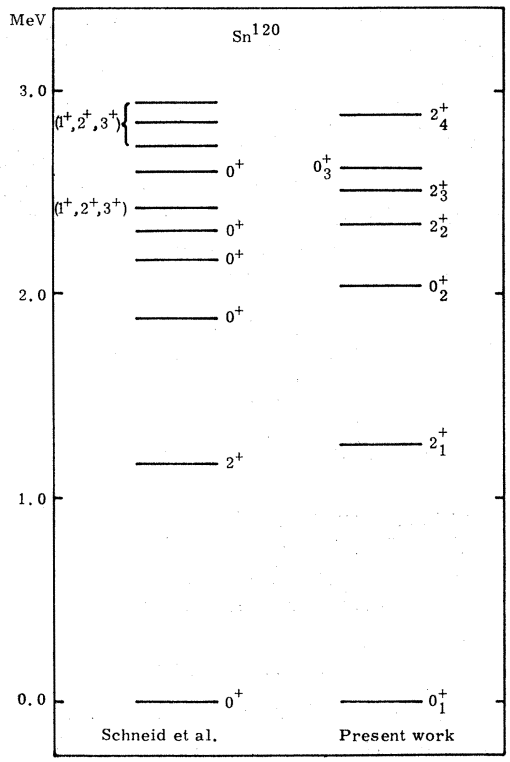

FIG. 6. Comparison of the QSTD energy levels 0^+ and 2^+ of Sn^{120} with the data of Ref. 14.

The detailed explicit expressions for the matrix element $\langle\|c^\dagger\|\rangle$ are given in Ref. 9. A quite similar expression defines the spectroscopic factor $S_{\mathcal{J}}{}^{(-)}(\mathcal{J}_n{}^\pi,0_1{}^+)$ for the pickup reaction on the same (A) even target to the odd $(A-1)$ isotope. In fact, $S_{\mathcal{J}}{}^{(-)}(\mathcal{J}_n{}^\pi,0_1{}^+)$ can be derived from the corresponding $S_{\mathcal{J}}{}^{(+)}(0_1{}^+,\mathcal{J}_n{}^\pi)$ of Eq. (2) by a simple transformation; explicit expressions for this case are given in Eqs. (2), (5), and (5′) of Ref. 9.

TABLE II. Calculated spectroscopic factors for stripping and pickup reactions on the even tin isotopes with $A=116$, 118, and 120 leading to the lowest-lying states (with spin-parity $\mathcal{J}^\pi$) of the corresponding residual odd-mass nuclei. Experimental values of Schneid *et al.*[a] and Yagi *et al.*[b] are also listed for comparison.

$Sn^{116}(d,p)Sn^{117}(\mathcal{J}_n{}^\pi)$			$Sn^{118}(p,d)Sn^{117}(\mathcal{J}_n{}^\pi)$	
$\mathcal{J}_n{}^\pi$	$S_{\mathcal{J}}{}^{(+)}(0_1{}^+,\mathcal{J}_n{}^\pi)$	Expt.[a]	$S_{\mathcal{J}}{}'(\mathcal{J}_n{}^\pi,0_1{}^+)$	Expt.[b]
$(\frac{1}{2})_1{}^+$	0.55	0.65	0.45	0.55
$(\frac{3}{2})_1{}^+$	0.68	0.55	0.31	0.50
$(\frac{11}{2})_1{}^-$	0.67	0.81	0.37	0.26
$(\frac{7}{2})_1{}^+$	0.31	0.13	0.67	0.96
$(\frac{5}{2})_1{}^+$	0.12	0.061	0.84	0.53
$Sn^{118}(d,p)Sn^{119}(\mathcal{J}_n{}^\pi)$			$Sn^{120}(d,t)Sn^{119}(\mathcal{J}_n{}^\pi)$	
$\mathcal{J}_n{}^\pi$	$S_{\mathcal{J}}{}^{(+)}(0_1{}^+,\mathcal{J}_n{}^\pi)$	Expt.[a]	$S_{\mathcal{J}}{}'(\mathcal{J}_n{}^\pi,0_1{}^+)$	
$(\frac{1}{2})_1{}^+$	0.47	0.59	0.54	
$(\frac{3}{2})_1{}^+$	0.63	0.52	0.36	
$(\frac{11}{2})_1{}^-$	0.59	0.56	0.46	
$(\frac{7}{2})_1{}^+$	0.26	0.14	0.71	
$(\frac{5}{2})_1{}^+$	0.09	0.006	0.84	
$Sn^{120}(d,p)Sn^{121}(\mathcal{J}_n{}^\pi)$				
$\mathcal{J}_n{}^\pi$	$S_{\mathcal{J}}{}^{(+)}(0_1{}^+,\mathcal{J}_n{}^\pi)$	Expt.[a]		
$(\frac{3}{2})_1{}^+$	0.58	0.43		
$(\frac{1}{2})_1{}^+$	0.39	0.39		
$(\frac{11}{2})_1{}^-$	0.51	0.21		
$(\frac{7}{2})_1{}^+$	0.21	0.19		
$(\frac{5}{2})_1{}^+$	0.02	0.065		

[a] Reference 14.
[b] Reference 15.

Our final results for these cases are summarized in Table II and in Figs. 7 and 8. We use the notation

$$S_{\mathcal{J}}{}'(\mathcal{J}_n{}^\pi,0_1{}^+)=\hat{\mathcal{J}}^{-2}S_{\mathcal{J}}{}^{(-)}(\mathcal{J}_n{}^\pi,0_1{}^+).$$

As seen from Table II and from Figs. 7 and 8, the general agreement with the data of Refs. 14 and 15 is generally good, even somewhat better than that of Refs. 2 and 8. The data on the pickup reaction with

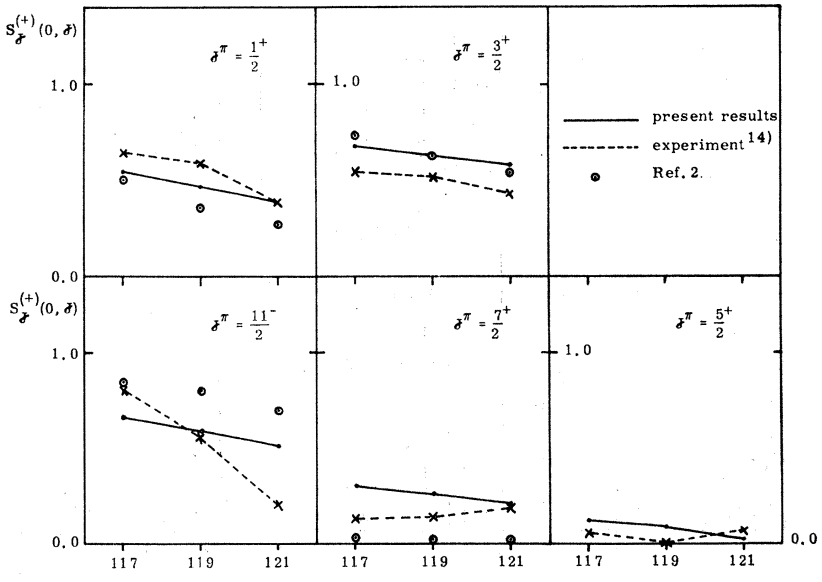

FIG. 7. Spectroscopic factors $S_{\mathcal{J}}{}^{(+)}(0_1{}^+,\mathcal{J}^\pi)$ for (d,p) reactions on the even tin targets with $A=116$, 118, 120 leading to the lowest state of spin $\mathcal{J}^\pi$ of the odd-mass isotopes. The present results are connected by a solid line. The experimental values (Ref. 14) are connected by a dashed line, and the results of Kuo *et al.* (Ref. 2) are indicated by ⊙.

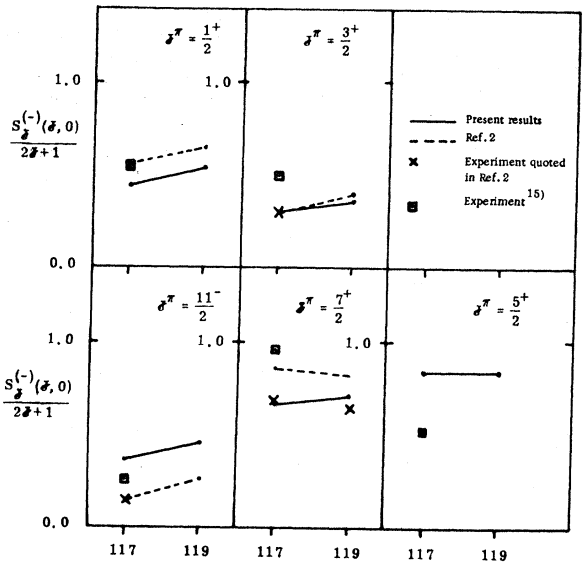

FIG. 8. Spectroscopic factors $Sg'(\mathcal{J}^\pi,0_1^+)$ for pickup reactions on the even tin targets with $A=118$ and 120 leading to the lowest state of spin $\mathcal{J}^\pi$ of the odd-mass isotopes. The present results are connected by a solid line; those obtained in Ref. 2 are connected by a dashed line; the data of Ref. 15 for Sn^{118} (p,d) Sn^{117} are marked ⊡; and the data for (d,t) reactions quoted in Ref. 2 are marked ×.

Sn^{119} in the final state refer to the reaction $Sn^{120}(d,t)Sn^{119}$ as quoted in Ref. 2 rather than to the corresponding (p,d) reaction.

A second series of our calculations concerns odd targets and even isotopes in the final states. In Fig. 9 we compare our calculated spectroscopic factors $S_{l=0}'(\frac{1}{2}_1^+,0_{1,2,3}^+)$ and $S_{l=2}'(\frac{1}{2}^+,2_{1,2,3}^+)$ for the $Sn^{117,119}(d,p)$ $Sn^{118,120}$ $(0_{1,2,3}^+,2_{1,2,3}^+)$ reactions with the corresponding data of Ref. 14. Again a reasonable over-all agreement with experiment is obtained. In Fig. 10 our calculated $S_{l=0}^{(-)}(0_{1,2,3}^+,\frac{1}{2}_1^+)$ and $S_{l=2}^{(-)}(2_{1,2,3}^+,\frac{1}{2}_1^+)$ for the pickup reactions $Sn^{117}(p,d)Sn^{116}$ $(0_{1,2,3}^+,2_{1,2,3}^+)$ are compared with the data of Ref. 15. A reasonable semiquantitative agreement with experiment is obtained.

The l-spectroscopic factors are here defined as

$$S_l'(\mathcal{J},J)=\frac{2J+1}{2\mathcal{J}+1}\sum_{j=l-1/2}^{l+1/2}S_j^{(+)}(\mathcal{J},J) \tag{3}$$

and

$$S_l^{(-)}(J,\mathcal{J})=\sum_{j=l-1/2}^{l+1/2}S_j^{(-)}(J,\mathcal{J}). \tag{4}$$

Explicit formulas for $S_j^{(+)}(\mathcal{J},J)$ and $S_j^{(-)}(J,\mathcal{J})$ are given in Eqs. (2), (6′), and (7′) of Ref. 9.

In Table III we give the exact numerical values of the calculated spectroscopic factors of Figs. 9 and 10.

IV. CONCLUSIONS

There are rather considerable uncertainties in the "experimental" spectroscopic factors for the one-nucleon stripping and pickup reactions, since the extraction of such factors from the data involves some assumptions about the reaction mechanism and distorted-wave Born-approximation (DWBA) amplitudes which are far from being well founded and understood.

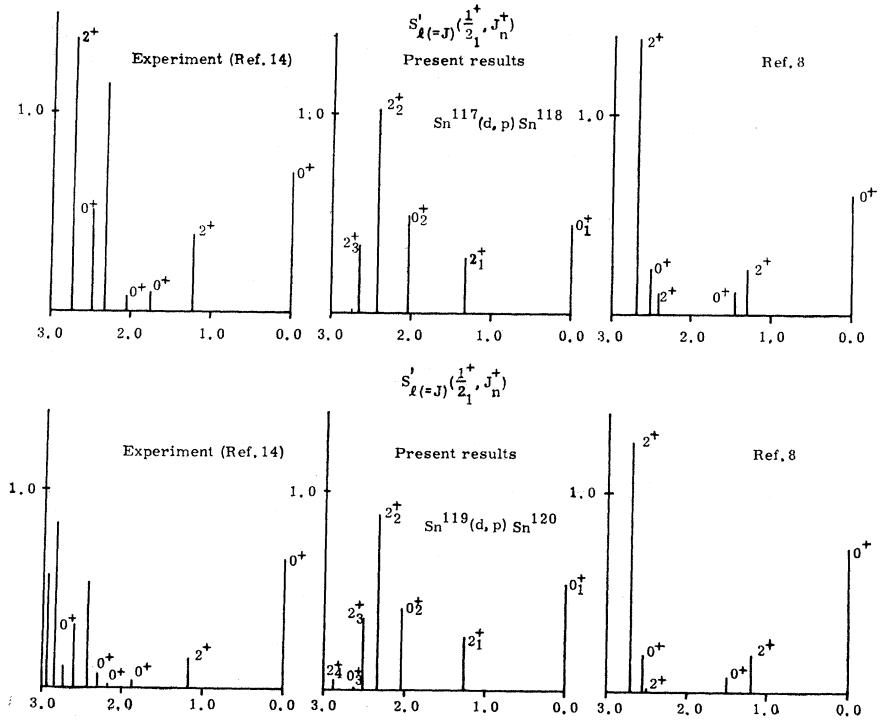

FIG. 9. Spectroscopic factors $S_{l(=J)}'(\frac{1}{2}_1^+,J_n^+)$ for (d,p) reactions on the odd tin targets with $A=117$ and 119 leading to the states of Sn^{118} and Sn^{120}, respectively, with $J^\pi=0^+$ and 2^+ up to 3 MeV of excitation energy. Present results are plotted in the center, results of Ref. 8 to the right. Among the experimental data of Ref. 14 plotted on the left, the states not labelled explicitly have an ambiguous $(1^+,2^+,3^+)$ spin assignment.

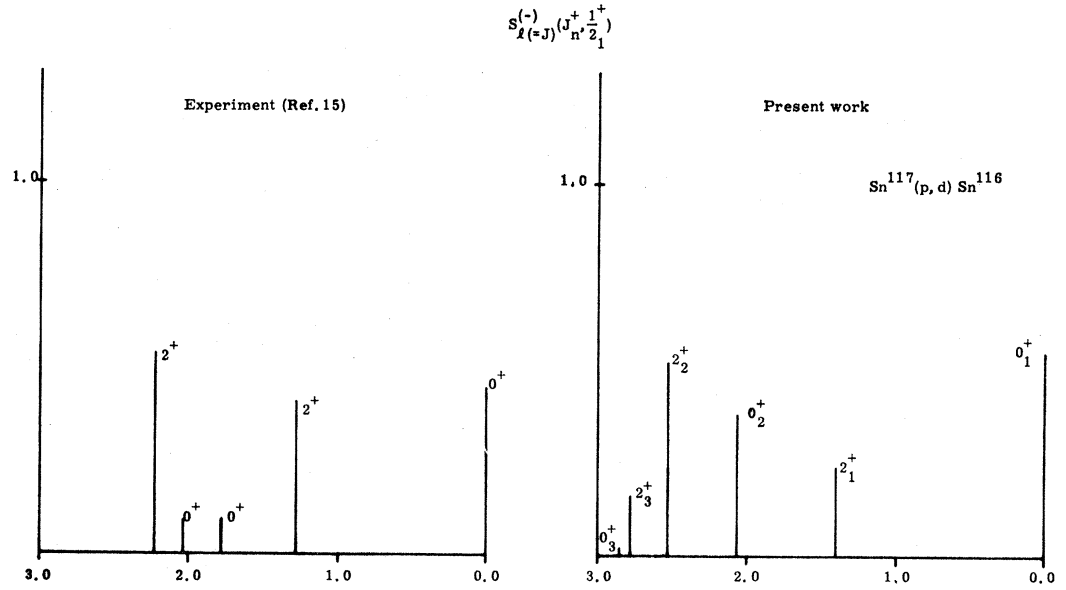

FIG. 10. Spectroscopic factors $S_{l(=J)}{}^{(-)}(J_n^+, \frac{1}{2}_1^+)$ for the (p,d) reactions on Sn^{117}, leading to the states of Sn^{116} with $J^\pi = 0^+$ and 2^+ up to 3-MeV excitation. Present results are plotted to the right and the experimental values of Ref. 15 to the left.

One should then be rather careful not to attach too much importance to a detailed quantitative agreement of calculated and observed spectroscopic factors. Still, it is gratifying to see that, *in addition* to fitting reasonably well the observed spectra of both the even and the odd isotopes of Sn, the calculated spectroscopic factors are generally consistent with those extracted from the stripping and pickup experiments. An analysis was previously performed for the two-neutron transfer reaction in even tin isotopes[17] [although the spurious parts of $|0\rangle$ are removed in the QSTD 0^+ states in Ref. 17, in contrast to the present calculation; it would be interesting to perform calculations on $Sn(t,p)$ and $Sn(p,t)$ reactions with the 0^+ QSTD eigenvectors as defined in the present paper and to compare the spectroscopic factors both with those of Ref. 17 and with experiment]. The comparison of our results for the one-neutron transfer reactions obtained with $\langle 0|\Psi_{sp4}'\rangle = 0$ with the corresponding ones calculated with $\langle 0|\Psi_{sp4}\rangle \neq 0$ given in the Appendix shows that numerical differences between the two variants are generally small. This shows that our results are not appreciably affected by the uncertainty in the treatment of the four-qp QSTD spurions.

TABLE III. Calculated spectroscopic factors for stripping and pickup reactions on odd tin targets with $A = 117$ and 119 leading to the first three 0^+ and 2^+ states in the corresponding residual even-mass isotopes.

J_n^+	$Sn^{117}(d,p)Sn^{118}$ $S_{l(=J)}'(\frac{1}{2}_1^+, J_n^+)$	$Sn^{119}(d,p)Sn^{120}$ $S_{l(=J)}'(\frac{1}{2}_1^+, J_n^+)$	$Sn^{117}(p,d)Sn^{116}$ $S_{l(=J)}{}^{(-)}(J_n^+, \frac{1}{2}_1^+)$
0_1^+	0.45	0.54	0.55
0_2^+	0.49	0.41	0.38
0_3^+	0.01	0.01	0.02
2_1^+	0.28	0.27	0.24
2_2^+	1.02	0.88	0.52
2_3^+	0.34	0.36	0.16

The realistic residual interaction potential used in our nuclear-structure calculation involves no adjustable parameter either in its "bare" or its core-polarization parts of the matrix elements. All the parameters involved in our theory are "externally" determined.

A calculation in which the single-qp input parameters are determined directly from the experimental level spacings of odd Sn isotopes by a combination of the inverse gap equations and QTD13 is now in preparation in collaboration with Gambhir.[18] Since there exist contaminations of our eigenvectors by higher-order spurious states due to the nucleon-number nonconservation and some ambiguities in the methods of projecting out the basic spurions, calculations based on exactly number-projected Tamm-Dancoff theories would be highly desirable. Unfortunately, such calculations will be much more complicated than the present ones. One calculation along this line by the Trieste group is already in preparation.

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APPENDIX: COMPONENT OF QUASIPARTICLE VACUUM IN FOUR-QP 0^+ QSTD SPURIONS

We compare the spectroscopic factors for the one-neutron stripping and pickup reactions on even and odd isotopes of tin calculated with the QSTD 0^+ eigenvectors obtained after projecting out the four-qp spurions $|\Psi_{sp4}'\rangle$ of Eq. (1) with those which result after projecting out the "usual" spurions $|\Psi_{sp4}\rangle$ of Eq. (22) of Ref. 3, i.e., including the qp-vacuum $|0\rangle$ components ($\langle 0|\Psi_{sp4}\rangle \neq 0$). From the comparison of the 0^+ QSTD energy levels of the present paper with those of Ref. 4 one can see that they are almost identical. The only difference is a very small depletion of the $|0\rangle$ component in $|0_1^+\rangle$ of the present paper.

In Table IV we compare the data of Refs. 14 and 15, respectively, with the spectroscopic factors $S_{\mathcal{J}}{}^{(+)}(0_1^+, \mathcal{J}_n{}^\pi)$ for the reaction $Sn^{116}(d,p)Sn^{117}$ and $S_{l=0}{}^{(-)}(0_n^+, \frac{1}{2}1^+)$ for the reaction $Sn^{117}(p,d)Sn^{116}$ calculated with the $|0_n^+\rangle$ eigenvectors of Sn^{116} corresponding to the two definitions of the four-qp spurions. We see that with the $|\Psi_{sp4}\rangle$ definition of the four-qp spurions ($\langle 0|\Psi_{sp4}\rangle \neq 0$) the calculated spectroscopic factors are in good over-all agreement with the data—not worse than that of the factors calculated with $|\Psi_{sp4}'\rangle(\langle 0|\Psi_{sp4}'\rangle = 0)$; in some cases of $\mathcal{J}_n{}^\pi$ the agreement is even better when one calculates with $|\Psi_{sp4}\rangle$.

TABLE IV. Comparison of the experimental spectroscopic factors of Schneid *et al.*[a] and Yagi *et al.*[b] for the reactions $Sn^{116}(d,p)$-Sn^{117} and $Sn^{117}(p,d)Sn^{116}$, with the spectroscopic factors calculated using the two different definitions of the four-qp spurions discussed in this paper.

$\mathcal{J}_n{}^\pi$	QSTD ($\langle 0\|\Psi_{sp4}'\rangle = 0$)	QSTD ($\langle 0\|\Psi_{sp4}\rangle \neq 0$)	Expt.	Reaction
$\frac{1}{2}1^+$	0.55	0.36	0.65	
$\frac{3}{2}1^+$	0.68	0.50	0.55	
$\frac{11}{2}1^-$	0.67	0.46	0.81	$Sn^{116}(d,p)Sn^{117}$
$\frac{7}{2}1^+$	0.31	0.16	0.13	
$\frac{5}{2}1^+$	0.12	0.07	0.061	
0_1^+	0.55	0.36	0.45	
0_2^+	0.38	0.38	0.087	$Sn^{117}(p,d)Sn^{116}$
0_3^+	0.02	0.02	0.092	

[a] Reference 14.
[b] Reference 15.

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High-Resolution Gamma-Ray Spectroscopic Study of the Decay $^{133}Ba \rightarrow {}^{133}Cs$†

D. P. DONNELLY, J. J. REIDY, AND M. L. WIEDENBECK
Department of Physics, University of Michigan, Ann Arbor, Michigan 48104

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The decay of ^{133}Ba to levels in ^{133}Cs was studied using Ge(Li) γ-ray spectrometers. The energies of the measured γ-ray transitions are 53.14(8), 160.68(10), 223.08(10), 276.45(8), 302.87(8), 355.99(7), and 383.92(7) keV. The corresponding relative γ-ray intensities of these transitions are 3.16(39), 0.99(10), 0.72(8), 11.62(82), 29.4(21), 100, and 14.3(10). γ-ray spectra in coincidence with the 53.1-, 79.6-, 81.0-, 276.4-, 302.9-, 356.0-, and 383.9-keV γ rays were obtained. The data are in quantitative agreement with the existing decay scheme. The K-shell internal-conversion coefficients were determined, using the present γ-ray relative intensities and published K-shell internal-conversion-electron relative intensities. These results and earlier lifetime measurements and mixing ratios were used to calculate γ-ray-transition retardation and enhancement factors.

1. INTRODUCTION

THE low-lying levels of ^{133}Cs are populated following the electron-capture decay of ^{133}Ba. From this decay, as well as from the decay of ^{133}Xe and Coulomb-excitation studies, much information has been obtained concerning the low-lying levels of ^{133}Cs. References to experimental studies of ^{133}Cs prior to 1967 have been given by Hennecke *et al.*[1]; more recent works are listed in Refs. 2–4. The main features of the decay scheme, consistent with recent work, are shown in Fig. 1. The levels were established in early work primarily on the basis of coincidence measurements. The first three levels have also been observed in Coulomb-excitation studies. The present spin assignments seem to be well established.

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