Isotopic-Spin Mixing in Direct and Compound-Nuclear Reactions

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(Received 12 April 1968)

The semidirect mechanism of isospin violation in deuteron-induced nuclear reactions, which was presented in an earlier article, is recapitulated. A number of reactions involving the same basic mechanism are discussed and the available data on them are briefly reviewed. This mechanism also leads to much stronger isospin mixing in certain levels in light nuclei than is predicted by the usual perturbative treatment of Coulomb interactions. Such mixing is discussed in terms of a simple two-level model. A suggestion by Levinson has led to a reexamination of the Wilkinson hypothesis that isospin-violating reactions proceed via formation of isotopically impure compound-nuclear levels at "intermediate" excitation energies. It is shown that the most important factor in determining the contribution of these levels is not their degree of isospin mixing, but rather the experimental energy resolution. If the resolution I is poor compared to the average spacing $\langle D_J\rangle$ of levels of given J^P, then in general the contribution of these levels is reduced by the ratio $\langle D_J\rangle/I$. When compound levels are so correlated as to give rise to intermediate structure, or a giant resonance, or an isobaric analog state, their contribution to the isospin-nonconserving amplitude need not be small. This implies that isospin-violating reactions are ideally suited to the study of intermediate structure. Because the actual level density in light nuclei at intermediate excitation is not well known, it is not clear whether Levinson's suggestion is relevant to the studies so far conducted on (a) $C^{12}(d,\alpha)B^{10*}(1.74, T=1)$, and (b) $O^{16}(d,\alpha)N^{14*}(2.31,T=1)$. It is surely relevant to heavier compound nuclei, however, and estimates based on a statistical level-density formula suggest that Levinson's idea is even relevant for the N^{14} and F^{18} compound nuclei entering reactions (a) and (b) above. Finally, the data of Meyer-Schützmeister *et al.* are analyzed for evidence of compound-direct interference. It is conjectured on the basis of a crude but suggestive calculation that the broad 3^- (with some possible 1^- admixture) resonance seen by them in N^{14} at $E_x\sim 18$ MeV is actually an intermediate-structure resonance whose "doorway state" is a "single-particle cluster resonance" in either the entrance or exit channels, or in both. A brief theoretical discussion of compound-direct interference is included for completeness.

I. INTRODUCTION

ISOTOPIC spin has been the subject of many experimental and theoretical investigations in low-energy nuclear physics. Adair[1] and Christy[2] pointed out long ago that if nuclear forces are charge-invariant, certain nuclear reactions are forbidden by a selection rule which we now recognize as conservation of isospin. Most early experimental investigations were concerned with establishing the charge invariance of nuclear forces.[3] Much theoretical effort has subsequently been devoted to showing that electromagnetism indeed accounts for such apparent violations as the Thomas-Ehrman shift in mirror nuclei,[4,5] and isospin mixing in nuclear reactions and in nuclear levels. (Since the major part of the electromagnetic interaction is the Coulomb repulsion between protons, whose r^{-1} behavior leads to certain well-known difficulties,[6] it is not always easy to show that an observation is consistent with charge symmetry.)

The current emphasis in studies of isospin-violating reactions is somewhat different: No one today seriously questions the charge symmetry of the nuclear Hamiltonian.[7] Rather, the interest is in the use of such reactions as a means for investigating nuclear properties that might otherwise be impossible to ascertain. In particular, various investigators[8–10] have hoped to explore compound levels at "intermediate" excitation in light nuclei. Wilkinson,[11] and Lane and Thomas,[12] had suggested that compound levels whose average spacing $\langle D_J\rangle$ and width $\langle\Gamma_J\rangle$ are small compared to the average Coulomb matrix element $\langle H_c\rangle$ should experience the strongest isospin mixing. (These criteria are presumably satisfied in certain light nuclei in the excitation region 10–20 MeV.) Moreover, since direct reaction times are typically much shorter than $[\langle H_c\rangle]^{-1}$, it has been commonly assumed that substantial isospin violation cannot occur in direct reactions.[13] (Closer examination of this generalization reveals that it is an oversimplification, especially for reactions involving deuterons.[14–16])

[1] R. K. Adair, Phys. Rev. **86**, 155 (1952).
[2] R. F. Christy, Phys. Rev. **89**, 839 (1953).
[3] Actually, the present evidence from nucleon-nucleon $^1S_0$ scattering lengths is that there is charge *symmetry* (i.e., $a_{nn}^{(0)}=a_{pp}^{(0)}$) but not charge *invariance* ($a_{np}^{(0)}\neq a_{nn}^{(0)}$). The size of the violation of charge invariance is at most a few percent of the nucleon-nucleon force. See, e.g., E. M. Henley, in *Proceedings of the Conference on Isobaric Spin in Nuclear Physics, 1966* (Academic Press Inc., New York, 1966), p. 3.
[4] R. G. Thomas, Phys. Rev. **81**, 148 (1951); **88**, 1109 (1952).
[5] J. B. Ehrman, Phys. Rev. **81**, 412 (1951).
[6] J. V. Noble, Phys. Rev. **148**, 1528 (1966).
[7] This is a slight exaggeration. The question is still occasionally raised in one form or another. See, e.g., E. G. Adelberger, C. L. Cocke, and C. N. Davids, Bull. Am. Phys. Soc. **12**, 1194 (1967).
[8] L. Meyer-Schützmeister, D. von Ehrenstein, and R. G. Allas, Phys. Rev. **147**, 743 (1966), hereafter referred to as MEA.
[9] J. Jobst, S. Messelt, and H. T. Richards (to be published), hereafter referred to as JMR.
[10] J. W. Hemsky, E. Bleuler, and D. J. Tendam, Nucl. Phys. **A105**, 665 (1967). A quite complete set of references to earlier work may be found in this article.
[11] D. H. Wilkinson, Phil. Mag. **1**, 379 (1956).
[12] A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 262 (1958).
[13] Reference 12, p. 346.
[14] J. V. Noble, Phys. Rev. **157**, 939 (1967).
[15] J. V. Noble, Phys. Rev. **161**, 945 (1967).
[16] J. V. Noble, Phys. Rev. **162**, 934 (1967), hereafter referred to as N.

In the past few years a number of studies have been made of (d,α) reactions on $T=0$ targets, leading to (known) $T=1$ states of the residual nuclei.[8–10] The major motivation of these investigations was the hope of studying compound levels uncontaminated by a direct component in the reaction mechanism. However, Meyer-Schützmeister, von Ehrenstein, and Allas[8] (MEA) have found unequivocal evidence of a direct T-violating mechanism in the reaction $C^{12}(d,\alpha)B^{10*}\times(1.74,T=1)$ at deuteron energies above 11.4 MeV. This surprising discovery prompted several theoretical attempts to unriddle the mechanism.[17,18]

In two previous papers,[14,15] I suggested that deuterons scattering on a massive, charged spinless target, with the nucleons interacting with the target by means of spin-dependent potentials, can undergo spin-flip transitions to their $^1S_0$, $T=1$ "state." (The $^1S_0$ state will subsequently be denoted by φ.) This idea was subsequently exploited[16] to account for the observations of MEA both qualitatively and quantitatively.

Levinson[19] has recently pointed out that when the experimental energy spread I exceeds the average compound-nuclear level spacing, the contribution of the Wilkinson[11] mechanism for uncorrelated levels is reduced by the ratio $\langle D_J\rangle/I$. The density of levels (of the same spin and parity) in light nuclei is not very well known, so that it is not completely clear whether Levinson's comment is relevant to either the study conducted by MEA, or to a similar study of the reaction

$$O^{16}(d,\alpha)N^{14}(2.31,T=1)$$

made by Jobst, Messelt, and Richards.[9] However, the level density in P^{30} and Sc^{42} is almost certainly great enough to make Levinson's comment pertinent to $Si^{28}\times(d,\alpha)Al^{26}$ $(T=1)$ and $Ca^{40}(d,\alpha)K^{38}$ $(T=1)$ studies which have been made in recent years.[10,20]

If Levinson's objection to the usually accepted Wilkinson mechanism is in fact relevant to the present regime of isospin-violating experiments, several interesting possibilities arise. First, the preferential spin-flip mechanism may well be the predominant form of isospin violation in *all* deuteron-induced reactions, whether compound or direct. The second and more interesting point is that T-forbidden reactions may be the ideal way to observe intermediate structure and "doorway states" (in the language of Feshbach[21]), since it is possible to show that when $\langle D_J\rangle$ is less than I, sets of highly correlated levels will contribute preferentially to the reaction cross section.

Finally, it is possible that the spin-flip mechanism may be the predominant form of isospin mixing for certain types of discrete levels. This would mean that hitherto unseen T-forbidden decays of certain low-lying particle-unstable levels may have branching ratios just beyond the current experimental limits, rather than being undetectable by many orders of magnitude (as would be predicted by the ordinary Coulomb mixing model).

This paper will discuss these questions in some detail. Section II briefly recapitulates the semidirect mechanism described in N. A number of isospin-violating reactions which can occur via this mechanism are described, and the available data on them is reviewed. A discussion of spin-flip isobaric mixing in stationary states is also given in Sec. II. The Wilkinson mechanism and purely compound reactions are considered in Sec. III, and Levinson's objection is discussed in detail. The intermediate regime in which both compound and direct effects can appear simultaneously is the topic of Sec. IV. Also in this section is a review of some evidence for the presence of doorway resonances in N^{14}. Finally, the contents of the paper are summarized in Sec. V.

II. SEMIDIRECT ISOSPIN VIOLATION

A. Recapitulation of Preferential Spin-Flip Mechanism

We consider reactions initiated by deuterons (considered as composite states) incident on $J^\pi=0^+$, $T=0$ targets. The scattering matrix, defined by

$$\langle\chi_f|T_{fi}^{(+)}(E)|\chi_i\rangle = \lim_{\eta\to 0+}\langle\chi_f|V_f[1+(E+i\eta-H)^{-1}V_i]|\chi_i\rangle, \quad (1)$$

gives the amplitude for transition from the initial state $|\chi_i\rangle$, containing the deuteron and target and with a plane-wave function describing their relative motion, to the final state $\langle\chi_f|$ containing the reaction products moving in relative plane waves.[22] As usual, we have assumed that the Hamiltonian can be partitioned in the forms

$$H=H_i+V_i=H_f+V_f, \quad (2)$$

and that

$$(H_i-E)|\chi_i\rangle=(H_f-E)|\chi_f\rangle=0. \quad (3)$$

Clearly T_{fi} satisfies the Lippmann-Schwinger[23] equation

$$T_{fi}(W)=V_f+T_{fi}(W)G_i(W)V_i, \quad (4)$$

where $G_i(W)=(W-H_i)^{-1}$.

Since we are at present interested in the specifically three-body features of the problem, we begin by projecting out the internal degrees of freedom of the target in the usual way.[24] To do this, we define the projection operator onto the target ground state $|0\rangle$, and its

[17] R. J. Drachman, Phys. Rev. Letters **17**, 1017 (1966).
[18] T. A. Griffy, Phys. Letters **21**, 693 (1966).
[19] C. Levinson (private communication); also, see Ref. 13.
[20] P. G. Bizzetti and A. M. Bizzetti-Sona, Nucl. Phys. **A108**, 274 (1968).
[21] H. Feshbach, A. K. Kerman, and R. H. Lemmer, Ann. Phys. **41**, 230 (1967).
[22] See, e.g., M. L. Goldberger and K. M. Watson, *Collision Theory* (John Wiley & Sons, Inc., New York, 1964), pp. ff.
[23] B. A. Lippmann and J. Schwinger, Phys. Rev. **79**, 469 (1950).
[24] H. Feshbach, Ann. Phys. **19**, 287 (1962).

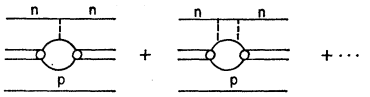


FIG. 1. Direct contributions to $V_n(W)$. There are also exchange contributions to $V_n(W)$ (not illustrated).

complement in the target internal space

$$\mathbf{P}=|0\rangle\langle 0|$$

and

$$\mathbf{R}=\mathbf{1}-\mathbf{P},$$

which obviously obey the relations

$$\mathbf{P}^2=\mathbf{P},\quad \mathbf{R}^2=\mathbf{R},\quad \mathbf{PR}=\mathbf{RP}=0.$$

Note also that $\mathbf{P}G_i=G_i\mathbf{P}$ and therefore $\mathbf{P}G_i\mathbf{R}=0$. We are interested in the object $T_{fi}\mathbf{P}$, which may easily be shown to satisfy the equation

$$T_{fi}\mathbf{P}=V_f\mathbf{P}+V_f(W-\mathbf{R}H\mathbf{R})^{-1}\mathbf{R}V_i\mathbf{P}+T_{fi}\mathbf{P}G_i[\mathbf{P}V_i\mathbf{P}+\mathbf{P}V_i\mathbf{R}(W-\mathbf{R}H\mathbf{R})^{-1}V_i\mathbf{P}]. \quad (5)$$

In the present model (with or without including exchange effects) the effective interaction

$$V_{\text{eff}}(W)=\mathbf{P}V_i\mathbf{P}+\mathbf{P}V_i\mathbf{R}(W-\mathbf{R}H\mathbf{R})^{-1}V_i\mathbf{P} \quad (6)$$

has the form $V_n(W)+V_p(W)+C(W)$, where $V_n(W)$ and $V_p(W)$ are (energy-dependent) two-body potentials between the neutron and target, and proton and target, respectively; and where $C(W)$ is an energy-dependent three-body force. (These properties are illustrated diagrammatically in Figs. 1 and 2.) Thus, as previously implied, Eq. (5) reduces at any energy resolution to the problem of three bodies, for which an infinite variety of mathematically acceptable (equivalent) solutions are now known.[14,25–28] What happens if we perform an energy average of Eq. (5), such as would occur in a poor-resolution experiment? Since $C(W)$ involves "off-diagonal" matrix elements of $V_{\text{eff}}(W)$, we would expect that in the spirit of the random-phase approximation (RPA), the energy average of $C(W)$ will be "small" in some sense, compared to that of $V_n(W)+V_p(W)$. Thus the energy average of $V_{\text{eff}}(W)$ is assumed to have the form

$$\langle V_{\text{eff}}(W)\rangle_{\text{av}}\simeq V_n{}^{\text{opt}}+V_p{}^{\text{opt}}, \quad (7)$$

where $V_{n,p}{}^{\text{opt}}$ are what we usually mean by nucleon-target optical potentials.

When the energy resolution is poor, we may construct $\langle T_{fi}\mathbf{P}\rangle_{\text{av}}$ by solving the three-body model of two nucleons scattering on a spin-0 featureless "target" A, in which the particles interact via well-behaved two-body (optical) potentials. Since the mathematical details of the solution of this problem have already been discussed fully in earlier publications,[14–16] we confine ourselves here to a qualitative discussion of the special features of the realistic problem that lead to direct isospin nonconservation. First, it is reasonable to assume that the optical potentials $V_n{}^{\text{opt}}$ and $V_p{}^{\text{opt}}$, defined as in (7), are similar to the empirical ones determined from nucleon-scattering experiments. Second, charge symmetry implies that $V_n{}^{\text{opt}}$ and $V_p{}^{\text{opt}}$ differ only by a proton-target Coulomb potential. It turns out that it is important to include the long-range (point-charge) part of this Coulomb potential *exactly*, i.e., to all orders in Ze^2. This is because the long-range nature of the Coulomb repulsion excludes the proton from the nuclear region relative to the neutron (when the neutron and the proton are not moving in a spatially correlated state such as the φ or the deuteron). Clearly this exclusion depends on the average energy of the proton in three-body intermediate states, and therefore vanishes asymptotically at infinite deuteron energy.

The exclusion of the proton from the nuclear region (relative to the neutron) is significant because the empirical nucleon-target interaction is spin-dependent. Transitions from the ${}^3S_1$, $T=0$ state of the n-p subsystem, to the ${}^1S_0$, $T=1$ state become possible; at some energies they occur with an amplitude determined by the full strength of the spin-dependent part of the nucleon-target potential. Figure 3 indicates clearly the lack of symmetry between the spin-flipping matrix elements of the nucleon-target interactions when the charged particles are constrained to move in Coulomb-modified plane waves rather than plane waves. The (Born approximation) diagrams shown in Fig. 3 would exactly cancel if the Coulomb repulsion were turned off—its presence keeps them from cancelling, and in fact at long wavelengths makes the proton spin-flip term exponentially small relative to the corresponding neutron spin-flip term. The sum of the diagrams in Fig. 3 yields the amplitude (on the energy shell)[16]

$$\langle \tfrac{1}{2}\tfrac{1}{2};00|\tfrac{1}{2}(\mathbf{K}'\times\mathbf{K})\cdot\boldsymbol{\sigma}_n|\tfrac{1}{2}\tfrac{1}{2};1\nu\rangle\tilde{V}_{\text{so}}(|\mathbf{K}'-\mathbf{K}|)A(K),$$

where $\boldsymbol{\sigma}_n$ is the neutron spin operator, $\mathbf{K}$ is the initial deuteron momentum (and ν is its spin z projection), $\mathbf{K}'$ is the final φ momentum, $\tilde{V}_{\text{so}}$ is the Fourier transform of the spin-orbit part of the empirical nucleon optical potential, and $A(K)$ is the Coulomb asymmetry factor

$$A(K)=1-(4\pi Ze^2/\hbar^2K)[\exp(4\pi Ze^2/\hbar^2K)-1]^{-1}.$$

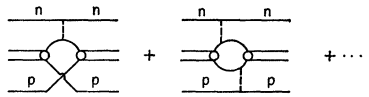


FIG. 2. Direct and exchange contributions to $C(W)$.

[25] L. D. Faddeev, *Mathematical Aspects of the Three-Body Problem in the Quantum Scattering Theory* (Israel Program for Scientific Translations, Jerusalem, 1965).
[26] S. Weinberg, Phys. Rev. **133**, B232 (1964).
[27] L. Rosenberg, Phys. Rev. **135**, B715 (1964).
[28] R. Sugar and R. Blankenbecler, Phys. Rev. **136**, B472 (1964).

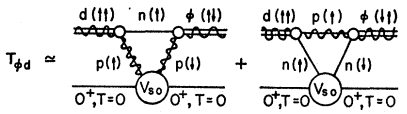


FIG. 3. Lowest-order contributions to the isospin-violating process $d+A \rightarrow \varphi+A$, correct to all orders in Ze^2. "The superposed wiggles on certain particle lines in Figs. 3 and 4(a) indicate that these are charged particles propagating in Coulomb-modified plane waves."

The net effect of the preferential spin-flip mechanism described above is to permit isospin-forbidden reactions to occur at low energies, in what is essentially the second nuclear Born approximation. Since the effect involves spin-flip, conservation of parity and angular momentum forces it to disappear when only S waves are present, i.e., at sufficiently low incident deuteron energies. The result is that the mechanism operates only over a limited region of incident energies, because it vanishes at $E_d=0$ and $E_d=\infty$.

Returning to the question of isospin-violating (d,α) reactions, we see that [assuming that $V_f\mathbf{P}$ in (5) is isospin conserving] they take place as indicated diagrammatically in Fig. 4(a). That is, the deuteron makes a preliminary transition to the $^1S_0$, $T=1$ state, and then it picks up two nucleons (in a relative $^1S_0$ state) from the target, forming an α particle and leaving the residual nucleus in a $T=1$ state. This latter two-nucleon pickup process occurs with essentially the same amplitude as the T-allowed (direct) (d,α) pickup process, as can be seen diagrammatically in Fig. 4(b).

B. Experimental Consequences of Spin-Flip Mechanism: Survey of Available Data

The preferential spin-flip model that was just described predicts direct isospin violation in a number of reactions. (It also allows isospin violation during the formation and decay of compound states, but we shall now consider only direct reactions.) In the first place, one would expect the effect to be independent of the target, so that it should appear in reactions such as $O^{16}(d,\alpha)N^{14}(2.31,\ T=1)$. Jobst, Messelt, and Richards[9] have conducted a high-resolution study of this reaction, with deuterons ranging from 3 to 15 MeV in energy. Even at the highest energies in their study, the compound contribution clearly overshadows any direct component of the cross section. Thus the evidence from this reaction is as yet ambiguous. Perhaps at higher incident energies the compound contributions will be sufficiently small to allow observation of the direct process, before it in turn is reduced too much to be seen.

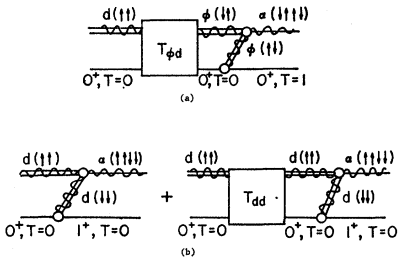


FIG. 4(a). Isospin-violating (d,α) reaction amplitude; (b) isospin-conserving (d,α) amplitude.

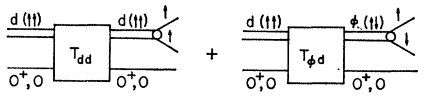


FIG. 5. Direct contributions to the (d,np) breakup amplitude.

The prediction that the direct amplitude decreases rapidly with increasing energy has been substantiated by the recently reported observations of Jänecke *et al.*[29] on $C^{12}(d,\alpha)B^{10}(1.74,T=1)$ at higher deuteron energies. (Evidently in this reaction there are no strong compound contributions at higher energies to complicate the observation of the direct process.)

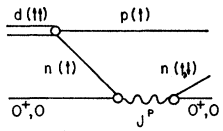
$$C^{12}(d,np)C^{12}$$

The suggestion was put forth in N that it might be possible to detect transitions to the $^1S_0$, $T=1$ state of the n-p system in the (d,np) breakup reaction. This transition is certainly implied by the spin-flip model (Fig. 5). An experiment was carried out to attempt to observe this transition using the procedure described in N.[30] Unfortunately, there is sufficient competition from sequential decay processes (Fig. 6) that the results are at present ambiguous. When the experiment is repeated, it may be possible to correct for the effects of competing sequential processes.

$$C^{12}(\alpha,d)N^{14*}(2.31,T=1)$$

Time-reversal invariance, coupled with the knowledge that $N^{14}(d,d')N^{14}(2.31,T=1)$ takes place with a respectable cross section[31] leads to the conclusion that $C^{12}(\alpha,d)N^{14*}(T=1)$ should occur also with a reasonable amplitude. An experiment by Zafiratos *et al.*[32] set an upper limit on the cross section (at $E_\alpha=42$ MeV) of 0.3% of the allowed cross section. We should note, however, that $E_\alpha=42$ MeV corresponds to a deuteron energy of 17.5 MeV, for which the ratio of forbidden to allowed cross sections is quite small in both $C^{12}(d,\alpha)B^{10*}$ and in $N^{14}(d,d')N^{14*}$. (In this latter case we may ex-

FIG. 6. Sequential contribution to (d,np).



[29] J. A. Jänecke, T. F. Yang, R. M. Polichar, and W. S. Gray, Bull. Am. Phys. Soc. **12**, 1194 (1967); and Phys. Rev. (to be published).
[30] R. Zurmuhle (private communication).
[31] J. Duray (private communication).
[32] C. D. Zafiratos, J. S. Lilley, and F. W. Slee, Phys. Rev. **154**, 887 (1967).

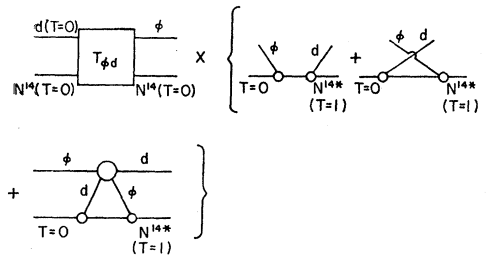


FIG. 7. Direct contribution to $N^{14}(d,d')N^{14*}$ $(T=1)$.

trapolate the ratio from the data of Duray.[31]) Thus the upper limit obtained by Zafiratos *et al.* is consistent with what is known from other reactions.

There are several other kinds of reactions in which semidirect isospin mixing is expected to play a role. For example, the reactions

$$N^{14}(d,d')N^{14*}(2.31,T=1),$$
$$N^{14}(\alpha,\alpha')N^{14*}(2.31,T=1),$$
$$C^{12}(Li^6,C^{12})Li^{6*}(3.56,T=1),$$

will proceed via the direct processes depicted in the diagrammatic representations of Figs. 7 and 8; many other reactions should take place via similar mechanisms.

$N^{14}(d,d')N^{14*}(2.31,T=1)$

The reaction $N^{14}(d,d')N^{14*}$ has been investigated for incident energies 5–10 MeV.[31] Because there is still considerable compound structure in O^{16} at these excitations, and because (as may be seen from Fig. 7) even the direct amplitude may be backward peaked because of heavy-particle exchange, examination of these differential cross sections does not reveal unequivocal direct structure. However, the ratio of forbidden to allowed cross sections rapidly decreases with energy, indicating the possibility of distinct compound and direct regions here also.

$N^{14}(\alpha,\alpha')N^{14*}(2.31,T=1)$

This reaction has been studied over the energy range 10 MeV$<E_\alpha<$13 MeV.[33] Again, the prominent structure in the differential cross section seems to result from several broad resonances, and does not seem at all direct in character.

$C^{12}(Li^6,C^{12})Li^{6*}(3.56,T=1)$

This reaction has not yet been investigated, to my knowledge, but may be studied in the near future.[34] A similar reaction, $C^{12}(Li^6,\alpha)N^{14*}(2.31,T=1)$ has been studied at rather low energies,[35] and evidently has a small but observable cross section. The latter reaction would proceed (in lowest *direct* order) as in Fig. 8(c).

C. Isospin Violation in Resonant States

The three-body model of deuteron scattering on charged $T=0$, $J^P=0^+$ targets predicts transitions from the $S=1$ state to the $S=0$ state (where S is the n-p total spin) if the nucleon-target forces are spin-dependent. The model also predicts isospin mixing in certain discrete states of the elementary core-plus-two-nucleon system. If we assume that the n-p interaction is adequately represented by one separable term in the $^3S_1$ state plus one separable term in the $^1S_0$ state with the parameters adjusted to reproduce low-energy n-p scattering,[36] then the deuteron elastic scattering amplitude satisfies a (two-channel) set of coupled two-body Lippmann-Schwinger equations[16] $(S,S'=0,1)$

$$\langle K'L'S'|T_J^{(+)}(E)|KLS\rangle$$
$$=\sum_{L''S''}\int_0^\infty dK''K''^2\langle K'L'S'|B_J^{(+)}(E)|K''L''S''\rangle$$
$$\times\{\delta(K''-K)\delta_{L''L}\delta_{S''S}-\tau_{S''}^{(+)}(E-(2M)^{-1}K''^2)$$
$$\times\langle K''L''S''|T_J^{(+)}(E)|KLS\rangle\}. \quad (8)$$

In Eq. (8), the partial-wave analysis has already been performed in L-S coupling, i.e., $\mathbf{J}=\mathbf{L}+\mathbf{S}$. The $T=1$, $^1S_0$ state of the n-p system has been treated as a particle on the same footing as the deuteron, even though it is not bound.[37] The restrictions imposed by conservation of angular momentum and parity reduce Eq. (8) to two independent sets of two coupled equations. The effective deuteron-A interaction, denoted by

$$\langle K'L'S'|B_J^{(+)}(E)|KLS\rangle,$$

may actually be obtained in closed form in the static model,[14] and is in any case defined in terms of the nucleon-A interactions.[14] [We should note that Eq. (8) is practically unchanged in the *non*static case, the only differences being a change in the deuteron reduced mass M appearing in the two-nucleon propagators $\tau_S^{(+)}\times(E-(2M)^{-1}K^2)$, and increased difficulty in determining the effective interaction B_J since it is no longer available in closed form when A can recoil.] Equation (8) implies that the states with different isospin which are coupled by the spin-flip mechanism are those with $L'=L=J>0$, always of course assuming that

$$\langle K'J1|B_J^{(+)}(E)|KL0\rangle$$

does not vanish identically.

We shall suppose that the scattering matrix, with $L'=L=J>0$, and with $\langle K'J1|B_J^{(+)}(E)|KL0\rangle$ (which we shall henceforth abbreviate as B_{10}) arbitrarily set

[33] C. M. Chesterfield and B. M. Spicer, in *Proceedings of the Conference on Isobaric Spin in Nuclear Physics, 1966* (Academic Press Inc., New York, 1966), p. 734.
[34] K. Bethge (private communication).
[35] R. R. Carlson and D. W. Heikenen, Phys. Letters 17, 305 (1965).
[36] Y. Yamaguchi, Phys. Rev. 95, 1628 (1954); 95, 1635 (1954).
[37] R. Aaron, R. D. Amado, and Y. Y. Yam, Phys. Rev. 136, B650 (1964).

to zero, has a resonance in the $S=1\rightarrow S=1$ channel, and also has a nearby resonance in the $S=0\rightarrow S=0$ channel. The isospin-violating scattering matrix T_{01} (in an obvious notation) satisfies the formal equation

$$T_{01}=B_{01}-B_{01}\tau_1 T_{11}-B_{00}\tau_0 T_{01}. \tag{9}$$

Equation (9) is just Eq. (8) with $L'=L=J, S'=0, S=1$; with the sum over discrete indices performed explicitly, and with the integration over intermediate momenta left implicit. The total and orbital angular-momentum labels have been dropped for simplicity. The corresponding equation satisfied by T_{11} may be used to formally eliminate T_{11} from (9) to give

$$T_{01}=(1+B_{00}\tau_0)^{-1}B_{01}(1+\tau_1 B_{11})^{-1}(1+\tau_1 B_{10}\tau_0 T_{01}). \tag{10}$$

In the vicinity of its resonance, the operator $(1+B_{SS}\tau_S)^{-1}$ has the form[38]

$$1+|v_S\rangle\eta_S(1-\eta_S)^{-1}N_S^{-1}\langle\bar{v}_S|\tau_S, \tag{11}$$

where $\eta_S(E)$ is a function whose real part becomes unity at the resonance, and whose imaginary part is positive-definite, and where N_S is a normalization factor. Putting Eq. (11) into Eq. (10) and taking the most singular part, we obtain the resonant contribution to T_{01}:

$$T_{01}^{\rm res}=|v_0\rangle\gamma\eta_0\eta_1(N_0N_1D)^{-1}\langle\bar{v}_1|, \tag{12}$$

where

$$D=(1-\eta_0)(1-\eta_1)-\gamma\bar{\gamma}(N_0N_1)^{-1} \tag{13}$$

and

$$\gamma=\langle\bar{v}_0|\tau_0B_{01}\tau_1|v_1\rangle,\quad \bar{\gamma}=\langle\bar{v}_1|\tau_1B_{10}\tau_0|v_0\rangle. \tag{14}$$

Similarly, we find that near the resonance in the $S=1$ channel the dominant contribution to T_{11} is

$$|v_1\rangle(N_1D)^{-1}[\eta_1(1-\eta_0)-\gamma\bar{\gamma}(N_1N_0)^{-1}]\langle\bar{v}_1|,$$

so that the ratio of forbidden to allowed transition amplitudes near resonance is

$$R\simeq[\langle K'|v_0\rangle/\langle K'|v_1\rangle]\times\gamma\eta_0\eta_1[N_0\eta_1(1-\eta_0)-\gamma\bar{\gamma}/N_1]^{-1}. \tag{15}$$

It is clear from (15) that near the resonance in the exit channel a considerable enhancement would be expected, since if $\eta_0\sim1$ and η_1 is not too small,[39] and assuming the final vertex functions $|v_0\rangle$ and $|v_1\rangle$ have essentially the same structure and magnitude, R can become of the order of $\eta_0\eta_1N_1/\bar{\gamma}$, which is the ratio

$$\langle\bar{\Gamma}_1|B_{11}|\Gamma_1\rangle/\langle\bar{\Gamma}_1|B_{10}|\Gamma_0\rangle, \tag{16}$$

where the $|\Gamma_S\rangle$ are resonance wave functions as defined by Weinberg.[40] Since B_{11} is the effective interaction for deuterons, one would expect it to have a strength and range consonant with the empirically determined deuteron optical potential. Similarly, the isospin-violating effective interaction B_{10} can become of the order of the empirical nucleon spin-orbit potential, so that the maximum value of R might be as large as 10–100. Away from the $S=0$ resonance, i.e., where η_0 is not close to unity, R will fall to approximately the inverse of (16), which would give a value of 10^{-2}–10^{-1}.

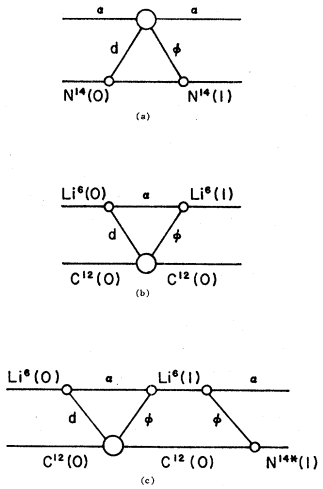


FIG. 8. (a) Direct contribution to $N^{14}(\alpha,\alpha')N^{14*}$ $(T=1)$; (b) direct contribution to $C^{12}(Li^6,C^{12})Li^{6*}$ $(T=1)$; and (c) direct contribution to $C^{12}(Li^6,\alpha)N^{14*}$ $(T=1)$.

The eigenfunction expansion of the kernel of Eq. (8) gives a way of defining wave functions of resonances as a logical extension from the wave functions of bound states.[40] The unperturbed eigenfunctions (that is, with $B_{10}=0$) are defined by

$$[\eta_S+\tau_SB_{SS}]|\Gamma_S\rangle=0,\quad S=0,1 \tag{17}$$

and

$$\langle\bar{\Gamma}_S|[B_{SS}\tau_S+\eta_S]=0,$$

and in terms of them we have

$$|v_S\rangle=B_{SS}|\Gamma_S\rangle,\quad N_S=\langle\bar{\Gamma}_S|B_{SS}|\Gamma_S\rangle. \tag{18}$$

We can similarly define eigenfunctions of the scattering kernel when B_{10} is not zero:

$$\lambda|\phi_S\rangle=-\textstyle\sum_{S'}\tau_SB_{SS'}|\phi_{S'}\rangle,\quad S=0,1. \tag{19}$$

The eigenfunctions of the pair of Eqs. (19) will be two-component functions. As the (isospin-mixing) perturbing interaction is turned on, we can see how the pure bound or resonant states become mixed. In the simple two-state model described previously, we may write

$$|\phi_S\rangle=\textstyle\sum_{S'}|\Gamma_{S'}\rangle\hat{O}_{SS'}, \tag{20}$$

where $\hat{O}$ is an orthogonal 2×2 matrix. (Since the effective interaction $B_{S'S}$ is *symmetric*, it may be diagonalized by an *orthogonal* matrix.) Assuming that the mixing is small, we find that the fraction of, say, $S=1$

[38] This follows from the biorthogonal expansion of the scattering kernel $B_{SS}\tau_S$, and from the assumption that there is only *one* resonance in each isospin channel.

[39] If $|\eta_1|\ll1$, we would clearly be far from the resonance in the $S=1$ channel, violating the hypothesis of close unperturbed levels.

[40] S. Weinberg, Phys. Rev. 131, 440 (1963).

mixed into the $S=0$ state is given in first order by

$$\epsilon \simeq \langle \bar{\Gamma}_1 | B_{10} | \Gamma_0 \rangle (E_0 - E_1)^{-1} \langle \bar{\Gamma}_0 | \Gamma_0 \rangle^{-1/2} \langle \bar{\Gamma}_1 | \Gamma_1 \rangle^{-1/2}, \quad (21)$$

which is in fact the analytic continuation to positive energies of the usual first-order perturbation result for bound states.[41] This result is to be compared with the usual perturbation theory of isospin mixing, in which the appropriate T-mixing matrix element is

$$\langle T=1 | H_c | T=0 \rangle,$$

where H_c is the Coulomb interaction.

Because the Coulomb interaction is spin-independent, the states of different isospin that it connects will have the same spin-wave functions and consequently will have rather different spatial dependence. Thus the Coulomb interaction (in first order) connects states lying relatively far apart in energy. Also the typical Coulomb matrix element will be rather small compared to the spacing of the levels it connects. (Wilkinson estimates $\langle H_c \rangle \lesssim 100$ keV in light nuclei.[11]) Thus perturbation theory in the *Coulomb potential* predicts relatively small isospin mixing. The long-range nature of the Coulomb potential leads to a number of instances in which simple perturbation theory breaks down; the preferential spin-flip mechanism for isospin violation is only the latest and most complex example of this.

We should expect a number of low-lying levels in light nuclei to be strongly mixed in isotopic spin, even though the spacing between $T=0$ and $T=1$ levels of given J^P is much larger than $\langle H_c \rangle$. The nuclei that are most likely to experience this effect are those to which our model applies, namely, nuclei with a neutron and a proton outside a closed-shell core. Examples are Li^6, B^{10}, N^{14}, F^{18}, and so forth. We should also expect the prediction to be better satisfied for nuclei in which the low-lying levels are reasonably pure L-S coupling states. Finally, only states in which $L=J$ and $J>0$ experience mixing. This automatically makes the ground states and low-lying $T=1$, 0^+ states of Li^6, B^{10}, N^{14}, and F^{18} isobarically pure, as far as this mechanism is concerned. However, the $2^+, T=1$ level at 5.35 MeV in Li^6, and the corresponding $2^+, T=0$ level at 4.5 MeV should mix strongly as they (a) are $L=2$ states, (b) are not too high above the Coulomb barrier, and (c) are not too far apart in energy compared to $\langle B_{10} \rangle$.

A search for deuterons emitted by the T-violating decay of the 5.35-MeV level in Li^6 is being conducted by Cocke.[42] The branching ratio for this decay is easily estimated from Eq. (15). Away from the resonance in the $S=1$, $T=0$ channel, we expect the branching fraction to be approximately the inverse absolute square of the ratio (16). Assuming that B_{11} has essentially the strength (~ 100 MeV) of the empirical deuteron optical potential, and that B_{01} has the strength of a typical nucleon spin-orbit interaction (~ 10 MeV), we see that the branching fraction might be as large as 1% (if the eigenfunctions $|\Gamma_1\rangle$ and $|\Gamma_0\rangle$ have essentially the same spatial dependence). Since the empirical spin-orbit force is surface-peaked, whereas B_{11} will be a volume interaction, the branching fraction could be considerably smaller than the approximate upper limit of 1%: The uncertainty is perhaps an order of magnitude in R, with the consequence that the branching ratio could be as small as 0.01% and thus probably undetectable.

[41] The continuation must be conducted with some care, since the resonance "wave functions" $|\Gamma_S\rangle$ are not normalizable.

[42] C. L. Cocke (private communication).

III. COMPOUND-NUCLEUS FORMATION AND THE WILKINSON MECHANISM

Let us consider the opposite extreme from that of Sec. II, namely, we assume that we can represent the reaction amplitude $T_{fi}\mathbf{P}$ in the form

$$T_{fi}\mathbf{P} = V_f\mathbf{P} + \sum_\lambda V_f|\lambda\rangle[W - \mathcal{E}_\lambda(W)]^{-1}\langle\lambda|V_i\mathbf{P}. \quad (22)$$

In (22), $\mathcal{E}_\lambda(W)$ is analytic in the cut W plane, and its cut structure is identical with that of $(W-H)^{-1}$. All the zeroes of $W-\mathcal{E}_\lambda(W)$, occurring when the energy W is in the scattering region, lie on unphysical sheets so that as expected, (22) has no physical poles when $\mathrm{Re}(W)$ is above the lowest threshold. Some of the indices λ in (22) are continuously distributed; these shall not concern us because we are only interested in the ones corresponding to compound resonances.

We now examine Eq. (22) to determine whether it indeed predicts isospin violation through formation of isotopically mixed compound-intermediate states. According to Wilkinson,[11] when the average level spacing becomes small compared to the average Coulomb interaction matrix element $\langle H_c \rangle$, and if the natural widths of the states are also small compared with $\langle H_c \rangle$, then substantial isospin mixing takes place. To illustrate what is going to happen, we consider first a simple model in which there are only two compound levels of a given J^P. We suppose that they originate from the Coulomb (or spin-flip) mixing of two states with natural isospin 0 and 1; they contribute an energy averaged amplitude of the form

$$\langle T_{CN}(E)\rangle = \sum_{\lambda=0}^{1} \langle \chi_f | V_f | \lambda \rangle \times [E+iI - \mathcal{E}_\lambda(E+iI)]^{-1} \langle \lambda | V_i | \chi_i \rangle, \quad (23)$$

where we have performed the energy average over the interval $(E-\frac{1}{2}I, E+\frac{1}{2}I)$, which just has the effect of replacing W by $E+iI$. Suppose that the beam spread I is large compared to $\Delta = |\mathcal{E}_0(E+iI) - \mathcal{E}_1(E+iI)|$. Then for E in the vicinity of $\mathrm{Re}\langle\mathcal{E}\rangle$, where

$$\langle\mathcal{E}\rangle = \tfrac{1}{2}[\mathcal{E}_0(E+iI) + \mathcal{E}_1(E+iI)],$$

we have, approximately,

$$\langle T_{CN}(E)\rangle \simeq (E+iI-\langle\mathcal{E}\rangle)^{-1} \sum_\lambda \langle \chi_f | V_f | \lambda \rangle \langle \lambda | V_i | \chi_i \rangle + (E+iI-\langle\mathcal{E}\rangle)^{-2} \sum_\lambda (\mathcal{E}_\lambda - \langle\mathcal{E}\rangle) \times \langle \chi_f | V_f | \lambda \rangle \langle \lambda | V_i | \chi_i \rangle. \quad (24)$$

The mixed states $|\lambda\rangle$ are related to the natural isospin states $|t\rangle$ by the (orthogonal) matrix which diagonalizes the T-mixing interaction:

$$|\lambda\rangle=\sum_t |t\rangle\hat{O}_{\lambda t}. \tag{25}$$

Thus $\sum_\lambda|\lambda\rangle\langle\lambda|\equiv\sum_t|t\rangle\langle t|$ and the first term of (24) conserves isospin. The isospin-violating part of (24) is of order Δ/I relative to the isospin-conserving term.

Let us now examine the more general many-level case. We suppose that there are $2N$ levels all with the same J^P, obtained from the mixing of N $T=0$ and N $T=1$ natural isospin levels. We consider an expression of the form

$$\sum_{\lambda=1}^{2N}\sum_{\nu'=1}^{N}\sum_{\nu=1}^{N}|\nu't'\rangle\hat{O}_{\lambda,\nu't'}D_\lambda^{-1}(E)\hat{O}_{\lambda,\nu t}\langle\nu,t|, \tag{26}$$

where $D_\lambda(E)=E+iI-\mathcal{E}_\lambda(E+iI)$, and $\hat{O}$ is the (orthogonal) T-mixing matrix. When I is greater than the average level spacing, the sum may be replaced by an integral

$$\sum_\lambda \hat{O}_{\lambda,\nu't'}\hat{O}_{\lambda,\nu t}D_\lambda^{-1}(E)\rightarrow$$

$$2N(b-a)^{-1}\int_a^b dx\frac{f_{\nu't',\nu t}(x)}{E+iI-x}, \tag{27}$$

where $b-a=\mathrm{Re}(\mathcal{E}_{2N}-\mathcal{E}_1)$. That is, as one moves away from a string of poles in the complex W plane, they look like a branch cut. Now the function $f_{\nu't',\nu t}(x)$ appearing in (27) has the property

$$\int_a^b dx\, f_{\nu't',\nu t}(x)=\delta_{\nu'\nu}\delta_{t't}; \tag{28}$$

this follows from the orthogonality of $\hat{O}_{\lambda,\nu t}$. Defining $\langle\mathcal{E}\rangle=\frac{1}{2}(b+a)$, we obtain the asymptotic expansion

$$\int_a^b dx f_{\nu't',\nu t}(x)[E+iI-x]^{-1}$$
$$=[E+iI-\langle\mathcal{E}\rangle]^{-1}\delta_{\nu'\nu}\delta_{t't}+[E+iI-\langle\mathcal{E}\rangle]^{-2}$$
$$\times\int_a^b dx(x-\langle\mathcal{E}\rangle)f_{\nu't',\nu t}(x)+\cdots. \tag{29}$$

Setting $t'\neq t$, we see that the first nonvanishing isospin-violating contribution to Eq. (27) is

$$2N(b-a)^{-1}(E+iI-\langle\mathcal{E}\rangle)^{-2}\int_a^b dx x f_{\nu't',\nu t}(x).$$

Since we expect $f_{\nu'1,\nu0}(x)$ to be a randomly fluctuating function whose sign changes on the average in a distance Δ (called the correlation distance), and whose average magnitude is $(2N)^{-1}$, this term will have the approximate magnitude

$$\tfrac{1}{2}\Delta(E+iI-\langle\mathcal{E}\rangle)^{-2},$$

which is reduced from that of the isospin conserving amplitude by at least the factor $\frac{1}{2}\Delta/I$.

The next question is how large is Δ? For completely uncorrelated compound levels, Δ will be on the order of the level spacing, and so isospin violation would be unobservable with poor energy resolution. Thus it would seem to be *a priori* impossible to use poor-resolution studies of isospin-violating reactions to examine the (hypothetical) region of strong isospin mixing. The only way isospin nonconservation can be observed is if the correlation distance is on the order of or greater than the resolution. In Sec. II we have already considered one case where (in a sense) Δ exceeds I, namely, the semidirect mechanism allowed by the (energy-averaged) effective interaction, $\langle V_{\mathrm{eff}}(E)\rangle_{\mathrm{av}}$.

There is another possibility we have not yet mentioned, namely, that the correlations between the compound levels give rise to an "intermediate structure" (IS) resonance.[21] In Sec. IV we shall therefore consider the effects of coherence, leading to intermediate structure and compound-direct interference.

Finally, we should note that isospin mixing that takes place during the formation or decay of a set of compound levels contributes through the first term of Eq. (29) and is thus not reduced by $\frac{1}{2}\Delta/I$. The preferential spin-flip mechanism, in fact, leads to isospin violation during the formation of compound levels by deuteron bombardment. This effect can be shown to be equivalent to the purely direct mechanism in the absence of intermediate structure, so there is no point in treating it as an independent phenomenon.

IV. COHERENCE, INTERMEDIATE STRUCTURE AND COMPOUND-DIRECT INTERFERENCE

A. General Considerations

In this section we shall be interested in the coherent superposition of compound levels which leads to intermediate structure and direct reactions.[21,43] This subject was briefly discussed in N. It is relevant to the present study of isospin-violating reactions, both because these reactions offer a nearly unique means of investigating compound-nuclear structure, and because, as we shall see, the MEA data on $C^{12}(d,\alpha)B^{10*}(T=1)$ seem to indicate the presence of intermediate structure and also of compound-direct interference.

Statistical treatments of nuclear spectra have had considerable success,[44] despite the fact that (because nuclei are described by a *definite* Hamiltonian) energies, partial widths, and other characteristics of nuclear levels are not distributed in a truly random fashion. On the other hand, we frequently observe nuclear behavior in which most of the degrees of freedom seem to have been suppressed, so that only a few are effective. Examples of this are the various collective states, giant

[43] G. E. Brown, Rev. Mod. Phys. 31, 893 (1959).
[44] C. E. Porter, *Statistical Theories of Spectra: Fluctuations* (Academic Press Inc., New York, 1965).

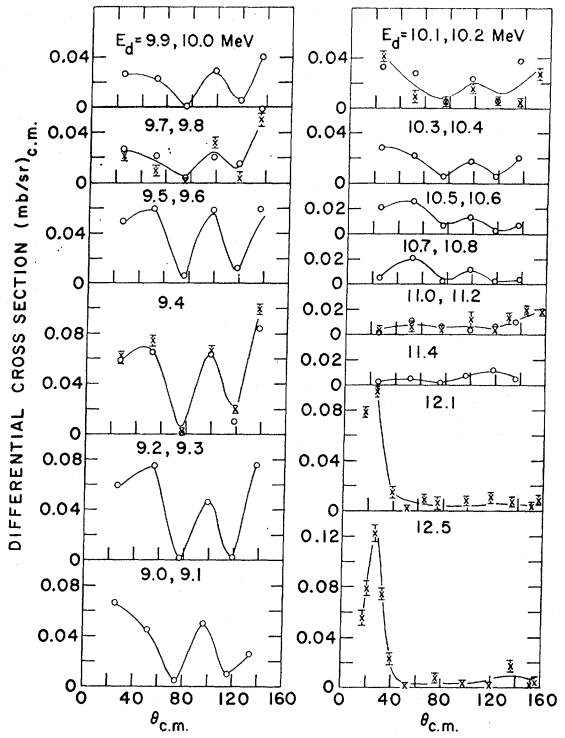


FIG. 9. Angular distributions for the isospin-forbidden α group $C^{12}(2)$ obtained at a number of deuteron energies. Both the differential cross section and the angles are given in the c.m. system. Crosses indicate measurements with the magnetic spectrograph, open circles represent data taken with the solid state detectors. The lines are given as guides for the eye. When the angular distribution was averaged over two runs of different deuteron energies, both energies are indicated. (This figure and its caption are taken unmodified from Ref. 8.)

single-particle resonances, isobaric analog states, and so forth. In a many-level formulation, the "simple" states appear when the nearby compound states are no longer statistically independent, but have correlated partial widths. Similarly, the ability to describe reactions of complex nuclei with optical-model interactions between only a few participating particles also represents a considerable reduction in the number of available degrees of freedom. The appearance of simple excitations in complex systems generally reflects the existence of an approximate Hamiltonian with greater symmetry than the exact Hamiltonian. It is easy, of course, to identify the approximate Hamiltonian and its additional symmetry in the case of isobaric analog states; sometimes (as in the case of the giant dipole resonance[45]) they can even be determined for collective states. At our present stage of sophistication, however, it does not seem possible to determine with certainty the underlying character of a general intermediate-structure resonance.

As was mentioned in the Introduction, the density of levels of like spin and parity at intermediate excitations in light nuclei is not very well known. Such factors as shell effects and intrinsic deformations probably cause the level density to vary considerably from one light nucleus to another. If we naively try to extrapolate the parameters of a statistical level-density formula such as[46]

$$\omega(J,P;E_x)=C(N,Z)(2J+1)\times\exp[2(aE_x)^{1/2}-J(J+1)/\Theta] \quad (30)$$

to small A and intermediate excitation energy, the results are extremely uncertain. For N^{14} and E_x between 17 and 20 MeV, $\omega(3^-)$ could be as large as 250 MeV^{-1} or as small as 10 MeV^{-1}, and a similar range of uncertainty is found for F^{18} in the region 12–16 MeV. Thus it is not clear whether the results of the preceding section apply to the MEA or JMR data.

B. Analysis of MEA Data

In Figs. 9 and 10 we reproduce the differential cross sections and total cross section, respectively, obtained by MEA for the reaction $C^{12}(d,\alpha)B^{10*}(1.74,0^+,T=1)$. Inspection of the differential cross sections (as a function of incident deuteron energy) reveals three distinct energy regions: the compound (C) region, 9.0–11.0 MeV, the direct (D) region, above 12.0 MeV, and a

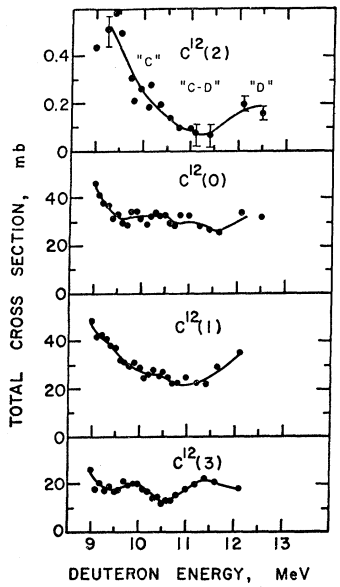


FIG. 10. Total cross sections for the $C^{12}(d,\alpha)B^{10}$ reaction to the lowest four levels in B^{10}. The curves represent the forbidden α group $C^{12}(2)$ and the allowed groups $C^{12}(0)$, $C^{12}(1)$, and $C^{12}(3)$, respectively, as functions of the deuteron energy. On the $C^{12}(2)$ cross section, the compound (C), direct (D) and compound-direct interference (C-D) regions have been located approximately. (This figure and part of its caption are taken from Ref. 8.)

[45] D. Brink, Nucl. Phys. 4, 215 (1957).

[46] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1962), p. 528. The level-density estimates were made by counting the levels of known spin at low excitation, in N^{14} and F^{18}, taking $a(A)=(\frac{1}{8}A)$ MeV^{-1}. This value is reasonably in accord with recent determinations such as that by U. Facchini, M. G. Marcazzan, L. Milazzo-Coli, and E. Saetta-Menichella, Phys. Letters **26B**, 278 (1968).

narrow compound-direct (C-D) region 11.0–~12.0 MeV. The energy resolution of the experiment was 20–30 keV. Thus if the average level spacing were as large as, say, 50–60 keV, one would expect rather large fluctuations in the total T-violating cross section, in the C region. There do not seem to be any distinctive fluctuations exceeding the statistical error limits, so we conclude tentatively that the average level spacing (for given J^P) is less than ~20 keV. Therefore, the considerations of Sec. III probably apply to this case.

An interesting feature of the MEA data is that the T-forbidden angular distributions in the C region appear to be derived from only one or two states of definite J^P. Since there are perhaps hundreds of compound levels of various J^P contained in this 2-MeV interval, that only those with $J^P=3^-$ (and possibly a small positive-parity admixture) actually contribute to the angular distribution is really quite remarkable. The other interesting feature of this data is the sharp minimum in the excitation function $\sigma_T(E_d)$ at $E_d\sim 11.3$ MeV. (See Fig. 10.) As is clear from Fig. 9, the region labelled CD in Fig. 10 is where transition from compound to direct reaction mechanisms takes place. In a many-level description, the dip in σ_T would be attributed to complicated phase relationships among the partial widths, leading to destructive interference in the amplitude that, by its very nature, could occur only over a narrow energy region. Another way of saying the same thing is that the minimum represents compound-direct interference of a rather special kind, namely, interference between an intermediate-structure resonance and a direct background.

It is straightforward to show that in the vicinity of an intermediate-structure resonance, the energy-averaged reaction amplitude has the form

$$\langle T_{fi}^{(+)}(E)\,\mathbf{P}\rangle \simeq \chi_f|\,V_f\mathbf{P}\Omega^{(+)}|\,\chi_i\rangle+(E+\tfrac{1}{2}i\Gamma-E_r)^{-1}\times\langle\chi_f|\,V_f\mathbf{R}|\,\Phi_r\rangle\langle\Phi_r|\,\mathbf{R}V_i\mathbf{P}\Omega^{(+)}|\,\chi_i\rangle. \quad (31)$$

[The assumptions included in the derivation of Eq. (31) will be discussed subsequently.] The wave operator appearing in (31) is just that associated with the energy-averaged effective interaction $\langle V_{\rm eff}(E)\rangle_{\rm av}$;

$$\Omega^{(+)}(E)=[1-\mathbf{P}G_i(E+iI)V_{\rm eff}(E+iI)]^{-1}. \quad (32)$$

Now the differential cross section is proportional to

$$|\langle T_{fi}^{(+)}(E)\,\mathbf{P}\rangle|^2.$$

Presumably the resonance in (31) has definite spin and parity, so that in the excitation function it interferes with only the corresponding part of the direct amplitude:

$$\sigma_{fi}(E)=2\pi C\int_0^\pi d\theta\,\sin\theta\,|\langle T_{fi}^{(+)}(E,\theta)\rangle|^2$$
$$=C\sum_{J,P}(2J+1)\,|\langle T_{fi}^{(+)}(E;J,P)\rangle|^2,$$

which may be rewritten

$$\sigma_{fi}(E)=C\sum_{J,P\neq J_0,P_0}{}'(2J+1)\,|\langle\chi_f|\,V_f\mathbf{P}\Omega^{(+)}|\,\chi_i\rangle_{J,P}|^2$$
$$+C(2J_0+1)\,|\langle\chi_f|\,V_f\mathbf{P}\Omega^{(+)}|\,\chi_i\rangle_{J_0,P_0}$$
$$+\langle\chi_f|\,V_f\mathbf{R}|\,\Phi_{J_0P_0}\rangle(E+\tfrac{1}{2}i\Gamma-E_r)^{-1}$$
$$\times\langle\Phi_{J_0P_0}|\,\mathbf{R}V_i\mathbf{P}\Omega^{(+)}|\,\chi_i\rangle|^2. \quad (33)$$

(In the preceding two expressions, C is a normalization constant involving kinematic factors which vary slowly with energy over the resonance.) Inspection of the second term of Eq. (33) reveals that the relative phase and magnitude of the interfering contributions is essentially determined by the sign and magnitude of $\mathbf{R}V_i\mathbf{P}$, the channel-coupling interaction. (The phase contributed by the resonance denominator is trivial.) The shape of the excitation function near the resonance is then given by the expression

$$f(x)=[(x+x_0)^2+\tfrac{1}{4}\Gamma^2](x^2+\tfrac{1}{4}\Gamma^2)^{-1}, \quad (34)$$

where $x=E-E_r$, and x_0 is the slowly varying expression

$$\langle\chi_f|\,V_f\mathbf{R}|\,\Phi_{J_0P_0}\rangle\langle\Phi_{J_0P_0}|\,\mathbf{R}V_i\mathbf{P}\Omega^{(+)}|\,\chi_i\rangle/\langle\chi_f|\,V_f\mathbf{P}\Omega^{(+)}|\,\chi_i\rangle_{J_0P_0},$$

which has the dimensions of energy and (as we pointed out above) is to a good approximation real. The maximum and minimum of (34) occur when

$$x=\tfrac{1}{2}[-x_0\pm(x_0^2+\Gamma^2)^{1/2}],$$

which is which depends on the sign of x_0. When x_0 is negative and $\Gamma>|x_0|$, the maximum lies to the left of the minimum at $\sim\frac{1}{2}(|x_0|-\Gamma)$; the situation is reversed when x_0 is positive.

Let us now examine the experimental excitation function for the reaction $C^{12}(d,\alpha)B^{10*}(1.74,T=1)$ (see Fig. 10), with the aim of fitting its shape by the function $f(x)+g$, where g is a constant representing the slowly varying background. We suppose $x_0<0$ and $\Gamma>|x_0|$. Then Γ is the distance between maximum and minimum, ~2 MeV. To the right of the minimum, $f(x)$ rapidly approaches unity, so that (from Fig. 10)

$$(1+g)/[f(\tfrac{1}{2}\Gamma+\tfrac{1}{2}|x_0|)+g]\simeq 2\pm 0.5.$$

Finally, the ratio of the maximum to the minimum is about 6, so

$$[f(\tfrac{1}{2}|x_0|-\tfrac{1}{2}\Gamma)+g]/[f(\tfrac{1}{2}|x_0|+\tfrac{1}{2}\Gamma)+g]\simeq 6\pm 1.$$

These equations, when solved for $|x_0|$ and g, give

$$g=0.5,\qquad |x_0|=\tfrac{3}{5}\Gamma\simeq 1.2\text{ MeV}. \quad (35)$$

The data are also consistent with

$$g=1.8,\qquad |x_0|=1.1\text{ MeV}. \quad (35')$$

TABLE I. Depth of square-well potential for cluster decompositions of N^{14}.

Cluster	V_0 (MeV)	V_0 (MeV) +18 (MeV)
$p+C^{13}$ $n+N^{13}$	49.5	67.5
$d+C^{12}$	73.8	91.8
H^3+C^{11} He^3+B^{11}	113	131
He^4+B^{10}	133	151

TABLE II. Square-well depths giving zero-energy bound states for different clusters and L values.

	V_L (MeV)						
Cluster	$L=1$	$L=2$	$L=3$	$L=4$	$L=5$	$L=6$	$L=7$
$p+C^{13}$ $n+N^{13}$	18.7 74.6	38.1 113	62.7 156.5	92.5	126		
$d+C^{12}$	10.2 40.7 91.5[c] 163	20.8 61.5 122.5 204	34.2 85.3[b] 157	50.4 112	69 141	90[a] 173	114
H^3+C^{11} He^3+B^{11}	7.4 29.6 66.4 118 184	15 44.5 89 148 222	24.8 62 113 180	36.5 81.2 140	50 102 170	65.5 125 200	82.5 151
He^4+B^{10}	6.1 24.4 54.8 97.5 152[c] 219	12.4 36.7 73.3 122 183	20.4 51 93.6 149[b] 215	30.1 67 116 177	41.2 84.5 140 206	54 103 165	68 124 192

[a] $L=6$ state near the physical value, but not seen, probably because of small width.
[b] $L=3$ states near the physical value.
[c] $L=1$ states near the physical value.

Since x_0 is in some sense given by an "average" matrix element of $\mathbf{R}V_i\mathbf{P}$, the fits (35) and (35′) are in accord with our assumption that $\mathbf{R}V_i\mathbf{P}$ should be weak and attractive.[47] Similarly, since g measures the ratio of the contribution of the direct mechanism in partial waves other than J_0P_0, to that in the resonant partial wave, the values (35) and (35′) are not unreasonable, although from the shape of the direct differential cross section the value $g=1.8$ is more reasonable than $g=0.5$. (The direct amplitude clearly does not consist predominantly of one partial wave.)

Having established that there may be an intermediate structure resonance in N^{14} with $J^P=3^-$, and $E_x\sim18$ MeV, $\Gamma\sim2$ MeV, it is interesting to speculate on its nature. One intriguing possibility is that it is a single-particle potential-well resonance appearing in the scattering of a cluster such as α or d (or a single nucleon, for that matter) on an appropriate target. In order to obtain some ideas on this question, we can perform the following extremely crude calculation: We assume that the scattering of, say, He^4 on B^{10} is described by a square well of radius $R=1.5A^{1/3}$. Then we determine the well depth V_0 by adding the binding energy of N^{14} [with regard to breakup into, say, He^4+B^{10*} (1.74, $T=1$)] to the well depth required to produce a zero-energy S-wave bound state. These values are listed for the cluster decompositions $p+C^{13}$, $d+C^{12}$, H^3+C^{11}, He^4+B^{10} in Table I. We then use the well-known formula[48]

$$j_{L-1}[R(2M_rV_0\hbar^{-2})^{1/2}]=0$$

to find those values of V_0 which produce zero-energy bound states with orbital angular momentum $L(>0)$. These values are listed in Table II for $L\leq7$. In column 3 of Table I are listed the "experimental" well depths plus 18 MeV; instead of reducing the values in Table II by 18 MeV to find the wells with an 18-MeV resonance, we have increased the experimental well depths to achieve the same end result. Comparing column 3 of Table I with Table II, we find that the only candidates at all near the actual resonance are an $L=3$ resonance in He^4+B^{10} at ~16 MeV, an $L=1$ resonance in He^4+B^{10} at ~19 MeV, an $L=3$ and an $L=6$ resonance in $d+C^{12}$ at ~16 MeV, and an $L=1$ resonance in $d+C^{12}$ at ~18 MeV. Despite the crudity of the calculation it is clear that there are no other nearby levels with $L\leq7$, a feature we would expect to persist in a more "accurate" calculation.

Although one might perhaps hesitate to describe the resonance(s) seen by MEA as a single-particle cluster state (SPCS) on the basis of these crude estimates, it is nevertheless tempting to do so, particularly since the SPCS's appear only in the entrance and exit channels. (This would explain why these states in particular are excited.) There is perhaps a smaller credibility gap associated with the He^4+B^{10} SPCS, since such states have already been observed in other reactions.[49]

C. Model of Compound-Direct Interference and Intermediate Structure-Direct Interference

We shall now show how compound-direct interference fits into the framework of the three-body model of deuteron scattering as expressed by Eqs. (5) and (6). Taking the zero of energy at the ground state of the target plus two noninteracting nucleons with no kinetic energy, we see that the projected Green's function $\mathbf{P}G_i\mathbf{P}$ has the form

$$\mathbf{P}G_i(W)\mathbf{P}\equiv|0\rangle(W-H_0-V_{np})^{-1}\langle0|\,, \qquad (36)$$

where H_0 is the kinetic energy of $n+p+$target, and V_{np} is the neutron-proton potential. [Clearly $(W-H_0-V_{np})^{-1}$ operates only in the three-body space.] Thus the analytic structure of $\mathbf{P}G_i(W)\mathbf{P}$ is particularly

[47] Note that both $\mathbf{R}$ and $\mathbf{P}$ are positive operators, and we expect V_i to be the sum of attractive potentials.
[48] E. Merzbacher, *Quantum Mechanics* (John Wiley & Sons, Inc., New York, 1961), p. 212.
[49] R. Middleton, B. Rosner, D. J. Pullen, and L. Polsky, Phys. Rev. Letters **20**, 118 (1968).

simple; it has only branch cuts corresponding to the elastic and breakup thresholds for deuteron scattering. (The elastic threshold is -2.225 MeV and the breakup threshold is 0.) All of the branch cuts associated with the presence of lower thresholds have been subsumed into the effective interaction, Eq. (6). Since (d,α) reactions are usually exoergic, it is quite possible that the elastic deuteron threshold lies above the elastic scattering threshold for α particles on the residual nucleus. To include this possibility while keeping the model as simple as possible, we assume the approximation

$$\mathbf{P}V_i\mathbf{R}(W-\mathbf{R}H\mathbf{R})^{-1}\mathbf{R}V_i\mathbf{P} \simeq\sum_\lambda|v_\lambda\rangle[W-\mathcal{E}_\lambda(W)]^{-1}\langle v_\lambda|, \quad (37)$$

where λ is now a purely discrete index, and where the $\mathcal{E}_\lambda(W)$ are analytic functions including those cuts of $(W-\mathbf{R}H\mathbf{R})^{-1}$ with thresholds lower than -2.225 MeV. We are now in a position to solve the integral equation (6). Suppose for some λ, $W-\mathcal{E}_\lambda(W)$ is very small, so that this term predominates in the sum on the right-hand side of (37). Then defining

$$U_{\lambda_0}=\sum_{\lambda\neq\lambda_0}|v_\lambda\rangle[W-\mathcal{E}_\lambda(W)]^{-1}\langle v_\lambda| \quad (38)$$

and

$$\Omega(W;\lambda_0)=[1-\mathbf{P}G_i(W)\{\mathbf{P}V_i\mathbf{P}+U_{\lambda_0}\}]^{-1}, \quad (39)$$

we find formally

$$T_{fi}\mathbf{P}=[V_f\mathbf{P}+V_f\mathbf{R}(W-\mathbf{R}H\mathbf{R})^{-1}V_i\mathbf{P}]\Omega(W,\lambda_0) +T_{fi}\mathbf{P}G_i|v_{\lambda_0}\rangle[W-\mathcal{E}_{\lambda_0}(W)]^{-1}\langle v_{\lambda_0}|\Omega(W,\lambda_0), \quad (39')$$

which gives

$$T_{fi}\mathbf{P}=\{V_f\mathbf{P}+V_f\mathbf{R}(W-\mathbf{R}H\mathbf{R})^{-1}V_i\mathbf{P}\}\Omega(W,\lambda_0) \times\{1+\mathbf{P}G_i(W)|v_{\lambda_0}\rangle[W-\mathcal{E}_{\lambda_0}(W) -\langle v_{\lambda_0}|\Omega(W,\lambda_0)\mathbf{P}G_i(W)|v_{\lambda_0}\rangle]^{-1} \times\langle v_{\lambda_0}|\Omega(W,\lambda_0)\}. \quad (40)$$

Approximating $V_f\mathbf{R}(W-\mathbf{R}H\mathbf{R})^{-1}V_i\mathbf{P}$ in analogy with Eq. (37) we see that Eq. (39′) becomes

$$T_{fi}P=V_fP\Omega(W,\lambda_0)+|\bar v_{\lambda_0}\rangle[W-\mathcal{E}_{\lambda_0}(W) -\langle v_{\lambda_0}|\Omega(W,\lambda_0)\mathbf{P}G_i(W)|v_{\lambda_0}\rangle]^{-1} \times\langle v_{\lambda_0}|\Omega(W,\lambda_0). \quad (41)$$

We now average Eq. (41) over the energy resolution by replacing W by $E+iI$, and then average over level subscripts λ_0 for which $|\mathrm{Re}[E-\mathcal{E}_{\lambda_0}(E+iI)]|<I$. We assume I is large enough that several levels are included. The wave operator $\Omega(E+iI;\lambda_0)$ is clearly very stable against these averaging processes, since the most significant contribution to $U_{\lambda_0}(E+iI)$ is an absorptive imaginary part.[50] We may thus replace $\langle\Omega(E+iI;\lambda_0)\rangle_{\lambda_0}$ by $\Omega^{(+)}(E)$ defined in (32). The average energy for all levels of given spin and parity lying within a distance I of E is defined as

$$\langle\mathcal{E}^{JP}(E+iI)\rangle=(N_{JP})^{-1}\sum_{\lambda_0}[\mathcal{E}_{\lambda_0}{}^{JP}(E+iI) +\langle v_{\lambda_0}|\Omega^{(+)}(E)G_i(E+iI)|v_{\lambda_0}\rangle]. \quad (42)$$

We once again use the same trick as in Eq. (24) or Eq. (29) to find

$$\sum_{J,P}(N_{JP})^{-1}\sum_{\lambda_0}|\tilde v_{\lambda_0}{}^{JP}\rangle[E+iI-\mathcal{E}_{\lambda_0}{}^{JP}(E+iI) -\langle v_{\lambda_0}{}^{JP}|\Omega^{(+)}(E)\mathbf{P}G_i(E+iI)|v_{\lambda_0}{}^{JP}\rangle]^{-1}\langle v_{\lambda_0}{}^{JP}| \simeq\sum_{J,P}[E+iI-\langle\mathcal{E}^{JP}(E+iI)\rangle]^{-1} \times\sum_{\lambda_0}|\tilde v_{\lambda_0}{}^{J,P}\rangle\langle v_{\lambda_0}{}^{J,P}|. \quad (43)$$

As we observed in Sec. III, the contribution of the right-hand side of (43) will be rather small (in fact of the order of the level spacing divided by the resolution) unless the partial widths are somehow correlated. If there is intermediate structure present, the expected form of

$$(N_{JP})^{-1}\sum_{\lambda_0}|\tilde v_{\lambda_0}{}^{JP}\rangle\langle v_{\lambda_0}{}^{JP}|$$

is $|\tilde V_{JP}\rangle\langle V_{JP}|$, where the V denote average vertex functions. The energy-averaged reaction amplitude in the vicinity of an intermediate-structure (IS) resonance thus has the form

$$\langle T_{fi}{}^{(+)}(E)\mathbf{P}\rangle_{\mathrm{av}}\simeq V_f\mathbf{P}\Omega^{(+)}(E) +|\tilde V_{JP}\rangle[E+iI-\langle\mathcal{E}^{JP}(E+iI)\rangle]^{-1}\langle V_{JP}|. \quad (44)$$

Equation (44) shows clearly the partition of the reaction amplitude into "direct" and resonant contributions. Comparison with Eq. (39′) gives

$$|\tilde V_{JP}\rangle=[V_f\mathbf{R}+V_f\mathbf{P}\Omega^{(+)}(E)\mathbf{P}G_i(E+iI)V_i\mathbf{R}] \times|\Phi_{JP}\rangle \quad (45a)$$

and

$$\langle V_{JP}|=\langle\Phi_{JP}|\mathbf{R}V_i\mathbf{P}\Omega^{(+)}(E). \quad (45b)$$

[We note that the wave operator $\Omega^{(+)}(E)$ is expected to introduce isospin mixing during the formation of the compound resonance(s). If no IS is present, this isospin mixing is included already in the direct term of Eq. (44). If IS is present in the form of a *deuteron* SPCS, then the near equality of the effective interactions B_{11} and B_{00} practically guarantees the existence of a nearby φ SPCS that will mix with it through the action of B_{10} as described in Sec. II C.] In deriving Eq. (31) we have made the further assumption that $\mathbf{P}G_i\mathbf{P}V_i\mathbf{R}$ is small and so

$$|\tilde V_{JP}\rangle\simeq V_f\mathbf{R}|\Phi_{JP}\rangle.$$

Using the value for x_0 determined in Eq. (35), we find that the operator norm $\|\mathbf{P}G_iV_i\mathbf{R}\|$ is on the order of $\frac{1}{3}$, so that this assumption is really quite reasonable.[51]

Finally, we note that for fine-resolution experiments Eq. (41) is already quite adequate in the vicinity of a compound resonance; also, $\Omega(E+iI;\lambda_0)$ may be replaced in this vicinity by $\Omega^{(+)}(E)$ without introducing excessive error.

[50] The real part of $U_{\lambda_0}(E+iI)$ will have a magnitude of the order of that of the imaginary part, which empirically is much smaller than $\mathbf{P}V_i\mathbf{P}$.

[51] The operator norm of $\mathbf{P}G_iV_i\mathbf{R}$ was estimated by assuming that $\mathbf{P}V_i\mathbf{R}$ is a central, surface-peaked potential with the usual range and diffuseness, and with strength $\sim 2|x_0|$.

V. SUMMARY

In this paper, I have tried to give a fairly complete and self-contained account of isobaric spin mixing in low-energy nuclear reactions, giving particular attention to the role of the preferential spin-flip mechanism in both direct and compound deuteron-induced reactions. This mechanism was briefly recapitulated in Sec. II, and its consequences for isospin violation in reactions and in stationary states were discussed there in some detail. It is appropriate to point out here that in Robson's language[52] the spin-flip mechanism is a boundary-condition effect, strongly dependent on the long range of the Coulomb interaction. It is well known that the sensitivity of the nucleon wave functions to boundary conditions can lead to much larger effects than simple photon absorption (isovector) can produce. As was shown in N, the effect is indeed considerably larger than that given by one-photon exchange in the case of $C^{12}(d,\alpha)B^{10*}(T=1)$.

Sections III and IV were devoted to the detailed investigation of isospin violation through the formation of isobarically-impure compound-intermediate states. (The considerations of these sections of course apply to reactions with other projectiles than deuterons.) The question of the actual level density in N^{14} and F^{18} at intermediate excitations was shown to be important in trying to understand the data, and arguments were given for the level spacing being somewhat less than the best available resolution. It was also argued that, at least in the MEA data, there was evidence for both compound-direct interference and intermediate structure. On the basis of suggestive numerology, presented in Tables I and II, the doorway states giving rise to the intermediate structure were conjectured to be cluster single-particle states in the entrance and exit channels. It is also worth mentioning that the isospin-violating (d,α) reactions on $J^P=0^+$, self-conjugate nuclei show a preference for compound intermediate states with $L=J$. Since most of the data involve reactions with spinless positive-parity particles in the exit channel, this is just a consequence of angular-momentum and parity conservation. It would be extremely interesting to try to find evidence for compound states with $J=L\pm1$ excited in the isospin-violating process $N^{14}(d,d')N^{14*}$ $(2.31,T=1)$. According to the model presented in this paper, these states cannot act as strongly mixed intermediate-structure resonances since they experience no isospin mixing from the spin-flip mechanism, and we thus expect the many fine-structure levels to be relatively uncorrelated. Therefore, we would expect that the compound isospin-violating reactions will not take place via $J=L\pm1$ intermediate states, and a search for such states would be a fairly critical test of the hypotheses of this paper.

Perhaps the most important conclusion to be drawn from this investigation is that the class of isospin-violating reactions considered here is especially suited to the investigation of intermediate structure. Since there seems to be hope that the level spacing in the interesting cases of N^{14} and F^{18} is not too much smaller than the best available energy resolution, it might be rewarding to try to improve the resolution and attempt to see whether there is in fact fine structure in, say, the broad 3^- resonance in N^{14}.

[52] D. Robson, in *Proceedings of the Conference on Isobaric Spin in Nuclear Physics, 1966* (Academic Press Inc., New York, 1966), p. 411.

ACKNOWLEDGMENTS

I am grateful to Dr. C. A. Levinson for pointing out the reduction of the Wilkinson mechanism in averages over uncorrelated levels, and thus providing the stimulus for part of this work. My sincerest thanks go to Dr. J. Duray, Dr. C. L. Cocke, Dr. H. T. Richards and Dr. J. A. Jänecke for discussing their results with me prior to publication; and to Dr. L. Meyer-Schützmeister, both for furnishing figures from the MEA paper, and for several helpful discussions. Finally, I am indebted to Dr. G. Bertsch, Dr. G. E. Brown, Dr. R. D. Amado, and Dr. A. Klein for discussing several aspects of this work with me.