Fermi-Liquid Effects on High-Frequency Wave Propagation in Simple Metals

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(Received 4 March 1968)

The propagation characteristics of high-frequency waves (HFW) which occur in simple metals near the Azbel'-Kaner cyclotron resonances are discussed in detail. In particular, it is emphasized that, while the existence of such wave propagation may be understood using a noninteracting-electron-gas model of the conduction electrons, the HFW are sensitive in principle to direct electron-electron interactions. Using the Landau-Silin phenomenological description of the Fermi liquid, it is shown that the HFW offer the possibility of direct measurement of all but the lowest moments of the spin-independent electron correlation function.

I. INTRODUCTION

IN the presence of a magnetic field the conduction electrons of pure, cold metals may collectively support a variety of relatively undamped wavelike electromagnetic excitations or plasma waves.[1] In this paper we wish to present an extensive but primarily theoretical discussion of a new class of plasma waves, first observed in 1965,[2,3] which propagate most readily perpendicular to the magnetic field in the vicinity of the fundamental cyclotron resonance and its harmonics [the Azbel'-Kaner cyclotron resonances (AKCR)]. Because these waves occur for $\omega \sim n\omega_c$, $n=1, 2, \cdots$ (where $\omega_c = eH/m^*c$ and m^* is the effective mass of a conduction electron), they will be referred to as high-frequency waves (HFW) as contrasted with the more familiar helicon or Alfvén modes of electromagnetic propagation in metals which require $\omega \ll \omega_c$.[1] While it proves to be possible to understand the existence and general features of the HFW in terms of an independent-particle representation of the conduction electrons (free-electron model), we wish to emphasize that the propagation characteristics (dispersion relations) are explicitly sensitive[4] to electron-electron interactions. Using the Landau-Silin[5] phenomenological description of a weakly excited Fermi liquid, it is shown that the HFW offer the possibility of experimental determination of the spin-independent moments of the electron-electron scattering or correlation function (Landau f function), though in practice the two lowest moments are not measurable in the presently accessible experimental regime. Their explicit sensitivity to many-body effects make the HFW complimentary to the recently discovered spin waves associated with the conduction-electron spin resonance of simple metals.[6,7] The spin waves provide quantitative information concerning the spin-dependent part of the electron-electron interactions.

Following a general description of the regime in which the HFW are observed and the corresponding limits in which the theory is to be developed (Sec. II), the dispersion equations are discussed and their relation to prior calculations of wave propagation in metallic and gaseous plasmas are indicated (Sec. III). The nonlocal magnetoconductivity tensor for the free-electron gas is then used to compute dispersion curves for the "ordinary wave" which are compared with experimental results for potassium (Sec. IV). Finally, modifications of the dispersion relations due to electron-electron correlations are treated using the Landau-Silin theory of interacting fermions (Sec. V). While no attempt is made to evaluate the moments of the interaction function from experiment, the general character of the predicted observable effects is discussed in some detail.

II. HFW REGIME

The HFW have thus far been most clearly observed in the alkali metals potassium[2,3] and sodium,[8] whose

[1] *Proceedings of the Symposium on Plasma Effects in Solids, Paris, 1964* (Academic Press Inc., New York, 1965); E. A. Kaner and V. G. Skobov, Usp. Fiz. Nauk **89**, 367 (1966) [English transl.: Soviet Phys.—Usp. **9**, 480 (1967)].

[2] W. M. Walsh, Jr., and P. M. Platzman, Phys. Rev. Letters **15**, 784 (1965).

[3] P. M. Platzman and W. M. Walsh, Jr., Phys. Rev. Letters **19**, 514 (1967); **20**, 89(E) (1968).

[4] By an *explicit* sensitivity to correlation effects we mean that the simple effective-mass renormalization procedure which is satisfactory for most observable phenomena in metals does not suffice to explain the HFW dispersion relations. The effective mass itself depends *implicitly* on many-body interactions.

[5] L. D. Landau, Zh. Eksperim. i Teor. Fiz. **30**, 1058 (1956) [English transl.: Soviet Phys.—JETP **3**, 920 (1956)]; V. P. Silin, *ibid.* **33**, 495 (1957) [English transl.: *ibid.* **6**, 945 (1958)].

[6] P. M. Platzman and P. A. Wolff, Phys. Rev. Letters **18**, 280 (1967).

[7] S. Schultz and G. Dunifer, Phys. Rev. Letters **18**, 283 (1967).

[8] W. M. Walsh, Jr., in *Proceedings of the Simon Fraser Summer School, "Electrons in Metals," Alta Lake, 1967* (Gordon and Breach Science Publishers, Inc., New York, to be published). The HFW have also been observed in rubidium [W. M. Walsh, Jr., and P. M. Platzman, in *Proceedings of the Tenth International Conference on Low Temperature Physics, Moscow, 1966* (VINITI, Moscow, 1967)] and, presumably, in the noble metals [S. Schultz (private communication)]. Both observation and analysis are seriously complicated in all but the simplest metals due to anisotropy of the conduction-electron velocity distribution.

Fermi surfaces are known to be single-sheeted and quite accurately spherical.[9] Wave propagation occurs under the same experimental conditions as are required for the observation of well-resolved cyclotron resonance spectra, i.e., samples of very high purity, liquid-helium temperatures, and sufficiently high excitation frequencies ($\omega\sim10^{11}$ rad/sec) that $\omega\tau\gg1$, τ being the mean electron scattering time due to lattice imperfections. The HFW occur in association with the AKCR phenomenon[10] which requires the magnetic field to be in or quite near the plane of a flat specimen.[11] Using conventional techniques, the magnetic-field derivative of the metal's total microwave power absorption versus field is recorded. The linearly polarized surface currents $\mathbf{J}$ which flow on both sample faces may be oriented either parallel or perpendicular to the slowly varying magnetic field $\mathbf{H}$. When plane-parallel samples of thickness $L\sim10^{-2}$ cm are examined, one finds, in addition to the AKCR spectrum of single-particle resonances, "extra" structure in the power absorption on the high-field side of the first few resonances. The variations of this extra structure with sample thickness and experimental frequency are consistent with the concept of wave excitation carrying microwave currents from one sample face to the other. The oscillations of the power absorption result from the varying phase of the transmitted currents relative to the primary driving currents.[12] The field-dependent phase variation yields the dispersion or variation of wavelength with field and frequency. Typically, the observed wavelengths prove to be comparable to maximal cyclotron orbit radii: $\lambda\sim R_c=V_F/\omega_c\sim10^{-3}$ cm for Fermi velocities $V_F\sim10^8$ cm/sec and magnetic fields of a few thousand G. Since the direction of propagation is along the sample normal, the wave vector $\mathbf{k}$ is usually perpendicular to $\mathbf{H}$ (propagation does, however, persist out to 10°–20° of tilt of $\mathbf{H}$ relative to the sample plane). In general, the most intense and easily interpreted evidence of HFW propagation is found for $\omega_c>\omega$ in the $\mathbf{J}\|\mathbf{H}$ polarization with $\mathbf{k}\perp\mathbf{H}$.[2] Similar "propagation windows" are also found on the high-field side of the first few subharmonic resonances.[8] Generally weaker and more complicated structure is observed in the $\mathbf{J}\perp\mathbf{H}$ polarization.[3,8]

Analysis of these magnetically induced transparencies is based on recognition that one is dealing primarily with wave propagation in the *bulk* of the metal. Almost all the incident radiation is reflected at the sample surfaces where intense fields and currents flow in the anomalous skin depth δ. To the extent that the specimen thickness is much greater than the surface region where the transmitted wave is excited ($L/\delta\sim10^2$ in practice), the propagation characteristics are those of the bulk metal. For the purpose of studying the HFW dispersion we need not, therefore, solve the complete and enormously difficult boundary-value problem but may restrict the analysis to the dielectric behavior of the unbounded medium. We thus ignore any questions concerning the amount of incident radiation which actually penetrates the metal slab in favor of studying the bulk response of the conduction-electron system.

An important aspect of the HFW is the particular range of frequencies and wavelengths in which they have been observed: An earlier theoretical investigation of helicon-wave propagation along the magnetic field[13] indicated that in the limit $kV_F/\omega\sim1$ it should be possible to observe explicit effects due to electron-electron interactions, i.e., to find deviations of the collective response from that predicted for noninteracting particles. The regime $kV_F/\omega=kR\sim1$ is very difficult to attain in the case of helicons but is precisely where the HFW are observed ($\lambda=2\pi/k\sim R_c\sim R$). It is, therefore, to be anticipated that the HFW dispersion behavior may differ significantly from that predicted for an electron gas and thus provide experimental information concerning many-body effects.

With this point in mind the remainder of the article is devoted to calculation of the dispersion relations for the HFW, first in the free-electron approximation and then allowing for interaction effects. The latter are treated within the framework of the Landau theory of Fermi liquids as extended by Silin for the case of conduction electrons.[5] This semiphenomenological theory provides a prescription for calculating all low-frequency, long-wavelength transport properties of a system of interacting quasiparticles. In this context frequencies are to be low with respect to E_F/h and ω_D, where E_F is the Fermi energy of the distribution and ω_D is the phonon Debye frequency. Wavelengths must be long compared to the Fermi-Thomas screening length.

Since the theory is semiphenomenological, the linear response of the metal, i.e., the electrical conductivity and the magnetic susceptibility, involves moments of an unknown function. This function, the Landau or quasiparticle quasihole scattering function $f(\mathbf{p},\sigma,\mathbf{p}',\sigma')$, is, in principle, calculable. In practice, however, because of the low density of conduction electrons in a metal, i.e., $r_s>1$, it can not be calculated with any real accuracy.[14] We may think of the HFW as an experimental means of directly measuring certain properties of this Landau scattering function. The question we will attempt to answer in the body of this paper is: What properties of the Landau function can be determined by carefully studying the properties of the HFW in very pure alkali metals? We shall show that these waves give informa-

[9] D. Shoenberg and P. J. Stiles, Proc. Roy. Soc. (London) **A281**, 62 (1964); M. J. G. Lee, *ibid.* **A295**, 440 (1966).

[10] M Ya Azbel' and E. A. Kaner, Zh. Eksperim. i Teor. Fiz. **32**, 896 (1957) [English transl.: Soviet Phys.—JETP **5**, 730 (1957)]; J. Phys. Chem. Solids **6**, 113 (1958).

[11] C. C. Grimes and A. F. Kip, Phys. Rev. **132**, 1991 (1963).

[12] A more extensive discussion of experimental details appears in *Proceedings of the Simon Fraser Summer School, "Electrons in Metals," Alta Lake, 1967* (Gordon and Breach Science Publishers, Inc., New York, to be published). The complete data for potassium and sodium will be published as a separate paper.

[13] P. M. Platzman and K. C. Jacobs, Phys. Rev. **134**, A974 (1964).

[14] T. M. Rice, Ann. Phys. (N. Y.) **31**, 100 (1965).

tion about the Landau function which is complementary to the information which is obtained from the dispersion of the recently discovered spin waves. The two sets of experiments can *in principle* be used to obtain a complete quantitative description of the functional form of the Landau scattering function.

III. DISPERSION RELATIONS

If, as discussed in the Sec. II, we confine our attention to the dispersion behavior of the HFW, rather than such aspects as their degree of coupling to the incident radiation, it is valid to ignore the complete boundary-value problem and concentrate on the bulk dielectric properties of the metal. Consider, therefore, the propagation characteristics of a weak electromagnetic plane-wave disturbance $\{\mathbf{E}\exp[i(\mathbf{k}\cdot\mathbf{r}-\omega t)]\}$ in an unbounded, translationally invariant metal. Maxwell's equations reduce to

$$\mathbf{k}\times(\mathbf{k}\times\mathbf{E})+k_0^2\boldsymbol{\epsilon}\cdot\mathbf{E}=0, \tag{1}$$

where the dielectric tensor

$$\epsilon_{\alpha\beta}(\mathbf{k},\omega)=\delta_{\alpha\beta}-4\pi\sigma_{\alpha\beta}(\mathbf{k},\omega,\mathbf{H})/i\omega \tag{2}$$

is the sum of a diagonal contribution, the displacement current, and a complex tensor contribution due to the magnetized conduction electrons. The quantity $\sigma_{\alpha\beta}$ is the frequency-, field-, and wave-number-dependent magnetoconductivity tensor which contains the physical essence of the problem.

In order to avoid gratuitous algebraic complications, we shall restrict the calculation to the cases of propagation accurately perpendicular to the magnetic field. Defining $\hat{z}\|\mathbf{H}$ and $\hat{x}\|\mathbf{k}$ as in Fig. 1, the off-diagonal components $\sigma_{xz}=\sigma_{zx}$ and $\sigma_{yz}=-\sigma_{zy}$ vanish. Setting the determinant of the coefficients in Eq. (1) equal to zero yields the familiar dispersion relation[15]

$$\left(\frac{k^2}{k_0^2}-\epsilon_{zz}\right)\left[\frac{k^2}{k_0^2}-\left(\epsilon_{yy}+\frac{\epsilon_{xy}^2}{\epsilon_{xx}}\right)\right]=0. \tag{3}$$

The first root

$$k^2/k_0^2=\epsilon_{zz} \tag{4}$$

is the so-called "ordinary wave"[15] which is purely transverse in character, i.e., $\mathbf{E}\|\hat{z}$. Because of its particular simplicity, it will be treated in detail below. The other root

$$k^2/k_0^2=\epsilon_{yy}+\epsilon_{xy}^2/\epsilon_{xx} \tag{5}$$

is known as the "extraordinary wave."[15] It differs from the ordinary wave in that it is not purely transverse in character but has a longitudinal component of electric field associated with it ($\mathbf{E}\cdot\mathbf{k}\neq 0$). In the low-density gas-plasma case, this solution of the wave equation

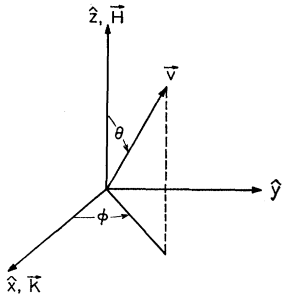

FIG. 1. Coordinate system used in the analysis of the text.

yields the longitudinal Bernstein modes[16] which manifest themselves both in absorption[17] and transmission[18] near cyclotron harmonics. In metals, however, the very high plasma or screening frequency $\omega_p=4\pi ne^2/m^*\sim 10^{16}$ rad/sec$\gg\omega$ (n being the density of conduction electrons) prevents any appreciable charge separation from developing.[19] As will become apparent, this reduces the physics of the extraordinary wave to the same level of interest as the algebraically simpler ordinary mode which will, therefore, receive most of our attention.

A. HFW Dispersion in the Free-Electron Model

The problem of wave propagation in bulk metals reduces essentially to the calculation of the conductivity tensor $\sigma_{\alpha\beta}(\mathbf{k},\omega,\mathbf{H})$. Rather than attempt a completely general treatment, we shall henceforth take a rather limited definition of a metal in which the electron velocity distribution is isotropic and relatively unaffected by the periodic potential due to the ion cores. Fortunately, nature has provided sodium and potassium, for which these statements appear to be valid.[9,11] It is the almost total absence of any band-structure effects which makes detailed studies of bulk wave-propagation phenomena profitable in these materials. They are the concrete embodiment of the hypothetical, uniform electron fluid or "jellium" for which the theory of interacting charged fermions has been developed. As such, they constitute unique arenas for the confrontation of experiment and the many-body theory of normal metals.

To begin, consider the conductivity tensor $\boldsymbol{\sigma}^0$ of a gas of noninteracting particles of charge e and mass m^* in a highly degenerate Fermi distribution. The tensor components have been evaluated previously[19,20] using a linearized Boltzmann theory. A typical component of present interest is

$$\sigma^0{}_{zz}=iN\sum_{n=0}^{\infty}\int_0^{\pi}\frac{aJ_n{}^2(b)\cos^2\theta\sin\theta d\theta}{(1+\delta_{n0})(a^2-n^2)}, \tag{6}$$

[15] W. P. Allis, S. J. Buchsbaum, and A. Bers, *Waves in Anisotropic Plasmas* (M. I. T. Press, Cambridge, Mass., 1963), pp. 90, 67.

[16] I. B. Bernstein, Phys. Rev. **109**, 10 (1958).

[17] S. J. Buchsbaum and A. Hasegawa, Phys. Rev. **143**, 303 (1966).

[18] F. W. Crawford and H. H. Weiss, J. Nucl. Energy **C8**, 21 (1966).

[19] M. Cohen, M. Harrison, and W. Harrison, Phys. Rev. **117**, 937 (1960).

[20] S. J. Buchsbaum and P. M. Platzman, Phys. Rev. **154**, 395 (1967).

where

$$N=3\omega_p^2/4\pi\omega_c,$$
$$a=(\omega+i/\tau)/\omega_c,$$
$$b=kV_F\sin\theta/\omega_c=kR_c\sin\theta.$$

In the range of parameters where the HFW are observed one has $\omega_p^2/\omega_c^2\sim10^{10}\gg k^2/k_0^2\sim10^5\gg1$. Because of the extremely high plasma frequency, the conduction-electron response completely dominates all other considerations, i.e., we may neglect both the displacement current in $\epsilon_{\alpha\beta}$ [Eq. (2)] and the k^2/k_0^2 in the dispersion relations Eqs. (4) and (5). These equations may be quite accurately solved by seeking the zeros of the appropriate conductivities:

$$\sigma^0{}_{zz}=0 \quad (J\|H), \tag{7}$$

$$\sigma^0{}_{yy}+\sigma^0{}_{xy}{}^2/\sigma^0{}_{xx}=0 \quad (J\perp H). \tag{8}$$

The physical content of these equations is that wave propagation, as opposed to nearly total reflection at a boundary, can only occur if for some combination of experimental frequency, wavelength in the medium, and magnetic-field value it is possible to make the conduction current essentially zero. In fact, of course, the current is not rigorously zero in a wave in a metal but the dispersion properties of the wave are given very accurately (to order k^2c^2/ω_p^2) by going to that limit. The apparently paradoxical requirement that the total conductivity become vanishingly small is only achievable because of the nonlocal nature of σ, i.e., its k dependence. This means that the total current is made up of a local contribution due to the electric field at a point in the medium plus a nonlocal contribution arising from electrons arriving at that point with velocity increments acquired at other points and earlier times. It is only when essentially exact cancellation between the local and nonlocal currents occurs that the possibility of wave propagation exists.

The idea that wave propagation might occur near the AKCR's was first discussed by Kaner and Skobov[21] but they envisaged appreciable coupling only to wavelengths comparable to the anomalous skin depth δ, i.e., much smaller than cyclotron orbit radii ($kR\gg1$). They, therefore, sought and found solutions to the dispersion relations Eqs. (4) and (5) in the short-wavelength limit. These Kaner-Skobov modes actually join smoothly with the HFW solutions of Eqs. (7) and (8) in the long-wavelength limit. Whereas little theoretical distinction exists between the two limits save for retention or neglect of the term k^2/k_0^2, it is important to recognize that only the long-wavelength limit has proved to be experimentally accessible.

The most useful form of the magnetoconductivity tensor components in the long-wavelength limit is an expansion in the parameter kR_c:

$$\sigma^0{}_{zz}=iNa\sum_{n=1}^{\infty}\frac{(kR_c)^{2(n-1)}}{(2n-1)(2n+1)(a^2-1)(a^2-4)\cdots[a^2-(n-1)^2]}, \tag{9}$$

$$\sigma^0{}_{yy}=\frac{iN}{a}\sum_{n=1}^{\infty}\frac{[a^2+2(n-1)n^2](kR_c)^{2(n-1)}}{(2n-1)(2n+1)(a^2-1)(a^2-4)\cdots(a^2-n^2)}, \tag{10}$$

$$\sigma^0{}_{xx}=\frac{iN}{a}\sum_{n=1}^{\infty}\frac{(kR_c)^{2(n-1)}}{(2n+1)(a^2-1)(a^2-4)\cdots(a^2-n^2)}, \tag{11}$$

$$\sigma^0{}_{xy}=-\sigma^0{}_{yx}=N\sum_{n=1}^{\infty}\frac{n(kR_c)^{2(n-1)}}{(2n+1)(a^2-1)(a^2-4)\cdots(a^2-n^2)}. \tag{12}$$

The resonances in these components occur at the cyclotron resonances $\omega=n\omega_c$ and have coefficients in the long-wavelength limit $kR_c\to0$ proportional to $(kR_c)^{2n}$ in the case of $\sigma^0{}_{zz}$ and $(kR_c)^{2(n-1)}$ for the others. Near the fundamental Azbel'-Kaner resonance the long-wavelength limits of the dispersion relations Eqs. (7) and (8) are

$$\sigma^0{}_{zz}\propto1+\frac{(kR_c)^2}{5(a^2-1)}+\cdots=0, \tag{13}$$

$$\sigma^0{}_{yy}+\frac{\sigma^0{}_{xy}{}^2}{\sigma^0{}_{xx}}\propto1+\frac{(kR_c)^4}{350(a^2-1)}+\cdots=0. \tag{14}$$

For $a^2\lesssim1$ ($\omega_c>\omega$), Eq. (13) has a solution quadratic in kR_c, whereas Eq. (14) predicts an extraordinary wave with a quartic dependence on kR_c for $\omega_c>\omega$. It is of interest to note that the coefficient of the latter quartic term, 1/350, is remarkably small. One is tempted to remark that the electron gas apparently resists exhibiting an appreciable singularity at the fundamental cyclotron resonance, a state of affairs noted earlier by Smith, Hebel, and Buchsbaum[22] in a treatment of

[21] E. A. Kaner and V. G. Skobov, Fiz. Tverd. Tela **6**, 1104 (1964) [English transl.: Soviet Phys.—Solid State **6**, 851 (1964)].

[22] G. E. Smith, L. C. Hebel, and S. J. Buchsbaum, Phys. Rev. **129**, 156 (1963).

nearly local conductivity. The situation is quite different on the second subharmonic for $J \perp H$, however, where

$$\sigma^0_{yy}+\frac{\sigma^0_{xy}{}^2}{\sigma^0_{xx}} \propto 1+\frac{6(kR_c)^2}{5(a^2-4)}+\cdots, \qquad (15)$$

a singularity quadratic in kR_c, appears analogous to the $\mathbf{J}\|\mathbf{H}$, $\omega_c \gtrsim \omega$ case.

The numerically computed solution of Eq. (4) in the region $\omega_c \gtrsim \omega$ with $\omega_p/\omega = 0.85\times 10^5$ and $V_F/c = 3\times 10^{-3}$ is shown in Fig. 2. A similar computation using the approximation of Eq. (7) does not differ appreciably from this curve until kR values of ~ 20 are reached. For $kR \lesssim 6$ the dispersion curve oscillates, a behavior which is characteristic of a sharp Fermi distribution. The oscillations in the attenuation of sound waves in pure metals as a function of a swept magnetic field (magneto-acoustic effect) has the same origins.[19] At still higher values of $kR \lesssim 10^2$, the dispersion curve eventually becomes monotonic and asymptotic to the $\omega_c/\omega = 1$ axis, which is just the behavior calculated by Kaner and Skobov using Eqs. (4) and (6) and the asymptotic forms of the Bessel functions for large values of their arguments.

Over the experimentally accessible region ($0 < kR \lesssim 6$), the dispersion curve is single-valued for $\omega_c/\omega < 1.77$, whereas in the interval $1.77 < \omega_c/\omega < 1.95$ it becomes multiple-valued, i.e., for a given magnetic field several waves of distinct k may be excited. Finally, for $\omega_c/\omega \gtrsim 1.95$, no solution of the wave equation is found and the metal is totally cut off.

Experimental data for a potassium slab are shown in Fig. 3. Under the existing antisymmetric excitation conditions, power-absorption minima are expected to occur for $L = (n+\frac{1}{2})\lambda$. Using this criterion, the extrema of the power-absorption derivative may be assigned kR values and plotted as in Fig. 4. Here the magnetic-field scale is determined by the effective mass ratio $m^*/m_0 = 1.21$, originally measured by Grimes and Kip.[11] Ambiguity as to the kR assignment may arise if only one such experimental curve is available. By examining the same specimen at several experimental frequencies or a series of samples of appreciably varying thickness, the "correct" wave-vector assignment may be determined.

The comparison of theory and experiment in Fig. 4 reveals quite good general agreement, though a small discrepancy exists at the $kR \to 0$ intercept where the experimental points tend to intercept at somewhat higher magnetic-field values than expected. This detail will be discussed later. At higher kR values, the general features of the theoretical dispersion appear to be reproduced: A large number of experiments have failed to reveal clear evidence of HFW waves for ω_c/ω appreciably greater than 1.7. We attribute this "washing out" to excitation of several distinct k vectors as the dispersion curve becomes multiple-valued. This would clearly lead to interference between the waves and a severe weakening of structure in the total power absorption. Furthermore, a weak but well defined and reproducible "break" (change of slope) in the experimental trace occurs for $\omega_c/\omega \simeq 1.9$. Presumably this reflects the "cutting off" of the metal as the limit of the propagation region is reached.

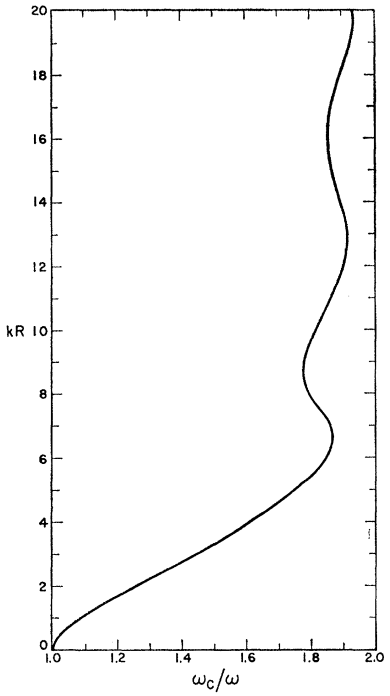

FIG. 2. The dispersion curve of the ordinary mode ($\mathbf{J}\|\mathbf{H}$) HFW associated with the fundamental cyclotron resonance of a non-interacting electron gas. The curve is a computed solution of Eq. (4) using the values $\omega_p/\omega = 0.85\times 10^5$ and $V_F/c = 3\times 10^{-3}$. The approximation of Eq. (7) yields a curve which is essentially indistinguishable from that shown here for $kR \lesssim 20$. At higher kR values the dispersion curve continues to oscillate weakly and finally, for $kR > 100$, returns monotonically to the $\omega_c/\omega = 1$ axis [Kaner-Skobov limit (see Ref. 21)].

Qualitatively similar evidence of HFW propagation is also found on the high-field side of the Azbel'-Kaner subharmonic resonances. A comparison of the observed dispersion for potassium and that computed from Eq. (7) near the second subharmonic is shown in Fig. 5. The over-all agreement is very satisfactory. In particular, there is no discrepancy in the $kR \to 0$ intercept in contrast to that found near the fundamental resonance.

IV. INTERACTION EFFECTS

Although there is good over-all agreement between the observed dispersion of the ordinary-mode HFW in potassium and that predicted by an independent-particle model, one can note modest discrepancies, such as the $kR \to 0$ intercept of the branch on the fundamental resonance which is found $\sim 3\%$ higher in field than expected. Since, as indicated earlier, explicit many-body interactions should appear in the frequency and wavelength range of the HFW, it is interesting to

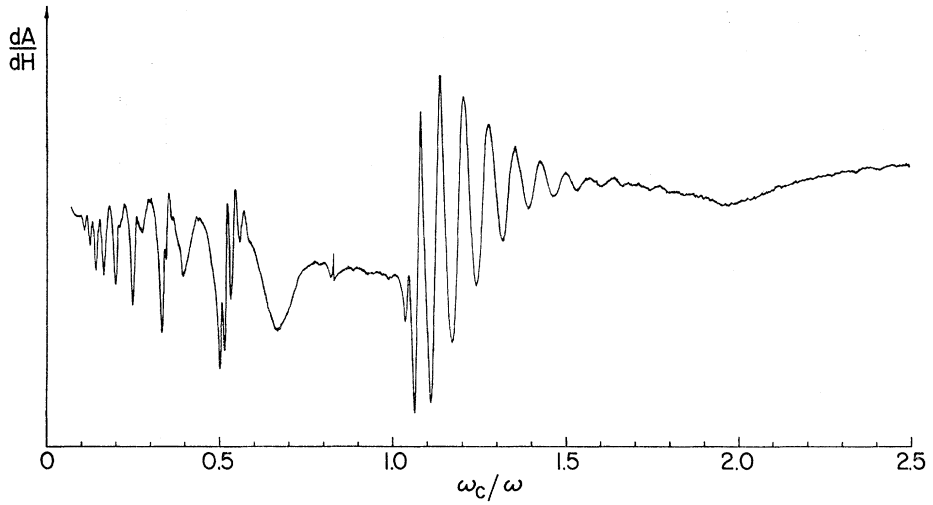

FIG. 3. Field derivative of the microwave power absorbed by a slab of pure potassium metal at 1.4°K versus a magnetic field lying in the sample plane. The experiment was performed in the $\mathbf{J}\|\mathbf{H}$ polarization with 11.985-GHz surface currents flowing on both faces of a sample 0.0141 cm thick. Oscillations due to ordinary-mode HFW propagation are clearly resolved on the high-field side of the fundamental and second Azbel'-Kaner cyclotron resonances. The "break" in slope at $\omega_c/\omega \simeq 1.95$ is attributed to the transition from a region of propagation to a cutoff condition (see Fig. 2).

examine the influence of such correlations on the dispersion relations.

In order to calculate the response of the system to slowly varying electromagnetic disturbances, we use the Landau theory of Fermi liquids.[5] By "slowly varying" we mean that the frequency ω and wave vector k of the external field satisfy $\hbar\omega/E_F \ll 1$, $\omega/\omega_D \ll 1$, and $k/k_{FT} \ll 1$. In this limit the response of the normal fermion system is completely described by a quasiparticle energy-momentum relationship $E(\mathbf{p},\mathbf{r},t,\sigma)$ and a quasiparticle density matrix $\rho(\mathbf{p},\mathbf{r},t,\sigma)$. In equilibrium, the quasiparticle energy $E^0(p)=p^2/2m^*$ and the quasiparticle distribution function $n_0(p)$ is a Fermi distribution. In nonequilibrium situations, the energy of the quasiparticles is a functional of its distribution function. To

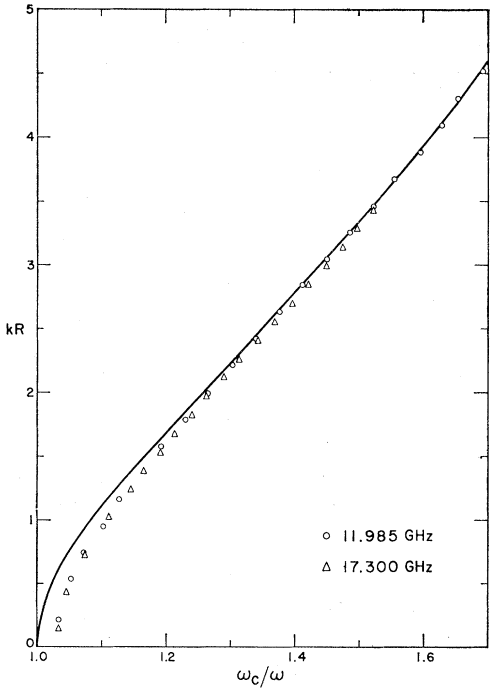

FIG. 4. A comparison of the free-electron ordinary-mode HFW dispersion curve with experimental data for postassium taken at two microwave frequencies using the sample of Fig. 3. While the agreement is qualitatively quite good, the $kR \to 0$ intercept of the experimental points lies $\sim 3\%$ above the resonance position. Such a deviation appears to result from explicit correlation effects as discussed in the text.

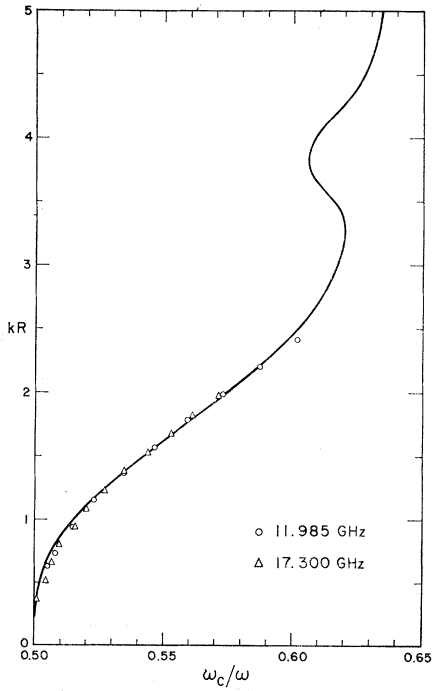

FIG. 5. A comparison of free-electron theory with experimental data for potassium in the region of the second-harmonic cyclotron resonance. In this case, the agreement is excellent, and therefore the discrepancy of Fig. 4 may not be removed simply by using a slightly larger value of the effective mass.

terms linear in deviations from equilibrium,

$$E(\mathbf{p},\mathbf{r},t,\boldsymbol{\sigma})=E^0(p)+\frac{1}{(2\pi)^3}\mathrm{Tr}_{\sigma'}\times\int d^3p' f(\mathbf{p},\boldsymbol{\sigma},\mathbf{p}',\boldsymbol{\sigma}')\delta\rho(\mathbf{p}',\mathbf{r},t,\boldsymbol{\sigma}'), \quad (16)$$

where

$$\delta\rho=\rho-n_0.$$

The function f is the basic phenomenological quantity characterizing the Fermi liquid. In microscopic terms, f is the forward quasiparticle-quasihole scattering amplitude where both momenta $\mathbf{p}$ and $\mathbf{p}'$ are confined to the Fermi surface. In the random-phase approximation (RPA), the f function for particles interacting via Coulomb forces is[14]

$$f\propto\frac{e^2}{|\mathbf{p}-\mathbf{p}'|^2+k_{FT}^2}\tfrac{1}{2}[1+\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'].$$

In general, for an isotropic system, f takes the form

$$f(\mathbf{p},\boldsymbol{\sigma},\mathbf{p}',\boldsymbol{\sigma})=f(\mathbf{p},\mathbf{p}')+\zeta(\mathbf{p},\mathbf{p}')\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}'. \quad (17)$$

Since $\mathbf{p}$ and $\mathbf{p}'$ are fixed on the Fermi surface, f and ζ may be expanded in a series of Legendre polynomials:

$$f(\mathbf{p},\mathbf{p}')=\sum_n f_n P_n(\cos\theta) \quad (18)$$

and

$$\zeta(\mathbf{p},\mathbf{p}')=\sum_n \zeta_n P_n(\cos\theta), \quad (19)$$

with

$$\cos\theta=\hat{p}\cdot\hat{p}'.$$

The set of dimensionless quantities

$$A_n=m^*p_F f_n/\pi^2(2n+1), \quad (20)$$

$$B_n=m^*p_F\zeta_n/\pi^2(2n+1) \quad (21)$$

is, along with m^*/m_0, the set of phenomenological parameters which describes the transport properties of a Fermi liquid.[23] In liquid He^3, for which the theory was originally envisaged, the effective mass is directly related to one of the Landau parameters ($m^*/m_0=1+A_1$), but in metals, such a simple relationship does not exist because of the presence of virtual phonon excitations.[24]

The parameters m^*, B_0, and B_1 are experimentally accessible: Azbel' has shown[25] that to an accuracy of order δ/R_c, the AKCR experiment yields m^* as does the electronic specific heat. The quantity B_0, as well as m^*, appears in the Pauli susceptibility of the conduction-electron spins[23]

$$\chi/\chi_0=(m^*/m_0)[1/(1+B_0)], \quad (22)$$

which is measurable, albeit with difficulty.[26] The entire set of B_n parameters is, at least in principle, determined by the dispersion of the spin waves which accompany conduction-electron spin resonance in very pure metals.[7] Thus far, B_0 and B_1 have been measured in sodium and potassium.[27]

The quantity A_0 is directly related to the compressibility of the interacting electron system.[23] The velocity of acoustic waves in metals should, therefore, yield information about the magnitude of A_0. Unfortunately, the electronic compressibility is only one contribution to the over-all compressibility and there does not appear to be a sufficiently reliable means of calculating the ionic-core contributions at this time. The parameter A_1 was shown by Azbel'[25] to produce a shift in the cyclotron resonance of quasiparticles, but only in the limit of uniform rf excitation over the entire orbit (diamagnetic resonance), the antithesis of the Azbel'-Kaner experiment. In an earlier publication[3] two of the authors (P. M. P. and W. M. W.) reported that both A_0 and A_1 measurably affect the dispersion of an extraordinary-mode HFW. However, as pointed out by McWhorter and Hamilton,[28] this result is incorrect. The following development illustrates the manner in which the A_n *can* influence the dispersion of HFW.

In the presence of external fields, the single quasiparticle density matrix satisfies a Boltzmann-like transport equation of the form[29]

$$\frac{\partial\rho}{\partial t}+\frac{i}{\hbar}[\varrho,\epsilon]+\tfrac{1}{2}\{\epsilon,\varrho\}+\tfrac{1}{2}\{\varrho,\epsilon\}=\frac{\partial\varrho}{\partial t}\Big|_{\text{collisions}}. \quad (23)$$

The symbol $\{\ ,\ \}$ is the Poisson bracket and $[\ ,\ ]$ is the commutator. The quantity ϵ is defined as

$$\epsilon=E(\mathbf{p}-(e/c)\mathbf{A})-(e\hbar/2m_0c)\boldsymbol{\sigma}\cdot\mathbf{H}+e\varphi, \quad (24)$$

where $\mathbf{A}$ and φ are the potentials describing the "total" (self-consistent) electromagnetic fields in the medium. The relaxation term on the right of Eq. (23) arises from collisions of quasiparticles with lattice defects and to the nonforward scattering of two quasiparticles. We are interested in pure materials at low temperatures and high frequencies where $(\partial\rho/\partial t)_{\text{collisions}}$ is negligible. We will, in this calculation, neglect it, although it may be included phenomenologically (relaxation-time approximation).

Defining the quantities

$$\epsilon_{lm}\equiv\epsilon_1\delta_{lm}+\boldsymbol{\epsilon}_2\cdot\boldsymbol{\sigma}_{lm}, \quad (25)$$

$$2\rho_{lm}\equiv n\delta_{lm}+\mathbf{u}\cdot\boldsymbol{\sigma}_{lm}, \quad (26)$$

where

$$n=\mathrm{Tr}_\sigma[\delta\rho], \quad (27)$$

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$$\mathfrak{y}=\mathrm{Tr}_\sigma[\boldsymbol{\sigma}\delta\rho], \tag{28}$$

$$\epsilon_1=E^0(p)+\frac{2}{(2\pi)^3}\int d^3p'\, f(\mathbf{p},\mathbf{p}')n(\mathbf{p}',\mathbf{r},t), \tag{29}$$

$$\mathfrak{e}_2=\frac{e\hbar}{2m_0c}\mathbf{H}+\frac{2}{(2\pi)^3}\int d^3p'\,\zeta(\mathbf{p},\mathbf{p}')\mathfrak{y}(\mathbf{p}',\mathbf{r},t), \tag{30}$$

we may easily carry out the manipulations implicit in Eq. (13) and obtain four equations for n and $\mathfrak{y}$. Since we are only concerned with the electrical properties of the medium, i.e., the induced current, we need only consider the equation for n:

$$\frac{\partial n}{\partial t}+e\mathbf{E}\cdot\frac{\partial n}{\partial\mathbf{p}}+\frac{e}{c}\left(\frac{\partial\epsilon_1}{\partial\mathbf{p}}\times\mathbf{H}\right)\cdot\frac{\partial n}{\partial\mathbf{p}}+\frac{\partial\epsilon_1}{\partial\mathbf{p}}\cdot\frac{\partial n}{\partial\mathbf{r}}-\frac{\partial\epsilon_1}{\partial\mathbf{r}}\frac{\partial n}{\partial\mathbf{p}}+\frac{e}{c}\left(\frac{\partial\epsilon_{2j}}{\partial\mathbf{p}}\times\mathbf{H}\right)\cdot\frac{\partial\mu_j}{\partial\mathbf{p}}+\frac{\partial\mu_j}{\partial\mathbf{r}}\cdot\frac{\partial\epsilon_{2j}}{\partial\mathbf{p}}-\frac{\partial\mu j}{\partial\mathbf{p}}\cdot\frac{\partial\epsilon_{2j}}{\partial\mathbf{r}}=0. \tag{31}$$

The last three terms in Eq. (31) are second order in the rf fields and, therefore, in an analysis of the linear response they may be neglected. This leads to a complete decoupling of the transport equations for n and $\mathfrak{y}$. The inclusion of spin-orbit coupling or nonlinear effects would reintroduce this coupling.

We next define the quantity g via

$$n=n_0+\frac{\partial n_0}{\partial E^0}g. \tag{32}$$

Ignoring collisions, the transport equation for g in the linear approximation becomes

$$\frac{\partial g}{\partial t}+\left[\mathbf{V}\cdot\nabla+\frac{e}{c}(\mathbf{V}\times\mathbf{H})\cdot\nabla_{\mathbf{p}}\right][g+\delta\epsilon_1]=e\mathbf{E}\cdot\mathbf{V}, \tag{33}$$

with

$$\delta\epsilon_1=\frac{2}{(2\pi)^3}\int d^3p'\,f(\mathbf{p},\mathbf{p}')g(\mathbf{p}',\mathbf{r},t).$$

The expression for the current becomes

$$j_\alpha=\frac{e}{(2\pi)^3}\int d^3p\,\frac{p_\alpha}{m^*}(g+\delta\epsilon_1)\delta(E_F-E^0(p)) \tag{34}$$

$$\equiv\frac{e}{(2\pi)^3}\frac{(1+A_1)}{m^*/m_0}\int d^3p\,\frac{p_\alpha}{m_0}g\delta(E_F-E^0(p)). \tag{35}$$

When the system is translationally invariant, i.e., the total momentum of the electrons is a good quantum number, $1+A_1=m^*/m_0$, which simplifies Eq. (35). In the presence of phonons this is no longer a valid conclusion and the factor $(1+A_1)/(m^*/m_0)$ must be retained.

A number of interesting features of the problem are immediately evident upon a cursory examination of the transport equation (33) and the associated expressions for the current, Eqs. (34) and (35). Equation (33) reduces to the ordinary Boltzmann equation when $\delta\epsilon_1\to 0$, i.e., in the absence of correlation effects. Only $\delta\epsilon_1$ or, equivalently, the parameters A_n enter. The function $\mathfrak{e}_2$ is not present; hence the B_n will be absent from the expressions for the conductivity.

The transport equation is composed of three terms: The first is of order ω, the second of order $\mathbf{k}\cdot\mathbf{V}_F$, and the third of order ω_c. Since we can use either expression (34) or (35) for the current, it is clear that if any one term of the three dominates (for example, if $\omega\gg\omega_c$ and $\omega\gg kV_F$), then correlations enter the conductivity only as a multiplicative factor depending on A_1. At Azbel'-Kaner resonances $\omega=n\omega_c$, the conductivity could in principle depend on all A_n. However, Azbel'[25] was able to show that because of the large values of kV_F which occur due to the skin effect, the term $\delta\epsilon_1$ may be neglected and the experiments simply yield the value of m^*/m. If, on the other hand, we have a bulk wave with $kV_F/\omega\sim 1$ and $\omega\sim\omega_c$, then *a priori* all the A_n could enter in a nontrivial way. The HFW have this property but, for example, helicons do not.[13]

The transport equation is a linear integral equation which is not easily solved for an arbitrary scattering function f. Choosing a disturbance of the form $e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ and a coordinate system as in Fig. 1, Eq. (34) becomes

$$-i\omega g+\left(i\mathbf{k}\cdot\mathbf{V}+\omega_c\frac{\partial}{\partial\varphi}\right)(g+\delta\epsilon_1)=e\mathbf{E}\cdot\mathbf{V}. \tag{36}$$

It is useful to expand g in a series of spherical harmonics, i.e.,

$$g=\sum_{n,m}\alpha_{n,m}Y_{n,m}(\theta,\varphi) \tag{37}$$

and

$$g+\delta\epsilon_1=\sum_{n,m}\alpha_{n,m}(1+A_n)Y_{n,m}. \tag{38}$$

As $k\to 0$, Eq. (36) becomes

$$\sum_{n,m}i(\omega-m\omega_c)(1+A_n)\alpha_{n.m}Y_{n,m}\sim eEY_{1,0} \tag{39}$$

for an electric field in the $\hat{z}$ direction. The "modes" of the system, i.e., the zeros of the left-hand side of Eq. (39), are given by

$$\omega=\omega_{nm}\equiv m\omega_c(1+A_n). \tag{40}$$

At $k=0$, the electric field in the $\hat{z}$ direction only couples to the $n=1$, $m=0$ mode, whereas fields in the $\hat{x}$ and $\hat{y}$ directions couple to the $n=1$, $m=\pm 1$ modes. As k increases, the $\mathbf{k}\cdot\mathbf{V}$ term mixes in the higher $\omega_{n,m}$ modes. Unlike the spin case,[7] we have no theory of the relative intensity in these modes, i.e., their oscillator strengths.

Before proceeding to discuss particular forms of the conductivity tensor components, it is important to

emphasize two quite general points originally made by McWhorter and Hamilton[28]: The parameter A_0 occurs in the transport equation multiplying $Y_{0,0}$, i.e., the net charge density produced by a disturbance. A_0 can, therefore, only influence waves possessing some degree of longitudinal character. This suggests that A_0 should enter explicitly into the extraordinary-wave dispersion relation, Eq. (5), and, indeed, each of the magneto-conductivity-tensor components σ_{xx}, σ_{yy}, and σ_{xy} does involve A_0. However, the particular combination of the components of Eq. (8) corresponds to a divergenceless current whence zero net charge is developed in the limit that displacement currents are negligible with respect to conduction currents ($\omega_p^2/\omega_c^2 \gg 1$). Since the latter approximation is extremely good at metallic densities, it follows that A_0 does not observably affect the HFW dispersion relations.

Similarly, the quantity A_1 enters the transport equation only as a coefficient of $Y_{1,m}$, the current associated with the disturbance. Since the essence of the HFW calculation is to seek zeros of the appropriate conductivities and, therefore, of their associated currents, it follows that A_1 cannot influence the HFW dispersion as long as $\omega_p^2/\omega^2 \gg k^2/k_0^2$, i.e., in the long-wavelength limit. A_1 might, however, become significant at short but perhaps attainable wavelengths as one passes into the Kaner-Skobov regime ($kR \lesssim 20$). It should also be noted that, inasmuch as A_1 enters the problem via the current in the wave, it could affect the degree of coupling, i.e., the actual amount of incident power which is transmitted through a slab versus ω_c/ω. In the absence of a solution of the boundary-value problem, however, any such coupling information is probably uninterpretable.

Turning now to explicit solutions of the nonlocal conductivity problem including the interaction or correlation effects, it would appear most logical to seek long-wavelength expansions in kR_c analogous to Eqs. (9)–(12). Using the spherical harmonic expansion for g, one can find such solutions by successive iterations of the transport equation. For example, the series expansion of σ_{zz} to order $(kR_c)^4$ is

$$\sigma_{zz} \sim \left\{ 1+(\tfrac{1}{2}kR_c)^2 \frac{1}{a}\left[\frac{C_{21}{}^{10}C_{10}{}^{12}}{a-(1+A_2)} + \frac{C_{2-1}{}^{10}C_{10}{}^{2-1}}{a+(1+A_2)} \right](1+A_1)(1+A_2) \right.$$

$$+(\tfrac{1}{2}kR_c)^4 \frac{1}{a}\left[\frac{C_{21}{}^{10}C_{32}{}^{21}C_{21}{}^{32}C_{10}{}^{21}(1+A_1)(1+A_2)^2(1+A_3)}{(a-(1+A_2))^2(a-2(1+A_3))} + \frac{1}{a}\left(\frac{C_{21}{}^{10}C_{30}{}^{21}}{a-(1+A_2)} + \frac{C_{2-1}{}^{10}C_{30}{}^{2-1}}{a+(1+A_2)} \right) \right.$$

$$\times \left(\frac{C_{21}{}^{30}C_{10}{}^{21}}{a-(1+A_2)} + \frac{C_{2-1}{}^{30}C_{10}{}^{21}}{a+(1+A_2)} \right)(1+A_1)(1+A_2)^2(1+A_3) + \frac{1}{a}\left(\frac{C_{2-1}{}^{10}C_{10}{}^{2-1}}{a+(1+A_2)} + \frac{C_{21}{}^{10}C_{10}{}^{21}}{a-(1+A_2)} \right)^2 (1+A_1)^2(1+A_2)^2$$

$$\left. \left. + \frac{C_{2-1}{}^{10}C_{3-2}{}^{2-1}C_{2-1}{}^{3-2}C_{10}{}^{2-1}(1+A_1)(1+A_2)^2(1+A_3)}{(a+(1+A_2))^2(a+2(1+A_3))} \right] \right\}, \tag{41}$$

where

$$C_{n+1,m+1}{}^{n,m} = -1/(2n+1), \qquad C_{n-1,m+1}{}^{n,m} = 1/(2n+1),$$

$$C_{n+1,m-1}{}^{n,m} = \frac{(n-m+2)(n-m+1)}{(2n+1)}, \quad C_{n-1,m-1}{}^{n,m} = -\frac{(n+m-1)(n+m)}{(2n+1)}. \tag{42}$$

Considering the earlier general statements as to the unimportance of the Landau parameter A_1 at long wavelengths, the reader may be justifiably startled by its frequent appearance in Eq. (41). This series-expansion form of σ_{zz} is, in fact, very misleading and cannot be used to compute the solution of the dispersion relation $\sigma_{zz}=0$. The difficulty arises because one is unwittingly dividing one power series by another, a characteristic feature of such calculations. The origin of the difficulty is most simply illustrated by examining a solution of the transport equation analogous to Eq. (6) for $\sigma^0{}_{zz}$ which can be obtained if one sets $A_1 \neq 0$ but all other $A_n = 0$, $n \geq 2$:

$$\sigma_{zz} = \frac{\sigma^0{}_{zz}}{1-[A_1/(1+A_1)]3a^2F(a,kR_c)}, \tag{43}$$

where

$$F(a,kR_c) = \sum_{n=0}^{\infty} \int_0^{\pi} \frac{J_n{}^2(b)\cos^2\theta d\theta}{(1+\delta_{n0})(a^2-n^2)} \tag{44}$$

$$= \frac{1}{a^2} \sum_{n=1}^{\infty} \frac{(kR_c)^{2(n-1)}}{(2n-1)(2n+1)(a^2-1)(a^2-2)\cdots[a^2-(n-1)^2]} \tag{45}$$

is functionally identical to σ^0_{zz} itself.[30] The presence of the A_1-dependent denominator in Eq. (43) does not affect the solution of $\sigma_{zz}=0$. In this severe truncation of the scattering function, the real roots are identical to those of $\sigma^0_{zz}=0$. The wavelength-dependent renormalization of $\sigma_{zz}/\sigma^0_{zz}$ due to A_1 can only become significant in the more general dispersion relation [Eq. (4)] when k^2/k_0^2 is no longer negligible with respect to σ_{zz}. If, however, both numerator and denominator of Eq. (43) are expressed as power series in kR_c and divided to obtain the "simple" series form of Eq. (41), terms involving A_1 will appear.

Despite this serious shortcoming, the series solution exhibits certain properties of general interest for σ_{zz}: The only modes $\omega_{n,m}$ which can appear are those for which $(n+m)$ is odd. This parity statement becomes evident if we note that

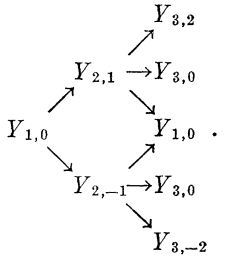
$$\sin\theta\cos\varphi Y_{n,m}\propto Y_{n\pm1,m\pm1}, \tag{46}$$

i. e., the $\mathbf{k}\cdot\mathbf{V}$ term changes both n and m by unity. Starting in the local, $k\rightarrow 0$, limit with $Y_{1,0}(n+m=1)$, the odd parity is preserved to arbitrary order in k. By a straightforward counting argument, one can show that for a given resonance the series term having the lowest power of k (which must dominate the position of the $k\rightarrow 0$ intercept of the dispersion curve) has its singularity shifted by the next-higher-order Landau parameter. That is, the intercepts no longer occur at $\omega=n\omega_c$ but at

$$\omega=n\omega_c(1+A_{n+1}). \tag{47}$$

Thus the ordinary-wave intercept on the fundamental resonance is shifted by A_2,[31] that on the second harmonic by A_3, etc. The experiments appear to bear out this picture of independent shifts of the intercepts in that the data cannot all be reconciled with the independent-particle model simply by a single change of the effective mass parameter. Referring to Figs. 4 and 5, one may estimate $A_2\simeq-0.03$ and $|A_3|<0.01$.

The form of the argument may be seen by noting that $\sigma_{zz}\propto\int gY_{1,0}$ and that each iteration of the transport equation brings successively higher order $Y_{n,m}$ into g, which starts as $Y_{1,0}$ in the local limit:



The first column is the k^0 or local contribution, the second is of order k, the third of order k^2, etc. The shortest path back to $Y_{1,0}$ for a given order in k produces the leading term for a particular resonance. Thus to order k^2 the path is via $Y_{2,1}$ whose amplitude is resonant at $\omega_{2,1}$ and the fundamental singularity becomes shifted by A_2. These are, of course, also higher-order terms shifting the fundamental singularity, e.g., in sixth-order $\omega_{4,1}$ will appear, leading to a fundamental singularity shifted by A_4. While this suggests the interesting possibility of new dispersion branches emerging from a series of intercepts near each resonance, we do not feel confident in interpreting the higher-order terms in the iterative solution in view of the difficulties mentioned earlier. Only the shift of the lowest-order term in k for a given resonance [Eq. (41)] is unambiguously given by the "simple" series. If a serious attempt is made to fit the experimental dispersion curves at finite k values, it appears necessary to truncate the expansion of the scattering function after a few terms and to solve the resulting set of coupled linear equations. The procedure, although laborious, is straightforward and the solution has been outlined by Fredkin and Wilson[32,33] for a similar problem. In practice, it is apparent that a considerable amount of numerical work will be required to establish a consistent scheme for actual evaluation of the A_n from the data. Clearly there is no conceptual difficulty in treating the extraordinary polarization by the same methods but the practical difficulties are far from trivial.

There is an alternative approximate procedure for solving the transport equation which does not depend on a truncation procedure. It amounts to assuming that $|A_n|\ll 1$. We can then expand the solution of Eq. (36), more specifically, the solution for σ_{zz} itself, as a power series in A_n and the leading term, i.e., the linear term, can be found. We solve, at least in principle, the $\frac{1}{2}n(n+1)$ coupled linear equations which result from assuming that the scattering function is composed of $n\geqslant 1$ spherical harmonics (n arbitrarily large).[31] The quantity σ_{zz} is then expressed as the ratio of two determinants,

$$\sigma_{zz}\sim|N|/|D|.$$

The terms which are linear in A_n can be found. In this approximation the equation $\sigma_{zz}=0$ reduces to

$$\Gamma_{10}{}^{10}(1+nA_n+ia\sum_{n,m}A_n\Gamma_{n,m}{}^{n,m})-ia\sum_{n,m}A_n\Gamma_{n,m}{}^{10}\Gamma_{1,0}{}^{n,m}=0, \tag{48}$$

with $n\geqslant 2$, $|m|\leqslant n$, and $m+n=$ odd. The four-index quantity $\Gamma_{k,l}{}^{n,m}$ is defined by

$$\Gamma_{k,l}{}^{n,m}\equiv\int F_{k,l}(\theta,\varphi)Y_{n,m}{}^*(\theta,\varphi)d\Omega, \tag{49}$$

[30] Equation (41) has been independently derived by D. Hamilton (private communication).

[31] S. C. Ying and J. J. Quinn [Phys. Rev. Letters **20**, 1007 (1968)] have computed a dispersion relation for the fundamental ordinary mode using $A_2\neq 0$ and $A_n=0$ for $n\geqslant 3$.

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where

$$F_{k,l}\equiv iY_{k,l}e^{-i(l\varphi+b\sin\varphi)}\sum_{q=-\infty}^{\infty}\frac{J_q(b)e^{i(l+q)\varphi}}{a-l-q}. \quad (50)$$

The quantity $\Gamma_{1,0}^{1,0}$ is proportional to the conductivity $\sigma^0{}_{zz}$ as given in Eq. (6).

V. CONCLUSIONS

The origin and general dispersion characteristics of the high-frequency waves have proved, at least in the case of the ordinary ($\mathbf{J}\|\mathbf{H}$) modes, to be explained quite well by an independent-particle or free-electron model of simple isotropic metals. Residual discrepancies exist, however, and should permit evaluation of several moments of the Landau scattering function which has been used to describe the effects of quasiparticle interactions. In practice, considerable care both in the experiments and their analysis will be required, since only moments V_n, $n\geqslant 2$, produce directly observable results and these higher moments are apparently quite small. We defer this quantitative problem to a later publication.

ACKNOWLEDGMENTS

We have benefited from discussions with numerous colleagues, particularly A. L. McWhorter and D. Hamilton of Lincoln Laboratory (MIT), and P. A. Wolff and S. J. Buchsbaum of Bell Telephone Laboratories.

PHYSICAL REVIEW VOLUME 172, NUMBER 3 15 AUGUST 1968

Lattice Heat Conductivity in Annealed and Quenched Gold-Platinum Alloys*

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(Received 11 March 1968)

The heat conductivity of dilute gold-platinum alloys, containing 0.11 and 1.02 at. % Pt, has been measured between 4.2 and 80°K. The influence of vacancies on the lattice heat conductivity K_g of the alloy with the higher platinum concentration was determined from measurements with specimens which were quenched and subsequently annealed. Above about 30°K, K_g in the annealed alloy Au+1.02 at. % Pt is found to be about 20% smaller than in pure gold. Whereas the lattice heat conductivity is clearly reduced at higher temperatures by quenched-in vacancies, it is practically unaffected by quenching below about 15°K. The phonon-scattering cross section of vacancies is estimated from the data, assuming a Rayleigh-type scattering law. It is found to be in reasonable agreement with the value obtained earlier from the phonon-drag thermopower of quenched and annealed pure gold.

INTRODUCTION

THE lattice heat conductivity and the phonon scattering by lattice defects has been studied in insulators in many experiments.[1] In metals, similar studies are rather complicated because of the dominant conductivity of heat by the conduction electrons. Since, at low temperatures, the lattice component of the thermoelectric power is of the same order of magnitude as or even larger than the electron-diffusion component, it appears to be advantageous to study the scattering of phonons by lattice defects from this property. For instance, it has been suggested from thermoelectric experiments with quenched platinum,[2] that the phonon scattering by vacancies in platinum is characterized by a resonance at low frequencies. On the other hand, it has been pointed out[3–5] that the phonon-drag thermopower may be complicated at low temperatures due to anisotropy of the relaxation times for electron scattering. An apparent anomaly in the phonon-drag thermopower at low temperature might be the result of either one of the mechanisms described.

Recently,[6] we have measured the thermoelectric power of annealed and quenched gold-platinum alloys between 4.2 and 300°K. For an unambiguous interpretation of these experiments it was necessary to study the effect of quenched-in vacancies on the lattice heat conductivity of the alloys. Therefore we measured the thermal conductivity of quenched and annealed gold-platinum alloys between 4.2 and 80°K. Preliminary results of the present investigation were reported earlier.[7]

* Based on work performed under the auspices of the U. S. Atomic Energy Commission.

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