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²⁴Eq. (37) can also be derived by expanding the exact \vec{S} of Eq. (16) to second order and dropping the constant terms and those which correspond to the redistributed, steady-state energy, that is, to the second term in Eq. (36).

Diffusion Coefficients and Nuclear Magnetic Susceptibility of Dilute He³-He II Solutions*

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Measurements of the spin-diffusion coefficient and the nuclear magnetic susceptibility of He³-He II solutions of molar concentrations between 0.030 and 3.0% have been made at temperatures above 0.4°K. The diffusion measurements show that the reciprocal of the spin-diffusion coefficient varies linearly with He³ concentration. The constant term is identified as the reciprocal of the binary diffusion coefficient. Results for the binary diffusion coefficient are in agreement with the results of other measurements where comparison can be made. Between 0.8 and 1.6°K, the binary diffusion coefficient varies exponentially with the reciprocal of the temperature, in a manner consistent with the roton excitation energy of 8.65°K. The susceptibility exhibits only small deviations from perfect-Fermi-gas behavior. A calculation of the effect on the susceptibility of the direct interaction between He³ quasiparticles made by utilizing an effective potential indicates that deviations from perfect-Fermi-gas behavior are small. There is reasonably good agreement between the measured and the calculated temperature dependence of the susceptibility.

I. INTRODUCTION

A very useful model for interpreting the properties of dilute solutions of He³ in superfluid He⁴ has been proposed by Landau and Pomeranchuk.¹ According to this model, the He³ atoms dissolved in the He II can be regarded as a gas of weakly interacting quasiparticles with an effective potential energy E_0 and an effective particle mass m_0^* .

The dilute He³-He II solutions can then be considered as a background superfluid and a mixture of gases of elementary excitations: (1) a gas of rotons, (2) a gas of phonons, and (3) a gas of He³ quasiparticles. In this paper, the model of gas mixture will be used to interpret results of nuclear magnetic measurements of diffusion and susceptibility obtained for He³-He II solutions at temperatures above 0.4°K.

At temperatures above 0.4°K, spin-diffusion measurements have been made by Garwin and Reich,² by Anderson *et al.*,³ and by Husa *et al.*⁴ Although the results of these measurements are generally in agreement, none of the measurements have been made at low enough He³ concentrations to give much information about the interaction of He³ quasiparticles with the other elementary excitations—the rotons or phonons. In this paper are presented the results of measurement of the spin-diffusion coefficient in He³-He II solutions for He³ concentrations between 0.03 and 3.0%. It is shown that these results clearly demonstrate the role of rotons in limiting the rate of spin-diffusion in very dilute solutions. The values for the binary diffusion coefficient of He³-He II solutions derived from the spin-diffusion data are compared with other experimental results and with theoretical results.

Susceptibility measurements made at temperatures above 0.4°K have been interpreted by Husa *et al.*⁵ in terms of an effective scattering length characterizing the interaction between He³ quasiparticles. Since the scattering cross section observed in spin-diffusion measurements²⁻⁴ is strongly temperature-dependent (varies nearly as $T^{3/2}$), it may be concluded that for temperatures above 0.4°K a scattering length is not sufficient to characterize the interaction. Emery⁶ has shown that a simple potential with a repulsive core and a short-range attractive region can be used to account for a number of the observed properties of He³-He II solutions. It is shown here that the same potential, when used in a calculation of the susceptibility, accounts for the fact that observed deviations from perfect-Fermi-gas susceptibility are small. The calculation of the nuclear magnetic susceptibility is made in the same manner as that for the susceptibility of He³ vapor at low densities.⁷

II. EXPERIMENTAL DETAILS

A pulsed resonance apparatus operating at 20 MHz was used to measure the spin-diffusion coefficient and relative values of the nuclear magnetic susceptibility of the He³-He II solutions. The basic NMR techniques of measurement are essentially the same as those used previously in experiments^{7,8} on He³ vapor; here only details related to properties of the sample cell and of the sample on which measurements were made are presented.

The sample cell is designed to confine all but a small fraction of the sample inside a well-defined cavity which is not part of a closed path for heat flow. The nylon sample container is sealed by a differential contraction seal to a copper adapter, which in turn is soldered to a liquid He³ evaporator. The sample is contained in a $\frac{1}{2}$ -in.-i.d. \times $\frac{1}{2}$ -in. cylindrical cavity, which is bored in the nylon from the bottom end. A threaded nylon plug, cemented to the body of the sample container with Eccobond 45 epoxy resin makes up a portion of the cavity bottom. Attached to the copper adapter by an indium-epoxy resin joint and extending to the top of the sample cavity

is a $\frac{1}{4}$ -in.-diam \times 2-in.-long sapphire rod that serves as a thermal link between the He³ evaporator and the sample. The thermal link is dielectric to avoid eddy current heating caused by rf pulses. The rf coils surrounding the sample container are supported in an evacuated space in such a way that they are not in thermal contact with the sample. In this manner the sample cell design minimizes temperature gradients arising from steady-state or transient heat flow through the sample.

Temperatures down to 0.4°K are obtained with a He³ cryostat employing a modified Welch 1402B Duo-Seal vacuum pump. A Cartesian monostat equipped with a 1-mm orifice reduces temperature fluctuations in the He³ evaporator to less than $\pm 1\%$ for a period of an hour. Temperatures are measured with modified radio-type Allen-Bradley resistors. Two resistors, nominally 47 and 6.8 Ω , with the original insulation removed, are cemented to the He³ evaporator with GE 7031 varnish. Resistances are measured by utilizing a standard resistor, a dc current source, and a Keithley Model 660 guarded dc differential voltmeter to observe the voltage drop across the standard resistor and the thermometer resistor. The calibration of the thermometers was made by utilizing He³ and He⁴ vapor-pressure thermometry. Recalibration of the thermometers following several experimental runs showed that the resistance thermometers, which at no time were immersed in liquid helium, were sufficiently stable to determine temperature with an error less than 1%.

In the course of making measurements on the He³-He II sample the level of liquid in the 0.020-in.-diam fill tube was maintained at some point between the thermal ground to the liquid He⁴ bath and the thermal ground to the He³ evaporator. Since the lower 4 cm of the fill tube was thermally grounded to the He³ evaporator, this procedure assured the existence of a column of solution with nonzero He³ concentration in the fill tube. The thermal conductivity of this column was sufficiently low so that nearly all of the heat leak into the He³ cryostat was carried through the thermal ground of the fill tube, rather than into the sample container and subsequently through the sapphire thermal link. That a nonzero He³ concentration can exist can be seen by considering the conditions required for quasiequilibrium⁹ of the liquid in the fill tube. When stationary conditions obtain in the absence of a pressure gradient, the relationship between the temperature T and molar concentration X is given by

$$R\nabla(TX) = -S_{4L}^0 \nabla T, \quad (1)$$

where R is the gas constant and S_{4L}^0 the entropy per mole of liquid He⁴. Integration of Eq. (1) between the limits of the temperature of the sample T_s and the temperature T_0 gives the result

$$T_0 X_0 - T_s X_s = - \int_{T_s}^{T_0} (1/R) S_{4L}^0 dT. \quad (2)$$

For a temperature $T_0 = 1.15^\circ\text{K}$ and a sample temperature less than 0.6°K , the integral in Eq. (2) varies only slightly with sample temperature T_S ; its approximate value for T_S less than 0.6°K is 0.003°K , a value obtained by utilizing data for S_{4L}^0 obtained by Kramers *et al.*¹⁰ Now if $T_S X_S$ is greater than 0.003°K , it is possible for a non-zero He^3 concentration to exist in the fill tube at temperatures up to 1.15°K , the temperature of the He^4 bath. If $T_S X_S$ is less than 0.003°K , this is no longer true, and the heat leak into the He^3 cryostat is determined by the film flow rate in the fill tube. If this heat leak is not too large, there is still a point in the fill tube below which the He^3 concentration is nearly equal to the sample concentration X_S . This is obviously true if a T_0 can be chosen such that (i) the integral in Eq. (2) is small compared to $X_S T_S$ and (ii) the heat flow across the thermal boundary resistance that is calculated by assuming that all the liquid above a certain point in the fill tube is at T_0 is greater than the total heat leak into the He^3 cryostat. In this experiment such a T_0 could be chosen for all sample concentrations with the heat leak assumed to be the maximum observed value of $140 \mu\text{W}$.

The accuracy of diffusion and susceptibility measurements is limited by possible errors arising from several sources. Errors in temperature measurements were estimated to be less than 1%. The concentration of gaseous He^3 - He^4 mixtures prepared by mixing together measured amounts of He^3 and He^4 gas was uncertain by an amount equal to 3% of the nominal He^3 concentration. Errors in determination of sample concentration arising from heat flush were estimated to be less than 1%. Measurements of signal amplitudes used to determine susceptibility could be made to within an accuracy of $\pm 0.5\%$. Measurements of the spin-diffusion coefficient were subject to a possible $\pm 5\%$ systematic error in the determination of the magnetic field gradient and a minimum

random error of $\pm 2\%$ in the determination of relative values of the spin-diffusion coefficient.

III. RESULTS AND DISCUSSION

A. Diffusion Coefficients

The experimental data for the spin-diffusion coefficient obtained in this experiment are presented in Table I. Analysis of the data shows that the measured spin-diffusion coefficient $D_S(X, T)$ can be represented accurately by the empirical relation

$$D_S^{-1} = D^{-1} + D_{ii}^{-1} \quad (3)$$

where the quantities D and XD_{ii} are independent of the molar concentration X . Figure 1 shows the kind of fit that can be made to the spin-diffusion data by using the method of least squares to calculate D and XD_{ii} as a function of temperature. At temperatures between 0.4 and 1.15°K , the quantity XD_{ii} can be represented to within 2% by the relation

$$XD_{ii} = (1 + 5.6^\circ\text{K}/T)10^{-5}\text{cm}^2\text{sec}^{-1}. \quad (4)$$

The temperature dependence of D is shown in Fig. 1.

The form of Eq. (3) is the same as that of an expression for the spin diffusion that can be derived by considering a Boltzmann-type equation for the mixture of gases of elementary excitations associated with the He^3 - He^4 solutions. The calculation is an extension of Khalatnikov's calculations¹¹ of various transport properties of He^3 - He^4 solutions; details of the calculation are discussed in a doctoral dissertation by one of the authors.¹² Important simplifications can be introduced in the calculation because of the

TABLE I. Measured values of the spin-diffusion coefficient D_S for several molar concentrations X .

T ($^\circ\text{K}$)	D_S ($10^{-5} \text{cm}^2/\text{sec}$)	T ($^\circ\text{K}$)	D_S ($10^{-5} \text{cm}^2/\text{sec}$)	T ($^\circ\text{K}$)	D_S ($10^{-5} \text{cm}^2/\text{sec}$)
$X = 0.030\%$		$X = 0.31\%$		$X = 0.92\%$	
1.137	945	1.638	64.5	0.944	683
0.930	4170	1.416	170	0.766	906
0.775	13750	1.142	669	0.746	877
0.604	26700	0.932	1693	0.610	1144
0.511	34800	0.781	2560	0.511	1358
$X = 0.102\%$		0.607	3360	0.399	1713
1.639	57.7	0.530	3900	0.392	1672
1.399	192	$X = 0.92\%$		$X = 3.0\%$	
1.150	783	2.59	5.38	1.635	40.7
1.130	820	2.04	11.43	1.412	83.0
0.946	2760	1.643	57.8	1.137	162.2
0.765	6760	1.640	54.7	0.929	219
0.624	9300	1.407	146.8	0.773	261
0.527	10860	1.135	378	0.601	328
0.506	11750	1.132	414	0.510	382
				0.398	495

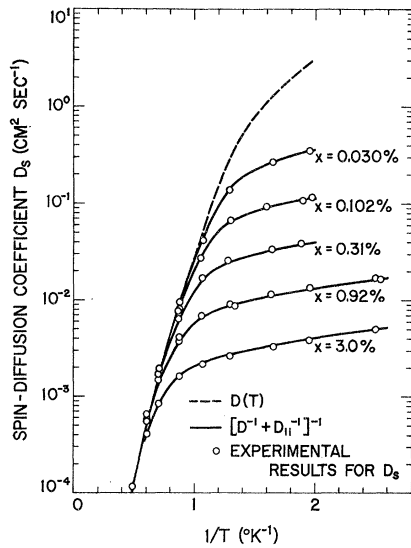


FIG. 1. Spin-diffusion coefficient of He³-He II solutions versus the reciprocal of the temperature T for five values of He³ molar concentration X . Open circles represent experimental data. The solid lines are a family of curves of the form indicated in the legend. The functions $D(T)$ and XD_{ii} , which are independent of the concentration X , are chosen to give the best fit to the experimental data in the manner described in the text. The dashed line represents the function $D(T)$.

spin-echo technique of measurement. In the course of a measurement of the spin-diffusion by this technique, the density of He³ atoms and the temperature remain spatially uniform. Hence the distribution function of the elementary excitations associated with the He II can be taken to be the distribution function when thermal equilibrium obtains. Also, there is no need to consider the background superfluid since there exist no macroscopic velocities of normal fluid or superfluid. If it is explicitly assumed that the gas of He³ quasiparticles is nondegenerate, then the solution of the Boltzmann equation to first order in the collision rate is of the same form as that of Eq. (3), with D and XD_{ii} being independent of the concentration X . The quantities D_{ii} and D are readily identified. The quantity D_{ii} is the spin-diffusion coefficient calculated for the gas of He³ quasiparticles by considering only the interaction between them. The relationship between D_{ii} and scattering cross sections characterizing the interaction has been given by Emery.⁶ The quantity D is the binary diffusion coefficient of a He³-He II solution in the limit of vanishing He³ concentration. This is the limit that, in the calculation of the binary diffusion coefficient, corresponds to taking the distribution functions of rotons and phonons to be the thermal-equilibrium functions.

The experimental results for XD_{ii} given by Eq. (4) are in general agreement with the results of other measurements.²⁻⁴ The gross features of the temperature dependence of the quantity XD_{ii} are accounted for by a potential for the He³ quasi-

particle interaction proposed by Emery.⁶ This potential, which will be referred to again in a discussion of the susceptibility, is a potential with a hard core extending to a distance of 1.9 Å and an attractive region extending to 2.6 Å.

Values of D determined from the spin-diffusion data are compared with binary diffusion coefficients determined in heat-conductivity experiments by Ptukha¹³ in Fig. 2. In the temperature range where data from both kinds of measurement are available the agreement is excellent. It is seen

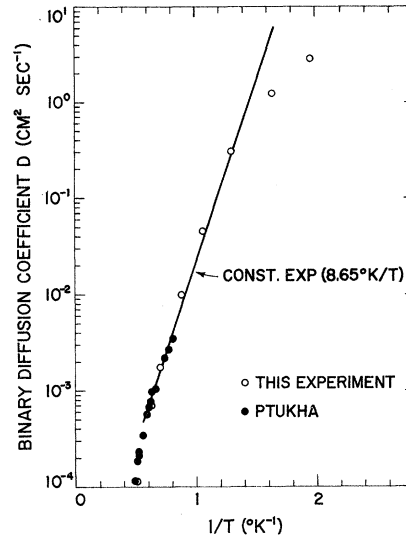


FIG. 2. Binary diffusion coefficient D of very dilute He³-He II solutions versus the reciprocal of the temperature T . Open circles represent diffusion data derived from the spin-diffusion data represented in Fig. 1. Closed circles represent Ptukha's data obtained in heat-conductivity measurements. The solid line shows the kinetic theory result for the temperature dependence of the diffusion coefficient.

that, by means of the spin-echo technique, it is possible to measure the binary diffusion coefficients at temperatures well below 1.2°K where the effective heat conductivity becomes relatively insensitive to the diffusion coefficient.

The binary diffusion coefficient of He³-He II solutions has been calculated by Khalatnikov and Zharkov^{13,14} on the basis of the Landau-Pomeranchuk model. In the calculation, the roton-He³ quasiparticle interaction energy was taken to be a δ function of the separation, and was treated as a perturbation. The final result for the diffusion coefficient for temperatures between 0.6 and 1.6°K in the limit of small He³ concentration is of the form

$$D^{-1} \propto T^{-1/2} n_r \quad (5)$$

The number density of rotons n_r is proportional to $T^{1/2} \exp(-\Delta_r/k_B T)$, where Δ_r is the roton excitation energy. Finally, D is given by

$$D = \text{const} \exp(\Delta_r/k_B T) \quad (6)$$

Since the constant in Eq. (6) must be evaluated from experimental diffusion data, it can be chosen here to give the best fit to the experimental data. In Fig. 2 the solid line represents Eq. (6). The value of $\Delta_\gamma/k_B = 8.65^\circ\text{K}$ is that obtained in neutron scattering experiments by Henshaw and Woods.¹⁵

From Fig. 2 it can be seen that, for temperatures between 0.8 and 1.6°K, the diffusion data can be characterized by an apparent activation energy which has essentially the same value as the roton excitation energy. It should be noted here that Garwin and Reich² found an apparent activation energy of 13.7°K in 1 and 2% He³-He II solutions at 19-atm pressure and at temperatures below 2.0°K. However, the discrepancy between their apparent activation energy and the roton excitation energy is largely removed if one restricts the analysis of their data to temperatures below 1.6°K.

Deviations of the experimentally determined quantity $D(T)$ from the theoretical result [Eq. (5)] are observed for temperatures lower than 0.8°K. Similar behavior, i. e., deviation from a simple $\exp(\Delta_\gamma/k_B T)$ dependence at low temperatures, has been observed by Meyer and Reif¹⁶ in a measurement of ion mobility in liquid helium. This behavior of ion mobility has been attributed to the scattering of ions by phonons by Meyer and Reif and by Abe and Aizu.¹⁷ In the case of the diffusion coefficient, however, Khalatnikov and Zharkov¹⁴ calculate a contribution from phonon-He³ quasiparticle scattering that is at least an order of magnitude too small to be significant. Because Khalatnikov and Zharkov's treatment of He³-phonon scattering gives an adequate explanation of the low-temperature heat conductivity of dilute He³-He II solutions (cf. Ref. 13) it cannot be ignored in relation to diffusion phenomena without reconsidering the problem of heat conductivity. A possible interpretation of the behavior of $D(T)$ at low temperatures is that the He³-roton scattering cross section is dependent on the energy of He³ quasiparticles in a way substantially different from that estimated by Khalatnikov and Zharkov.

Unfortunately, the points shown in Fig. 2 for the two lowest temperatures are related to the difference between two nearly equal quantities $1/D_S$ and $1/D_{ii}$, and depend critically on measured values of D_S for $X=0.030\%$. It is just these measured values that may be subject to unexpectedly large errors because of diminished signal amplitude and heat flush; hence the deviation from the theoretically calculated temperature dependence may arise from undetected experimental errors. Accurate measurements at lower He³ concentrations, which would have served to clarify these results, were not feasible with the apparatus in its existing state.

The relaxation-time data that were accumulated in the course of the spin-echo measurements exhibited no simple relationship between relaxation times and the diffusion coefficient. Measured values of the relaxation time T_1^* (representative of wall relaxation effects and distinct from a true spin-lattice relaxation time) varied from a low value of 14 sec in the 0.03% solutions at 0.5°K to

a value of 200 sec in the 3.0% solutions at 1.6°K. Values of T_1^* generally increased with decreasing values of the spin-diffusion coefficient, as expected; in all cases the diffusion rate is sufficiently large to account for the T_1^* results, if all the relaxation of magnetization occurs at the sample container walls. None of the diffusion data showed any effects of a transverse relaxation time T_2 . The diffusion data were obtained with the aid of a magnetic field gradient of 1.1 G/cm. A transverse relaxation time less than 0.5 sec should have been observable. Figure 3 shows a typical plot of the logarithm of the ratio (e_2/e_1) of successive spin-echo amplitudes versus the time separation τ to the third power. In Fig. 3, the data correspond to a temperature of 0.94°K and a concentration of 3.0%. A relaxation time of 80 msec as observed by Husa *et al.*⁴ would manifest itself in an easily detectable curvature in the line generated by the experimental points.

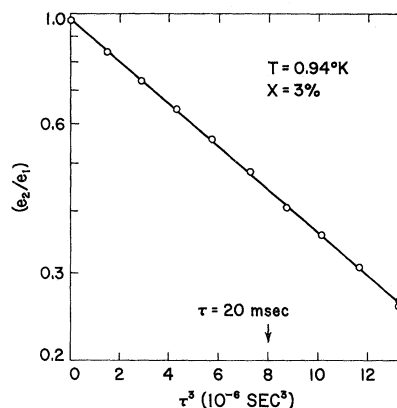


FIG. 3. Logarithm of the ratio of the amplitudes (e_2/e_1) of successive spin echoes versus the separation in time τ to the third power for a typical spin-diffusion measurement. Open circles represent experimentally observed ratios.

B. Nuclear Magnetic Susceptibility

The results of susceptibility measurements in a 3.0% He³-He II solution are indicated in Fig. 4. Only relative values of susceptibility are measured, and therefore the choice of the constant C is arbitrary. It is evident that the results are approximated quite well by the dashed line representing the susceptibility of a perfect gas of fermions each having mass $m_0^* = 2.33m_{\text{He}^3}$. (Our measurements of susceptibility for He³ concentrations of 0.92, 0.31, and 0.102% are also consistent with the same effective mass whose value was taken from Emery's paper.⁶ At these concentrations the measurements are quite insensitive to effective mass values; they do serve, however, as a check on the temperature measurements.) A similar value of effective mass has been derived in normal-fluid density measurements by Grigor'ev *et al.*¹⁸ Now it is not obvious *a priori* that the susceptibility should vary with temperature as the susceptibility of a perfect Fermi gas, even though the He³ quasiparticles

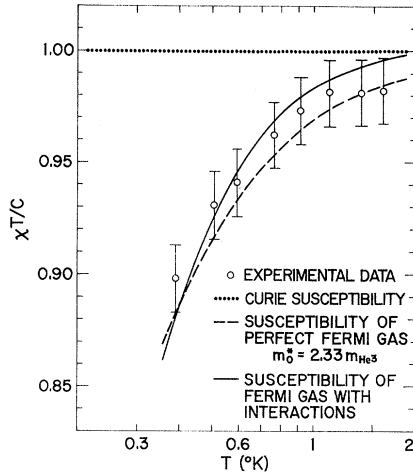


FIG. 4. Nuclear magnetic susceptibility of a 3.0% He³-He II solution versus temperature. Open circles represent experimental data. The choice of the constant C is arbitrary. The dashed line represents perfect-Fermi-gas susceptibility; the solid line represents the susceptibility calculated by including the effects of interactions in the manner described in the text.

exhibit a gas-like behavior. In particular, the susceptibility of He³ vapor⁷ does not exhibit a perfect-Fermi-gas behavior. It needs to be demonstrated that an interaction potential that is consistent with the spin-diffusion data is also consistent with the susceptibility data.

In our previous paper,⁷ we derived an expression for the susceptibility χ of a Fermi gas that included the effects of interaction to the first order in density. This expression can be written as

$$\chi/\chi_{FD} = 1 + 2^{5/2} \lambda^3 (1 - \Delta) (N/V), \quad (7)$$

where χ_{FD} is the susceptibility of a perfect Fermi gas, N/V is the number density of fermions, and λ is given by

$$\lambda = (\hbar^2/2\pi m k_B T)^{1/2}, \quad (8)$$

where m is the mass of a fermion; the quantity $1 - \Delta$ can be written in terms of the wave number k and the phase shifts η_l as

$$1 - \Delta = \frac{8\lambda^2}{\pi^2} \int_0^\infty [\sum_{l=0,1,2} (-1)^l (l+1)(\eta_{l+1} - \eta_l)] \times \exp(-k^2 \lambda^2/2\pi) k dk. \quad (9)$$

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In Fig. 4, the solid line represents the values of susceptibility calculated by using Eqs. (7)–(9). Phase shifts calculated for the potential proposed by Emery⁶ were used to evaluate the quantity $1 - \Delta$. The effect of the interaction as calculated in this manner is quite small, and therefore it may be concluded that the interaction potential which has been used in analysis⁶ of diffusion is also consistent with the susceptibility data.

Measurements of the susceptibility as a function of solution concentration would be preferable to measurements as a function of temperature described here. Such measurements would do much to eliminate the problem of arbitrary normalization of data, since it is expected that at low densities the fractional deviation from Curie's law is proportional to the density. An order-of-magnitude improvement in accuracy is required before susceptibility measurements of the kind described here can give much useful new information about interaction between the fermions.

IV. CONCLUSIONS

Analysis of the results of spin-diffusion measurements shows that the reciprocal of the spin-diffusion coefficient varies linearly with the concentration of He³-He II solutions. The constant term is identified as the reciprocal of the binary diffusion coefficient. At temperatures where comparison can be made, there is excellent agreement between diffusion coefficients determined in this manner and those determined by heat-conductivity measurements. The diffusion data obtained in this experiment at temperatures between 0.8° and 1.6°K are in agreement with a roton excitation energy of 8.65°K.

Susceptibility measurements made on a 3.0% solution are in satisfactory agreement with values of susceptibility calculated by using a simple He³-quasiparticle interaction potential originally proposed in connection with the interpretation of diffusion measurements. The effect of direct He³-He³ interactions on the susceptibility, calculated with the aid of an effective potential, is small; this affords a plausible explanation for the observed nearly perfect-Fermi-gas behavior of the susceptibility of He³-He II solutions at temperatures above 0.4°K.

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Recombination, Attachment, and Ambipolar Diffusion of Electrons in Photo-Ionized NO Afterglows*

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Photo-ionized plasma afterglows of NO have been studied by combined microwave and mass-spectrometric techniques. Nitric-oxide-neon mixtures (5–65 mTorr NO, 2–7 Torr Ne) are contained in a 10-cm resonant cavity where they are ionized by a “single pulse” of Lyman- α radiation. A temporal spectrum of ions diffusing to the wall is obtained by a differentially pumped mass spectrometer and multichannel analyzer. Analysis of the electron-density decay curves obtained by microwave techniques to obtain an electron-ion recombination coefficient for NO^+ is complicated by the conversion of NO^+ to the dimer ion, $(\text{NO})_2^+$. At sufficiently low densities of nitric oxide the $(\text{NO})_2^+$ concentration becomes negligible, and the NO^+ wall current tracks the electron-density decay. From comparisons of experimental electron-density decay curves obtained under recombination-controlled conditions, with computer solutions of the electron-continuity equation, the values $\alpha(\text{NO}^+) = (7.4 \pm 0.7)$, (4.1 ± 0.3) , and $(3.1 \pm 0.2) \times 10^{-7} \text{ cm}^3/\text{sec}$, at $T = 200, 300,$ and 450°K , respectively, are obtained. From analysis of electron-density decay curves at higher densities of NO, where $(\text{NO})_2^+$ is the dominant ion, the value $\alpha(\text{NO})_2^+ = (1.7 \pm 0.4) \times 10^{-6} \text{ cm}^3/\text{sec}$ at $T = 300^\circ \text{K}$ is obtained. The three-body electron-attachment and ambipolar diffusion coefficients have been measured in pure NO (0.1–5 Torr) and are found to be $K = (1.3 \pm 0.1) \times 10^{-31} \text{ cm}^6/\text{sec}$ and $D_a \rho = 80 \pm 16 \text{ cm}^2 \text{ Torr}/\text{sec}$, respectively, at $T = 300^\circ \text{K}$.

I. INTRODUCTION

The principal interest in electron removal processes in nitric oxide plasmas stems from the important role the NO^+ ion plays in the earth's ionosphere. Although the neutral NO molecule is a minor species in the upper atmosphere, NO^+ is an important atmospheric ion.^{1,2} Since NO has the lowest ionization potential of the atmospheric gases, the NO^+ ion usually cannot react further with the ionic and neutral species present, but must wait to recombine with a free electron (or, possibly, a negative ion) in order to be neutralized.

The rate of recombination between NO^+ ions and electrons may be expected to vary with the temperature of the ions and of the electrons. Over the ionospheric regions of interest, the neutral-gas and ion temperatures vary from approximately 200 to over 1000°K .³ The electron temperature above about 150 km has been shown to be greater

than the ion temperature by as much as several hundred degrees.⁴ As a result, laboratory experiments should include an extensive range of ion and electron temperatures in order that the measured rates will be applicable to all regions of the ionosphere where recombination of electrons with NO^+ ions is important.

The NO^+ ion in the ionosphere is expected to be in its ground electronic state. In the *D* region, where it is produced by Lyman- α photoionization, this is a certainty. In the *E* and *F* regions, where the NO^+ ion is produced by ion-molecule reactions, energy considerations require that it be in its ground electronic state, unless the primary ion (e.g., N_2^+ or O_2^+) in the reaction is electronically or vibrationally excited. However, there is no evidence for such an excited-ion population in the ionosphere. Unfortunately, at present nothing can be said about the vibrational state of the NO^+ ion in the ionosphere.