Two-Pion-Exchange Theory. Nucleon-Nucleon Scattering

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Two-pion-exchange contributions to NN and $\bar{N}N$ scattering at around 2.6 BeV/c are calculated. It is found that the two-pion-exchange mechanism accounts for roughly half of each of the absorptive forward elastic amplitudes, and that the imaginary part of the forward amplitude for the charge-exchange process $np \rightarrow pn$ can be attributed entirely to two-pion exchange. The method of calculation entails a spinless approximation, the purpose of which is to clarify the physical analysis at the expense of little loss of numerical accuracy. As a byproduct of this method, the distorted-wave one-pion-exchange prediction for the reaction $\pi N \rightarrow f^0 N^*(1238)$ is very simply verified.

I. INTRODUCTION

In the absence of a general calculational scheme for nuclear interactions, many specialized techniques, some more phenomenological than others, have been devised which meet with reasonable success in their limited domains. In this paper, we discuss one such technique, namely, the pion-exchange theory of peripheral interactions. Recently a number of two-body reactions have been very successfully described in terms of the one-pion-exchange (OPE) theory,[1] which theory has as its basis the supposition that the longest-range part of the nuclear force necessarily results from the exchange of the lightest hadronic quanta. It is the purpose of this paper to give a physically meaningful discussion of this theory in its extension to the next-longest-range nuclear force, namely, that resulting from two-pion exchange (TPE).

The TPE theory of the type to be considered here was originally proposed by Amati, Fubini, and Stanghellini.[2] In this extensive analysis it was shown that asymptotically the TPE mechanism yields features characteristic of "Regge behavior," namely, poles (and possibly cuts) in the angular momentum plane, and factorizability of scattering amplitudes, etc. However, no specifically quantitative predictions of the theory were given for the relatively moderate machine energies presently available. Following this original work, some more quantitative predictions were obtained by Berman and Drell[3] and by Smrž and von Baeyer.[4] But these analyses were somewhat inconclusive, in that they relied primarily on the above-mentioned factorizability property, which property is not unique to TPE. On the other hand, the TPE contribution to the process $\gamma p \rightarrow \rho^0 p$ at a few BeV/c has been rather extensively studied by Peaslee[5] and by Maor and Yock.[6] At these energies it is possible to include explicitly all possible intermediate states, thus allowing for a stringent test of the theory. But in these analyses it was assumed that the process $\gamma p \rightarrow \omega p$ was well described by OPE.[6,7] Although early experimental results[8] tended to confirm this, with the advent of more detailed data[9] it became apparent that the polarization of the photoproduced ω's was inconsistent with that predicted by OPE. We note here that various derivations[10,11] given of the OPE theory fail for the reaction $\gamma p \rightarrow \omega p$ because of its partially electromagnetic nature. Finally, we refer the reader to the work of Amaldi, Biancastelli, and Francaviglia,[12] in which TPE contributions to NN scattering are calculated in a manner quite similar to that which we shall follow, but in a region of lower projectile energies.

In this paper, a calculation is made of the TPE contribution to nucleon-nucleon diffraction scattering at an energy especially chosen so that the contributions from the various possible intermediate states are calculable. The method is extendable to other reactions and, in principle at least, to higher energies. Special emphasis is given to the physical interpretation of the theory.

II. TWO-PION-EXCHANGE THEORY. NUCLEON-NUCLEON SCATTERING

We consider elastic proton-proton scattering and, for simplicity, treat the protons as if they were spinless. In what follows, the generalization to an arbitrary two-body process is quite straightforward. Moreover, the inclusion of spin would not alter drastically any of the predictions to be obtained, since the model

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[1] For a general survey of the OPE theory, see J. D. Jackson, Rev. Mod. Phys. **37**, 484 (1965). Further references are given with this paper.

[2] D. Amati, S. Fubini, and S. Stanghellini, Nuovo Cimento **26**, 896 (1962).

[3] S. M. Berman and S. D. Drell, Phys. Rev. **133**, B791 (1964).

[4] P. Smrž and H. C. von Baeyer, Nuovo Cimento **51**, 889 (1967).

[5] D. C. Peaslee, Nuovo Cimento **44**, 784 (1966).

[6] U. Maor and P. C. M. Yock, Phys. Rev. **148**, 1542 (1966).

[7] G. Kramer and K. Schilling, Z. Physik **191**, 51 (1966).

[8] Cambridge Bubble Chamber Group, Phys. Rev. Letters **13**, 636 (1964); Phys. Rev. **155**, 1468 (1967).

[9] Aachen-Berlin-Bonn-Hamburg-Heidelberg-München Collaboration, Nuovo Cimento **41**, 270 (1966); **46**, 795 (1966). The author thanks Dr. G. Wolf for making this group's data available prior to publication.

[10] K. Gottfried and J. D. Jackson, Nuovo Cimento **34**, 735 (1964).

[11] P. C. M. Yock, Nuovo Cimento **52**, 952 (1967).

[12] U. Amaldi, Jr., R. Biancastelli, and S. Francaviglia, Nuovo Cimento **47**, 85 (1967).

involves primarily forward inelastic scattering which, in OPE theory, is helicity-preserving.[13]

The various interaction mechanisms that give rise to scattering may be classified according to their ranges, which, for elastic or kinematically nearly elastic reactions, is in each case the inverse of the mass of the exchanged system.[14] Hence, the longest-range forces are one-, two-, and three-pion exchange with ranges, respectively, μ^{-1}, $\frac{1}{2}\mu^{-1}$, and $\frac{1}{3}\mu^{-1}$. In the language of S-matrix theory this follows, as we shall shortly see, from the unitarity condition and the correspondence between particles and poles.

If the projectile momentum p satisfies the condition

$$p \gg 3\mu\,, \tag{1}$$

so that the de Broglie wavelength $\lambda\!\!\!^{-} \ll \frac{1}{3}\mu^{-1}$, then it is possible to construct spatially localized wave-packet states with impact parameters definitely falling either inside or outside the range of the three-pion-exchange force.[15] Then, if the scattering amplitude is written in the angular momentum representation,[16]

$$f(\theta)=\sum_l (l+\tfrac{1}{2})T^l P_l(\cos\theta)\,, \tag{2}$$

the amplitudes T^l with $l>L$, where

$$L\equiv p/3\mu\,, \tag{3}$$

may be associated with impact parameters greater than the range of the three-pion-exchange force. Hence, for $l>L$,

$$T^l \simeq T^{l,\mathrm{OPE}}+T^{l,\mathrm{TPE}}. \tag{4}$$

Equation (4) is basic to the subsequent analysis.

To calculate the TPE amplitudes, we make use of the nonlinear unitarity condition satisfied by the symmetric partial-wave amplitudes. This is

$$2\,\mathrm{Im}T_{i,f}{}^l=\sum_n (T_{i,n}{}^l)^* T_{n,f}{}^l\,, \tag{5}$$

which implies that

$$2\,\mathrm{Im}T_{i,f}{}^{l,\mathrm{TPE}}= \sum (T_{i,n}{}^{l,\mathrm{OPE}})^* T_{n,f}{}^{l,\mathrm{OPE}}\,, \tag{6}$$

which equation has been explicitly verified[17] in perturbation theory. Numerically, the bilinear product of OPE amplitudes appearing on the right-hand side of Eq. (6) decreases with l approximately as though it resulted from the single exchange of a particle of mass 2μ. This is in contrast to the other contributions to the right-hand side of Eq. (5). Hence, we conclude that the range associated with the TPE force really is $\frac{1}{2}\mu^{-1}$, and that all other interaction mechanisms (except, of course, OPE) are of shorter range. For an analytic discussion of the l dependence of the right-hand side of Eq. (6) we refer the reader to the recent work of Ino, Kikugawa, and Yonezawa[18] on the shrinkage of the TPE diffraction process.

To make the simplest possible use of Eq. (6), we limit the energy so that only a few intermediate states can contribute to the sum, but at the same time bear in mind the lower bound in energy implied by Eq. (1). For the higher partial waves, two-particle intermediate states may be expected to dominate the sum (6). In NN scattering, the lightest two-body intermediate states are NN, NN^*, N^*N^*, NN^{**}, etc.[19] If we set $p_{NN}\simeq 6\frac{1}{2}\mu$ (corresponding to a projectile momentum of 2.6 BeV/c in the laboratory), then $p_{NN^*}\simeq 5\frac{1}{2}\mu$, and $p_{N^*N^*}\simeq p_{NN^{**}}\simeq 3\mu$. Hence, at 2.6 BeV/$c$, the NN and NN^* states alone should dominate the sum (6) for $l>L$. Extra support for this assertion is provided by noting that the contribution from the N^*N^* state (and likewise other high-mass states) is further limited, since the effective OPE interaction range is significantly reduced from μ^{-1} at 2.6 BeV/c because of the mass inelasticity of the channel $NN\rightarrow N^*N^*$. Alternatively, the assertion may be regarded as a consequence of the not unreasonable assumption that, at a few hundred MeV, πN scattering proceeds via N and N^* states alone, for nearly physical pions. Above 2.6 BeV/c

[13] In more detail, the argument for $NN\rightarrow NN^*$ goes as follows: The kinematics of the process require that, for small-angle scattering, the exchanged pion go forward. Since the pion carries no spin, this implies that the OPE mechanism is helicity-preserving. This means that we need consider only two helicity amplitudes, namely, $++\rightarrow\frac{1}{2}+$ and $+-\rightarrow\frac{1}{2}-$, where the notation is self-explanatory. At the energy we shall be considering, the validity of this result may be numerically checked in the region of non-negligible amplitudes, namely, $p^2\theta^2\lesssim\mu^2$. Because OPE amplitudes factorize, the two amplitudes above are equal (in magnitude). Also, in the region of non-negligibility, the spin-dependent vertex functions are nearly constant. Finally, in doing the partial-wave decomposition, we note that in this same region of non-negligibility, since $p^2\theta^2\lesssim\mu^2 \Longrightarrow \theta\ll 1$, we can make use of the relation $d_{11}{}^J\simeq d_{00}{}^J=P^J$ for $J\gg 1$. Combining these facts, we conclude that the J dependence of the helicity-preserving amplitudes, which are the only nonvanishing ones and which are themselves equal (in magnitude), is correctly given in spinless approximation in the region $J\gg 1$. Remembering that the cross section is defined by averaging over initial spin states, this last result is all that we require for the following analysis. The argument can, however, be extended in the following way: The energy dependence of $d\sigma/dt$ for any spinless OPE reaction is identical to that for any spin-inclusive OPE reaction (provided initial and final spin states are summed over). This means that energy dependence is correctly given in spinless approximation, so that one phenomenological coupling constant for each vertex ($N\pi N$, $N\pi N^*$, etc.) is all that is required to define a spinless model over a range of energies. Such a model could be considered as a simplified version of the one used by the author [Ph.D. thesis, M.I.T., 1965 (unpublished)] in *hand calculations* of the absorptive OPE predictions for the reactions $\gamma p\rightarrow\rho^0 p$ and $\gamma p\rightarrow\pi^- N^{*++}$.

[14] We use units in which $\hbar=c=1$. M denotes the nucleon mass and μ the pion mass.

[15] For a detailed discussion of relativistic wave-packet states, we refer to the paper of D. I. Blokhintsev, International Centre for Theoretical Physics Report No. IC/67/36 (unpublished).

[16] The dimensionless amplitude f is normalized, so that $d\sigma/d\Omega=p^{-2}|f|^2$.

[17] S. Mandelstam, Phys. Rev. **115**, 1741 (1959).

[18] T. Ino, M. Kikugawa, and M. Yonezawa, Nuovo Cimento **50**, 960 (1967). We refer in particular to their Eq. (4).

[19] We denote by N^* and N^{**} the pion-nucleon resonances at 1238 and 1512 MeV, respectively. We neglect in this paper the possible existence of a πN resonance at about 1450 MeV [L. D. Roper, Phys. Rev. Letters **12**, 340 (1964)]. Because of its relatively high mass, its inclusion would not have a large effect on the partial waves $l>L$ at the energy considered here.

(beginning at about 3.0 BeV/c), other intermediate states contribute significantly to (6), and below this energy (below about 2.3 BeV/c) it is probably not possible to make the dynamical impact-parameter angular-momentum correspondence used in deriving Eq. (4). Certainly, the Fourier-Bessel representation of the scattering amplitude that we are about to utilize is unreliable below this energy.

Now, the OPE mechanism gives rise to real amplitudes. Hence, with our previous assumptions, we have,[20] for $l>L$ and at 2.6 BeV/c,

$$2\,\mathrm{Im}T_{pp\to pp}{}^{l}\simeq 2\,\mathrm{Im}T_{pp\to pp}{}^{l,\mathrm{TPE}} \simeq (T_{pp\to pp}{}^{l,\mathrm{OPE}})^2+(T_{pp\to pN^{*+}}{}^{l,\mathrm{OPE}})^2 +(T_{pp\to nN^{*++}}{}^{l,\mathrm{OPE}})^2+(T_{pp\to N^{*+}p}{}^{l,\mathrm{OPE}})^2 +(T_{pp\to N^{*++}n}{}^{l,\mathrm{OPE}})^2. \quad (7)$$

To proceed further, the OPE amplitudes must be calculated. The $N\pi N$ and $N\pi N^*$ vertex functions may be replaced by constants for the evaluation of the OPE amplitudes with $l>L$, since deviations from constant behavior are associated with threshold masses$=3\mu$ and consequently affect the lower partial waves ($l<L$) only. Denoting these constant vertices by $Mg_{N\pi N}$ and $Mg_{N\pi N^*}$, respectively, the amplitudes for $l>L$ are[21]

$$T_{pp\to NN^*}{}^{l,\mathrm{OPE}}\simeq\left(\frac{M^2}{4p^2}\right)\left(\frac{g_{p\pi N}g_{p\pi N^*}}{4\pi}\right)K_0\left(\frac{l\mu}{p}\right) \quad (8)$$

and

$$T_{pp\to pp}{}^{l,\mathrm{OPE}}\simeq\left(\frac{M^2}{4p^2}\right)\left(\frac{g_{p\pi p}{}^2}{4\pi}\right)K_0\left(\frac{l\mu}{p}\right), \quad (9)$$

where K_0 denotes the modified Bessel function. The renormalized phenomenological scalar coupling constants $g_{N\pi N}$ and $g_{N\pi N^*}$ may be determined in a variety of ways. For example, we may proceed as follows. First, note that both experimentally[22] and in OPE theory,[23,24] $d\sigma(\pi p\to\rho p)/d\sigma(\pi^+p\to\rho^0N^{*++})\simeq\frac{1}{3}$ for large s and small t. Hence, we conclude that

$$\frac{g_{p\pi p}{}^2}{4\pi}\simeq\frac{1}{3}\left(\frac{g_{p\pi N^{*++}}{}^2}{4\pi}\right). \quad (10)$$

Next, note that the total (unsymmetrized) single-isobar production cross section is, according to Eq. (8),

$$\sigma_{pp\to NN^*}\simeq\frac{2\pi}{p^2}\left(\frac{M^4}{16p^4}\right)\left(\frac{g_{p\pi N}{}^2}{4\pi}\right)\left(\frac{g_{p\pi N^*}{}^2}{4\pi}\right)\int_{p/2\mu}^{\infty} l\left[K_0\left(\frac{l\mu}{p}\right)\right]^2 dl =\frac{\pi M^4}{32p^4\mu^2}\left(\frac{g_{p\pi N}{}^2}{4\pi}\right)\left(\frac{g_{p\pi N^*}{}^2}{4\pi}\right)\int_1^\infty x[K_0(\tfrac{1}{2}x)]^2dx \simeq\frac{M^4}{9p^4\mu^2}\left(\frac{g_{p\pi N}{}^2}{4\pi}\right)\left(\frac{g_{p\pi N^*}{}^2}{4\pi}\right). \quad (11)$$

Because OPE is known[24,25] to dominate this process, we may determine the product $(g_{p\pi N}{}^2/4\pi)\cdot(g_{p\pi N^*}{}^2/4\pi)$ from Eq. (11) by demanding that it yield the observed production cross section. If we set

$$g_{p\pi p}{}^2/4\pi=\tfrac{2}{3}, \quad (12)$$

and

$$g_{p\pi N^{*++}}{}^2/4\pi=2, \quad (13)$$

consistent with Eq. (10), then the predicted cross sections for the reactions $pp\to nN^{*++}$ at 5.5 BeV/c and $pp\to pN^{*+}$ at 2.85 and 6 BeV/c are 1.2, 2.0, and 0.3 mb, respectively. These compare closely with the observed values[25–27] of 1.5, 1.9, and 0.2 mb, respectively.

Note that it would have been erroneous to calculate the scalar coupling $g_{N\pi N}{}^2/4\pi$ by attempting to equate the spinless OPE prediction for the differential cross section of the reaction $np\to pn$ to its observed value, since the inclusion of spin has a large effect in this process in which the initial- and final-state particles have identical masses.[28] Similarly, it would have been incorrect to calculate the scalar coupling $g_{N\pi N^*}{}^2/4\pi$ by requiring the decay width of the spinless N^* to be that of the actual N^*, namely, 120 MeV. This is because, in contrast to the situation prevailing in the spinless model, the actual decay is p-wave and, as such, is depressed. In fact, a simple calculation shows that Eq. (13) yields a width of about 200 MeV for the scalar N^*.

Substituting Eqs. (8) and (9) into Eq. (7), we obtain the following expression for the imaginary parts of the elastic proton-proton partial-wave amplitudes for $l>L$ at around 2.6 BeV/c:

$$2\,\mathrm{Im}T_{pp\to pp}{}^{l}\simeq\left(\frac{M^4}{16p^4}\right)\left(\frac{g_{p\pi p}{}^2}{4\pi}\right) \times\left\{\left(\frac{g_{p\pi p}{}^2}{4\pi}\right)+\frac{16}{3}\left(\frac{g_{p\pi N^{*++}}{}^2}{4\pi}\right)\right\}\left[K_0\left(\frac{l\mu}{p}\right)\right]^2, \quad (14a)$$

[20] In this approximate analysis, the protons are treated as distinguishable and all l values (even and odd) are included. Justification for this procedure has recently been given by A. D. Krisch, Phys. Rev. Letters **19**, 1149 (1967).

[21] See, for example, Ref. 11. In Eq. (8), the small mass difference in the final state has been neglected. Its inclusion would have little effect, even at the rather low energy considered here.

[22] Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Phys. Rev. **138**, B897 (1965); Aachen-Berlin-CERN Collaboration, Phys. Letters **19**, 608 (1965).

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[24] P. C. M. Yock and D. Gordon, Phys. Rev. **157**, 1362 (1967).

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[26] I. M. Blair, A. E. Taylor, W. S. Chapman, P. I. P. Kalmus, J. Litt, M. C. Miller, D. B. Scott, H. J. Sherman, A. Astbury, and T. G. Walker, Phys. Rev. Letters **17**, 789 (1966).

[27] E. W. Anderson, E. J. Bleser, G. B. Collins, T. Fujii, J. Menes, F. Turkot, R. A. Carrigan, Jr., R. M. Edelstein, N. C. Hein, T. J. M. McMahon, and I. Nadelhaft, Phys. Rev. Letters **16**, 855 (1966).

[28] Compare G. A. Ringland and R. J. N. Phillips, Phys. Letters **12**, 62 (1964), and P. C. M. Yock, Nuovo Cimento **44**, 777 (1966).

which, with the previously determined values of the coupling constants, reduces to

$$\operatorname{Im} T_{pp\to pp}{}^{l} \simeq 0.2\left(\frac{M^4}{p^4}\right)\left[K_0\left(\frac{l\mu}{p}\right)\right]^2. \tag{14b}$$

The second term in the curly brackets of Eq. (14a) largely dominates the first. This means that the NN^* intermediate states dominate the sum (6), so that the validity of the TPE calculation rests almost entirely on the accuracy of the N^* production amplitudes, (8). The fact that these were made by construction to agree with experiment constitutes a stringent test on the self-consistency of the analysis.[29]

Just to make doubly sure, however, we now consider briefly the reactions $\bar{p}p \to \bar{N}^{*++}N^{*++}$ and $\pi N \to f^0 N^*$. First, the double-isobar production process.

In OPE theory, we have

$$\sigma_{\bar{p}p\to\bar{N}^{*++}N^{*++}} \simeq \frac{2\pi}{p^2}\left(\frac{M^4}{16p^4}\right)\left(\frac{g_{p\pi N^{*++}}{}^2}{4\pi}\right)^2 \int_{p/2\mu}^{\infty} l\left[K_0\left(\frac{l\mu}{p}\right)\right]^2 dl,$$

which, with Eq. (13), yields the result

$$\sigma_{\bar{p}p\to\bar{N}^{*++}N^{*++}} \simeq 4M^4/9p^4\mu^2. \tag{15}$$

At momenta equal to 3.4, 5.7, and 6.9 BeV/c, Eq. (15) predicts cross sections of, respectively, 3.9, 1.5, and 1.0 mb. These compare very reasonably with the reported values[30–32] of 2.8, 2.1, and 1.5 mb, respectively, the slight systematic error being in this case due to the non-negligibility of the kinematic inelasticity of this reaction.[33] Second, we consider the "distorted-wave OPE"[24] prediction for $\pi N \to f^0 N^*$, a process which heretofore has not been so analyzed, presumably because of the complexity associated with the high particle spins. In the spinless approximation, we can write down the solution

$$\frac{d\sigma}{dt}(\pi N \to f^0 N^*) = \frac{g_{N\pi N^*}{}^2}{g_{N\pi N}{}^2}\frac{d\sigma}{dt}(\pi N \to f^0 N). \tag{16}$$

Comparison of the data[22,34] for these reactions at 8 BeV/c confirms Eq. (16) easily within experimental errors (in the region $|t| \lesssim 15\mu^2$). Hence, remembering that distorted-wave OPE theory has been verified for $f^0 N$ production,[24] we surmise that it correctly predicts $f^0 N^*$ production also. This conclusion is further supported by the fact that the observed f^0 decay distribution in $f^0 N^*$ production at 4 BeV/c is in agreement with that predicted by OPE.[22]

Returning to the TPE theory, we note that the derivation leading to Eq. (14) holds for the high partial waves ($l > L$) only, so that it is best checked by consideration of the forward amplitude. According to Eq. (14), we have

$$\operatorname{Im} f(0) \simeq 0.2\left(\frac{M^4}{p^4}\right)\int_L^{\infty} l\left[K_0\left(\frac{l\mu}{p}\right)\right]^2 dl + \sum_{l=0}^{L}(l+\tfrac{1}{2})\operatorname{Im} T^l$$

$$\simeq \frac{0.06M^4}{p^2\mu^2} + \sum_{l=0}^{L}(l+\tfrac{1}{2})\operatorname{Im} T^l.$$

Unitarity requires that $0 \leqq \operatorname{Im} T^l \leqq 2$, so that the last term may be crudely bounded as follows:

$$0 \leqq \sum_{l=0}^{L}(l+\tfrac{1}{2})\operatorname{Im} T^l \leqq \sum_{l=0}^{L} 2(l+\tfrac{1}{2}) \simeq L^2 = p^2/9\mu^2.$$

Hence we have

$$\frac{0.06M^4}{p^2\mu^2} \leqq \operatorname{Im} f(0) \leqq \frac{0.06M^4}{p^2\mu^2} + \frac{p^2}{9\mu^2}, \tag{17a}$$

which, according to the optical theorem, reads

$$\frac{3M^4}{4p^4\mu^2} \leqq \sigma_{pp}{}^{\text{tot}} \leqq \frac{3M^4}{4p^4\mu^2} + \frac{4\pi}{9\mu^2}, \tag{17b}$$

which inequality is supposed to be valid at about 2.6 BeV/c. We thus conclude that, at this energy,

$$20\text{ mb} \leqq \sigma_{pp}{}^{\text{tot}} \leqq (20+30)\text{ mb}, \tag{18}$$

where the contribution of 20 mb corresponds to TPE and the possible remainder of 30 mb to shorter-range interactions. That is, at least two-fifths of the absorptive forward elastic amplitude results from two-pion exchange.

Analogous analyses of pn, $\bar{p}p$, and $\bar{p}n$ scattering yield the results

$$16\text{ mb} \leqq \sigma_{pn}{}^{\text{tot}} \leqq (16+30)\text{ mb}, \tag{19}$$

$$16\text{ mb} \leqq \sigma_{\bar{p}p}{}^{\text{tot}} \leqq (16+30)\text{ mb}, \tag{20}$$

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[29] Strictly speaking, we have verified that the NN^* contribution dominates in the spinless model only. As we have previously mentioned, this model is not reliable for estimating the contribution of the NN intermediate state. We can, however, refer to the spin-inclusive absorptive OPE calculations (see, for example, Ringland and Phillips, Ref. 28) of nucleon-nucleon scattering for verification of the smallness of the NN contribution. Alternatively, the smallness of this contribution can be seen by a direct spin-inclusive calculation of it.

[30] T. Ferbel, A. Firestone, J. Sandweiss, H. D. Taft, M. Gailloud, J. W. Morris, W. J. Willis, A. H. Bachman, P. Baumel, and R. M. Lea, Phys. Rev. **138**, B1528 (1965).

[31] V. Alles-Borelli, B. French, A. Frisk, and L. Michejda, Nuovo Cimento **48**, 360 (1967).

[32] T. Ferbel, A. Firestone, J. Johnson, H. Kraybill, J. Sandweiss, and H. D. Taft, Nuovo Cimento **38**, 19 (1965).

[33] G. Wolf, Phys. Rev. Letters **19**, 925 (1967); J. T. Donohue, CERN Report No. TH844, 1967 (unpublished).

[34] J. A. Poirier, N. N. Biswas, N. M. Cason, I. Derado, V. P. Kenny, W. D. Shephard, E. H. Synn, H. Yuta, W. Selove, R. Ehrlich, and A. L. Baker, Phys. Rev. **163**, 1462 (1967).

and

$$20\text{ mb}\leqq\sigma_{\bar{p}n}^{\text{tot}}\leqq(20+30)\text{ mb}. \qquad (21)$$

The limits implied by Eqs. (18), (19), and (20) may be compared with the following observed[35] total cross sections at around 2.6 BeV/c:

$$\sigma_{pp}^{\text{tot}}\simeq 43\text{ mb}, \qquad (22)$$

$$\sigma_{pn}^{\text{tot}}\simeq 39\text{ mb}, \qquad (23)$$

and

$$\sigma_{\bar{p}p}^{\text{tot}}\simeq 75\text{ mb}. \qquad (24)$$

Considering the approximations made (both mathematical and physical), the results are in as good agreement with experiment as could be expected. The conversion of the partial-wave sum to an integral for $l<L$ suffices by itself to account for the discrepancies.

III. DISCUSSION

Our aim in the foregoing analysis has been to give a physically meaningful discussion of that part of the nucleon-nucleon force which may be associated with a range $>\frac{1}{2}\mu^{-1}$. To do this, we considered a particular process which is largely dependent on this force. Insofar as that process is not entirely dominated by forces of range $>\frac{1}{2}\mu^{-1}$, it is doubtful that a numerically more accurate treatment of the long-range part of the force would shed more light on the problem. This will certainly be true until the effect of short-range forces (for example, those resulting from ρ and ω exchange, and from one- and three-particle intermediate states) can be reliably incorporated into the theory. Until such is the case, we feel that the foregoing analysis is of sufficient numerical accuracy, and that it does provide some insight into the scattering process. We are, however, fully cognizant of the rather obvious improvements that could possibly be made by including effects of spin, exact partial-wave summations, form factors, N^*N^* states, and the like into the calculation. A more subtle area for improvement lies in the problem of giving a more realistic description of the sharp-cutoff distinction that we have made in Eq. (4) between TPE forces and those of shorter range.

As a further test of the theory, we can consider small-angle scattering as opposed to the strictly forward scattering we have so far analyzed. This will suffice to bear out our previous remarks concerning the shortcomings of the model in its neglect of short-range forces. The l dependence of the TPE amplitudes is given by $[K_0(l\mu/p)]^2$. Therefore, for small angles and neglecting partial waves $l<L$, we have

$$\left(\frac{d\sigma}{dt}\right)_{t=-p^2\theta^2}\bigg/\left(\frac{d\sigma}{dt}\right)_{t=0} =\left\{\int_L^\infty l\left[K_0\left(\frac{l\mu}{p}\right)\right]^2 J_0(l\theta)dl\bigg/\int_L^\infty l\left[K_0\left(\frac{l\mu}{p}\right)\right]^2 dl\right\}^2. \qquad (25)$$

Expanding the Bessel function $J_0(l\theta)$ about $\theta=0°$, this yields

$$\frac{d\sigma}{dt}\bigg/\left(\frac{d\sigma}{dt}\right)_{t=0}=1+At+O(t^2), \qquad (26)$$

where

$$A\simeq\tfrac{1}{3}\mu^{-2}=18\ (\text{BeV}/c)^{-2}. \qquad (27)$$

This may be compared with the experimentally observed slope of about 10 $(\text{BeV}/c)^{-2}$. Clearly the discrepancy arises primarily from our neglect of the short-range effects. This is borne out by the fact that $A_{\text{TPE}}>A_{\text{expt}}$. In fact, since the TPE force accounts for about one-half of the complete amplitude, we would expect to find, as we have done, that $A_{\text{TPE}}\approx 2A_{\text{expt}}$, because short-range scattering is *a priori* roughly isotropic.

Our next remark concerns the $\bar{N}N$ cross section. According to the theory, it would appear that at around 2.6 BeV/c it receives a considerably larger contribution from the lower partial waves than does NN scattering. This observation could have been anticipated on the grounds that the many annihilation channels open to $\bar{N}N$ scattering proceed through central collisions, since these allow maximum overlap of the incident nucleonic and antinucleonic wave packets.

Concerning the energy dependence of the scattering, we note that it would be incorrect to attempt to ascribe this, except possibly in the immediate neighborhood of 2.6 BeV/c, to that given by the TPE contribution of Eq. (14). This is because (i) the model gives no information on the central collision processes which presumably are energy-dependent, and (ii) as the energy is increased, more intermediate states must be included. Thus, although each intermediate state gives a contribution to the TPE amplitude which decreases asymptotically with energy, the complete TPE amplitude may or may not decrease with energy, depending on the spectrum of intermediate states available. This phenomenon is well illustrated in Fig. 6 of Ref. 2.

Although the preceding theory is phenomenological in the sense that the static properties (masses and widths) of the hadrons are inserted as input, we wish to emphasize that it does have the three advantages of being (i) at all stages capable of physical interpretation, (ii) derivable without *ad hoc* assumptions, and (iii) unique in the sense that it contains no adjustable parameters.[36]

[35] See W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. H. Phillips, A. L. Read, and R. Rubenstein, Phys. Rev. **138**, B913 (1965), and other references quoted therein.

[36] It is not evident that any of the various versions of "Regge-pole" theory share any of these three properties.

It is possible to apply the TPE theory to other reactions and at higher energies. As a particularly simple example, consider the charge-exchange process $np \to pn$. Isospin invariance requires that the imaginary part of the forward charge-exchange amplitude is, apart from kinematical factors, equal to the difference $\sigma_{pp}^{tot}-\sigma_{pn}^{tot}$. This difference is observed to be $\simeq 4$ mb [see Eqs. (22) and (23)], which is itself the difference in the TPE contributions to these cross sections [see Eqs. (18) and (19)]. Hence, at 2.6 BeV/c, it is possible to attribute the imaginary part of the forward charge-exchange amplitude completely to TPE processes. Evidently TPE dominates $np \to pn$ but not $pp \to pp$, because in the former case competition from competing channels necessarily depletes (via the unitarity condition) the low-l partial waves, whereas just the opposite is true in the elastic process.

In conclusion, it should be stressed that two-pion exchange as considered here cannot, of course, be simulated by elementary ρ exchange.

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The author takes pleasure in thanking Professor B. T. Feld for stimulating his interest in peripheral interactions.

Errata

Perturbed Bound-State Poles in Potential Scattering. II, Y. S. Kim and Kashyap V. Vasavada [Phys. Rev. **150**, 1236 (1966)]. The following paragraph should be added at the end of Sec. II.

In the language of elementary quantum mechanics, we are interested in calculating the first-order energy shift

$$\delta E=(\phi,\delta V\phi),$$

where the wave function ϕ is to be determined by strong interactions and is therefore determined approximately. There are, however, good approximations and bad approximations. If the wave function decreases exponentially for large r, we say it is a good approximation. If, on the other hand, the wave function increases exponentially, it is fair to say that it is a bad approximation. See Fig. 4. We have shown above [see also Ref. 1] that the approximation of Dashen and Frautschi in general corresponds to

$$\delta E^{\text{bad}}=(\phi^{\text{good}},\delta V\phi^{\text{bad}}).$$

Their infrared divergence comes from this bad approximation.

In the following section [Sec. III], we present an approximation scheme corresponding to

$$\delta E^{\text{good}}=(\phi^{\text{good}},\delta V\phi^{\text{good}}).$$

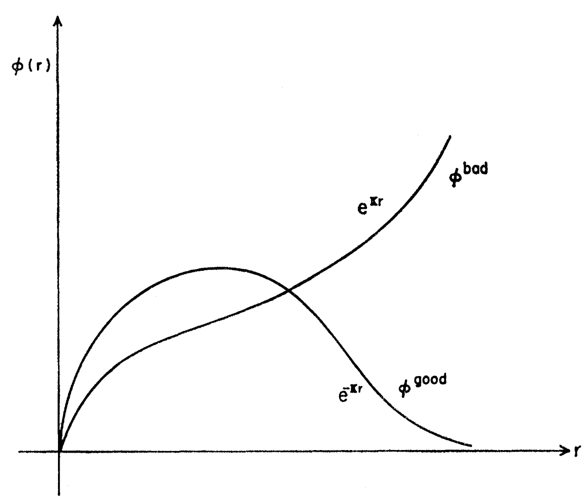

Fig. 4. Wave functions corresponding to good and bad approximations. The bad-approximation wave function comes from the failure to cut off the incoming wave component during the process of analytic continuation from scattering states.

Generalized Pole-Dominance Hypothesis, Current Algebra, and S-Wave K^+p Scattering, Probir Roy [Phys. Rev. **162**, 1644 (1967)]. The following misprints should be corrected:

1. Reference 12: The third expression within the curly bracket in the right-hand side of the equation should read:

$$ig_3(k^2)\epsilon^{\sigma\alpha\beta\gamma}\epsilon_{\gamma\rho\tau\mu}p_\alpha p_\beta k^\rho k^\tau.$$

2. Page 1648, line 5 should read:

$$-\frac{2}{3}\frac{g_A(Y_1^*)^2}{M_{Y_1^*}-M_N}\left(1+\frac{M_K}{2M_N}+\frac{m_K M_N}{M_{Y_1^*}{}^2}+\frac{m_K}{6M_{Y_1^*}}\right)\Big],$$

and

$$\left(\frac{df_0^R}{dq}\right)_{q=0}=0.$$