$SU(3)$-Symmetry Breaking in a Bethe-Salpeter Bootstrap Model*

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We study in detail a bootstrap model which contains the $\frac{1}{2}^+$, $\frac{3}{2}^+$ baryon and 0^- meson $SU(3)$ multiplets. The Bethe-Salpeter equation is used to ensure vertex symmetry for this multichannel system. First, we solve the model in the $SU(3)$ limit, obtaining results in satisfactory agreement with experiment. Then, we allow the mesons to interact with their correct isospin-invariant masses. This leads to driving shifts in baryon variables which agree impressively with experiment. These baryon shifts, fed back into the meson channel in a Fermi-Yang-type model, lead to a qualitatively correct meson mass spectrum and in particular imply a deeply bound pion. Finally, we examine the feedback properties of the model, and find that dominant dissymmetry modes transforming as **8** and **27** are present.

1. INTRODUCTION

OVER the past decade, it has become apparent that baryons and mesons are composite entities. This belief is based on firm experimental evidence, e.g., the electromagnetic form factors of the nucleon,[1] $N^*(1236)$ resonance,[2] and pion,[3] the importance of production processes in scattering reactions, and the very existence of hadron excited states, a typical property of composite systems. However, the nature of the compositeness is still a matter of serious debate.[4] One promising approach to this question is the bootstrap theory, in which the entire hadron spectrum is taken to be generated entirely by the strong interactions. A very appealing feature of this theory is its consistency with commonly accepted dynamical principles. By starting with the idea that the longest-range part of the potential is given by single-particle-exchange processes, one can generate self-consistent models of particles which agree qualitatively with experiment. In particular, such general features of the hadron spectrum as the existence of excited states,[5] and the appearance of higher symmetries,[6] have been shown to be in accord with the bootstrap formalism.

Inherent in bootstrap theory is the occurrence of self-interacting matter, a situation which by virtue of its nonlinearity could be related to the breakdown of $SU(3)$ symmetry.[7] The possibility of spontaneous breakdown of symmetries associated with bootstrap systems was pointed out some time ago by Cutkosky and Tarjanne.[8] They argued that the symmetry solution could be unstable with respect to certain types of perturbations and emphasized the importance of dynamics in identifying the main dissymmetry mode. Later, Wali and Warnock showed how a simple N/D bootstrap model of the $\frac{3}{2}^+$ baryon decuplet can give rise to qualitatively acceptable mass shifts if the unitarity-cut phase space is evaluated using correct particle masses, i.e., they calculated with effects induced by the right-hand cut.[9] The author studied a multichannel analysis of many long-range forces occurring in meson-baryon scattering, using correct particle masses and $SU(3)$-invariant coupling constants.[10] This calculation, concentrating on effects induced by the left-hand cut, obtained results entirely in qualitative agreement with experiment. Subsequently, the most extensive program to gain understanding of the breakdown of $SU(3)$ has been carried out by Dashen, Frautschi, and collaborators in a series of papers.[11,12] Not only do they attempt to determine reasonably accurate masses and coupling constants, but they also attempt to show explicitly that the phenomenon of octet dominance is a consequence of bootstrap dynamics. They base their findings on a first-order perturbative N/D model and, working in the static model, study splittings occurring in the elastic $\frac{1}{2}^+$ baryon-0^- meson scattering matrix. Their approach emphasizes an analysis of the "feedback matrix" rather than a careful study of the "driving terms" (see Secs. 3 and 4 of this paper for definitions of these terms), so that their results are scaled by fitting several of the unknown variables to experiment. Although the approximations the authors use have been the subject of a certain amount of con-

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troversy,[13,14] the results they obtain are apparently successful in that the octet dominance appears naturally, and the mass shifts have the correct behavior. However, the situation is not yet entirely clear. Recently a careful phenomenological analysis of Kp data by Kim[15] has determined the ΛpK^- coupling constant to be $g^2(\Lambda pK^-)/4\pi=16\pm2$, more than a factor of 2 larger than that predicted in Ref. 12. This discrepancy, which remains for reasonable variations in the D/F ratio, may be a symptom of the lack of a careful treatment of the driving terms, the neglect of higher-order perturbations, or the severe truncation made in a model handling only $\frac{1}{2}^+$ baryon–0^- meson scattering.

In this paper we study the problem of $SU(3)$-symmetry breaking with a bootstrap model based on the Bethe-Salpeter equation as developed by Cutkosky and collaborators.[16] The apparent virtues of this approach lie in its use of many two-particle channels and its symmetric treatment of the various particles (vertex symmetry). In particular, we are interested in seeing how the imposition of vertex symmetry affects the instability properties of a multichannel system. Because we wish to determine the response of our $SU(3)$-invariant model to various types of perturbations, we employ a first-order perturbative approach and study the eigenvalues and eigenvectors of a "feedback matrix."[8,11,12] In addition, a major motivation for the work described here is our desire to study correlations between the spectra of the pseudoscalar mesons and the $\frac{1}{2}^+$, $\frac{3}{2}^+$ baryons. We calculate not only the effect on baryon variables produced by meson mass splittings, but also examine the reaction of the baryon shifts back on the mesons. Therefore, particular emphasis is directed in this paper towards a careful study of the driving terms.

We now give a summary of the contents. In Sec. 2, we study an $SU(3)$-invariant bootstrap model of the $\frac{1}{2}^+$, $\frac{3}{2}^+$ baryon and 0^- meson multiplets as previously formulated by Lin and Cutkosky.[16] Section 3 concerns the shifts induced in baryon parameters by splitting of the meson masses from $SU(3)$ degeneracy. In Sec. 4, we examine the "feedback effect" and in Sec. 5, we list our conclusions. There is also an Appendix, in which certain technical details of our calculation are exhibited.

2. $SU(3)$-SYMMETRIC BOOTSTRAP MODEL

The calculation described in this section consists of a bootstrap model containing the $\frac{1}{2}^+$ baryon octet (B), $\frac{3}{2}^+$ baryon decuplet (D), and 0^- meson octet (P), all

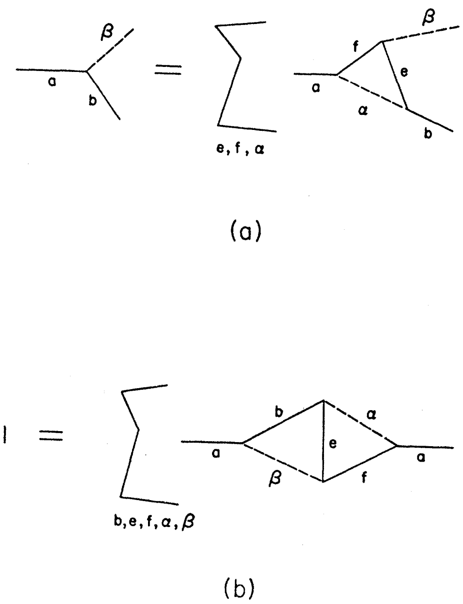

FIG. 1. (a) Vertex and (b) normalization equations. Solid (dashed) lines represent baryons (mesons).

interacting with $SU(3)$-invariant coupling constants. We briefly preview the results by noting that the model is successful in predicting reasonable values both of the mass difference between the baryon $\frac{3}{2}^+$ decuplet D and $\frac{1}{2}^+$ octet B, and the scale of their coupling constants. This success suggests to the author that the $SU(3)$ symmetry is dynamical in origin. That is, particles that interact with the forces described below, have approximately the correct (experimentally observed) properties if their masses and coupling constants agree with the $SU(3)$ values.

The general framework of our calculation has been developed by Lin and Cutkosky, who previously studied the type of model discussed in this section.[16] Below, we exhibit essentially the same bootstrap equations used by Lin and Cutkosky, both for the convenience of the reader and because they differ in the approximations made for the functions defined in Eqs. (2a), (2b). The bootstrap equations are inferred from the Bethe-Salpeter equation and are of two types: vertex [see Fig. 1(a)],

$$f_{b\beta}{}^{a}=\sum_{e,f,\alpha} f_{f\alpha}{}^{a} f_{b\alpha}{}^{e} f_{f\beta}{}^{e} D_{ab}{}^{ef}\,, \tag{1a}$$

and normalization [see Fig. 1(b)],

$$1=\sum_{b,e,f,\alpha,\beta} f_{f\alpha}{}^{a} f_{f\beta}{}^{e} f_{b\alpha}{}^{e} f_{b\beta}{}^{a} W_{bf}{}^{ae}\,, \tag{1b}$$

where the vertex dynamical factor $D_{ab}{}^{ef}$ is given by

$$D_{ab}{}^{ef}=\frac{1}{12\pi^2}\int_{\bar{\mu}}^{\Lambda}\frac{k^3dw}{(w+M_e-M_b)(w+M_f-M_a)} \tag{2a}$$

[13] G. L. Shaw and D. Y. Wong, Phys. Rev. 147, 1028 (1966). This paper questions the choice of D function used in Ref. 12.

[14] F. Ernst, K. Wali, and R. Warnock, Phys. Rev. 141, 1354 (1966). This paper contends that it may be incorrect to try to relate $SU(3)$ parameters to physical parameters by means of a first-order perturbative calculation.

[15] J. K. Kim, Phys. Rev. Letters 19, 1074 (1967). See also C. H. Chan and F. T. Meiere, *ibid.* 20, 568 (1968).

[16] K. Y. Lin and R. E. Cutkosky, Phys. Rev. 140, B205 (1965), and references cited therein.

and the normalization dynamical factor

$$W_{bf}{}^{ae}=K_a{}^b\partial D_{ab}{}^{ef}/\partial M_a. \tag{2b}$$

In the above, $f_{b\beta}{}^a$ gives the coupling of baryon a to baryon b and meson β, M_i is the mass of baryon i, μ_α is the mass of meson α, k is the meson momentum, related to its energy w by $k=(w^2-\mu_\alpha{}^2)^{1/2}$ and $K_a{}^b$ is a two-particle Green's function. The cutoff Λ in (2a), which represents the fact that all particles involved have structure, is fixed by requiring the πNN coupling constant to have the correct numerical value (this pins down the scale of the bootstrap model) and the lower limit $\bar{\mu}$ is the average meson mass. For convenience in solving Eqs. (1a) and (1b), we work in the static model, and also expand the expressions in (2a), (2b) to first order in the $\frac{3}{2}^+$, $\frac{1}{2}^+$ baryon mass difference, $M(D)-M(B)$. The use of $SU(3)$ symmetry implies that the model contains only four independent coupling constants, $f_{8-8}{}^A$, $f_{8-8}{}^S$, f_{10-8}, and f_{10-10}, corresponding to transitions $B\to(BP)_A$, $B\to(BP)_S$, $D\to BP$, and $D\to DP$. The symbols A,S represent the antisymmetric and symmetric couplings of the B,P octets to the B octet. The momentum and spin dependence of each vertex for the coupling of baryon a to the composite baryon b: meson α is given by

$$if_{b\alpha}{}^a\frac{\mathbf{S}_{ab}\cdot\mathbf{k}}{(2w_k)^{1/2}}\nu(k), \tag{3}$$

where $\mathbf{S}_{ab}$ is an appropriate spin operator (a Pauli matrix if both baryons have $J^P=\frac{1}{2}^+$). In this P-wave model, we assume that all vertices have a common structure $\nu(k)$. When we perform the expansion in baryon mass difference mentioned above, the couplings and the mass difference are described naturally in the following dimensionless units:

$$g_D=D_2{}^{1/2}f_{8-8}{}^S,\quad g_F=(5)^{1/2}D_2{}^{1/2}f_{8-8}{}^A,$$
$$g_1=D_2{}^{1/2}f_{10-8},\quad g_2=(2)^{-1/2}D_2{}^{1/2}f_{10-10},$$

and

$$\chi=(D_3/D_2)[M(D)-M(B)].$$

The quantities D_n are given by

$$D_n=\frac{1}{12\pi^2}\int_{\bar{\mu}}^{\Lambda}\frac{k^3dw}{w^n}. \tag{4}$$

With these definitions, the bootstrap equations have the form

$$\begin{aligned}
g_D&=\tfrac{1}{10}g_D{}^3+(1/30)g_Dg_F{}^2+\tfrac{4}{3}g_Dg_1{}^2C(-\chi)+\tfrac{2}{3}g_Fg_1{}^2C(-\chi)+\tfrac{5}{6}g_1{}^2g_2C(-2\chi),\\
g_F&=(-1/30)g_F{}^3+\tfrac{1}{6}g_D{}^2g_F+(10/3)g_Dg_1{}^2C(-\chi)+(25/6)g_1{}^2g_2C(-2\chi),\\
g_1&=g_1[(4/15)g_D(g_D+g_F)C(\chi)+\tfrac{1}{12}g_1{}^2+\tfrac{1}{3}g_2(g_D+g_F)+\tfrac{2}{3}g_2{}^2C-(\chi)],\\
g_2&=(11/10)g_2{}^3+\tfrac{1}{6}(g_D+g_F)g_1{}^2C(2\chi)+\tfrac{2}{3}g_1{}^2g_2C(\chi),
\end{aligned}\tag{5a}$$

for the vertex equations, while the normalization equations are

$$\begin{aligned}
&\tfrac{1}{10}g_D{}^4-(1/150)g_F{}^4+(1/15)g_F{}^2g_D{}^2+g_1{}^2[\tfrac{2}{3}W(0,-\chi)g_D(g_D+g_F)+\tfrac{4}{3}g_D(g_D+g_F)W(-\chi,0)\\
&\quad+(5/24)g_1{}^2W(-2\chi,0)+(5/3)g_2(g_D+g_F)W(-\chi,-\chi)+(5/3)g_2{}^2W(-2\chi,-\chi)]=2.2g_2{}^4\\
&\quad+g_1{}^2[(4/15)g_D(g_D+g_F)W(2\chi,\chi)+\tfrac{1}{12}g_1{}^2W(2\chi,0)+\tfrac{2}{3}W(\chi,\chi)g_2(g_D+g_F)+\tfrac{2}{3}g_2{}^2W(0,\chi)+\tfrac{4}{3}g_2{}^2W(\chi,0)].
\end{aligned}\tag{5b}$$

In the above, we have defined the dynamical functions C and W as

$$\begin{aligned}
C(\chi)&=1+\chi &&\text{if}\quad \chi>0\\
&=1/(1-\chi) &&\text{if}\quad \chi<0
\end{aligned}\tag{6a}$$

and

$$\begin{aligned}
W(\chi,y)&=1+\chi(1+\tfrac{1}{2}\eta)+\eta y &&\text{if}\quad \chi(1+\tfrac{1}{2}\eta)+\eta y>0\\
&=\frac{1}{1-(1+\frac{1}{2}\eta)\chi-\eta y} &&\text{if}\quad \chi(1+\tfrac{1}{2}\eta)+\eta y<0,
\end{aligned}\tag{6b}$$

where $\eta=D_2D_4/D_3{}^2$. The forms given to C and W are so chosen as to give good approximations to the exact forms in (2a) and (2b).

The nonlinear algebraic Eqs. (5) are solved numerically. The only self-consistent solution gives $g_D=0.693$, $g_F=1.25$, $g_1=0.613$, $g_2=0.544$, $\chi=0.285$ for the dynamical variables and a value of $\Lambda=1.03$ GeV for the cutoff in meson energy. This solution agrees rather well with experiment. The predicted $\frac{3}{2}^+$, $\frac{1}{2}^+$ mass difference is 225 MeV with the correct sign, whereas the experimental mass difference is 232 MeV. The ratio of the two distinct BBP couplings gives the F/D mixing angle as $\tan\theta=f_{8-8}{}^A/f_{8-8}{}^S=0.81$ or $\theta=39°$. In terms of the more familiar f,d parameters[17] (with $f+d=1$), the model predicts $f=0.378$ and $d=0.672$. The relation between the parameters θ and f is $\tan\theta=3f/5^{1/2}(1-f)$. Finally, the predicted ratio of DBP and BBP couplings can be compared to the experimentally known $N^*N\pi$ and $NN\pi$ couplings. We have, in the dimensionless units,

$$g(NN\pi)=(3/\sqrt{20})g_D+\tfrac{1}{2}g_F=0.779,$$
$$g(N^*N\pi)=(1/\sqrt{2})g_1=0.434,$$

leading to a predicted ratio (hence dividing out the common factors $D_2{}^{1/2}$) $f^2(N^*N\pi)/f^2(NN\pi^0)=3.1$. The

[17] For instance, see P. A. Carruthers, *Introduction to Unitary Symmetry* (Interscience Publishers, Inc., New York, 1966).

TABLE I. Baryon coupling constants. Column A gives the $SU(3)$-invariant couplings as obtained from the model described in Sec. 2. Column B gives these couplings as corrected by the meson-mass-shift driving terms (see Sec. 3). Units are dimensionless as described in Sec. 2. The coupling describing a particular spin orientation of the baryons is obtained from the above by multiplication with the relevant $SU(2)$ Clebsch-Gordan coefficient.

	A	B		A	B
$\Sigma^+ \to P\bar{K}^0$	0.151	0.168	$\Lambda \to \Sigma^+\pi^-$	0.310	0.296
$\Lambda \to PK^-$	0.435	0.514	$Y_1^+ \to \Sigma^+\pi^0$	0.177	0.167
$Y_1^+ \to P\bar{K}^0$	0.250	0.302	$Y_1^+ \to \Lambda\pi^+$	0.307	0.309
$Y_1^+ \to N^{++}K^-$	0.384	0.421	$Y_1^+ \to Y_1^+\pi^0$	0.314	0.309
$\Xi^0 \to \Sigma^0\bar{K}^0$	0.430	0.380	$\Xi^{*0} \to \Xi^0\eta$	−0.307	−0.252
$\Xi^0 \to \Lambda\bar{K}^0$	−0.125	−0.110	$\Xi^0 \to \Xi^0\pi^0$	−0.107	−0.083
$\Xi^{*0} \to \Sigma^0\bar{K}^0$	0.177	0.167	$\Xi^{*0} \to \Xi^0\pi^0$	0.177	0.102
$\Xi^{*0} \to \Lambda\bar{K}^0$	0.307	0.307	$\Xi^{*0} \to \Xi^{*0}\pi^0$	0.157	0.091
$\Xi^{*0} \to Y_1^{*0}\bar{K}^0$	0.314	0.290	$P \to P\eta$	−0.125	−0.138
$N^{++} \to \Sigma^+K^+$	−0.434	−0.443	$N^{++} \to N^{++}\eta$	−0.272	−0.272
$\Omega^- \to \Xi^0K^-$	0.433	0.331	$\Sigma^+ \to \Sigma^+\eta$	−0.310	−0.279
$\Omega^- \to \Xi^{*0}K^-$	0.384	0.288	$\Lambda \to \Lambda\eta$	0.310	0.367
$P \to P\pi^0$	0.430	0.531	$Y_1^+ \to Y_1^+\eta$	0.0	0.049
$N^{++} \to P\pi^+$	0.434	0.527	$\Xi^0 \to \Xi^0\eta$	0.435	0.376
$N^{++} \to N^{++}\pi^0$	0.471	0.547	$\Xi^{*0} \to \Xi^{*0}\eta$	0.272	0.278
$Y_1^+ \to \Sigma^+\eta$	−0.307	−0.305	$\Omega^- \to \Omega^-\eta$	0.544	0.412
$\Sigma^+ \to \Sigma^+\pi^0$	0.324	0.300	$Y_1^+ \to \Xi^0K^+$	−0.250	−0.213

experimentally observed value of $f(NN\pi^0)$ expressed in pseudovector coupling [see Eq. (3)] is $f=(4\pi)^{1/2}\times(0.08)^{1/2}$, whereas using the formula

$$\frac{f^2}{4\pi}=\frac{\frac{1}{2}\Gamma}{k^3}\frac{6M_R}{E+M}, \quad (7)$$

we find $f^2(N^*N\pi)=4.1$. In (7), Γ is the resonance width, MR its mass, k the decay momentum, M the decay baryon mass, and E the decay baryon energy. Therefore, experiment implies $f^2(N^*N\pi)/f^2(NN\pi^0)=4.1$, in reasonable agreement with the predicted value 3.1. This latter result exposes the somewhat fortuitous nature of the excellent $\frac{3}{2}^+$, $\frac{1}{2}^+$ mass difference already given. A more detailed tabulation of the $SU(3)$-invariant coupling constants which are predicted by the model is given in column A of Table I.

3. MESON INPUT PERTURBATIONS

A. Shifts in Baryon Variables

The π, η, K, and $\bar{K}$ mesons, which constitute the pseudoscalar octet, do not all have the same mass. Consider how this lack of degeneracy affects the vertex and normalization equations of Sec. 2. Because we wish to study the instability properties of our bootstrap model under perturbations, we work to first order in the meson mass splittings. As a practical matter, this approach is almost necessary, for without the assumption of $SU(3)$ symmetry, our model contains 41 variables, 34 coupling constants, and 7 mass differences. With a system of this size, the solution of nonlinear equations like (1a) and (1b) becomes prohibitively difficult. We begin by examining the vertex dynamical function $D_{ab}{}^{ef}$, defined in (2a). It is important to exhibit its dependence on the mass μ_α of a meson α, internal to a triangle graph [Fig. 1(a)]. We partially suppress the baryon labels "a,b,e,f" in the following:

$$D^{(E)}(\mu_\alpha{}^2)=\frac{1}{12\pi^2}\int_{\bar{\mu}}^{\Lambda}\frac{k^3dw}{(w+M_e-M_b)(w+M_f-M_a)},\quad k=(w^2-\mu_\alpha{}^2)^{1/2} \quad (8a)$$

or

$$D^{(P)}(\mu_\alpha{}^2)=\frac{1}{12\pi^2}\int_0^{\Lambda'}\frac{k^4dk}{(w+M_e-M_b)(w+M_f-M_a)},\quad w=(k^2+\mu_\alpha{}^2)^{1/2}. \quad (8b)$$

In Eqs. (8a) and (8b), we integrate over the energy and momentum respectively, giving the corresponding momentum and energy explicitly as a function of μ_α. In each case, the cutoff, which depends implicitly on the short-range forces (ignored in the model of Sec. 2) is constant, independent of μ_α. The form in (8a) corresponds to a cutoff fixed in energy (hence the superscript E) whereas the form in (8b) has a cutoff fixed in momentum (hence the superscript P). Expanding $D^{(E)}(\mu_\alpha{}^2)$, $D^{(P)}(\mu_\alpha{}^2)$ in powers of the $\frac{3}{2}^+$, $\frac{1}{2}^+$ baryon mass difference, we generate quantities

$$D_n{}^{(E)}(\mu_\alpha{}^2)=\frac{1}{12\pi^2}\int_{\bar{\mu}}^{\Lambda}\frac{(w^2-\mu_\alpha{}^2)^{3/2}dw}{w^n}, \quad (9a)$$

$$D_n{}^{(P)}(\mu_\alpha{}^2)=\frac{1}{12\pi^2}\int_0^{\Lambda'}\frac{k^4dk}{(k^2+\mu_\alpha{}^2)^{(n+1)/2}}, \quad (9b)$$

for fixed energy and momentum cutoffs, respectively. It is the functions (9a) and (9b) whose dependence on the internal meson mass μ_α we study. The variation of each such function with μ_α^2 is given by

$$\delta D_n^{(E)}(\mu_\alpha^2) = -\frac{\delta\mu_\alpha^2}{8\pi^2}\int_{\bar{\mu}}^{\Lambda}\frac{(w^2-\mu_\alpha^2)^{1/2}}{w^n}dw = -\frac{\delta\mu_\alpha^2}{8\pi^2}\int_{\bar{\mu}}^{\Lambda}\frac{k}{w^n}dw \tag{10a}$$

and

$$\delta D_n^{(P)}(\mu_\alpha^2) = -\frac{(n+1)}{24\pi^2}\delta\mu_\alpha^2\int_0^{\Lambda'}\frac{k^4dk}{w^{n+3}} = -\frac{(n+1)}{24\pi^2}\delta\mu_\alpha^2\int_0^{\Lambda'}\frac{k^3dw}{w^{n+2}}. \tag{10b}$$

In each case, an increase of μ_α^2 decreases $D_n(\mu_\alpha^2)$ because the phase space integrated over is effectively decreased. The dynamical functions W_n which appear in the baryon mass-difference expansion of the normalization equations (1b) behave in a similar manner. Consequently, the effect of the meson mass shifts upon the bootstrap equations [(1a) and (1b)] can be expressed as

$$\delta f_{b\beta}{}^a = \sum_{e,f,\alpha} f_{b\alpha}{}^a f_{b\alpha}{}^e f_{f\beta}{}^e \frac{\partial D_{ab}{}^{ef}}{\partial\mu_\alpha^2}(\mu_\alpha^2)\delta\mu_\alpha^2,$$
$$\delta N_a = \sum_{b,e,f,\alpha,\beta} f_{b\beta}{}^a f_{b\alpha}{}^e f_{f\beta}{}^e f_{f\alpha}{}^a \frac{\partial W_{bf}{}^{ae}}{\partial\mu_\alpha^2}(\mu_\alpha^2)\delta\mu_\alpha^2. \tag{11}$$

We call these quantities the "meson driving terms." It is clear from our discussion that they are not unique to the extent that we may use the expressions in (10a) or (10b) to evaluate them, and that this problem arises from our inability to handle the short-range forces. The ambiguity is not a serious one since both approaches give quantitatively similar results. For definiteness, all numerical results given in this paper follow from the fixed momentum expressions (10b).

The input meson mass shifts $\delta\mu_\alpha^2$ can be determined from experiment in the following manner. First, we compute the $SU(3)$ average, $\bar{\mu}^2 = \frac{1}{8}(4\mu_k^2+\mu_\eta^2+3\mu_\pi^2) = 0.1676$ GeV^2. The fractional displacement of each meson mass from the average is then

$$\delta\mu_\pi^2/\bar{\mu}^2 = -0.886,\ \delta\mu_\eta^2/\bar{\mu}^2 = 0.796,\ \delta\mu_k^2/\bar{\mu}^2 = 0.466. \tag{12}$$

Although taking $SU(3)$ breaking into account, we assume that the electromagnetic interactions are turned off and so we work exclusively with isospin-invariant quantities. The meson shifts (12), fed into the Eqs. (11), lead to the shifts in baryon coupling constants shown in column B of Table I. In order to get a better idea of the effect of the baryon shifts, we have added them to the $SU(3)$-invariant coupling constants. Agreement of these numbers with those values known experimentally is very good. For instance, consider the $D \rightarrow BP$ transitions. Using Eq. (7), we have for the *squares* of coupling constants the ratios

$N^*N\pi$:	$Y_1^*\Lambda\pi$:	$Y_1^*\Sigma\pi$:	$\Xi^*\Xi\pi$,	
1:	0.375:	0.194:	0.222,	experiment
1:	0.500:	0.334:	0.500,	pure $SU(3)$
1:	0.348:	0.202:	0.202,	pure $SU(3)$ plus meson driving terms.

This is to be compared with ratios predicted by Ref. 9, namely, 1: 0.303: 0.072: 0.067 and by Ref. 12, namely, 1: 0.360: 0.102: 0.152. A comparison with $B \rightarrow BP$ transitions is limited because so very few of them have been determined with any accuracy. However, Kim has recently done a careful analysis of low-energy K^-p data and has deduced the values $g^2(p \rightarrow \Lambda K^+)/4\pi = 16 \pm 2.5$ and $g^2(p \rightarrow \Sigma^+ K^0)/4\pi = 0.3 \pm 0.5$, expressed in pseudoscalar coupling.[15] Our $SU(3)$ coupling constants, corrected by the meson driving terms, and scaled to $g^2(PP\pi^0)/4\pi = 15$, give $g^2(P \rightarrow \Lambda K^+)/4 = 14.1$ and $g^2(P \rightarrow \Sigma^+ K^0)/4\pi = 1.5$. A value of the $P \rightarrow \Lambda K^+$ coupling equalling or exceeding that of $p \rightarrow p\pi^0$ has been predicted by the model of Ref. 14 but not by that of Ref. 12.

The mass shift of a given baryon does not come directly out of the perturbed bootstrap equations (11). However, the shift in the normalization equation of each baryon behaves in the same way as a mass shift, i.e., the tensorial properties are the same, so that it is of interest to examine the former. From the scale invariance of bootstrap theory, only the shift in normalization relative to one of the particles, taken here as the nucleon, is observable. The following ratios come out of our numerical model (Eq. 11) and are shown with the experimental mass ratios:

Ω-N:	Ξ^*-N:	Y_1^*-N:	N^*-N:	Ξ-N:	Σ-N:	Λ-N:	
1:	0.71:	0.42:	0.13:	0.77:	0.48:	0.35,	numerical model
1:	0.71:	0.43:	0.14:	0.75:	0.50:	0.35,	experiment.

Again, the predictions agree very well with experiment. In conclusion, it is clear that the introduction of the meson mass shifts into the baryon dynamical system gives splittings which have the correct behavior. This strengthens our suggestion that there is a definite correlation between the $SU(3)$ breaking of the baryon and meson spectra. We stress that aside from the phenomenological meson mass splittings, all other parameters in the model are determined from the dynamics and that, in particular, the $\frac{1}{2}^+$, $\frac{3}{2}^+$ baryon multiplets are dealt with simultaneously as a natural consequence of the vertex symmetry in our model.

TABLE II. Coupling of pseudoscalar mesons to baryon-antibaryon channels. Column A gives the $SU(3)$-invariant squared coupling constants whereas column B gives the squared $SU(3)$-invariant couplings corrected by the meson driving terms (see Sec. 3). Units are dimensionless as described in Sec. 2. The baryon-antibaryon states are summed over for a given coupling, e.g., $\pi^+ \to NN^*$ represents $\pi^+ \to P\bar{N}^{*0}$ plus $\pi^+ \to n\bar{N}^{*+}$.

	A	B		A	B		A	B
$\pi^+ \to N\bar{N}$	0.037	0.564	$\eta \to N\bar{N}$	0.031	0.038	$K^+ \to N\bar{\Lambda}$	0.190	0.264
$\Lambda\bar{\Sigma}$	0.096	0.088	$\Lambda\bar{\Lambda}$	0.096	0.135	$N\bar{\Sigma}$	0.034	0.042
$\Sigma\bar{\Lambda}$	0.096	0.088	$\Sigma\bar{\Sigma}$	0.288	0.234	$\Lambda\bar{\Xi}$	0.016	0.012
$\Sigma\bar{\Sigma}$	0.210	0.180	$\Xi\bar{\Xi}$	0.378	0.283	$\Sigma\bar{\Xi}$	0.555	0.433
$\Xi\bar{\Xi}$	0.023	0.014	$\Sigma\bar{Y}_1^*$	0.282	0.279	$N\bar{Y}_1^*$	0.094	0.137
$N\bar{N}^*$	0.251	0.370	$\Xi\bar{\Xi}^*$	0.188	0.127	$\Lambda\bar{\Xi}^*$	0.094	0.094
$\Lambda\bar{Y}_1^*$	0.094	0.096	$Y_1^*\bar{\Sigma}$	0.282	0.279	$\Sigma\bar{\Xi}^*$	0.094	0.084
$\Sigma\bar{Y}_1^*$	0.063	0.037	$\Xi^*\bar{\Xi}$	0.188	0.127	$\Xi\bar{\Omega}$	0.188	0.110
$\Xi\bar{\Xi}^*$	0.063	0.056	$N^*\bar{N}^*$	0.296	0.296	$N^*\bar{\Sigma}$	0.376	0.392
$N^*\bar{N}$	0.251	0.370	$Y_1^*\bar{Y}_1^*$	0.000	0.007	$Y_1^*\bar{\Xi}$	0.094	0.068
$Y_1^*\bar{\Lambda}$	0.094	0.095	$\Xi^*\bar{\Xi}^*$	0.148	0.155	$N^*\bar{Y}_1^*$	0.296	0.354
$Y_1^*\bar{\Sigma}$	0.063	0.056	$\Omega\bar{\Omega}$	0.296	0.170	$Y_1^*\bar{\Xi}^*$	0.296	0.252
$\Xi^*\bar{\Xi}$	0.063	0.037				$\Xi^*\bar{\Omega}$	0.148	0.083
$Y_1^*\bar{Y}_1^*$	0.197	0.191						
$N^*\bar{N}^*$	0.492	0.663						
$\Xi^*\bar{\Xi}^*$	0.049	0.032						

B. Qualitative Model of the Mesons

A natural extension of the baryon model described in this paper is the Fermi-Yang meson model.[18] Arguing that crossing symmetry motivates one to adopt this approach, Cutkosky and Jacobs have studied the properties of meson spectra generated by $B\bar{B}$ composites of $SU(6)$ multiplets.[19] Among their findings is that the effect of $SU(6)$-symmetry breaking in the baryon channel leads to qualitatively correct splittings in the meson channel if the Fermi-Yang dynamical approach holds. We reinforce their ideas in this section by studying a semiquantitative model of the pseudoscalar mesons in which the square of a meson mass is assumed to be inversely proportional to the squares of coupling constants of all possible baryon-antibaryon channels. That is, if μ_α is the mass of meson α, then

$$\mu_\alpha^{-2} \propto \sum_{i,j} (g_{ij}^\alpha)^2 \tag{13}$$

summed over all relevant baryons i,j. In the limit of $SU(3)$ invariance, Eq. (13) predicts a degenerate octet, as expected. This situation is shown in column A of Table II. Upon using the sum of the $SU(3)$ value and the meson driving term contribution for each coupling constant, we obtain the results shown in column B of Table II. These imply the correct qualitative pseudoscalar meson mass spectrum, $\mu_\eta^2 > \mu_K^2 > \mu_\pi^2$. To get an idea of the extent to which each meson mass is shifted from the $SU(3)$ degenerate value, we compute ratios of the total squared coupling of each meson minus the total $SU(3)$-invariant squared couplings. We find

$$\frac{\sum_{ij} g_{ij}^2(\pi) - (g_0)_{ij}^2(\pi)}{\sum_{ij} g_{ij}^2(\eta) - (g_0)_{ij}^2(\eta)} = -1.5\,,$$

$$\frac{\sum_{ij} g_{ij}^2(\pi) - (g_0)_{ij}^2(\pi)}{\sum_{ij} g_{ij}^2(K) - (g_0)_{ij}^2(K)} = -2.5\,,$$

$$\frac{\sum_{ij} g_{ij}^2(\eta) - (g_0)_{ij}^2(\eta)}{\sum_{ij} g_{ij}^2(K) - (g_0)_{ij}^2(K)} = 1.6\,,$$

whereas the corresponding experimental numbers taken from mass values are -1.1, -1.9, and 1.7. In the above, g and g_0 are the total and $SU(3)$-invariant coupling constants, respectively. Several features of Table II should be pointed out here: (i) Meson and baryon splittings from $SU(3)$ are consistent in the combined static and Fermi-Yang models and (ii) it is apparently not possible to obtain a viable model of the mesons without including strange particles as well as members of the $\frac{3}{2}^+$ decuplet. This explains in part the difficulty encountered by Ball *et al.* in their attempt to understand mesons with only $N\bar{N}$ composites[20]; (iii) it is probably possible to obtain a very deeply bound pion. On the basis of our admittedly qualitative considerations, we feel there is no reason to single out the pion as "aristocratic" simply because of its light mass.[21]

4. FEEDBACK MATRIX

The strength and range of the forces in bootstrap theory depend upon the particle masses and coupling constants, which, on the other hand, emerge as output

[18] E. Fermi and C. N. Yang, Phys. Rev. **76**, 1739 (1949).
[19] R. E. Cutkosky and M. Jacobs, Phys. Rev. **162**, 1416 (1967).
[20] J. S. Ball, A. Scotti, and D. Y. Wong, Phys. Rev. **142**, 1000 (1966).
[21] G. F. Chew, Comments Nucl. Part. Phys. **1**, 187 (1967).

from the theory. Any external perturbations on a bootstrap system generated, say, by meson mass deviations from $SU(3)$ degeneracy or by effects from electromagnetism, perturb these output values and, consequently, produce changes in the strengths and ranges of the strong-interaction forces. The latter effect is called the "feedback phenomenon" and is the subject of discussion in this section.[8,11] Analogous to our discussion of "driving terms," we limit ourselves here to an examination of first-order shifts in baryon variables, and for simplicity, examine in detail the effect of feedback only on the vertex equations (1a). Under a perturbation, these equations become

$$\delta f_{b\beta}{}^{a}=\sum_{e,f,\alpha}\delta(f_{f\alpha}{}^{a}f_{b\alpha}{}^{e}f_{f\beta}{}^{e})D_{ab}{}^{ef}+f_{f\alpha}{}^{a}f_{b\alpha}{}^{e}f_{f\beta}{}^{e}\delta D_{ab}{}^{ef}+\text{driving terms},\quad(14)$$

the first term relating coupling shift to coupling shift, and the second, mass shift to coupling shift. Equations (14) are linear whereas the unperturbed vertex equations (1a) are nonlinear. This fact makes solution of the former appreciably simpler than that of the latter. We now evaluate the quantity $\delta D_{ab}{}^{ef}$ in (14) as follows. From (2a), we have

$$\delta D_{ab}{}^{ef}=-\frac{1}{12\pi^2}\int_{\bar{\mu}}^{\Lambda}\frac{k^3dw}{(w+M_e-M_b)(w+M_f-M_a)}\times\left[\frac{\delta(M_e-M_b)}{(w+M_e-M_b)}+\frac{\delta(M_f-M_a)}{(w+M_f-M_a)}\right]\quad(15)$$

and expanding in powers of the baryon mass differences, we find after some algebra

$$\delta D_{ab}{}^{ef}=-D_2\{[1+\eta(\chi_b-\chi_e+2\chi_a-2\chi_f)](\delta\chi_f-\delta\chi_a)+[1+\eta(\chi_a-\chi_f+2\chi_b-2\chi_e)](\delta\chi_e-\delta\chi_b)\},\quad(16)$$

where the D_n are defined in Eq. (4), $\eta=(D_2D_4)/D_3{}^2$, and the mass of particle i, χ_i, is given in dimensionless units by $\chi_i=(D_3/D_2)M_i$. Multiplying Eq. (14) by $D_2{}^{1/2}$ and using Eq. (16), we obtain the final form of the perturbed vertex equation,

$$\delta g_{b\alpha}{}^{a}=\sum_{e,f,\alpha}\delta(g_{f\alpha}{}^{a}g_{b\alpha}{}^{e}g_{f\beta}{}^{e})(1+\chi_a+\chi_b-\chi_e-\chi_f)-g_{f\alpha}{}^{a}g_{b\alpha}{}^{e}g_{f\beta}{}^{e}\{[1+\eta(\chi_b-\chi_e+2\chi_a-2\chi_f)](\delta\chi_f-\delta\chi_a)+[1+\eta(\chi_a-\chi_b+2\chi_b-2\chi_e)](\delta\chi_e-\delta\chi_b)\},\quad(17)$$

where $g_{b\alpha}{}^{a}$ gives the coupling of some baryon a to a meson-baryon composite αb in dimensionless units. There is one such perturbed vertex equation for each independent $SU(2)$-invariant coupling constant in the model. We may express the corresponding linear set symbolically as a matrix equation:

$$\delta g=A^{gg}\delta g+A^{g\chi}\delta\chi+\text{driving terms.}\quad(18)$$

The normalization equations (1b) are handled similarly:

$$0=\sum_{b,e,f,\alpha,\beta}\delta(f_{f\alpha}{}^{a}f_{b\alpha}{}^{e}f_{b\beta}{}^{e}f_{b\beta}{}^{a})W_{bf}{}^{ae}+f_{f\alpha}{}^{a}f_{b\alpha}{}^{e}f_{b\beta}{}^{e}f_{b\alpha}{}^{a}\delta W_{bf}{}^{ae}+\text{driving terms.}\quad(19)$$

Evaluation of $\delta W_{bf}{}^{ae}$ in the second term of (19) follows from its defining Eq. (2b) and after considerable algebra we find the complicated expression:

$$\delta W_{bf}{}^{ae}=[2(1+\eta)-2\eta(2+3\beta)(\chi_f-\chi_a)-2\eta(1+\beta)(\chi_e-\chi_b)+4\eta(\chi_a-\chi_b)]\delta\chi_a+[\eta-2+\eta(4-\beta)(\chi_f-\chi_a)+2\eta(1-\beta)(\chi_e-\chi_b)+2\eta(\chi_a-\chi_b)]\delta\chi_b-\eta[1-2\beta(\chi_e-\chi_b)-\beta(\chi_f-\chi_a)+2(\chi_a-\chi_b)]\delta\chi_e-\eta[2-6\beta(\chi_f-\chi_a)-2\beta(\chi_e-\chi_b)+4(\chi_a-\chi_b)]\delta\chi_f,\quad(20)$$

where $\beta=(D_2D_5)/(D_3D_4)$. Expressing Eqs. (19) in matrix form,

$$0=A^{\chi g}\delta g+A^{\chi\chi'}\delta\chi+\text{driving terms},\quad(21a)$$

and adding a column vector $\delta\chi$ to each side of (21a), we obtain a matrix equation for mass shifts:

$$\delta\chi=A^{\chi g}\delta g+A^{\chi\chi}\delta\chi+\text{driving terms.}\quad(21b)$$

Equations (18) and (21b) can be put collectively into the form

$$\delta\eta=A\,\delta\eta+P,\quad(22)$$

where the column vector $\delta\eta$ represents the shifts in all the baryon variables and the column vector P gives the driving terms. The solution to (22) is given simply by

$$\delta\eta=(1-A)^{-1}P,\quad(23)$$

which indicates that to first order, the shift in baryon variables depends not only on the external perturbation P, but also on the feedback matrix A.

Since A depends only on the strong interactions, which for the unperturbed state in this model are $SU(3)$-invariant, the eigenvectors of A transform according to irreducible representations of $SU(3)$. Hence, analysis of the baryon shifts can be carried out with $SU(3)$ tensors. Certain details of this, including useful coupling constant sum rules, are discussed in the Appendix. The most important aspect of an eigenvalue analysis of A is the determination of the eigenvalues nearest unity, for these should dominate the behavior of the solution to Eq. (23). We obtained the eigenvalues of A and found that the model described in this paper does not have the correct feedback properties.[22] In fact, the eigenvalue nearest unity, $\lambda=1.1$, has an eigenvector which transforms as **27** although there are also nearby eigenvalues, $\lambda=1.5$, 0.9, 0.83, 0.64, etc., which transform as **8**. In order to eliminate the possibility that approximations made in evaluating the normalization equations (1b) and their shifts, Eq. (20), could be at fault here, we evaluated the eigenvalues and eigenvectors of the submatrix A^{gg} which relates coupling shifts to each other.

[22] All eigenvalues of A are found to be real. The matrix A, although real, is not symmetric.

Again, we found that the main mode transforms as **27**, with several nearby modes transforming as **8**. With regard to the over-all aim of this calculation, solution of Eq. (23) therefore becomes meaningless because octet dominance is not a unique property of the model. That is, although analysis of the meson mass driving terms supports the conjectured dynamical breakdown of $SU(3)$ symmetry, the presence of the dominant **27** mode in the baryon feedback matrix is inconsistent with the concept of spontaneous symmetry breakdown. A suggestion for relaxing the assumptions on which the model is based is given in the next section.

5. CONCLUSIONS

The main assumptions underlying our model are (i) static kinematics, i.e., inclusion of only baryon-exchange processes in the dynamics, (ii) truncation in number of channels to include only the $\frac{1}{2}^+$, $\frac{3}{2}^+$ baryon and 0^- meson multiplets, and (iii) vertex symmetry, the symmetrical treatment of all channels. Apparently, these assumptions are sufficient to ensure a successful model of $SU(3)$-invariant interactions.[16] The model of Sec. 2 gives very acceptable values for both mass differences and coupling constants. When we allow the mesons to interact with their actual isospin-invariant masses, the shifts consequently generated in the baryon variables are in impressive agreement with experiment, especially in comparison with previous calculations appearing in the literature. Furthermore, the baryon shifts induce acceptable meson mass shifts if the mesons are taken as Fermi-Yang composites. The success of this driving-term analysis suggests that a correlation exists between baryon and meson channels and that a simultaneous dynamical description of each should be possible. In particular, it suggests that a calculation of the pion bound state within the realm of conventional dynamics is possible if enough channels are included. Contrary to the driving terms, the feedback properties of our model are not correct because instabilities other than octet type are present. This property is also true of the subproblem in which only coupling-constant shifts are calculated. Our result is somewhat surprising since the major difference between this model and those claiming a dynamical understanding of octet dominance lies in our use of a larger number of channels and our adherence to vertex symmetry. It would be of great interest to see whether the success of these previous calculations remains as more channels are introduced and the requirements of crossing symmetry are more strictly obeyed. There are several possibilities to consider in determining what additional features our model requires for us to attain an understanding of octet dominance. Perhaps, the suggestion of Ernst, Wali, and Warnock[14] that first-order effects do not properly mirror the symmetry-breaking properties of a bootstrap system is valid and an entirely different approach must be tried. However, in our opinion, a more likely possibility is that the assumption of static kinematics must be relaxed and that vector mesons should be admitted into the model. This will not affect the pseudoscalar-meson driving terms appreciably and will make the strong-interaction dynamics more complete by taking into account the most important of the shorter-range forces, vector-meson exchange.

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APPENDIX

In this Appendix, we discuss several features of the tensorial analysis used in our calculation. In order to give meaning to our results, we list the phase used in defining the baryon (B) and decuplet (D) wave functions, $B \to BP$, $B \to DP$, $D \to BP$, $D \to DP$, where P stands for the pseudoscalar meson octet. The relevant $SU(3)$ isoscalar factors[23] are

$$
\begin{aligned}
&B \to (BP)_D:\\
&\quad N_D = (1/\sqrt{20})[3N\pi + N\eta - 3\Sigma K + \Lambda K],\\
&\quad \Lambda_D = (1/\sqrt{5})[(1/\sqrt{2})N\bar{K} + \sqrt{3}\Sigma\pi + \Lambda\eta - (1/\sqrt{2})\Xi K],\\
&\quad \Sigma_D = (1/\sqrt{10})[\sqrt{3}N\bar{K} - \sqrt{2}\Sigma\eta - \sqrt{2}\Lambda\pi + \sqrt{3}\Xi K],\\
&\quad \Xi_D = (1/\sqrt{20})[\Lambda\bar{K} + 3\Sigma\bar{K} - 3\Xi\pi + \Xi\eta];\\
&B \to (BP)_F:\\
&\quad N_F = \tfrac{1}{2}[N\pi - N\eta + \Sigma K + \Lambda K],\\
&\quad \Lambda_F = (1/\sqrt{2})[N\bar{K} + \Xi K],\\
&\quad \Sigma_F = (1\sqrt{6})[2\Sigma\pi + \Xi K - N\bar{K}],\\
&\quad \Xi_F = \tfrac{1}{2}[\Sigma\bar{K} - \Lambda\bar{K} + \Xi\pi + \Xi\eta];\\
&D \to BP:\\
&\quad Y_1^* = \tfrac{1}{2}[\Lambda\pi - \Sigma\eta + (\sqrt{\tfrac{2}{3}})N\bar{K} + (\sqrt{\tfrac{2}{3}})\Sigma\pi - (\sqrt{\tfrac{2}{3}})\Xi K],\\
&\quad N^* = (1/\sqrt{2})[N\pi - \Sigma K],\\
&\quad \Omega = \Xi\bar{K},\\
&\quad \Xi^* = \tfrac{1}{2}[\Sigma\bar{K} + \Lambda\bar{K} + \Xi\pi - \Xi\eta];\\
&B \to DP: \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{(A1)}\\
&\quad N = (1/\sqrt{5})[-2N^*\pi - Y_1^*K],\\
&\quad \Sigma = (1/\sqrt{5})[(\sqrt{\tfrac{2}{3}})Y_1^*\pi - Y_1^*\eta - 2(\sqrt{\tfrac{2}{3}})N^*\bar{K} + (\sqrt{\tfrac{2}{3}})\Xi^*K],\\
&\quad \Lambda = (1/\sqrt{5})[-\sqrt{3}Y_1^*\pi - \sqrt{2}\Xi^*K],\\
&\quad \Xi = (1/\sqrt{5})[\Xi^*\pi - Y_1^*\bar{K} - \Xi^*\eta + \sqrt{2}\Omega K];\\
&D \to DP:\\
&\quad N^* = (1/\sqrt{8})[\sqrt{2}Y_1^*K - N^*\eta + (\sqrt{5})N^*\pi],\\
&\quad Y_1^* = (1/\sqrt{3})[\Xi^*K + Y_1^*\pi + N^*\bar{K}],\\
&\quad \Xi^* = (1/\sqrt{8})[\Xi^*\eta + \sqrt{2}\Omega K + 2Y_1^*\bar{K} + \Xi^*\pi],\\
&\quad \Omega = (1/\sqrt{2})[\Xi^*\bar{K} + \Omega\eta].
\end{aligned}
$$

[23] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).

To determine the $SU(3)$ Clebsch-Gordan coefficient pertaining to a coupling of three specific particles, multiply the relevant isoscalar factor by the relevant $SU(2)$ or isospin Clebsch-Gordan coefficient.

Now we consider the problem of expressing an isoscalar factor in the presence of $SU(3)$-breaking effects. To preserve hypercharge and isospin invariance, the breaking of $SU(3)$ can only transform as a $Y=I=0$ member of the **1**, **8**, **27**, **64**, **125** dimensional multiplets (no higher-dimensional multiplets are relevant for the model discussed in this paper).

An isospin-invariant coupling,

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} & \big| & \mathbf{C}_i \\ \alpha & \beta & \big| & \gamma \end{pmatrix},$$

perturbed from its $SU(3)$-invariant value by a symmetry-breaking effect of type $\mathbf{Z}$, is in general described by[12,24]

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} & \big| & \mathbf{C}_i \\ \alpha & \beta & \big| & \gamma \end{pmatrix}_{\mathbf{Z}} = \sum_{\mathbf{N}} \chi_{\mathbf{N}}(\mathbf{Z}) \begin{pmatrix} \mathbf{C} & \mathbf{Z} & \mathbf{N} \\ \gamma & 0 & \gamma \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{N} \\ \alpha & \beta & \gamma \end{pmatrix}. \quad \text{(A2)}$$

The sum goes over all allowed representations. For instance, in the case of $D\to DP$ we find for an octet perturbation $\mathbf{N}=\mathbf{8}, \mathbf{10}, \mathbf{27}, \mathbf{35}$, so that

$$\begin{aligned} \Omega\to\Omega\eta &= (1/\sqrt{2})\chi_1(8)+\tfrac{1}{2}\chi_{10}(8)+(1/\sqrt{2})\chi_{35}(8)\,, \\ N^*\to N^*\pi &= (\sqrt{\tfrac{5}{8}})\chi_1(8)-(\sqrt{\tfrac{5}{8}})\chi_{10}(8) \\ &\quad +\tfrac{3}{16}(\sqrt{5})\chi_{27}(8)-(\sqrt{\tfrac{5}{16}})\chi_{35}(8)\,, \end{aligned} \quad \text{(A3)}$$

and so forth. In the presence of several types of perturbation, we must sum over these to get

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} & \big| & \mathbf{C}_i \\ \alpha & \beta & \big| & \gamma \end{pmatrix}_{\text{tot}} = \sum_{\mathbf{Z}} \begin{pmatrix} \mathbf{A} & \mathbf{B} & \big| & \mathbf{C}_i \\ \alpha & \beta & \big| & \gamma \end{pmatrix}_{\mathbf{Z}}, \quad \text{(A4)}$$

where the quantity on the left-hand side of the above represents the total coupling. Listed below are the various $SU(3)$ representations $\mathbf{Z}$ allowed for the couplings considered in this paper:

$$\begin{aligned} B\to BP&: \quad \mathbf{Z}=\mathbf{1},\, \mathbf{8},\, \mathbf{27},\, \mathbf{64}; \\ D\to BP&: \quad \mathbf{Z}=\mathbf{1},\, \mathbf{8},\, \mathbf{27},\, \mathbf{64}; \\ D\to DP&: \quad \mathbf{Z}=\mathbf{1},\, \mathbf{8},\, \mathbf{27},\, \mathbf{64},\, \mathbf{125}. \end{aligned} \quad \text{(A5)}$$

Consider a particular type of $SU(3)$ breaking, say $\mathbf{Z}=\mathbf{8}$, and a particular set of coupling constants, say, $D\to DP$. There are ten $D\to DP$ couplings, described in the case of $SU(3)$ invariance plus octet breaking by five tensors, $\chi_1(8)$, $\chi_8(8)$, $\chi_{10}(8)$, $\chi_{27}(8)$, $\chi_{35}(8)$, in the notation of (A3). Hence five linear relations or "sum rules" between the $D\to DP$ couplings are possible. These sum rules are extremely useful in the course of our calculation for checking (i) the $SU(3)$ isotropy of the A matrix and (ii) the generation of driving terms of a particular tensorial type. We exhibit below those that we derived in the course of this calculation. The cases of octet perturbation for BBP and DBP coupling constants [corresponding to specific particle couplings and not to the $SU(2)$-invariant couplings we consider in this paper] have been previously exhibited in the literature.[24,25] We caution that the following sum rules hold only with the phase convention used in our equation (A1).

[24] V. Gupta and V. Singh, Phys. Rev. **136**, B782 (1964).

Coupling-Constant Sum Rules

$B\to BP$ with **8** breaking:

$$\begin{aligned} &\Sigma\Sigma\eta-NN\eta-\tfrac{1}{3}NN\pi+(4/\sqrt{6})\Sigma N\bar{K} \\ &\qquad +(1/\sqrt{6})\Sigma\Sigma\pi=0\,, \\ &-\Sigma\Sigma\eta+\Xi\Xi\eta-\tfrac{1}{3}\Xi\Xi\pi-\tfrac{4}{3}\Xi\Sigma\bar{K} \\ &\qquad +(1/\sqrt{6})\Sigma\Sigma\pi=0\,, \\ &-\Sigma\Sigma\eta+\Lambda\Lambda\eta-(4/9)NN\pi+(4/9)\Xi\Xi\pi \\ &-(8/3\sqrt{6})\Sigma N\bar{K}-(8/9)\Xi\Sigma\bar{K}+(2/\sqrt{3})\Lambda\Sigma\pi=0\,, \\ &\tfrac{2}{3}NN\pi+\tfrac{2}{3}\Xi\Sigma\bar{K}-(1/\sqrt{2})\Lambda N\bar{K}-(1/\sqrt{3})\Lambda\Sigma\pi \\ &\qquad -(1/\sqrt{6})\Sigma N\bar{K}-(1/\sqrt{6})\Sigma\Sigma\pi=0\,, \\ &-\Xi\Lambda\bar{K}-(1/\sqrt{3})\Lambda\Sigma\pi-\tfrac{2}{3}\Xi\Xi\pi+(2/\sqrt{6})\Sigma N\bar{K} \\ &\qquad -\tfrac{1}{3}\Xi\Sigma\bar{K}+(1/\sqrt{6})\Sigma\Sigma\pi=0. \end{aligned}$$

$B\to BP$ with **27** breaking:

$$\begin{aligned} &-5(\sqrt{6})\Sigma N\bar{K}+5(\sqrt{6})\Sigma\Sigma\pi-(10/\sqrt{2})\Lambda N\bar{K} \\ &\qquad -8\Lambda\Lambda\eta+6\sqrt{3}\Lambda\Sigma\pi-4\Xi\Xi\eta-8\Xi\Xi\pi \\ &\qquad +6\Xi\Lambda\bar{K}-2\Xi\Sigma\bar{K}=0\,, \\ &-6\Xi\Xi\eta-2\Xi\Xi\pi-\Xi\Lambda\bar{K}-3\Xi\Sigma\bar{K}+3\Lambda\Lambda\eta \\ &\qquad -\sqrt{3}\Lambda\Sigma\pi+5NN\pi-5NN\eta=0. \end{aligned}$$

$B\to BP$ with **64** breaking:

$$\begin{aligned} &\Sigma\Sigma\eta+\Xi\Xi\eta+\Xi\Lambda\bar{K}-NN\eta \\ &\qquad -(1/\sqrt{2})\Lambda N\bar{K}+(1/\sqrt{3})\Lambda\Sigma\pi=0\,, \\ &2(NN\eta)+\sqrt{2}\Lambda N\bar{K}-\sqrt{3}\Lambda\Sigma\pi+\Lambda\Lambda\eta=0. \end{aligned} \quad \text{(A6)}$$

$D\to BP$ with **8** breaking:

$$\begin{aligned} &\sqrt{2}N^*N\pi+2\Xi^*\Xi\pi \\ &\qquad =\tfrac{1}{2}[6Y_1^*\Lambda\pi+(\sqrt{6})Y_1^*\Sigma\pi]\,, \\ &-\sqrt{2}N^*\Sigma K+2\Xi^*\Sigma\bar{K} \\ &\qquad =\tfrac{1}{2}[-6Y_1^*\Sigma\eta+(\sqrt{6})Y_1^*\Sigma\pi]\,, \\ &(\sqrt{6})Y_1^*N\bar{K}+\Omega\Xi\bar{K}=3\Xi^*\Lambda\bar{K}+\Xi^*\Sigma\bar{K}\,, \\ &-(\sqrt{6})Y_1^*\Xi K+\Omega\Xi\bar{K}=-3\Xi^*\Xi\eta+\Xi^*\Xi\pi\,, \\ &(\sqrt{6})Y_1^*\Sigma\pi+\Omega\Xi\bar{K}=2(\Xi^*\Xi\pi+\Xi^*\Sigma\bar{K})\,, \\ &\sqrt{2}N^*N\pi+2\Xi^*\Sigma\bar{K}=\sqrt{6}[Y_1^*N\bar{K}+Y_1^*\Sigma\pi]\,, \\ &-\sqrt{2}N^*\Sigma K+2\Xi^*\Xi\pi=\sqrt{6}[-Y_1^*\Xi K+Y_1^*\Sigma\pi]. \end{aligned}$$

[25] M. Muraskin and S. Glashow, Phys. Rev. **132**, 482 (1963).

$D \to DP$ with **8** breaking:

$$(1/\sqrt{5})N^*N^*\pi - \Xi^*Y_1^*\bar{K} - \Xi^*\Xi^*\pi + \Omega\Xi^*\bar{K} = 0,$$
$$\Omega\Omega\eta - \Omega\Xi^*\bar{K} - 2\Xi^*\Xi^*\eta + \Xi^*Y_1^*\bar{K} + Y_1^*Y_1^*\eta = 0,$$
$$\Omega\Xi^*\bar{K} - 2\Xi^*Y_1^*\bar{K} + (\sqrt{\tfrac{3}{2}})Y_1^*N^*\bar{K} = 0,$$
$$\Xi^*\Xi^*\pi - \Xi^*\Xi^*\eta + 2Y_1^*Y_1^*\eta - N^*N^*\eta - (1/\sqrt{5})N^*N^*\pi = 0,$$
$$2\Xi^*\Xi^*\pi - \Xi^*Y_1^*\bar{K} + (\sqrt{\tfrac{3}{2}})[-Y_1^*Y_1^*\pi + Y_1^*N^*\bar{K}] = 0.$$

As noted in Ref. 24, one of the $D \to BP$ octet breaking sum rules can be checked experimentally as follows. Relating resonance width to coupling constant with Eq. (7), we find

$$f(N^*N\pi): f(Y_1^*\Lambda\pi): f(Y_1^*\Sigma\pi): f(\Xi^*\Xi\pi) = 1: 0.612: 0.440: 0.471,$$

so that the first of the DBP octet sum rules in (A6) divided by $f(N^*N\pi)$ becomes $\sqrt{2}(1)+2(0.471) = 0.5[6(0.612)+(\sqrt{6})(0.44)]$ or $2.36=2.37$, a remarkable agreement wihch reinforces the deduction from the mass spectrum that octet dominance governs the baryon resonance region. It is amusing to note that these $D \to BP$ octet sum rules imply that if any one of the couplings $Y_1^*N\bar{K}$, $\Omega\Xi\bar{K}$, $\Xi^*\Sigma\bar{K}$, or $\Xi^*\Xi\bar{K}$ is determined from experimental analysis, then the other three follow from relations (A6). The fact that all the $J^P=\frac{1}{2}^+$ baryons are bound states makes determination of the $B \to BP$ coupling constants much more difficult and, at present, only $NN\pi$, $\Lambda N\bar{K}$, $\Sigma N\bar{K}$ have been determined with any kind of precision.[15] Consequently, the $B \to BP$ octet sum rules in (A6) have not proved as useful to this time.

We conclude the Appendix by displaying the relations used to check the normalization equation shifts of the particle mass shifts, and for baryon b of multiplet B ($B=\mathbf{8}$ or $\mathbf{10}$) we have for the normalization N in the presence of perturbations transforming as irreducible representations $\mathbf{Z}$ of $SU(3)$:

$$\delta N(\mathbf{B},b) = \sum_{\mathbf{Z}} Y_{\mathbf{Z}}(\mathbf{B},b) \begin{pmatrix} \mathbf{B} & \mathbf{Z} & | & \mathbf{B} \\ b & 0 & | & b \end{pmatrix}, \quad \text{(A7)}$$

where

$$\begin{pmatrix} \mathbf{B} & \mathbf{Z} & | & \mathbf{B} \\ b & 0 & | & b \end{pmatrix}$$

is an $SU(3)$ isoscalar factor and the sum goes over all allowed multiplets. The perturbation is constrained to conserve hypercharge isospin and so transforms as the $Y=I=0$ member of $\mathbf{Z}$. The values of $\mathbf{Z}$ are **1**, **8**, **27** for the baryon octet, and **1**, **8**, **27**, **64** for the decuplet. For instance, the nucleon normalization is

$$N(\text{nucleon}) = Y_1 - (\sqrt{5}/10)Y_{8_1} + \tfrac{1}{2}Y_{8_2} + (1/3\sqrt{5})Y_{27}$$

whereas for the Ω^-,

$$N(\Omega^-) = Y_1 - (Y_8/\sqrt{2}) + (Y_{27}/\sqrt{7}) - (1/2\sqrt{14})Y_{64}. \quad \text{(A8)}$$

In the $SU(3)$-invariant limit, the particles have the same normalization. Because of the scale invariance of bootstrap theory, only normalization shifts relative to one of the particles has meaning, so all shifts were measured relative to that of the nucleon. The following sum rules were generated:

8 splitting:

$$3N(\Lambda-N)+N(\Sigma-N)=2N(\Xi-N),$$
$$N(\Omega-N)-N(\Xi^*-N)=N(\Xi^*-N)-N(Y_1^*-N)=N(Y_1^*-N)-N(N^*-N),$$
(equal-spacing rule).

27 splitting:

$$N(\Lambda-N)=3N(\Sigma-N),$$
$$N(\Xi-N)=0, \quad \text{(A9)}$$
$$N(N^*-N)-N(\Xi^*-N)=N(\Omega-N)-N(N^*-N).$$

64 splitting:

$$N(Y_1^*-N)=N(\Omega-N),$$
$$N(\Xi^*-N)-N(\Omega-N)=2N(N^*-N)-2N(Y_1^*-N).$$