A Model of Baryons Made of Quarks with Hidden Spin

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The quark model of baryons is investigated in an attempt to combine its excellent agreement with experiment with a consistent foundation. It is found that effective Bose statistics for the spin states of identical quarks (quark statistics) is sufficient to explain the properties of baryons *as if* the three-quark wave functions were in the **56** representation of $SU(6)$. A new internal degree of freedom is suggested as a mechanism for achieving quark statistics with fermion quarks. This mechanism also can produce quark saturation (at 3) and allows for the possibility of light quarks ($\sim\frac{1}{3}M_p$) that have a large effective mass if removed from baryons. Baryon magnetic moments and mass differences are analyzed in the quark model with the primary purpose of determining the required properties of quarks. Sum rules that are independent of quark moments in a static s-wave model are derived for baryon magnetic moments. Relativistic effects on magnetic moments are considered and are shown to be a likely mechanism for the deviation of the proton-neutron magnetic-moment ratio from the static prediction of $-\frac{3}{2}$. Higher-orbital states are also considered and a general formula is derived for orbital contributions to baryon magnetic moments.

I. INTRODUCTION

EVER since the introduction of quarks by Gell-Mann[1] and by Zweig,[2] theoreticians have been tempted to take them more and more seriously. The temptress has been the remarkable agreement with experiment almost wherever quarks have been taken seriously, if not too seriously. Yet the snowballing success of quark models[3] has been in the face of contradictions and inconsistencies (not to mention that quarks have never been seen) that would have crushed lesser theories.

The main, well-known infirmities of the quark model are the following: (1) Most predictions of the quark model really make sense only for light quarks with moderate binding, yet quarks have never been seen; (2) quarks seem to want Bose statistics for fermions; and (3) quark binding has to stop at three for baryons and two for quark-antiquark mesons. In this paper, we study the quark model of baryons and attempt to resolve the contradictions in a way that does not violate established principles or common sense. We also attempt, in each encounter with experiment, to answer the question, what are the minimal assumptions of symmetry, beyond the existence of quarks, required to explain the results? The answer is, usually, none.

The usual procedure in quark- or group-based models has been to start with complete $SU(6)$ symmetry and then use the quark model to study "symmetry-breaking perturbations." This does not seem to be the appropriate order in which to do things, considering the almost 2:1 mass ratio in the baryon "multiplet" and the 7:1 mass ratio in the meson multiplet. As an alternative approach, we use a baryon model based on three distinct (nonidentical) quarks which are the usual quarks, but

assume no symmetry until it is necessary to produce an actually observed symmetry of the baryons. It turns out that we do not require any $SU(6)$- or $SU(3)$-symmetry assumptions for the quarks or their interactions. This approach is somewhat similar to that of Rubinstein, Scheck, and Socolow.[4] They use nonsymmetric quark interactions, but their baryon wave functions are assumed to be fully symmetric in the quark indices, i.e., are assigned to the **56** representation of $SU(6)$. It turns out, however, that the weaker starting assumption of quark statistics (which we define to mean symmetrization of the spin wave function of only identical quarks) is sufficient to derive all the properties of the baryons *as if* they were in the **56** representation of $SU(6)$.

In Sec. II, we discuss physical methods of achieving quark statistics and are led to the introduction of a new internal degree of freedom as an effective way of resolving the many conceptual and practical problems involved. In Sec. III, we introduce baryon wave functions based on quark statistics. In Sec. IV, we discuss the connection between charge independence and isotopic spin for the quarks and for the baryons. Sections V and VI review the quark-model predictions for the magnetic moments and mass differences of baryons. In Sec. VII, we summarize the major new points discussed in this paper. In the Appendix, we consider all possible orbital contributions to the magnetic moments of spin-$\frac{1}{2}$ baryons.

II. QUARK STATISTICS

We consider a model starting with a $\mathcal{P}$ quark of charge $+\frac{2}{3}$ (in units of the positron charge), an $\mathfrak{N}$ quark of charge $-\frac{1}{3}$, and a λ quark of charge $-\frac{1}{3}$. All three quarks have spin $\frac{1}{2}$ and are distinct, nonidentical particles, as can be manifested by their having different masses $m_{\mathcal{P}} \neq m_{\mathfrak{N}} \neq m_\lambda$ and different quark-quark inter-

[1] M. Gell-Mann, Phys. Letters 8, 214 (1964).

[2] G. Zweig, CERN Report Nos. TH. 401, TH. 412, 1964 (unpublished).

[3] R. H. Dalitz [in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, Cal., 1967), pp. 215–236] has an extensive quark bibliography.

[4] H. R. Rubinstein, Phys. Rev. Letters 17, 41 (1966); H. R. Rubinstein, F. Scheck, and R. H. Socolow, Phys. Rev. 154, 1608 (1967).

actions. The physical baryons are made up out of these quarks, held together by attractive two-body q-q interactions, where the spin wave function must be symmetric for identical quarks (e.g., $\mathcal{P}\mathcal{P}$ must be in a triplet spin state but $\mathcal{P}\mathfrak{N}$ can be either a triplet or singlet or mixed state). The quark statistics correspond to effective Bose statistics for identical quarks (while the baryons obey the usual Fermi statistics). Quark statistics must be explained by some mechanism that, at the same time, should also solve the "saturation problem" (exclusion of low-lying states of more than three quarks). If possible, we would like to do this without three- or four-body forces. These provide simple solutions to the saturation problem but diminish the significance of the many interesting results that follow from the assumption of the dominance of two-body quark forces.

We discuss several means of achieving quark statistics and saturation. One method is the use of parastatistics[5] and s-wave orbitals for the three quarks making up a baryon. This simply defines the problem away. The first three quarks obey effective Bose statistics in s states. Any further quarks must be in higher orbital states and, presumably, lie much higher. The saturation requirement would be harder to satisfy for distinct quarks. Parastatistics would not forbid four (or more) quark states like ($\mathcal{P}\mathfrak{N}\lambda\lambda$) all in s states. Thus the usefulness of parastatistics seems to be linked to considering all quarks as identical, which puts in more symmetry at the start than we shall see that we need.

The other general method is the use of Fermi statistics for the quarks with an additional degree of freedom in which the wave function is antisymmetric, so that the spin wave function is required to be symmetric for the over-all Fermi statistics. We mention two types of such additional degrees of freedom. The first, which is well known, is the use of the orbital degree of freedom. If all quark pairs are in odd orbital-angular-momentum states, then they must be symmetric with respect to spin interchange of identical quarks to achieve Fermi statistics. There are difficulties with this "orbital" model, which have been discussed by Dalitz[3] and by Mitra and Majumdar.[6] The basic difficulty with respect to its use with a model based on two-body quark forces is that the lowest-lying orbital state would be expected to be the completely antisymmetric state with $l=L=1$ with $l+L=1$. (We use the Dalitz notation that l is the orbital angular momentum of two quarks in their barycentric system and L is the orbital angular momentum of the third quark with respect to the c.m. of the other two quarks.) This would require, effectively, three-body spin-orbit forces that would conspire to combine the spin-$\frac{1}{2}$ and -$\frac{3}{2}$, three-quark combination with $l+L=1$, so that the state with $l+L+S=S$ would be bound and other, unwanted combinations would lie

[5] O. W. Greenberg, Phys. Rev. Letters **13**, 598 (1964).
[6] A. N. Mitra and Rabi Majumdar, Phys. Rev. **150**, 1194 (1966).

much higher. The other way out would be to construct the completely antisymmetric $l+L=0$ state,[3] but this requires a combination of $l(L)$ of both 1 and 3 and would lie higher than $l+L=1$ with $l=L=1$ if there were only two-body forces.

Another weakness of the orbital model is that it requires the thus far ad hoc hypothesis that the spin-spin (or space-exchange[3]) forces are of such a nature that they are weak or repulsive when identical quarks are in the singlet state. That is, the quark statistics are dynamical and not forced. This is required to remove low-lying, unobserved s states in the orbital model. It would presumably remain for some theory of quark forces, say, due to meson exchange to produce the required spin-spin dependence.

The orbital model with $l+L=0$ leads to very few observable differences from the parastatistics model, unless detailed calculations involving radial wave functions are attempted (we do not attempt this). Mass formulas are the same and, since $l+L=0$, orbital contributions to magnetic moments cancel out. Because of the difficulty of reconciling the orbital model with a model based on the dominance of two-body quark forces, we prefer to think in terms of models in which quark statistics are achieved without using the spatial degree of freedom. For the same reason, we do not consider any of a class of models that could achieve quark statistics through strong attractive $SU(6)$-symmetric three-body forces [in the face of large $SU(6)$ violations for the two-body forces] and repulsive four-body forces to solve the saturation problem.

Another type of additional degree of freedom which we can use is one which is as yet hidden and has not yet had any physical consequences, other than increasing the number of states. This would be the case in atomic physics, for instance, if there were no spin-orbit interaction. In that case, there would be two electrons for every state, because the electron has spin $\frac{1}{2}$, with no other immediate consequences. For a quark model, the required number is 3 and the obvious candidate is spin 1. The model takes the following form: (1) All quarks have "hidden spin" $H=1$; (2) for some reason, the three-quark state with $H=0$ lies lowest and corresponds to the baryons. Since the $H=0$ state of three $H=1$ particles is completely antisymmetric, we get effective parastatistics for the three quarks in a baryon. There will be possible differences from true parastatistics which we note below.

We first must suggest "some reason" for the $H=0$, three-quark state to lie lowest. We list several different possibilities: (1) For some dynamical reason, the q-q interaction is attractive and strong in the $H=1$ state and weak or repulsive in the $H=0$ and 2 states. This is the same type of assumption for the quark-quark interaction as was required in the orbital model. Then, since the $H=0$ state of three quarks is pure $H=1$ for any pair of quarks, it will be the lowest-lying state. More than three quarks could not all be pure $H=1$ for

any pair, and this solves the saturation problem. This is just like Chew's original model for the ω meson composed of three $I=1$ pions.[7] It could also possibly lead to a stable diquark as proposed by Lichtenberg, Tassie, and Keleman,[8] with the possibility that the lowest-mass state of nonintegral charge might include $Q=\frac{4}{3}$ as originally suggested by Gell-Mann[1] and by de Swart.[9] The existence or nonexistence of this state is a blessing or drawback of this model as with most other quark models.

(2) There is something in nature that "abhors" $H\neq0$ for physically realizable states. This could take the form of a very large effective moment of inertia in H space or perhaps some long-range interaction that could be responsible for states of $H>0$ lying much higher. It also could just be a new physical principle that does not seem to be in conflict with known physical principles. This suggestion provides, in effect, the infinite barrier suggested by Gell-Mann[10] to prevent the physical appearance of free quarks. In this case, the quarks, having $H=1$, would not appear physically. Also, the mass of quarks *inside* physical particles need not be very large, since the large "mass" contribution due to their H spin would either be cancelled or shielded by the other quarks. This case does leave the possibility of a diquark having $H=0$, which, because of Fermi statistics, would make it look like the antisymmetric 15 representation of $SU(6)$. It is the only nonintegral charged particle that could be expected in this version of the hidden-spin model. There could be a saturation problem for this case, especially if the $H=0$ diquark were found to exist. Then many quark systems could be expected with total $H=0$ unless repulsive many-body forces provided saturation. One very attractive feature of the model is the possibility that the $H=0$, q-q force is weak or repulsive, so that there is no bound diquark. Then there need be no free particles of nonintegral charge, even though real quarks could exist inside baryons and mesons.

(3) We have been led to the combination of (1) and (2). The quark-quark force is attractive in the $H=1$ state and repulsive in the $H=0$ and 2 states, and physical states of $H\neq0$ either cannot exist or lie considerably higher. This would provide appropriate quark statistics in s states, solve the saturation problem, and quarks could have moderate mass inside baryons, while it would either be impossible to produce isolated particles of nonintegral charge or they would lie considerably higher than would be expected from their mass within baryons. In a q-$\bar{q}$ model of mesons, the q-$\bar{q}$ force would have to be attractive for $H=0$ and repulsive

[7] G. F. Chew, Phys. Rev. Letters 4, 142 (1960).
[8] D. B. Lichtenberg, L. J. Tassie, and P. J. Keleman, Phys. Rev. 167, 1535 (1968).
[9] J. J. de Swart, Phys. Rev. Letters 18, 618 (1967).
[10] M. Gell-Mann, in *Proceedings of the Thirteenth International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967), p. 5.

for $H=1$ to solve the saturation problem effectively for states like $(qq\bar{q})$.

The possibility of having moderately light $(\sim\frac{1}{3}M_p)$ quarks inside a baryon is an important feature of the hidden-spin model. Most of the achievements of quark models are difficult to understand if this is not the case. The mass formulas make more sense if the interactions required are not enormous. High-energy cross-section results require quark-additivity assumptions that seem based on weak or moderate binding, and magnetic-moment predictions, as we discuss later, really only make sense for quarks with $m_q\sim\frac{1}{3}M_p$.

We believe that fermion quarks with hidden spin offer the best foundation for a quark model of hadrons. However, most of the results of the following sections depend only on quark statistics, independently of how they are achieved.

III. BARYON WAVE FUNCTIONS

We now proceed using effective Bose statistics for the spin wave functions of identical quarks, pointing out differences between the various models for achieving this when they occur. The allowed three-quark states are

$$p=\mathcal{PPR}T,$$
$$n=\mathcal{RRP}T,$$
$$\Lambda=\mathcal{PR}\lambda S,$$
$$\Sigma^+=\mathcal{PP}\lambda T,$$
$$\Sigma^0=\mathcal{PR}\lambda T,$$
$$\Sigma^-=\mathcal{RR}\lambda T,$$
$$\Xi^0=\lambda\lambda\mathcal{P}T,$$
$$\Xi^-=\lambda\lambda\mathcal{R}T,$$

(1)

and

$$N^{*++}=\mathcal{PPP}X,$$
$$N^{*+}=\mathcal{PPR}X,$$
$$N^{*0}=\mathcal{RRP}X,$$
$$N^{*-}=\mathcal{RRR}X,$$
$$Y^{*+}=\mathcal{PP}\lambda X,$$
$$Y^{*0}=\mathcal{PR}\lambda X,$$
$$Y^{*-}=\mathcal{RR}\lambda X,$$
$$\Xi^{*0}=\lambda\lambda\mathcal{P}X,$$
$$\Xi^{*-}=\lambda\lambda\mathcal{R}X,$$
$$\Omega^-=\lambda\lambda\lambda X,$$

(2)

where the spin wave functions T, S, and X are given explicitly by

$$T=3^{-1/2}[2^{1/2}\uparrow\uparrow\downarrow-2^{-1/2}(\uparrow\downarrow+\downarrow\uparrow)\uparrow],$$
$$S=2^{-1/2}(\uparrow\downarrow-\downarrow\uparrow)\uparrow,$$

and

$$X=\uparrow\uparrow\uparrow.$$

The spin function T corresponds to the first two quarks being in a triplet state which is combined with the third quark to give total spin $\frac{1}{2}$. S corresponds to the first two quarks being in a singlet state, so that the total spin must be $\frac{1}{2}$. X is the completely symmetric spin-$\frac{3}{2}$ function and requires any two quarks to be in a triplet state. T, S, and X are representative spin functions corresponding to maximal z-axis projections. To get the other z-axis projections, the usual spin-lowering operations can be used.

Since the quarks are nonidentical particles, the order in which we put them in a wave function does not matter. This means that many of the gross results like mass differences and lowest-order magnetic moments will be the same as if we completely symmetrized with respect to quark symbol and spin projection simultaneously, which would give $SU(6)$-symmetric wave functions. Although the practical differences are small, we believe that there are fundamental distinctions between using nonsymmetric and $SU(6)$-symmetric wave functions. Although the quark order does not matter, once we have given the spin functions in a definite order the quark symbols of Eqs. (1) and (2) must keep the order in which they have been written. In the wave functions, the appropriate letter is used to denote the physical baryon, the letters $\mathcal{P}$, $\mathcal{N}$, and λ denoting their quark constituents. In most cases, there are at least two identical quarks, and the quark statistics discussed in Sec. II dictate the choice of spin function. This is not the case for Σ^0, Λ, and Y^{*0}, each of which is composed of $\mathcal{P}\mathcal{N}\lambda$ quarks. Given the $\mathcal{P}\mathcal{N}\lambda$ system, there are three possible ways to add up the quark spins, and we have chosen to do this as given in Eqs. (1) and (2). There is no question that $Y^{*0}=\mathcal{P}\mathcal{N}\lambda X$, because it has spin $\frac{3}{2}$. Which quark combination corresponds to Σ^0 and which to Λ is ultimately an experimental question which will be discussed later.

At this point we note, of course, that the allowed three-quark wave functions comprise 18 orthogonal states corresponding to the well-known baryon spin-$\frac{1}{2}$ octet and spin-$\frac{3}{2}$ decuplet. This has been arrived at with no particular reference to $SU(3)$ or $SU(6)$. If they have crept in, it is because the quarks happen to form their fundamental representations, but this need not imply any $SU(3)$ or $SU(6)$ symmetry for the system. The particular charges of the quarks are chosen to produce the baryon charge spectrum and, also, happen to put in some $SU(3)$ symmetry.

Given quark statistics, the baryon wave functions of Eqs. (1) and (2) are unique. This means that we can calculate strong and medium-strong effects to any order using these internal and spin wave functions. There are also "radial" wave functions which are to be understood in Eqs. (1) and (2), and these will change from order to order. However, if we limit ourselves to results which do not depend on the details of radial wave functions, these results will hold to all orders, and not just in lowest-order perturbation theory, as long as

quark statistics hold for really identical quarks. This explains why many perturbationlike formulas can be remarkably well satisfied, even though, as "perturbations," they are hardly small. We emphasize again that, since we have introduced no symmetry, none is being "broken," and we do not have to consider any parameter of smallness, beyond which something like quark statistics would be expected to break down.

If the quark-quark interaction were described by a Hamiltonian with only two-body interactions, then the wave functions (with suitable radial parts) of Eqs. (1) and (2) can be shown to be eigenstates of that Hamiltonian, except that the states that we have labelled Λ and Σ^0 would be slightly mixed by the electromagnetic difference between the $\mathcal{P}$ and $\mathcal{N}$ interactions. The fact that these states could be considered eigenstates of a strong two-body Hamiltonian reinforces the preceding remarks about their nonperturbative character. The electromagnetic mixing of Λ and Σ^0 would affect mass formulas to second order (and be negligible) but would affect the Λ magnetic moment to first order in the electromagnetic interaction.

IV. CHARGE INDEPENDENCE

It is well known that the observed degeneracy structure (without electromagnetic effects) of the baryons can be inferred from charge independence and elegantly described by the isotopic-spin formalism. We trace how we are led to this in a quark model. Going down the list of wave functions in Eqs. (1) and (2), we first observe that the p-n degeneracy would result if the $\mathcal{P}$ and $\mathcal{N}$ quarks had equal masses and the quark forces were charge-symmetric, that is, if the $\mathcal{N}$-$\mathcal{N}$ force were the same as the $\mathcal{P}$-$\mathcal{P}$ force. The Σ^+, Σ^0, Σ^- degeneracy requires, in addition, that the $\mathcal{P}$-$\mathcal{N}$ force in the triplet spin state also equal the $\mathcal{P}$-$\mathcal{P}$ and $\mathcal{N}$-$\mathcal{N}$ forces. The Σ degeneracy also requires that the $\mathcal{P}$-λ interaction be the same as the $\mathcal{N}$-λ interaction for both the singlet and triplet spin states when the spin wave functions are recoupled in terms of the second and third (or first and third) instead of the first two quarks. The other degeneracies of the octet and decuplet then follow.

Given this experimental deduction of the charge independence of quark forces along with the degeneracy of the $\mathcal{P}$ and $\mathcal{N}$ quarks, we could now introduce the isotopic-spin formalism. This would involve forming isotopic spin eigenstates of the nucleon-quark systems and extending quark statistics to the $\mathcal{P}$-$\mathcal{N}$ quark combination, now considered as different I_3 states of the same quark. This is not necessary, however, for most of the baryon properties discussed in this paper, and we continue to use the simple wave functions of Eqs. (1) and (2), treating the $\mathcal{P}$ and $\mathcal{N}$ quarks as distinct particles. The Λ and Σ^0 baryons are already distinguished by the quark-spin combinations, and we do not require an isotopic-spin distinction between them. In the following

sections, we do not assume charge independence or use the isotopic-spin formalism, except where we explicitly so state. We do not mean to imply that we do not believe in isotopic spin, but just that we shall use it only where it is useful.

In a discussion of meson properties in the type of quark model discussed here, the use of the isotopic-spin formalism does seem more necessary. Such a model of the mesons as q-$\bar{q}$ bound states leads to the usual nonets of pseudoscalar and vector mesons. Then, in order to distinguish the π^0 and the η (or η') mesons, isotopic spin, or something similar, would have to be used, since they have the same quark-spin states. A similar situation would hold for the ρ^0 and ω (or φ) mesons. These would be the only low-lying states for which isotopic spin would be necessary to distinguish the states.

V. MAGNETIC MOMENTS

We discuss magnetic moments first because this can be done, at first, making no assumptions about quark interactions. The baryon magnetic moments in a quark model are given as the sum of the individual quark moments and any orbital contribution. In the models discussed here, there is either no orbital angular momentum or its magnetic-moment contribution cancels out, as when $l+L=0$. We first discuss the implications of no orbital contribution to the magnetic moments, then in the Appendix, look at possible orbital effects.

We start with three different quark magnetic moments $\mu_{\mathcal{O}}$, $\mu_{\mathcal{N}}$, and μ_λ. Then the wave functions of Eqs. (1) and (2) lead to

$$\mu_p = \tfrac{1}{3}(4\mu_{\mathcal{O}}-\mu_{\mathcal{N}}),$$
$$\mu_n = \tfrac{1}{3}(4\mu_{\mathcal{N}}-\mu_{\mathcal{O}}),$$
$$\mu_\Lambda = \mu_\lambda,$$
$$\mu_{\Sigma^+} = \tfrac{1}{3}(4\mu_{\mathcal{O}}-\mu_\lambda),$$
$$\mu_{\Sigma^0} = \tfrac{1}{3}(2\mu_{\mathcal{O}}+2\mu_{\mathcal{N}}-\mu_\lambda),$$
$$\mu_{\Sigma^-} = \tfrac{1}{3}(4\mu_{\mathcal{N}}-\mu_\lambda),$$
$$\mu_{\Xi^0} = \tfrac{1}{3}(4\mu_\lambda-\mu_{\mathcal{O}}),$$
$$\mu_{\Xi^-} = \tfrac{1}{3}(4\mu_\lambda-\mu_{\mathcal{N}}),$$
$$\mu_{\Omega^-} = 3\mu_\lambda \tag{3}$$

for the "stable" baryons, and

$$\mu(N^{*+}p) = \tfrac{2}{3}\sqrt{2}\,(\mu_{\mathcal{O}}-\mu_{\mathcal{N}}),$$
$$\mu(Y^{*+},\Sigma^+) = \tfrac{2}{3}\sqrt{2}\,(\mu_{\mathcal{O}}-\mu_\lambda),$$
$$\mu(\Sigma^0\Lambda) = \tfrac{1}{3}\sqrt{3}\,(\mu_{\mathcal{N}}-\mu_{\mathcal{O}}) \tag{4}$$

for representative transition moments. Equation (3) gives nine baryon moments in terms of three quark moments, implying six relations among the baryon moments. These can be taken as

$$\mu_{\Sigma^+} = (1/15)(16\mu_p+4\mu_n-5\mu_\Lambda) = 2.71\pm0.05, \tag{5a}$$

$$\mu_{\Sigma^-} = (1/15)(4\mu_p+16\mu_n-5\mu_\Lambda) = -1.10\pm0.05, \tag{5b}$$

$$\mu_{\Sigma^0} = \tfrac{1}{3}(2\mu_p+2\mu_n-\mu_\Lambda) = 0.83\pm0.05, \tag{5c}$$

$$\mu_{\Xi^0} = (1/15)(-4\mu_p-\mu_n+20\mu_\Lambda) = -1.59\pm0.20, \tag{5d}$$

$$\mu_{\Xi^-} = (1/15)(-\mu_p-4\mu_n+20\mu_\Lambda) = -0.65\pm0.20, \tag{5e}$$

$$\mu_{\Omega^-} = 3\mu_\Lambda = -2.19\pm0.48. \tag{5f}$$

The transition moments satisfy

$$\mu(N^{*+}p) = -2(\sqrt{\tfrac{2}{3}})\mu(\Sigma^0\Lambda) = \tfrac{2}{3}\sqrt{2}\,(\mu_p-\mu_n). \tag{5g}$$

All these magnetic-moment formulas follow directly from the procedure of Rubinstein, Scheck, and Socolow,[4] except that they use the equality $\mu_{\mathcal{O}}=-2\mu_{\mathcal{N}}$ from the start. All the formulas in Eq. (5) are independent of quark moments. We have written Eq. (5) in the form of quark-model predictions in terms of the fairly accurately known p, n, and Λ moments, for which we have taken[11] (in units of the proton magneton $=e/2M_p$)

$$\mu_p = 2.79, \quad \mu_n = -1.91, \quad \mu_\Lambda = -0.73\pm0.16. \tag{6}$$

The Σ^+ magnetic moment has been measured in three experiments,[12] which give, individually, 1.5 ± 1.1, 3.5 ± 1.5, and 3.0 ± 1.2. These results are in agreement with the prediction, but more accuracy is required for a real test. Measurements of μ_{Ξ^-} and μ_{Ξ^0} may also be available soon. The numerical predictions of Eq. (5) do not differ appreciably from quark-model predictions based on $\mu_{\mathcal{O}}=-2\mu_{\mathcal{N}}$ (or $\mu_p=-\tfrac{3}{2}\mu_n$), because that assumption is close to the experimental result. An accurate, definitive determination of μ_{Σ^+} (or μ_{Ξ^-} or μ_{Ξ^0}) is important because Eq. (5) is independent of quark moments and is a prerequisite for believing the better-known quark magnetic-moment predictions which depend on assumptions about quark moments.

We now look at the well-known magnetic-moment ratios, for which the experimental values are

$$-\mu_p/\mu_n = 1.46, \tag{7a}$$

$$\mu_\Lambda/\mu_n = 0.38\pm0.08. \tag{7b}$$

It has been widely noted that $-\mu_p/\mu_n=1.5$ would follow from the assumption that $\mu_{\mathcal{O}}/\mu_{\mathcal{N}}=-2$ or that (at least for these two quarks) the quark moments are proportional to their charges. This is, in fact, a surprising assumption to make for a strongly interacting particle (even if "elementary"), where anomalous contributions to magnetic moments would be expected to be large. The quark charges can retain neat $(2:-1:-1)$ ratios under strong interactions because the charge is

[11] A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, P. Soding, and C. G. Wohl, Rev. Mod. Phys. **40**, 77 (1968).

[12] V. Cook, T. Ewart, G. Masek, R. Orr, and E. Platner, Phys. Rev. Letters **17**, 223 (1966), (1.5±1.1); C. R. Sullivan, A. D. McInturff, D. Kotelchuck, and C. E. Roos, *ibid.* **18**, 1163 (1967), (3.0±1.2); D. Kotelchuck, E. R. Goza, C. R. Sullivan, and C. E. Roos, *ibid.* **18**, 1166 (1967), (3.5±1.5).

related to a conserved vector current, but this mechanism is not available for the magnetic moments.

The assumption that the $\mathcal{P}$, $\mathfrak{N}$ magnetic moments are proportional to their charges only becomes more than a coincidence if, for some reason, the strong anomalous contributions vanish for quarks inside a baryon. There is a mechanism for this provided by the saturation property already required for quark models. Quark-meson intermediate states, which might be expected to be the heaviest contributors to quark anomalous moments, would lead to $\bar{q}qqqq$ states which should be forbidden inside baryons by the saturation requirement. In this way, quark moments inside baryons would not have strong anomalous contributions. It would also mean that isolated quarks (in models that permit them) would not have the magnetic moments expected from their contributions to baryon moments. This would solve the anomalous-moment problem for the quarks but would require the Dirac moment for each quark. Equations (3) and (6) can be solved for the "experimental" quark moments, giving

$$\mu_{\mathcal{P}}=1.85, \quad \mu_{\mathfrak{N}}=-0.97, \quad \mu_{\lambda}=-0.73\pm0.16. \quad (8)$$

If these are assumed to be Dirac moments, $\mu_i=q_i/2m_i$, this leads to the quark-mass predictions

$$m_{\mathcal{P}}=0.361M_p, \quad m_{\mathfrak{N}}=0.344M_p,$$
$$m_{\lambda}=(0.46\pm0.10)M_p. \quad (9)$$

These might be considered reasonable quark masses, except that quarks of this mass have never been seen. This would pose no problem in the hidden-spin model for which the effective mass[13] of an isolated quark could be much higher than its mass inside a baryon. Equation (9) also indicates that only the nucleons would be lighter than their quark constituents for the above quark masses. This, too, would be permissible in the hidden-spin model, since the effective binding energy would depend on the high effective mass of isolated quarks. Also, with quarks of this mass, the many high-energy cross-section predictions of independent quark models[14] would be more reasonable, since the quark binding would not be excessively strong.

The light apparent mass indicated by the Dirac moments of the quarks could also arise from relativistic effects for ultraheavy quarks if the very strong attractive interaction then required transformed like a scalar. We can indicate how this could happen without doing a detailed calculation by considering what the Dirac moment would be for a Dirac quark of mass m in a very deep potential with an external magnetic field **H**. Then, eliminating the small components from the Dirac equation leads to

$$(E-m-V^S-V^V)\psi=(E+m+V^S-V^V)^{-1}$$
$$\times(P^2+q\boldsymbol{\sigma}\cdot\mathbf{H})\psi, \quad (10)$$

where ψ is a two-component spinor (the large components), V^S is the scalar component of the potential, and V^V is the part that transforms like the fourth component of a four-vector. We have kept only relevant terms and consider the case for which the kinetic energy is small compared with the total energy, so that, in that sense, the motion is nonrelativistic. This is the usual nonrelativistic quark model for very heavy quarks and requires

$$E-m-V^S-V^V\ll E. \quad (11)$$

If the potential transforms like a four-vector ($V^S=0$), this leads to an effective quark moment

$$\mu_q\approx q/2m, \quad (12)$$

which is the usual result. However, if the potential transforms like a scalar ($V^V=0$), we get

$$\mu_q\approx q/2E\approx q/\tfrac{2}{3}M, \quad (13)$$

since $E\approx\tfrac{1}{3}M$ for a baryon of mass M.

Thus a model with very heavy quarks could lead to the experimental quark moments as Dirac moments if the very deep attractive potential required in such a model were predominantly a scalar.[15] In this sense, the effect of H spin could be interpreted as resulting in such an effective scalar potential. A scalar potential also has the nice feature of having the same sign (attractive) for q-q and q-$\bar{q}$ which is required for the major interaction in a very heavy quark model. On the other hand, a sizeable vector interaction would also be required to make the mesons so much lighter than the baryons.

The preceding discussion also implies that, with quarks of relatively light real or apparent masses, relativistic effects could affect the magnetic moments to an appreciable degree. Without attempting a rigorous

[13] This use of "effective mass" reverses the usual terminology (say, in the theory of metals) because we take the point of view that quarks inside of hadrons are free of one-body H-spin interactions, while quarks outside of hadrons would have effective one-body H-spin interactions which could be interpreted as an effective-mass contribution.

[14] E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma Redaktsiya 2, 105 (1965) [English transl.: Soviet Phys.—JETP Letters 2, 65 (1965)]; H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966); J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento 42A, 711 (1966); H. J. Lipkin, Phys. Rev. Letters 16, 952 (1966); J. J. J. Kokkedee, ibid. 22, 88 (1966); C. A. Levinson, N. S. Wall, and H. J. Lipkin, ibid. 17, 1122 (1966); J. J. J. Kokkedee and L. Van Hove, Nucl. Phys. B1, 169 (1967). The above references assume additivity of quark amplitudes, which would be questionable for tightly bound quarks. However, see also Victor Franco, Phys. Rev. Letters 18, 1159 (1967), who suggests that the cross-section results might not require additivity of quark amplitudes.

[15] The opposite effects of vector and scalar potentials on magnetic moments has been observed in connection with relativistic corrections to the deuteron magnetic moment by G. Breit, Phys. Rev. 71, 400 (1947); R. G. Sachs, ibid. 72, 91 (1947); H. Primakoff, ibid. 72, 118 (1947). This effect has been noted for quark models by N. N. Bogolubov, B. Striminski, and A. Tavkhelidze, Report Nos. JINR D-1968, Dubna, 1965 (unpublished); JINR D-2015, Dubna, 1965 (unpublished); JINR P-2141, Dubna, 1965 (unpublished); H. J. Lipkin and A. Tavkhelidze, Phys. Letters 17, 331 (1965).

three-body calculation, we use the suggestion from Eq. (10) that the apparent mass appropriate to the Dirac moment for the quark is

$$m^a = \tfrac{1}{2}(E + m + \bar{V}^S - \bar{V}^V), \tag{14}$$

where $\bar{V}$ represents an average value for the potential.

We now consider a model with equal-mass $\mathcal{P}$ and $\mathfrak{N}$ quarks, but for which relativistic effects lead to two different apparent masses ($m_0{}^a$ and $m_1{}^a$), depending on whether the quark is in a state of total spin $S = 0$ or 1 with another quark. That is, we are considering only two-body interactions, as will be discussed in Sec. VI. These assumptions then lead to

$$\mu_p = (e/72)(27/m_1{}^a + 9/m_0{}^a),$$
$$\mu_n = -(e/72)(15/m_1{}^a + 9/m_0{}^a). \tag{15}$$

From the experimental p and n moments we can determine the apparent masses

$$m_1{}^a = 0.410 M_p = 385 \text{ MeV},$$
$$m_0{}^a = 0.309 M_p = 290 \text{ MeV}. \tag{16}$$

The difference of the apparent masses is given by

$$m_1{}^a - m_0{}^a = 95 \text{ MeV} = \tfrac{1}{2}[(E_1 - E_0)$$
$$+ (\bar{V}_1{}^S - \bar{V}_0{}^S) - (\bar{V}_1{}^V - \bar{V}_0{}^V)]. \tag{17}$$

In Sec. VI [Eq. (27)], we relate the difference $E_1 - E_0$ to the N^*-n mass difference and find $E_1 - E_0 = 197$ MeV. It would normally be expected that

$$\bar{V}_1 - \bar{V}_0 > E_1 - E_0, \tag{18}$$

and this requires a combination of scalar and vector, spin-dependent potentials with

$$(\bar{V}_1{}^S - \bar{V}_0{}^S) - (\bar{V}_1{}^V - \bar{V}_0{}^V) = -7 \text{ MeV}. \tag{19}$$

This relativistic effect would thus be able to account for the experimental μ_p/μ_n ratio with equal mass, Dirac $\mathcal{P}$ and $\mathfrak{N}$ quarks, provided that a combination of scalar and vector potentials contributed to the spin dependence of the q-q interactions. Note that, because of the form of Eq. (15), a relatively large relativistic effect (15% in the apparent masses) leads to a relatively small change (3%) in the proton-neutron magnetic-moment ratio. This fact helps to resolve the puzzle of why the nonrelativistic quark model gives such a good μ_p/μ_n ratio.

VI. BARYON MASSES

Rubinstein, Scheck, and Socolow[4] have derived sum rules for baryon mass differences in the quark model with two-body interactions. The assumption of two-body interactions leads to the general mass formula

$$M = \Sigma(m_i + D_{ij}{}^S), \tag{20}$$

where the sum is over the quark constituents of each baryon as determined by the wave function of Eqs. (1)

and (2), and $D_{ij}{}^S$ is the interaction energy of two quarks of type i and j in a state of total spin S. Recoupling of the spin states is required for the interaction of the third quark.

For completeness, we list their mass formulas here:

$$m_{\mathfrak{N}} - m_{\mathcal{P}} + D_{\mathfrak{N}\mathfrak{N}}{}^1 - D_{\mathcal{P}\mathcal{P}}{}^1 = n - p \quad (1.29)$$
$$= \Sigma^- - \Sigma^+ + \Xi^0 - \Xi^- \quad (1.6 \pm 0.7)$$
$$= N^{*0} - N^{*+}$$
$$= Y^{*-} - Y^{*+} + \Xi^{*0} - \Xi^{*-} \quad (0.9 \pm 3.8)$$
$$= \tfrac{1}{3}(N^{*-} - N^{*++}) \quad (2.6 \pm 2.3), \tag{21}$$

$$D_{\mathcal{P}\mathcal{P}}{}^1 + D_{\mathfrak{N}\mathfrak{N}}{}^1 - 2D_{\mathcal{P}\mathfrak{N}}{}^1 = \Sigma^+ + \Sigma^- - 2\Sigma^0 \quad (1.8 \pm 0.1)$$
$$= Y^{*+} + Y^{*-} - 2Y^{*0}$$
$$= N^{*++} + N^{*0} - 2N^{*+} \quad (2.1 \pm 0.9)$$
$$[N^{*+} \text{ mass from Eq. (21)}] \tag{22}$$

for the "electromagnetic" mass differences, and

$$m_\lambda - m_{\mathcal{P}} + D_{\lambda\lambda}{}^1 - D_{\mathcal{P}\mathcal{P}}{}^1 = \tfrac{1}{3}(\Omega^- - N^{*++}) \quad (145 \pm 1)$$
$$= \Xi^{*0} - Y^{*+} \quad (147 \pm 1)$$
$$= \Xi^0 - \Sigma^+ \quad (122 \pm 1), \tag{23}$$

$$2\bar{D}_{\lambda\mathfrak{N}}{}^1 - \bar{D}_{\mathfrak{N}\mathfrak{N}}{}^1 - D_{\lambda\lambda}{}^1 = \bar{Y}^* + \bar{\Xi}^* - \bar{N}^* - \Omega^- \quad (7 \pm 4)$$
$$= 3\Lambda + \bar{\Sigma} - 2\bar{N} - 2\bar{\Xi} \quad (25.6 \pm 0.8) \tag{24}$$

for the strong mass splittings. The bar over the symbols in Eq. (24) means they are to be averaged over all members of that isotopic-spin multiplet. This prescription is that derived by Rubinstein, Scheck, and Socolow from the mass operator, Eq. (20). These are all well-known formulas and their relation to other approaches has been discussed by Rubinstein et al. As they emphasize, they depend on no symmetry, not even charge independence. They are in quite good agreement with experiment (given in parentheses in MeV for each equation). The only small disagreement is in the last of Eqs. (23), which also accounts for the entire error in Eq. (24). That is, Eqs. (22)–(24) can be combined to give the equivalent sum rule

$$2\bar{\Xi}^* - \bar{N}^* - \Omega^- \quad (155 \pm 4) = 3\Lambda - 2\bar{N} - \bar{\Xi}$$
$$(150.6 \pm 0.5), \tag{25}$$

which is well satisfied. The agreement of all the other mass formulas is so remarkable that even the small disagreement of Eq. (23) is of interest. It can be attributed solely to the Σ mass, but its explanation seems to be outside the scope of a quark model with two-body interactions.

Our concern with the mass formulas is primarily to use them to study two-quark interactions and wave functions. Because of the existence of the sum rules, the mass differences are not all independent, and we can only find certain differences of interaction energies. From the small experimental value of Eq. (24) (corresponding to the well-known equal decuplet spacing and the Gell-Mann–Okubo mass formula, which are thus linked as pointed out by Rubinstein et al.), there is the

near equality

$$\bar{D}_{\lambda\mathfrak{N}}{}^1 \cong \tfrac{1}{2}(\bar{D}_{\mathfrak{N}\mathfrak{N}}{}^1 + D_{\lambda\lambda}{}^1) \qquad (26)$$

for the triplet interactions. Because Eq. (26) also holds for the quark-mass contributions to these two-quark energies, it is possible to make a variety of assumptions, ranging from $SU(3)$ symmetry for the quark masses to $SU(3)$ symmetry for the triplet interactions, to produce the decuplet equal spacing and the Gell-Mann–Okubo mass formula, and this has been done in many places. Because of the large decuplet spacing in Eq. (23), it is, of course, not possible to assume $SU(3)$ symmetry for both quark masses and their interactions. If we take the estimate of quark masses in Eq. (9) based on magnetic moments, then $m_\lambda - m_{\mathfrak{N}} \sim 100$ MeV, which would indicate [from Eq. (23)] that the triplet quark interaction-energy differences are rather small.

The singlet interaction energies do not appear in mass formulas, but single relations for them can be found from the N^*-N and Σ-Λ mass differences

$$D_{\mathcal{P}\mathfrak{N}}{}^1 - D_{\mathcal{P}\mathfrak{N}}{}^0 = \tfrac{2}{3}(N^{*0} - n) = 197, \qquad (27)$$

$$\bar{D}_{\lambda\mathfrak{N}}{}^1 - \bar{D}_{\lambda\mathfrak{N}}{}^0 = \tfrac{2}{3}(N^{*0} - n) - (\Sigma^0 - \Lambda) = 122. \qquad (28)$$

It can be seen that the singlet interaction energies are quite different from the triplet energies. If we use the masses suggested by the quark moments, then from, say, $N^{*++} = 3m_{\mathcal{P}} + 3D_{\mathcal{P}\mathcal{P}}{}^1 = 1236$, we can estimate that $D_{\mathcal{P}\mathcal{P}}{}^1 \sim 70$ MeV, and from this can find all the interaction energies (in MeV) (neglecting electromagnetic differences):

$$\bar{D}_{\mathcal{P}\mathcal{P}}{}^1 \sim 70,$$

$$D_{\mathcal{P}\mathfrak{N}}{}^0 \sim -130,$$

$$D_{\lambda\lambda}{}^1 \sim 110 \pm 100,$$

$$\bar{D}_{\mathcal{P}\lambda}{}^1 \sim 90 \pm 50,$$

$$\bar{D}_{\mathcal{P}\lambda}{}^0 \sim -30 \pm 50. \qquad (29)$$

These estimates indicate moderate, but quite nonsymmetric interactions. As noted earlier, the quark-moment estimate of the quark masses leads to most baryons being heavier than their quark constituents, the binding then coming in the H-spin model from the over-all effect of H spin.

The electromagnetic mass differences can be used as a probe of the quark radial wave functions if we make the assumption that the only difference between $\mathcal{P}$ and $\mathfrak{N}$ interaction energies is the electromagnetic energy due to the quark charges and magnetic moments. Then

$$(D_{\mathcal{P}\mathcal{P}}{}^S - D_{\mathfrak{N}\mathfrak{N}}{}^S) = (D_{\mathfrak{N}\mathfrak{N}}{}^S - D_{\mathcal{P}\mathfrak{N}}{}^S)$$
$$= \tfrac{1}{3}e^2\{\langle 1/r_{\mathfrak{N}}\rangle_S - (2\pi/3m^a s^2)$$
$$\times [2S(S+1) - 3]|\psi_S(0)|^2\}, \qquad (30)$$

where $\langle 1/r_{\mathfrak{N}}\rangle_S$ is the mean value of $1/r_{\mathfrak{N}}$ ($r_{\mathfrak{N}}$ being the separation distance between nucleon quarks) in the quark-quark state of spin S, $m^a s$ is the apparent quark mass as determined by Eq. (16), and $\psi_S(0)$ is the quark-

quark wave function for $r_{\mathfrak{N}} = 0$. The first term on the right-hand side of Eq. (30) is the Coulomb energy. The second term comes from a magnetic contact term, $-(8\pi/3)\mathbf{\mu}_1 \cdot \mathbf{\mu}_2 \delta(\mathbf{r})$, in the interaction energy of point dipoles. We have assumed an s-wave model. A somewhat similar term involving $\langle 1/r_{\mathfrak{N}}{}^3\rangle_S$ would result in an orbital model. We could take Eq. (30) as defining the parameters $\langle 1/r_{\mathfrak{N}}\rangle_S$ and $|\psi_S(0)|^2/(m_S{}^a)^2$ without necessarily relating them to physical parameters. In particular, $m_S{}^a$ need not necessarily refer to the physical quark mass but can be taken as a measure of the quark magnetic moment.

The first equality of Eq. (30) follows just from saying that the electromagnetic interactions are proportional to the products of the quark-quark charges. From this assumption and Eqs. (21) and (22) we can deduce the nucleon-quark mass difference

$$m_{\mathfrak{N}} - m_{\mathcal{P}} = n - p + \tfrac{1}{3}(\Sigma^+ + \Sigma^- - 2\Sigma^0)$$
$$= 1.9 \pm 0.1 \text{ MeV}, \qquad (31)$$

so that there still must be an $\mathfrak{N}$-$\mathcal{P}$ mass difference with $m_{\mathfrak{N}} > m_{\mathcal{P}}$. The quark model, then, correlates electromagnetic mass differences but does nothing to "explain" the n-p (or $\mathfrak{N}$-$\mathcal{P}$) mass difference.

Substituting the full content of Eq. (30) into Eq. (22), we find

$$e^2\langle 1/r_{\mathfrak{N}}\rangle_1 - [2\pi/3(m^a{}_1)^2]|\psi_1(0)|^2$$
$$= \Sigma^+ + \Sigma^- - 2\Sigma^0 = 1.8 \pm 0.2 \text{ MeV}. \qquad (32)$$

At this point, we consider possible relations between $\langle 1/r\rangle$ and $|\psi(0)|^2$, both of which depend mostly on the inner part of the wave function. We consider three different types of inner quark-quark potentials: repulsive hard core (outside the charge "radius" of a quark); smooth, for which we use the simple harmonic oscillator (SHO) potential to relate $\langle 1/r\rangle$ and $|\psi(0)|^2$; and strongly attractive, for which we use a Coulombic ($1/r$) potential. We can characterize these types of inner potentials by the relations

$$|\psi(0)|^2 = 0 \quad \text{(hard core)} \qquad (33a)$$

$$= \tfrac{1}{8}\langle 1/r\rangle^3 \quad \text{(SHO)} \qquad (33b)$$

$$= (1/\pi)\langle 1/r\rangle^3 \quad \text{(Coulombic)}. \qquad (33c)$$

The exact values of the coefficients of $\langle 1/r\rangle^3$ are unimportant, but they indicate the manner in which we would expect $|\psi(0)|^2$ and $\langle 1/r\rangle$ to be related. This makes it possible to estimate $\langle 1/r_{\mathfrak{N}}\rangle_1$ from Eq. (32), with the result

$$\langle 1/r_{\mathfrak{N}}\rangle_1 = 1/(0.8 \pm 0.1 \text{ F}) \quad \text{(hard core)} \qquad (34a)$$

$$= 1/(0.7 \pm 0.2 \text{ F})$$
$$\text{or } 1/(0.4 \pm 0.1 \text{ F}) \quad \text{(SHO)} \qquad (34b)$$

$$= \text{impossible} \quad \text{(Coulombic)}. \qquad (34c)$$

This last result occurs because there is too much cancellation between the electric and magnetic effects

and indicates that the quark-quark interaction cannot be so singular as $1/r$ if the "small" mass differences are electromagnetic in origin. In fact, as the magnetic interaction becomes larger, the two solutions of Eq. (34b) each approach 0.53 F as a limiting radius. For stronger magnetic interactions the over-all electromagnetic energy cannot be made large enough to account for the Σ mass splitting.

We can make the same electromagnetic assumptions for the λ quark interactions. Then we would have

$$D_{\mathfrak{N}\lambda}{}^S - D_{\mathcal{P}\lambda}{}^S = \tfrac{1}{3}e^2\{\langle 1/r_\lambda\rangle_S - (2\pi/3\bar{m}_{\mathfrak{N}}m_\lambda)$$
$$\times[2S(S+1)-3]|\psi_S(0)|^2\}, \quad (35)$$

where r_λ is the separation distance between the λ quark and the nucleon quark in the state S. In this case, we use the λ-quark mass as measured by the Λ magnetic moment and an average value for the nucleon-quark mass. The general mass formula, Eq. (10), then leads to

$$e^2\{\langle 1/r_\lambda\rangle_1 - (2\pi/3\bar{m}_{\mathfrak{N}}m_\lambda)|\psi_1(0)|^2\}$$
$$= \tfrac{3}{2}(m_{\mathcal{P}} - m_{\mathfrak{N}} + \Xi^{*-} - \Xi^{*0}) = 5.4 \pm 3.3 \text{ MeV}$$
$$= \tfrac{3}{2}(m_{\mathcal{P}} - m_{\mathfrak{N}} + p - n + Y^{*-} - Y^{*0})$$
$$= 3.9 \pm 3.9 \text{ MeV}, \quad (36)$$

$$e^2\{\langle 1/r_\lambda\rangle_0 + (2\pi/\bar{m}_{\mathfrak{N}}m_\lambda)|\psi_0(0)|^2\}$$
$$= \tfrac{1}{2}[m_{\mathcal{P}} - m_{\mathfrak{N}} + 4(\Xi^- - \Xi^0) + \Xi^{*0} - \Xi^{*-}] = 7.8 \pm 1.2$$
$$= \tfrac{1}{2}[3(p - n + m_{\mathcal{P}} - m_{\mathfrak{N}}) + 4(\Sigma^- - \Sigma^+)$$
$$+ (Y^{*-} - Y^{*0})] = 8.0 \pm 1.3. \quad (37)$$

The sum rules implied by Eqs. (36) and (37) are not independent of those already given in Eqs. (21) and (22) without the electromagnetic assumption. It turns out that the electromagnetic assumption for the small mass differences does not make it possible to derive any new sum rules. The accuracy of Eq. (36) is not sufficient to estimate $\langle 1/r_\lambda\rangle_1$, but we can use Eq. (37) and the estimates of $|\psi(0)|^2$ from Eq. (33) to find

$$\langle 1/r_\lambda\rangle_0 = 1/(0.18 \pm 0.03) \text{ F} \quad \text{(hard core)} \quad (38a)$$
$$= 1/(0.43 \pm 0.03) \text{ F} \quad \text{(SHO)} \quad (38b)$$
$$= 1/(0.53 \pm 0.07) \text{ F} \quad \text{(Coulombic)}. \quad (38c)$$

Although there is no reason for the λ and nucleon quarks to have the same radial wave functions (especially since there are mesons like the pion that the λ quark cannot exchange), we would not expect their "radii" to be anomalously different. The inference, then, from Eqs. (34) and (38) is that magnetic effects are important in the electromagnetic mass differences, that the quark-quark radial wave functions correspond to smooth inner potentials (neither singular like $1/r$ nor hard-core outside the quark charge radius), and that the average inverse quark-quark separation is of the order of $1/0.5$ F. We should point out that the average inverse radius $\langle 1/r\rangle$ is a different measure of the radial wave function than the average radius $\langle r\rangle$ (which is related to the binding energy) would be. The value of $\langle 1/r\rangle$

depends more sensitively on the inner part of the wave function, so that it relates more to details of the potential such as its range or the radius of a hard core. It also turns out that the estimate of "size" is always smaller from $\langle 1/r\rangle$ than from $\langle r\rangle$. Thus we see that the quarks should not be very close together to understand the electromagnetic mass differences.

One point should be emphasized here. Since the assumption that the small mass differences within an "isotopic multiplet" are electromagnetic in nature (with a small unexplained $m_{\mathcal{P}}\text{-}m_{\mathfrak{N}}$ difference) does not lead to any new mass relations, it is possible that these small mass differences could be unrelated to electromagnetic effects. None of the experimental sum rules of the quark model would be affected by dropping this electromagnetic assumption; however, the smallness of these mass differences would be a bit of a puzzle. The fact that the electromagnetic-energy differences (which would be expected for real quarks in any event) do give the right magnitude for the mass differences with reasonable inverse radii then suggests that the electromagnetic energy can be used as the complete (except for the small $m_{\mathcal{P}}\text{-}m_{\mathfrak{N}}$ difference) source of the small mass splittings. It also is significant that in the one case, Eq. (37), for which the electromagnetic assumption requires a positive sign (because the electric and magnetic contributions add for the singlet-spin case), the two experimental results are quite definitely positive.

VII. SUMMARY

In the interests of continuity and to make this paper more or less self-contained, we have included much that is not new. Here we list the major points that we think are new.

(1) The symmetrization of the spin wave functions of only identical quarks (quark statistics) is all that is required to produce baryon wave functions that act *as if* they were in the 56-dimensional representation of $SU(6)$ in determining the properties of baryons. This removes just about the last vestige of $SU(6)$- or $SU(3)$-symmetry assumptions from the quark model.

(2) Because of (1), the quark-model predictions for magnetic moments and mass differences (assuming only two-body interactions) are seen to be not perturbative, but true to all orders, as long as quark statistics (which is not a symmetry) holds. This explains why the "linear" predictions of the quark model can be so well satisfied.

(3) Quark statistics in a model without many-body forces can best be achieved with fermion quarks and an additional (as yet unseen) internal degree of freedom. This is similar to parastatistics, but has important differences which we have discussed.

(4) The particular choice of H spin as the internal degree of freedom, with the assumptions that quarks have $H=1$ but that physical $H=1$ states cannot exist (or must have very large masses), explains quark

statistics and saturation, and permits a quark model with light quarks and moderate binding.

(5) The sum rules given in Eq. (5) are independent of assumptions about quark moments and, if accurately tested, are prerequisites for a belief in the better-known quark-model magnetic-moment ratios.

(6) Relativistic effects on baryon magnetic moments have been investigated and shown to be a reasonable cause for the deviation of the proton-neutron magnetic-moment ratio from the static-quark-model prediction of $-\frac{3}{2}$.

(7) A general formula has been derived (in the Appendix) for orbital contributions to baryon magnetic moments.

The agreement of quark predictions with experiment could be described as spectacular, and yet virtually every prediction really makes sense only if quarks are not too heavy or if the binding is not too strong. Quark statistics and saturation also must be explained. Most quark-model papers have been written in the spirit, "Look at the wonderful results; so what if it does not make sense?" In this situation, we have come to the view that some sense must underlie all these results. For this reason, we propose the H-spin model, or a similar type of internal degree of freedom, as a mechanism that removes the contradictions from such a "successful" model.

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APPENDIX

We consider a quark model that gives the prediction μ_0 for a baryon magnetic moment in the absence of orbital contributions. This would be the total moment if there were no orbital angular momentum, or if $l_0+L_0=0$, where l_0 and L_0 are the Dalitz orbital angular momenta of the baryon state. There could be a mixture of several l_0 (and L_0) in the wave function. We define the "no orbital" μ_0 to be that gotten with all l_0 of the same parity, so that the quark statistics are not mixed up. For spin-$\frac{1}{2}$ baryons, the total spin of the three quarks is $S=\frac{1}{2}$ for the no-orbital case. Orbital effects on the spin-$\frac{1}{2}$ baryon magnetic moments could then come from the following six admixtures to the wave function:

(1) $\quad L=l, \quad l+L=1, \quad (-)^l=(-)^{l_0}, \quad S=\frac{1}{2};$

(2) $\quad L=l, \quad l+L=1, \quad (-)^l=(-)^{l_0}, \quad S=\frac{3}{2};$

(3) $\quad L=l, \quad l+L=2, \quad (-)^l=(-)^{l_0}, \quad S=\frac{3}{2};$

(4$\pm$) $\quad L=l\pm2, \quad l+L=2, \quad (-)^l=(-)^{l_0}, \quad S=\frac{3}{2};$

(5) $\quad L=l, \quad l+L=0, \quad (-)^l=-(-)^{l_0}, S=\frac{1}{2};$

(6) $\quad L=l, \quad l+L=1, \quad (-)^l=-(-)^{l_0}, S=\frac{1}{2}.$

The last two have reversed quark statistics. For the proton, neutron, charged sigmas, and cascade, reversed quark statistics allow only $S=\frac{1}{2}$, because the $\mathcal{P}\mathcal{P}$, $\mathfrak{N}\mathfrak{N}$, or $\lambda\lambda$ quarks must then be in singlet spin states. For the lamda and Σ^0, there already is the possibility of some mixing of the $\mathcal{P}$-$\mathfrak{N}$ singlet and triplet states, so that the orbitally reversed statistics give nothing new. Interference contributions are possible for most of the above states and also between (1) and the state

(0) $\quad L=l\neq0, \quad l+L=0, \quad (-)^l=(-)^{l_0}, \quad S=\frac{1}{2},$

which, by itself, would not change the magnetic moment. If the state (0) or (1) is a major component of the ground state, as in the orbital model, then the (0)-(1) or (1)-(2) interference could be large. If the ground state is predominantly $l_0=0$, then the interference terms will be no larger than any other terms. Experimentally, the success of the mass-formula predictions is an indication that the reversed statistics and $S=\frac{3}{2}$ states (2)–(6) are unimportant.

We take for the magnetic-moment operator[16]

$$\mathbf{\mu}=\mathbf{\mu}_S+\mathbf{\mu}_l+\mathbf{\mu}_L,$$

with

$$\mathbf{\mu}_S=\frac{\mu_S}{S}\mathbf{S}=\sum_{i=1}^{3}\mu_i\mathbf{\sigma}_i, \quad \mathbf{\mu}_l=\mu_l\mathbf{l}, \quad \mathbf{\mu}_L=\mu_L\mathbf{L}. \quad (A1)$$

The operator $\mathbf{\mu}_S$ is the total spin contribution to the baryon magnetic moment; μ_i is the magnetic moment of the ith quark and $\mathbf{\sigma}_i$ is its Pauli spin operator. The operator $\mathbf{\mu}_l$ is the orbital-magnetic-moment operator for the c.m. system of the first two quarks [in the wave functions of Eq. (1)], and $\mathbf{\mu}_L$ is the orbital-magnetic-moment operator for the c.m. system of the third quark and an effective diquark composed of the first two quarks. The spin-moment magnitude μ_S is normalized to maximum z projection of spin, but the orbital-moment magnitudes μ_l and μ_L are normalized to $m_z=1$. In most quark models, we would have

$$\langle lm_z|\mu_{l_z}|lm_z\rangle$$
$$=[q_1/2m_1^2+q_2/2m_2^2)(m_1m_2/(m_1+m_2)]m_z \quad (A2)$$
$$=\mu_l m_z$$

and

$$\langle LM_z|\mu_{L_z}|LM_z\rangle$$
$$=[q_d/2m_d^2+q_3/2m_3^2)(m_dm_3/(m_d+m_3)]M_z \quad (A3)$$
$$=\mu_L M_z,$$

where q_i and m_i are the charge and mass[17] of the ith

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[16] In general, there are other terms in the three-body magnetic-moment operator, but these can be shown not to contribute for the wave functions of Eq. (1) (conservation of isotopic spin would be required for this to apply to the Λ).

[17] Equations (A2) and (A3) are for the nonrelativistic, weak binding case. In the general case, the m_i would be apparent masses determined in a manner similar to that discussed in Sec. V for the relativistic corrections to the quark magnetic moments.

quark, and m_d and q_d are the total mass and charge, respectively, of the first two quarks (considered as a diquark). We note that the quantities μ_l and μ_L are normalized so that they do not depend on the magnitudes of l or L.

For the state $l+L=L_T$ with z projection M_T, the orbital contribution will be

$$\delta\mu = |\alpha_{lLL_TM_T}|^2 \langle lLL_TM_T|\mu_z|lLL_TM_T\rangle, \qquad (A4)$$

where $\alpha_{lLL_TM_T}$ is the coefficient of $|lLL_TM_T\rangle$ in the expansion of the total wave function, and $|lLL_TM_T\rangle$ is given by the usual Clebsch-Gordan addition

$$|lLL_TM_T\rangle = \sum_m C(lLL_T; m, M_T-m)$$
$$\times Y^m(\Omega_l) Y_L^{M_T-m}(\Omega_L). \qquad (A5)$$

Then the orbital contribution for this state is given by

$$\delta\mu = \tfrac{1}{2}|\alpha_{lLL_TM_T}|^2 M_T\{\mu_l+\mu_L+ (\mu_l-\mu_L)$$
$$\times [l(l+1)-L(L+1)]/[L_T(L_T+1)]\}. \qquad (A6)$$

To calculate noninterference orbital contributions, we now just have to use Eq. (A6) in adding L_T to S to get the final spin $\tfrac{1}{2}$.

The interference between cases (1) and (2) can be calculated just like the calculations of octet-decuplet transition moments without orbital corrections. The interference between cases (0) and (1), or (2) and (3), or (5) and (6) is given by

$$\delta\mu = 2(\mu_l-\mu_L)\ \mathrm{Re}(\alpha_{L_T}{}^*\alpha_{L_T+1})f(lL_TS), \qquad (A7)$$

where

$$f(lL_TS) = \sum_M C(L_T,S,\tfrac{1}{2}; M,\tfrac{1}{2}-M)C(L_T+1,S,\tfrac{1}{2}; M,\tfrac{1}{2}-M)\times\sum_m mC(l,l,L_T; m,M-m)C(l,l,L_T+1; m,M-m). \qquad (A8)$$

After some algebra, we can write the total orbital contribution

$$\delta\mu = \sum_l\{ -\tfrac{2}{3}\sqrt{3}(\mu_l-\mu_L)[l(l+1)/3]^{1/2}\ \mathrm{Re}(\alpha_0{}^*\alpha_1)+\tfrac{1}{3}(-\mu_{1/2}+\mu_l+\mu_L)|\alpha_1|^2-\tfrac{4}{3}\langle\chi_{1/2,1/2}|\mu_{S_z}|\chi_{3/2,1/2}\rangle\ \mathrm{Re}(\alpha_1{}^*\alpha_2)$$
$$+ (1/18)(10\mu_{3/2}-3\mu_l-3\mu_L)|\alpha_2|^2-[(2l+3)(2l-1)/3]^{1/2}(\mu_l-\mu_L)\ \mathrm{Re}(\alpha_2{}^*\alpha_3)+\tfrac{1}{2}(-\tfrac{2}{3}\mu_{3/2}+\mu_l+\mu_L)$$
$$\times (|\alpha_3|^2+|\alpha_{4+}|^2+|\alpha_{4-}|^2)+\tfrac{1}{6}(\mu_l-\mu_L)[(2l-1)|\alpha_{4+}|^2-(2l+3)|\alpha_{4-}|^2]+\mu_{1/2}'|\alpha_5|^2-\tfrac{2}{3}[l(l+1)]^{1/2}$$
$$\times (\mu_l-\mu_L)\ \mathrm{Re}(\alpha_5{}^*\alpha_6)+\tfrac{1}{3}(-\mu_{1/2}'+\mu_l+\mu_L)|\alpha_6|^2\}, \qquad (A9)$$

where α_i is the l-dependent coefficient for state (i) in the expansion of the total wave function, $\mu_{1/2}'$ is the reversed-quark-statistics spin moment, and $\langle\chi_{1/2,1/2}|\mu_{S_z}|\chi_{3/2,1/2}\rangle$ is the spin transition moment. Equation (A9) gives the orbital contribution for any quark model. If we assume a model with equal-mass quarks having Dirac moments, Eq. (A9) applied to the proton and neutron leads to (in units of $e/2m_q$)

$$\mu_p = 1+\tfrac{1}{9}\sum_l\{ -4[l(l+1)]^{1/2}\ \mathrm{Re}(\alpha_0{}^*\alpha_1)-10|\alpha_1|^2-8\sqrt{2}\ \mathrm{Re}(\alpha_1{}^*\alpha_2)-5|\alpha_2|^2-2[3(2l+3)(2l-1)]^{1/2}\ \mathrm{Re}(\alpha_2{}^*\alpha_3)-9|\alpha_3|^2$$
$$+ (2l-10)|\alpha_{4+}|^2-(2l+12)|\alpha_{4-}|^2-12|\alpha_5|^2-4[l(l+1)]^{1/2}\ \mathrm{Re}(\alpha_5{}^*\alpha_6)-6|\alpha_6|^2\}, \qquad (A10)$$

$$\mu_n = -\tfrac{2}{3}+\tfrac{1}{9}\sum_l\{ 4[l(l+1)]^{1/2}\ \mathrm{Re}(\alpha_0{}^*\alpha_1)+8|\alpha_1|^2+8\sqrt{2}\ \mathrm{Re}(\alpha_1{}^*\alpha_2)+6|\alpha_2|^2+2[3(2l+3)(2l-1)]^{1/2}\ \mathrm{Re}(\alpha_2{}^*\alpha_3)+6|\alpha_3|^2$$
$$- (2l-7)|\alpha_{4+}|^2+(2l+9)|\alpha_{4-}|^2+12|\alpha_5|^2+4[l(l+1)]^{1/2}\ \mathrm{Re}(\alpha_5{}^*\alpha_6)+4|\alpha_6|^2\}. \qquad (A11)$$

This model is of interest because it predicts the no-orbital ratio $\mu_p/\mu_n = -\tfrac{3}{2}$. If we make the additional assumption of charge symmetry, so that the α's are the same for n and p, we can use Eqs. (A10) and (A11) to vary the no-orbital ratio in either direction, depending on which orbital effects we emphasize, but there are too many parameters to say very much.[18] We can note that Eqs. (A10) and (A11) rule out the version of the orbital model with the completely antisymmetric state $l=L=1$, $l+L=1$. This corresponds to pure state (1) ($\alpha_1=1$), which would imply $\mu_n/\mu_p = -2$, with the wrong sign for each moment.

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[18] If we assume that $|\alpha_{4-}| \geq |\alpha_{4+}|$, which seems reasonable, then all noninterference orbital effects [and the (1)-(2) interference] act to increase the ratio $-\mu_p/\mu_n$. Since experimentally $-\mu_p/\mu_n = 1.46 < \tfrac{3}{2}$, this suggests that admixtures of higher orbital states are unimportant. Interference between states (0)-(1), (2)-(3), or (5)-(6) cannot be ruled out, however, and could change this conclusion.