Determination of Cabibbo Parameters from Baryon Decays

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A comparison is made of Cabibbo theory to the experimental data on semileptonic hyperon decays. With a confidence level of 89%, the results are that the Cabibbo angle θ is 0.206±0.009, the weak D/F ratio α is 0.662±0.018, and the strangeness-changing axial-vector renormalization ξ is 1.42±0.10.

I. INTRODUCTION

THE octet-current hypothesis, combined with the generalized universality principle for weak interactions, has been very successful in describing semileptonic weak decays.[1] The intrinsic parameter in this theory is the so-called Cabibbo angle θ, which defines the direction of the weak hadron current in 3-dimensional unitary space. Although there is only one angle parameter in the theory, its phenomenological value may change for various processes, because the $SU(3)$ symmetry-breaking interactions may renormalize it differently.

This undesirable situation is to a large extent alleviated by the Ademollo-Gatto theorem.[2] This theorem states that there is no renormalization of the strangeness-changing weak vector coupling constants to first order in $SU(3)$ summetry breaking. If one believes that the second- and higher-order $SU(3)$ symmetry-breaking effects are negligible, as indicated by all available evidence, one may reasonably assume that all vector transitions, strangeness-conserving as well as strangeness-changing, can be described by a single *phenomenological* Cabibbo angle θ_V, which is, incidentally, well approximated by the "bare" Cabibbo angle θ.

For the axial-vector current, however, there is no counterpart of the Ademollo-Gatto theorem. In view of the limited experimental information available, what one should do is adopt a realistic and economical parametrization. For example, one can assume[3,4] a single *phenomenological* axial-vector Cabibbo angle $\theta_A{}^{(B)}$ for all baryon decays, and another one, which we denote by $\theta_A{}^{(M)}$, for the pseudoscalar-meson decays. In fact, Brene *et al.*[4] have carried out an analysis of all the hadron-leptonic decay data under the further assumption

$$\theta_A{}^{(B)}=\theta_A{}^{(M)}\equiv\theta_A\,. \tag{1}$$

They found that

$$\theta_V=0.212\pm0.004\,,\qquad \theta_A=0.268\pm0.001\,. \tag{2}$$

It seems to us, however, that this set of values does not properly reflect the present status of the baryon leptonic decay data. First, the assumption $\theta_A{}^{(B)}=\theta_A{}^{(M)}$ seems too restrictive, and their value for θ_A, quoted above, actually represents the $K_{\mu2}$ and $\pi_{\mu2}$ data. Secondly, the theoretical uncertainty in calculating radiative corrections to the Fermi coupling constant in $0^+\to0^+$ beta decays was not realistically allowed in their use of the relation

$$\cos\theta_V=G_V/G^{(\mu)}\,, \tag{3}$$

where $G^{(\mu)}$ is the coupling constant for muon decay and G_V is the Fermi coupling constant for $0^+\to0^+$ beta decays. Finally, the K_{l3} data are still inconsistent with the $\Delta T=\frac{1}{2}$ rule and the $\Delta Q=\Delta S$ rule, which, however, are required by the octet-current hypothesis of the Cabibbo theory.

In view of this, it seems desirable to have a determination of the phenomenological Cabibbo parameters using only the baryon leptonic decay data. In the spirit of the Cabibbo theory, which assumes a single angle parameter for both vector and axial-vector hadron currents, and the Ademollo-Gatto theorem, we shall use the parameters θ and ξ. While θ is the phenomenological vector Cabibbo angle and, according to the Ademollo-Gatto theorem, well approximates the "bare" Cabibbo angle, the deviation from 1 of the parameter ξ represents the renormalization effect on the strangeness-changing axial-vector vertices. The relationship between this set of parameters and the set previously used by Brene *et al.*, i.e., $\theta_V{}^{(B)}$ and $\theta_A{}^{(B)}$, is the following:

$$\theta_V{}^{(B)}=\theta\,,\qquad \tan\theta_A{}^{(B)}=\xi\tan\theta\,. \tag{4}$$

In Sec. II we shall give the matrix elements for the semileptonic decay processes, and describe the form factors used in determining the fit. The numerical formulas for the decay rates are also given. The final section discusses the fitting procedure and the results obtained.

II. MATRIX ELEMENTS AND DECAY RATES

According to the Cabibbo theory,[1] the Lagrangian for semileptonic weak interactions is of the form

$$\mathcal{L}_{\rm int}=(G/\sqrt{2})J_\lambda(x)l_\lambda(x)+\text{Herm. adj.}\,, \tag{5}$$

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[1] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[2] M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964); C. Bouchiat and Ph. Meyer, Nuovo Cimento 24, 1122 (1964); S. Fubini and G. Furlan, Physics 1, 229 (1964).

[3] H. T. Nieh, Phys. Rev. Letters 15, 902 (1965); Phys. Rev. 146, 1012 (1966).

[4] N. Brene, L. Veje, M. Roos, and C. Cronström, Phys. Rev. 149, 1288 (1966).

where l_λ is the leptonic current,

$$l_\lambda=\bar{\mu}\gamma_\lambda(1+\gamma_5)\nu_\mu+\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e\,, \tag{6}$$

and J_λ is the hadron current,

$$J_\lambda=\cos\theta(J_\lambda{}^1+iJ_\lambda{}^2)+\sin\theta(J_\lambda{}^4+iJ_\lambda{}^5)\,,\quad J_\lambda{}^\alpha\equiv V_\lambda{}^\alpha+A_\lambda{}^\alpha\,. \tag{7}$$

The superscript α $(=1,2,\cdots,8)$ is the $SU(3)$ index.

We express the matrix elements of the currents between two baryons in the form $(\alpha=1,2,4,5)$:

$$\langle B^\beta|V_\lambda{}^\alpha|C^\gamma\rangle=(2\pi)^{-3}\bar{u}(p_B)\left\{f^{\alpha\beta\gamma}F_1{}^{(\Delta S)}(q^2)\gamma_\lambda+[\alpha_M d^{\alpha\beta\gamma}+(1-\alpha_M)f^{\alpha\beta\gamma}]\frac{\mu_P-\mu_n}{2M_N}\sigma_{\lambda\nu}q_\nu\right\}u(p_C)\,,$$

$$\langle B^\beta|A_\lambda{}^\alpha|C^\gamma\rangle=(2\pi)^{-3}\bar{u}(p_B)\{\alpha d^{\alpha\beta\gamma}+(1-\alpha)f^{\alpha\beta\gamma}\}\times\beta^{(\Delta S)}G_1{}^{(\Delta S)}(q^2)\{\gamma_\lambda\gamma_5-ih^{(\Delta S)}\gamma_5 q_\lambda\}u(p_C)\,,$$

$$q=p_B-p_C\equiv-Q\,. \tag{8}$$

We use the following criteria for the form factors: We assume the nucleon electromagnetic form factor for $F_1{}^{(\Delta S=0)}(q^2)$ in accordance with the conserved-vector-current theory of Feynman and Gell-Mann,[5] and we assume the dipole K^* dominance for the strangeness-changing vector form factor $F_1{}^{(\Delta S=1)}(q^2)$. Dipole A_1 and K_A dominance are used for the $\Delta S=0$ and $\Delta S=1$ axial-vector form factors, respectively. We assume that only the strangeness-changing axial-vector vertices are renormalized by $SU(3)$ symmetry-breaking interactions and use the recently modified version[6] of the Goldberger-Treiman relation[7] for induced pseudoscalar couplings. Second-class coupling terms[8] are omitted. With the above choices, the form factors can be written as follows:

$$F_1{}^{(\Delta S=0)}(q^2)=\left[1+\frac{q^2}{0.7\ (\text{GeV})^2}\right]^{-2}\simeq1-2\frac{q^2}{0.7\ (\text{GeV})^2}\,, \tag{9}$$

$$F_1{}^{(\Delta S=1)}(q^2)=[1+q^2/M_{K^*}{}^2]^{-2}\simeq1-2q^2/M_{K^*}{}^2\,, \tag{10}$$

$$\alpha_M=\frac{3}{2}\frac{1}{1-\mu_P/\mu_n}\simeq0.774\,, \tag{11}$$

$$\beta^{(\Delta S=0)}=(G_A/G_V)_{n\to pe\bar{\nu}}=1.18\pm0.025, \tag{12}$$

$$\beta^{(\Delta S=1)}=\xi\beta^{(\Delta S=0)}\,, \tag{13}$$

$$G_1{}^{(\Delta S=0)}(q^2)\simeq(1+q^2/M_{A_1}{}^2)^{-2}\simeq1-2q^2/M_{A_1}{}^2\,, \tag{14}$$

$$G_1{}^{(\Delta S=1)}(q^2)=(1+q^2/M_{K_A}{}^2)^{-2}\simeq1-2q^2/M_{K_A}{}^2\,, \tag{15}$$

$$h^{(\Delta S=0)}=\frac{M_B+M_C}{M_\pi{}^2}\left(1-\frac{M_\pi{}^2}{M_{A_1}{}^2}\right), \tag{16}$$

$$h^{(\Delta S=1)}=\frac{M_B+M_C}{M_K{}^2}\left(1-\frac{M_K{}^2}{M_{K_A}{}^2}\right). \tag{17}$$

With the hadron-current matrix elements given by Eqs. (8)–(17), the decay rates for the various baryon leptonic decays can be computed using the numerical table of Nieto.[9] We list in the following the decay rates (in $\sec^{-1}$) in terms of the parameters θ, α, and ξ:

$$\Gamma(\Sigma^-\to\Lambda e\bar{\nu})=\cos^2\theta(1.022\times10^5)\alpha^2\,, \tag{18a}$$

$$\Gamma(\Sigma^+\to\Lambda e^+\nu)=\cos^2\theta(6.179\times10^5)\alpha^2\,, \tag{18b}$$

$$\Gamma(\Lambda\to pe\bar{\nu})=\sin^2\theta(2.467\times10^7)\times[1+3.944(1-\tfrac{2}{3}\alpha)^2\xi^2]\,, \tag{18c}$$

$$\Gamma(\Lambda\to p\mu\bar{\nu})=\sin^2\theta(4.156\times10^6)\times[1+3.858(1-\tfrac{2}{3}\alpha)^2\xi^2]\,, \tag{18d}$$

$$\Gamma(\Sigma^-\to ne\bar{\nu})=\sin^2\theta(9.731\times10^7)\times[1+4.081(1-2\alpha)^2\xi^2]\,, \tag{18e}$$

$$\Gamma(\Sigma^-\to n\mu\bar{\nu})=\sin^2\theta(4.640\times10^7)\times[1+3.891(1-2\alpha)^2\xi^2]\,, \tag{18f}$$

$$\Gamma(\Xi^-\to\Lambda e\bar{\nu})=\sin^2\theta(5.067\times10^7)\times[1+4.096(1-\tfrac{4}{3}\alpha)^2\xi^2]\,. \tag{18g}$$

III. RESULTS

As mentioned earlier, we attempt a fit to the semileptonic experimental data using three parameters (θ,α,ξ). For input we use nine pieces of experimental data.

The first seven are the decay widths:

$$\Gamma(\Sigma^-\to\Lambda e\bar{\nu})\,, \tag{19a}$$

$$\Gamma(\Sigma^+\to\Lambda e^+\nu)\,, \tag{19b}$$

$$\Gamma(\Lambda\to pe\bar{\nu})\,, \tag{19c}$$

$$\Gamma(\Lambda\to p\mu\bar{\nu})\,, \tag{19d}$$

$$\Gamma(\Sigma^-\to ne\bar{\nu})\,, \tag{19e}$$

$$\Gamma(\Sigma^-\to n\mu\bar{\nu})\,, \tag{19f}$$

$$\Gamma(\Xi^-\to\Lambda e\bar{\nu})\,, \tag{19g}$$

which are taken from the total decay widths and branching ratios given in the most recent tables by Rosenfeld *et al.*[10,11] These data are compared to the general theoretical partial decay widths given by Eqs. (18).

[5] R. Feynman and M. Gell-Mann, Phys. Rev. **109**, 129 (1958).
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[7] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).
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[10] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).
[11] An experiment by L. I. Gershwin *et al.* [University of California Radiation Laboratory Report No. UCLR 17740 (unpublished)] on polarized Σ^- decay yields a result that is inconsistent with Cabibbo theory. However, since they report only 36 events, we do not use the conclusions of this paper because of the poor statistics. We would like to thank Professor J. Cole, Professor P. Franzini, and Professor J. Lee-Franzini for discussions on this point.

The eighth input is

$$\beta^{(\Delta S=0)}(1-\tfrac{2}{3}\alpha)\xi=-(G_A/G_V)_{\Lambda\to pe\bar{\nu}}=1.14_{-0.23}^{+0.33}, \quad (19\text{h})$$

where $\beta^{(\Delta S=0)}$ is given in Eq. (12), and the value of $(G_A/G_V)_{\Lambda\to pe\bar{\nu}}$ is given in Ref. 10.

Our final input is[12]

$$\cos\theta=0.978\pm0.006, \quad (19\text{i})$$

which is taken from Sirlin.[13]

The values of $\theta=0.206$, $\alpha=0.662$, $\xi=1.42$ give a χ^2 fit of 1.71 (89% confidence level). In Fig. 1 we show the projection of the surface bounding one standard deviation ($\chi^2\leq 2.71$) in the three-dimensional (θ,ξ,α) space. This figure represents the correlated errors of the fit. For practical purposes we give the standard deviations as those which represent one standard deviation along each coordinate from the χ_{min}^2 point. We thus have

$$\theta=0.206\pm0.009,$$
$$\alpha=0.662\pm0.018, \quad (20)$$
$$\xi=1.42\pm0.10.$$

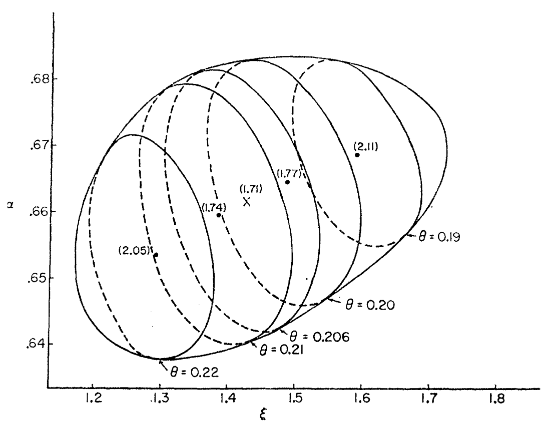

FIG. 1. The surface which bounds one standard deviation ($\chi^2\leq 2.71$) around the minimum ($\chi_{\text{min}}^2=1.71$). χ_{min}^2 is marked by an $\times$ and is located at $\theta=0.206$, $\alpha=0.662$, $\xi=1.42$. (The θ direction is perpendicular to the paper.) The cross cuts represent intersections of the surface with planes of $\theta=$ const. The position of the smallest χ^2 in each of these planes is marked by a point, and the χ^2 values of these points are listed beside them.

[12] Because we have nine experimental inputs and three parameters, we should ordinarily have five degrees of freedom in the χ^2 fit. We assume this even though the ninth input is only a function of θ.

[13] See the discussion in A. Sirlin, Phys. Rev. Letters **16**, 872 (1966), and the references quoted therein.

Combining Eq. (4) and Eq. (20) yields

$$\theta_A^{(B)}=0.288\pm0.024. \quad (21)$$

As a check of the sensitivity of the results on the assumed form factors, a fit was made with the dipole form factors of Eqs. (10), (14), and (15) being changed to monopole form factors. This gives a fit that is only slightly different from Eqs. (20) and (21):

$$\theta=0.2075\pm0.0090,$$
$$\alpha=0.665\pm0.018,$$
$$\xi=1.44\pm0.10,$$

with $\chi^2=1.83$ (87% confidence level), and

$$\theta_A^{(B)}=0.294\pm0.026.$$

Note added in proof. (1) After the submission of the present paper for publication, we learned of a similar work by N. Brene, M. Roos, and A. Sirlin, Nucl. Phys. (to be published). We wish to thank these authors for sending us a copy of their paper. (2) Doubts have been expressed by several authors concerning the validity of the assumption that second-order $SU(3)$ symmetry breaking effects on weak vector vertices are negligible. We wish to point out that all phenomenological *vector* Cabibbo angles determined from hyperon decays (as from our analysis), K_{l3} decay, and $0^+\to 0^+$ nuclear β decays agree with each other to within a few percent. This indicates that $SU(3)$ symmetry breaking effects on weak vector vertices are indeed small.

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