Static $SU(6)$ and Fundamental Substitution*

SHIN ISHIDA† AND P. ROMAN
Department of Physics, Boston University, Boston, Massachusetts
(Received 6 March 1968)

A prescription is given for avoiding the two major defects of the standard static $SU(6)$ theory, viz., its inability to treat processes which vanish in the static limit, and the contradictory results which arise from the channel dependence of $SU(6)$ symmetry. One crucial ingredient in our framework is what we call fundamental substitution by which we mean the substitution of a constituent quark by the corresponding antiquark, following the usual substitution law. The other essential assumption of our framework is the suppression of constituent quark-antiquark pair effects in the static limit. After formulating the framework rigorously and illustrating the procedure on a simple model, we apply it first to the analysis of the Yukawa vertex of the 56-baryon and the 36-meson multiplet. The major results are (i) A Sachs-type coupling of the 1^- mesons and $\frac{1}{2}^+$ nucleons is obtained. (ii) The electric-type vector-meson–nucleon interaction is pure F, and the magnetic-type is $F+D$ with $D/F=\frac{3}{2}$, so that the experimentally observed relations between the electric and magnetic form factors of the nucleons are reproduced within the framework of the vector-meson-dominance model. (iii) The $0^- - \frac{1}{2}^+$ interaction is $D+F$, with $D/F=\frac{3}{2}$. (iv) Resonably well-satisfied coupling-constant sum rules are obtained. The method is then applied to the trilinear meson vertex, and leads to a well-satisfied sum rule. Finally, some related problems are discussed.

I. INTRODUCTION

It is well known that, in spite of many attempts in this direction,[1] the relativistic extension of $SU(6)$ symmetry meets extremely serious difficulties. In this paper we therefore take the customary viewpoint of static $SU(6)$, which stipulates that $SU(6)$ is meaningful only in the static limit when the linear momenta of all participating particles are set equal to zero and assumes that all physical vertices (which themselves are relativistic) satisfy $SU(6)$ symmetry in this limit. However, this viewpoint *per se* has two defects. The first is that we cannot treat processes which vanish in the static limit. Even though Gürsey, Pais, and Radicati[2] obtained very interesting results regarding the Yukawa vertex of the meson 35-plet and the baryon 56-plet, the theoretical basis of these results is not clear. The baryon scattering channel of the vertex, shown[3] in Fig. 1(a), vanishes in the static limit, and the above expressed viewpoint does not permit one to derive any results.

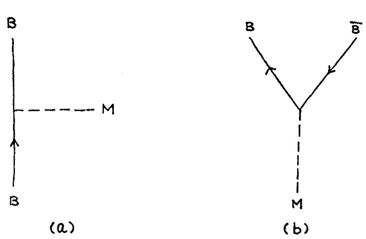

FIG. 1. Channels of a Yukawa vertex for meson and baryon.

If, on the other hand, we start with the pair creation (or annihilation) channel of the vertex, shown in Fig. 1(b) (which channel does have a nonvanishing static limit), then unsatisfactory results arise.[4] For example, pure F coupling is obtained for pseudoscalar mesons and both F and D coupling for the "electric-type" interaction of the vector mesons, contrary to experimental evidence. Another difficulty arises for the trilinear meson vertex, which has no nonvanishing static limit in any channel, so that the above viewpoint of static $SU(6)$ has prevented so far the derivation of satisfactory results for this vertex. The second defect is the following. The requirement of $SU(6)$ symmetry is channel-dependent[5] (as exemplified by the baryon-meson vertex), and this results in different and sometimes contradictory results. Within the framework of $SU(6)$ theory alone, there is no criterion for choosing a particular channel where the symmetry is to be required.

The purpose of the present paper is to give a prescription for avoiding these defects. We propose a unified method of treating $SU(6)$ symmetry which is also applicable to processes with vanishing static limit while it still preserves the above expressed viewpoint of static $SU(6)$. The clue to such a unified method comes from considering the background of static $SU(6)$. The origin of $SU(6)$ symmetry can be well visualized in the framework of the nonrelativistic quark model, and actually Sakita[6] proposed $SU(6)$ on the basis of this model. Apart from this, many interesting results have been derived from the quark model with realistic quarks.[7] In this paper, however, we shall use the non-

* Research supported by the U. S. Air Force under Grant No. AF-AFOSR-385-67.
† On leave of absence from Nihon University, Tokyo.
[1] For a review, see, for example, F. J. Dyson, *Symmetry Groups in Nuclear and Particle Physics* (W. A. Benjamin, Inc., New York, 1966).
[2] F. Gürsey, A. Pais, and L. A. Radicati, Phys. Rev. Letters **13**, 29 (1964).
[3] In all graphs of this paper, the direction of time is vertically upwards.
[4] H. Ruegg, W. Rühl, and T. S. Santhanam, Helv. Phys. Acta **40**, 9 (1967).
[5] Y. Ohnuki and A. Toyoda, Nuovo Cimento **36**, 1405 (1965); see also F. Gürsey, in *Non-Compact Groups In Particle Physics*, edited by Y. Chow (W. A. Benjamin, Inc., New York, 1966).
[6] B. Sakita, Phys. Rev. **136**, B1756 (1964).
[7] For a recent review, see, for example, R. H. Dalitz, in *Proceedings of the Thirteenth International Conference on High-Energy Physics* (University of California Press, Berkeley, 1967), p. 215.

relativistic quark model only in the heuristic sense, without assuming any details of the model.

The prescription for avoiding the first defect discussed above is to consider for any channel with a vanishing static limit a corresponding "fundamentally substituted channel." By "fundamental substitution" we mean the substitution of a constitutent quark by the corresponding antiquark (or vice versa), following the usual substitution law (crossing). The fundamentally substituted channel will have a nonvanishing static limit, because a quark and its antiquark have opposite parities. Hence, we can now require static $SU(6)$ symmetry in this fundamentally substituted channel. This then necessarily leads to certain restrictions between the types of vertex function for the original process.

In order to eliminate the second defect, it is only necessary to pursue the physical picture of the nonrelativistic quark model somewhat further. In this model it is natural and quite plausible to assume that quark-antiquark pair effects are rather small. Thus we assume that quark-antiquark pair effects are rather small. Thus we assume that, in the static limit, the quark number and the antiquark number are separately conserved. (Here we emphasize that this conservation is assumed *only* in the static limit.) With this assumption, the second defect is eliminated. For example, the channel of Fig. 1(b) vanishes in the static limit because of our assumption, so that the requirement of static $SU(6)$ does not lead to any statement for this channel.

In Sec. II we illustrate our basic procedure in detail on the simple example of a Yukawa vertex of the mesons and a fictitious particle with quark number 3. The method is applied in Sec. III to derive the Yukawa vertex of the **56**-representation-baryon and the **36**-representation-meson vertex [Eqs. (3.10) and (2.11)], which appears to have several pleasing features. In particular, it is shown that: (i) The Sachs-type coupling of the vector meson with the nucleon has, in our scheme, a more fundamental meaning than the usual Dirac and Pauli-type coupling. (ii) The "electric-type" interaction of the vector meson to the nucleon is pure F coupling, whereas the "magnetic-type" interaction is a combination of F and D coupling, with $D/F=\frac{3}{2}$, which then leads to the experimentally verified behavior (3.12) of the electromagnetic form factors of the nucleons. (iii) The pseudoscalar meson-nucleon interaction is also $D+F$ coupling, with $D/F=\frac{3}{2}$. (iv) Sum rules [Eq. (3.11)] are obtained among the coupling constants, which seem to be fairly well satisfied experimentally. In Sec. IV we apply our method to the trilinear meson vertex and obtain the well-satisfied coupling-constant sum rule (4.8). In Sec. V some related problems, such as the vertex with two baryon and two meson lines, are briefly discussed. Some comments are also given concerning the nonet assumption of Okubo[8] and the selection principle of Iizuka.[9]

II. SUBSTITUTION RULE AND THE MESON-QUARK INTERACTION

In this section we formulate rigorously our basic procedure and illustrate it on the simple example of the Yukawa vertex for the 36-plet meson[10] and a fictitious particle which has quark number 3 (i.e., baryon number 1), and spin $\frac{1}{2}$, and which belongs to a unitary triplet. The purpose of considering this model is simply to display the essential features of our procedure while avoiding the *kinematical* complexities which occur in the realistic problems which are discussed in the subsequent sections.

As already noted in Sec. I we take the viewpoint of static $SU(6)$ expressed by:

Postulate 1: All effective vertices satisfy $SU(6)$ symmetry in the static limit where the linear momenta of all participating particles is set zero.

We further adopt[11] the following:

Postulate 2: In the static limit (and *only* in this limit) the quark number and the antiquark number are separately conserved.

Because of the substitution law (crossing property), a relativistic vertex function describes, in a unified way, all processes in all possible channels. At this point, it will be useful to classify relativistic vertices into two categories. We shall call a relativistic vertex a *static vertex* if it possesses a channel which happens to conserve the quark number and the antiquark number. If there is no such channel then we call the relativistic vertex *nonstatic*. For the analysis of a static vertex, postulates 1 and 2 supply a complete frame. An interesting example of a static vertex is given by the two-baryon two-meson four-point function.[12] However, our main interest in this paper is directed toward the analysis of nonstatic vertices. Their discussion necessitates the introduction of a further postulate.

Postulate 3: The mesons are nonrelativistic bound states of a quark-antiquark pair. For the constituent quark field the standard substitution law is valid.

We shall call the application of the substitution law relative to the constituent quark field the "fundamental

[8] S. Okubo, Phys. Letters **5**, 165 (1963).

[9] J. Iizuka, Progr. Theoret. Phys. (Kyoto), Suppl. Nos. **37** and **38** (1966).

[10] Throughout the paper, we treat the singlet and the 35-plet mesons as *one* multiplet, of 36 dimensions. As will be discussed in Sec. V B, this treatment is a desirable extension of the nonet assumption by Okubo (Ref. 8). It implies that the mesons always appear in unitary spin space as nonets, so that the unitary spin index is trivial. Hence, we shall often suppress it.

[11] We mention that in a recent paper by K. Kikkawa [Phys. Rev. **165**, 1753 (1968)], the same assumption is used in another context.

[12] This vertex will be discussed briefly in Sec. V A, where we shall see that, even for a static vertex, our procedure gives a more clearcut view than the usual $SU(6)$ treatment.

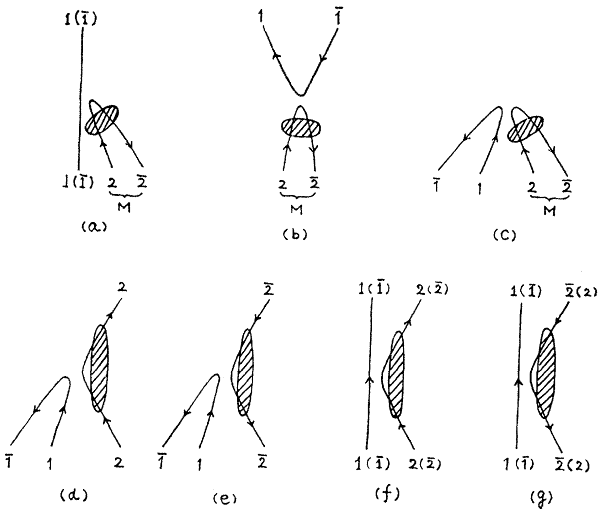

FIG. 2. Channels of a Yukawa vertex for composite meson obtained from fundamental substitution.

substitution." We note here that, since the baryon number is proportional to the quark number, it will not be necessary to assume a fundamental substitution law for baryons. The usual substitution (treating the baryon as a single entity) will suffice.

We now make the crucial observation that from the viewpoint of the fundamental substitution law, *all* physical vertices become static vertices, and thus we can apply our postulate 1 for *any* vertex in a unified manner.

Let us now consider the Yukawa vertex for the **36** multiplet of mesons and the above-defined **6** multiplet of fictitious particles with quark number 3. This is a simple example of a nonstatic vertex, and the realistic examples of the subsequent sections can be treated in a completely analogous manner. None of the standard channels of the vertex shown[13] in Figs. 2(a), 2(b), and 2(c) has a nonvanishing static limit, because of[14] postulate 2. If we now adopt postulate 3, the situation changes, inasmuch as we now have the additional channels shown in Figs. 2(d) and 2(e) as well as in Figs. 2(f) and 2(g). The first two have vanishing static limits, but the last two *do not vanish* in the static limit.

[13] In Fig. 2, the particle lines labeled 1 and 2 represent the fictitious particle q_1 and the constituent quark q_2 of the composite meson, respectively. We also note that, since we assume time-reversal invariance, we indicated in Fig. 2 only one of a pair of channels which arise from each other by time reversal.

[14] The channel represented by Fig. 2(a) vanishes in the static limit also in consequence of parity conservation. The channel of Fig. 2(b) which, without postulate 2, would have a nonvanishing static limit, was essentially used as the physical starting point of the "relativistic completion" attempt of M. A. B. Bég and A. Pais [Phys. Rev. **137**, B1514 (1965)]. However, in their framework these authors could not construct a Lorentz-invariant vertex function with desired properties for the Yukawa interaction between mesons and baryons; see their "*Note added in proof*" on p. B1521, following their Eq. (47). It may be also worthwhile to point out that the *matrix element* corresponding to the channel shown in Fig. 2(c) does not have to vanish in general theory.

The shaded areas in Fig. 2 indicate that the corresponding parts of the diagrams have the same transformation properties since these are not subjected to any change by the application of the fundamental substitution.

In analogy with the classification of vertices, it will be advantageous to classify channels into two categories. We shall call a channel a *static channel* if it conserves the quark number and the antiquark number. In the opposite case we call it a *nonstatic channel.* Then it follows from postulates 1 and 2 that in the static channels [Figs. 2(f) and 2(g)] the vertex function becomes $SU(6)$ invariant in the static limit, whereas in the nonstatic channels [Figs. 2(a) through 2(e)] it must vanish in the static limit.

Our postulate 3 on fundamental substitution implies that all shaded parts of the diagrams in Fig. 2 must be analytically expressed in a unified way by the quantity

$$\bar{t}^{\alpha}(x_f)\Omega_k t_\beta(x_i) = \begin{pmatrix} \bar{t}^{+\alpha}(x_f)\Omega_k t_\beta{}^+(x_i), & \bar{t}^{-\alpha}(x_f)\Omega_k t_\beta{}^+(x_i) \\ \bar{t}^{+\alpha}(x_f)\Omega_k t_\beta{}^-(x_i), & \bar{t}^{-\alpha}(x_f)\Omega_k t_\beta{}^-(x_i) \end{pmatrix}, \quad (2.1a)$$

with

$$\Omega_k = (1, \gamma_\mu, -\tfrac{1}{2}i[\gamma_\mu,\gamma_\nu], \gamma_5\gamma_\mu, \gamma_5) \quad \text{for} \quad k=(S,V,T,A,P). \quad (2.1b)$$

Here $t(x)$ represents the constituent quark field of the meson, the superscripts $+$ and $-$ refer to positive- and negative-frequency parts, $\bar{t}^{\pm}$ is the adjoint of $t^{\pm}$, x is a space-time coordinate, and α and β are unitary spin indices. The diagonal terms in (2.1a) pertain to the processes of Figs. 2(d) through 2(g), while the off-diagonal elements pertain to the processes of Figs. 2(a) through 2(c). In the spirit of the quark model,[15] the off-diagonal terms in (2.1a) are related to the meson fields as follows:

$$\int d^4x\, F(x) \begin{pmatrix} \bar{t}^{-\alpha}(X+\tfrac{1}{2}x)\Omega_k t_\beta{}^+(X-\tfrac{1}{2}x) \\ \bar{t}^{+\alpha}(X+\tfrac{1}{2}x)\Omega_k t_\beta{}^-(X-\tfrac{1}{2}x) \end{pmatrix} = K \begin{pmatrix} \bar{t}^{-\alpha}(X)\Omega_k t_\beta{}^+(X) \\ \bar{t}^{+\alpha}(X)\Omega_k t_\beta{}^-(X) \end{pmatrix} = \begin{pmatrix} \Phi_{k,\beta}{}^{+\alpha}(X) \\ \Phi_{k,\beta}{}^{-\alpha}(X) \end{pmatrix}, \quad (2.2a)$$

where

$$K \equiv \int d^4x\, F(x). \quad (2.2b)$$

Here X and x denote the center-of-mass and the relative coordinates, respectively. $F(x)$ represents the wave function of the meson. $\Phi_k{}^+$ ($\Phi_k{}^-$) is the positive- (negative-) frequency part of the meson fields which, in terms of the pseudoscalar nonet $P_\beta{}^\alpha$ and the vector

[15] We cannot tell whether our derivation of (2.2) and our substitution rule is self-consistent or not, because we do not know the equation of motion of the constituent quarks. The physical content of Eq. (2.2) is similar to the usual assumption in the nonrelativistic quark model, except that here we pay attention to the Lorentz transformation property of the c.m. motion of the mesons.

nonet $V_{\mu,\beta}{}^{\alpha}$, are specified as

$$\Phi_{k,\beta}{}^{\alpha}=(0,\, iV_{\mu,\beta}{}^{\alpha},\, (1/m)\partial_{[\mu}V_{\nu],\beta}{}^{\alpha},\, (1/m)\partial_{\mu}P_{\beta}{}^{\alpha},\, -P_{\beta}{}^{\alpha}) \quad (2.3)$$

for $k=(S,V,T,A,P)$ and where m denotes the central mass of the 36-plet. The subscript square brackets mean antisymmetrization of the indices μ and ν. In deriving (2.2a) and (2.3), the nonrelativistic nature of the bound system was assumed and accordingly the dependence of the quark field on the relative coordinate was neglected. The correspondence expressed by (2.3) is obvious from the transformation properties, and the relative magnitudes are determined in the rest frame of the mesons. In summary, we reached the following substitution rule[15]: The quantities

$$\bar{t}^{+\alpha}(X)\Omega_k t_{\beta}{}^{+}(X),\quad \bar{t}^{-\alpha}(X)\Omega_k t_{\beta}{}^{-}(X),$$
$$\bar{t}^{\mp\alpha}(X)\Omega_k t_{\beta}{}^{\pm}(X)=(1/K)\Phi^{\pm}{}_{k,\beta}{}^{\alpha}(X) \quad (2.4)$$

must be treated in a unified way.

We can now discuss all processes represented in Fig. 2 in a systematic way. Expanding[16] the vertex function in terms of the complete set of the Dirac matrices *which act on the constituent quark,* we can write

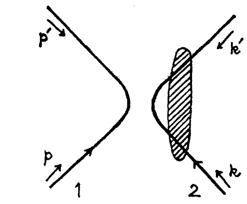

FIG. 3. Momentum assignments.

$$A(p,p';k,k')=\sum_i [\bar{t}^{\alpha}(1,p')H_i{}^{(1)}(p,p';k,k')t_{\beta}(1,p)] \times[\bar{t}^{\beta}(2,k')\Omega_i{}^{(2)}t_{\alpha}(2,k)], \quad (2.5)$$

where $t(1)$ and $t(2)$ are the spinorial wave functions of the fictitious particle and the constituent quark, with momentum assignments as indicated in Fig. 3, and we must choose positive- or negative-energy spinors according to the physical content of the corresponding channel. The momentum-dependent $H_i{}^{(1)}$ is a matrix in the spinor space of the fictitious particle. Assuming Wigner time-reversal invariance, the right-hand side of (2.5) has the general form

$$\sum_i H_i{}^{(1)}\Omega_i{}^{(2)}=C^S 1^{(1)}1^{(2)}+(C_1{}^V P_\mu+C_2{}^V\gamma_\mu{}^{(1)}+D^V\epsilon_{\mu\nu\kappa\lambda}Q_\nu P_\kappa\gamma_5{}^{(1)}\gamma_\lambda{}^{(1)})\gamma_\mu{}^{(2)}$$
$$+(C_1{}^T\epsilon_{\mu\nu\kappa\lambda}P_\kappa\gamma_5{}^{(1)}\gamma_\lambda{}^{(1)}+C_2{}^T\sigma_{\mu\nu}{}^{(1)}+D_1{}^T\epsilon_{\mu\nu\kappa\lambda}P_\kappa Q_\lambda\gamma_5{}^{(1)}+D_2{}^T P_\mu Q_\nu)\sigma_{\mu\nu}{}^{(2)}$$
$$+(C_1{}^A\gamma_5{}^{(1)}\gamma_\mu{}^{(1)}+C_2{}^A Q_\mu\gamma_5{}^{(1)}+D^A\epsilon_{\mu\nu\kappa\lambda}P_\nu Q_\kappa\gamma_\lambda{}^{(1)})\gamma_5{}^{(2)}\gamma_\mu{}^{(2)}+(C^P\gamma_5{}^{(1)}+D^P\epsilon_{\mu\nu\kappa\lambda}P_\mu Q_\nu\sigma_{\kappa\lambda}{}^{(1)})\gamma_5{}^{(2)}. \quad (2.6)$$

We point out that when deriving (2.6), we used the Dirac equation only for the fictitious particle, but of course we did not use it for the quark, which is a meson constituent. Furthermore, since apart from energy-momentum conservation we have the additional restriction $k-k'=0$, there are only two independent momenta, which we chose to be

$$P_\mu\equiv p_\mu-p_\mu',\quad Q_\mu\equiv-(p_\mu+p_\mu')=k_\mu+k_\mu'. \quad (2.7)$$

This is so because we are not treating a general vertex function with two external constituent quark lines and two external fictitious particle lines, but rather one subject to the restriction corresponding to the shaded areas.

If we neglect the Q^2 dependence[17] of the C and D coefficients in (2.6) (i.e., if we neglect form-factor effects), then it can be easily seen that all terms in (2.6) which have a C coefficient possess a nonvanishing static limit in at least one of the channels shown in Fig. 2. On the other hand, the static limit of all terms which have a D coefficient vanishes in all channels. Thus, the constants C will be subject to constraints which follow from our postulates. We now explore these relations.

In the static channels the vertex function must have the $SU(6)$-invariant static limit[10]

$$C_0{}^{qq(q\bar{q})}(\bar{t}^A(1)t_B(1))(\bar{t}^B(2)t_A(2))$$
$$=C_0{}^{qq(q\bar{q})}(\bar{t}^{\alpha}(1)t_{\beta}(1))(\bar{t}^{\beta}(2)t_{\alpha}(2))[\langle\bar{\chi}(1)\chi(1)\rangle\langle\bar{\chi}(2)\chi(2)\rangle + \langle\bar{\chi}(1)\sigma_i\chi(1)\rangle\langle\bar{\chi}(2)\sigma_i\chi(2)\rangle]. \quad (2.8)$$

Here $C_0{}^{qq}$ ($C_0{}^{q\bar{q}}$) is a constant corresponding to the channel Fig. 2(f) [Fig. 2(g)]; $t_A\equiv t_\alpha\chi_i$, where $t_\alpha(\chi_i)$ is a tensor component in unitary (ordinary) spin space; σ_i is the Pauli spin matrix; and $\langle\cdots\rangle$ indicates taking the trace. Comparing now the static limit of (2.6) with the required form (2.8), we find that

$$\pm C^S+2i\kappa C_1{}^V+C_2{}^V=\pm(-2\kappa C_1{}^T+C_2{}^T)-C_1{}^A = C_0{}^{qq} \text{ or } C_0{}^{q\bar{q}}, \quad (2.9)$$

where the upper (lower) sign corresponds to $C_0{}^{qq}$ ($C_0{}^{q\bar{q}}$) and to channel Fig. 2(f) [Fig. 2(g)]. The mass of quark 1 is denoted by κ. The other requirement following from our postulates (i.e., the vanishing of the static limit of the vertex in the nonstatic channels) leads to the relation

$$C_2{}^V\pm C_2{}^T=C_1{}^A-2i\kappa C_2{}^A\mp C^P=0, \quad (2.10)$$

where the upper (lower) sign corresponds to channel Fig. 2(b) [Fig. 2(c)]. The channels Figs. 2(a), 2(d),

[16] M. L. Goldberger, Y. Nambu, and R. Oehme, Ann. Phys. (N. Y.) **2**, 226 (1957).

[17] As a matter of fact, when we wrote down (2.6), we already treated the C's as constants. Otherwise, for example, because of the identity $\epsilon_{\mu\nu\kappa\lambda}Q_\nu P_\kappa\gamma_5{}^{(1)}\gamma_\lambda{}^{(1)}=(4\kappa^2+Q^2)\,\gamma_\mu{}^{(1)}+2i\kappa P_\mu$ (where κ is the quark mass), only two out of the three terms which multiply $\gamma_\mu{}^{(2)}$ could be considered as independent.

and 2(e) do not give any further restrictions since, because of parity conservation, they do not have nonvanishing static limits.

The solution of (2.9) and (2.10) gives the relations

$$2i\kappa C_1{}^V=-C_1{}^A=-2i\kappa C_2{}^A=C_+{}^{qM}, \quad (2.11a)$$

$$C^S=-2\kappa C_1{}^T=C_-{}^{qM}, \quad (2.11b)$$

$$C_2{}^V=C_2{}^T=C^P=0, \quad (2.11c)$$

i.e., the constants C can be expressed in terms of the two parameters

$$C_\pm{}^{qM}\equiv\tfrac{1}{2}(C_0{}^{qq}\pm C_0{}^{q\bar{q}}). \quad (2.11d)$$

The next step consists in the application of the substitution rule (2.4) [with the identifications as given by (2.3)] to the determination of the effective Yukawa vertex function of the meson and our fictitious sextuplet particle system. We find with ease that

$$A(p,p';Q)=A_C(p,p';Q)+A_D(p,p';q), \quad (2.12)$$

where

$$A_C=[\bar{t}^\alpha(p')(G_1{}^V(P_\mu/2i\kappa) + G_2{}^V(1/2\kappa)\epsilon_{\mu\nu\kappa\lambda}Q_\nu P_\kappa\gamma_5\gamma_\lambda)t_\beta(p)]iV_{\mu,\alpha}{}^\beta(Q) + [\bar{t}^\alpha(p')iG_1{}^P\gamma_5\gamma_\mu(1-Q^2/4\kappa^2)t_\beta(p)]Q_\mu P_\alpha{}^\beta(Q) \quad (2.12')$$

and

$$A_D=[\bar{t}^\alpha(p')(J_1{}^V\epsilon_{\mu\nu\kappa\lambda}Q_\nu P_\kappa\gamma_5\gamma_\lambda + J_2{}^VQ^2P_\mu)t_\beta(p)]iV_{\mu,\alpha}{}^\beta(Q) + [\bar{t}^\alpha(p')(J^P\epsilon_{\mu\nu\kappa\lambda}P_\mu Q_\nu\sigma_{\kappa\lambda})t_\beta(p)]iP_\alpha{}^\beta(Q). \quad (2.12'')$$

Here A_C and A_D originate from terms of (2.6) with coefficients C and D, respectively. The G constants in A_C obey, in consequence of (2.11), the relations

$$G_1{}^V=-mG^P=C_+{}^{qM}/K, \quad mG_2{}^V=C_-{}^{qM}/K. \quad (2.13)$$

On the other hand, obviously, no relations arise for the J coefficients[18] in A_D. However, it appears plausible to amend our framework with another postulate.

Postulate 4: A vertex function which does not have a nonvanishing static limit in at least one channel is identically zero.

This postulate receives support from the experimental consequences it leads to, as will be discussed briefly in Sec. V C. At any rate, if we adopt it, all D constants in (2.6) and hence all J constants in (2.12″) vanish and we have $A_D=0$. Thus, we finally obtain the effective interaction vertex. It is given by (2.12′), and the coupling constants are restricted by the relation (2.13).

III. MESON-BARYON INTERACTION

In this section we apply our method of Sec. II to explore the effective vertex function for the Yukawa interaction of the $SU(6)$ baryon 56-plet and the meson 36-plet,[10] which is surely the most interesting example of nonstatic vertices.

A. Derivation of the Vertex Function

Except for kinematical complexity, the vertex under consideration is essentially the same as the one studied in Sec. II. We must consider systematically all processes of Fig. 2, where now the particle line labeled 1 denotes the baryon B, and the line labeled 2 represents the quark constituent q of the meson M. With the assumption of Wigner time-reversal invariance, the general form of the vertex function is

$$\begin{aligned}A(p,p';k,k')=\sum_{K=F,D,S}\{&(\bar{N}(p')C_K{}^SN(p))_K(\bar{t}(k')t(k))+[\bar{N}(p')(C_{1,K}{}^VP_\mu+C_{2,K}{}^V\gamma_\mu)N(p)]_K\bar{t}(k')\gamma_\mu t(k)\\&+[\bar{N}(p')(C_{1,K}{}^T\epsilon_{\mu\nu\lambda\delta}P_\lambda\gamma_5\gamma_\delta+C_{2,K}{}^T\sigma_{\mu\nu})N(p)]_K\bar{t}(k')\sigma_{\mu\nu}t(k)+[\bar{N}(p')(C_{1,K}{}^A\gamma_5\gamma_\mu+C_{2,K}{}^AQ_\mu\gamma_5)N(p)]_K\bar{t}(k')\gamma_5\gamma_\mu t(k)\\&+(\bar{N}(p')C_K{}^P\gamma_5N(p))_K\bar{t}(k')\gamma_5t(k)\}+\{[C_1{}^T\epsilon_{\mu\nu\kappa\lambda}P_\kappa(\bar{D}_\lambda(p')N(p))+C_2{}^T(\bar{D}_\mu(p')\gamma_5\gamma_\nu N(p))\\&+C_3{}^T\epsilon_{\mu\nu\kappa\lambda}(\bar{D}_\kappa(p')\gamma_\lambda N(p))+C_4{}^TQ_\mu(\bar{D}_\nu(p')\gamma_5N(p))+C_5{}^TQ_\mu\epsilon_{\nu\kappa\lambda\delta}(\bar{D}_\kappa(p')\sigma_{\lambda\delta}N(p))]\bar{t}(k')\sigma_{\mu\nu}t(k)\\&+[C_1{}^A(\bar{D}_\mu(p')N(p))+C_2{}^A\epsilon_{\mu\nu\kappa\lambda}P_\nu(\bar{D}_\kappa(p')\gamma_5\gamma_\lambda N(p))]\bar{t}(k')\gamma_5\gamma_\mu t(k)+\text{H.c.}\}+\cdots. \quad (3.1)\end{aligned}$$

Here t_α ($N_\beta{}^\alpha$) is the Dirac spinor wave function of the constituent quark (octet baryon), and $(D_\mu)_{\alpha\beta\gamma}$ is the Rarita-Schwinger spin-vector wave function (with definite helicity) of the decuplet baryon. For these wave functions, positive or negative energy types will have to be chosen in accord with the physical content of the channel. The omitted terms, indicated by dots, correspond to the $\bar{D}DM$ vertex in which, for practical reasons, we are not interested. The rest of the notation is explained as follows:

$$\begin{aligned}(\bar{N}N)_{F,\beta}{}^\alpha&=\bar{N}_\gamma{}^\alpha N_\beta{}^\gamma-\bar{N}_\beta{}^\gamma N_\gamma{}^\alpha,\\(\bar{N}N)_{D,\beta}{}^\alpha&=\bar{N}_\gamma{}^\alpha N_\beta{}^\gamma+\bar{N}_\beta{}^\gamma N_\gamma{}^\alpha-\tfrac{2}{3}\delta_\beta{}^\alpha\langle\bar{N}N\rangle,\\(\bar{N}N)_{S,\beta}{}^\alpha&=\langle\bar{N}N\rangle\delta_\beta{}^\alpha,\\(\bar{D}N)_\beta{}^\alpha&=\epsilon_{\beta\gamma\epsilon}\bar{D}^{\alpha\gamma\delta}N_\delta{}^\epsilon,\\\langle\bar{N}N\rangle&=\bar{N}_\delta{}^\gamma N_\gamma{}^\delta.\end{aligned} \quad (3.2)$$

[18] Of course A_D has no effect on static problems.

The momenta of the particles referred to are assigned as in Fig. 3, and P_μ and Q_μ are defined by (2.7).

In the derivation of (3.1) we used the Dirac or Rarita-Schwinger equation for the baryon, but we did not use the Dirac equation for the constituent quark. Anticipating the use of fundamental substitution and our postulate 4, we indicated only those terms which have a nonvanishing static limit in at least one channel.

We again neglect the Q^2 dependence of the C coefficients and set out to find the restrictions which our scheme imposes onto them.

Let us represent the baryon multiplet as

$$B_{ABC}=D_{\alpha\beta\gamma}d_{ijk}+(1/3\sqrt{2})[\epsilon_{\alpha\beta\delta}\epsilon_{ij}N_\gamma{}^\delta n_k+\epsilon_{\beta\gamma\delta}\epsilon_{jk}N_\alpha{}^\delta n_i+\epsilon_{\gamma\alpha\delta}\epsilon_{ki}N_\beta{}^\delta n_j]. \quad (3.3)$$

Here we wrote out unitary and ordinary spin space factors separately, using the symbols $N_\beta{}^\alpha$ ($D_{\alpha\beta\gamma}$) and n_i (d_{ijk}) for the corresponding tensors of the octet (decuplet) baryon. Now we recall that in the static channels the vertex function must have an $SU(6)$-invariant form. With the representation (3.3), this invariant form can be readily written as[10]

$$\begin{aligned}C_0{}^{Bq(B\bar{q})}\bar{B}^{ACD}B_{BCD}\bar{t}^Bt_A=C_0{}^{Bq(B\bar{q})}\{&[(\bar{N}N)_F(\bar{t}t)\\&\times(\tfrac{1}{6}\langle\bar{n}n\rangle\langle\bar{\chi}\chi\rangle+(2/9)(\bar{n}n)_j{}^i(\bar{\chi}\chi)_i{}^j)\\&+(\bar{N}N)_D(\bar{t}t)\tfrac{1}{3}(\bar{n}n)_j{}^i(\bar{\chi}\chi)_i{}^j+\langle\bar{N}N\rangle\langle\bar{t}t\rangle\\&\times(\tfrac{1}{9}(\bar{n}n)_j{}^i(\bar{\chi}\chi)_i{}^j+\tfrac{1}{6}\langle\bar{n}n\rangle\langle\bar{\chi}\chi\rangle)]\\&+[(\bar{D}N)(\bar{t}t)\tfrac{1}{3}\sqrt{2}(\bar{d}n)_i{}^j(\bar{\chi}\chi)_i{}^j+(\bar{N}D)(\bar{t}t)\\&\times\tfrac{1}{3}\sqrt{2}(\bar{n}d)_j{}^i(\bar{\chi}\chi)_i{}^j]+\cdots\}. \quad (3.4)\end{aligned}$$

Here $C_0{}^{Bq}$ ($C_0{}^{B\bar{q}}$) are constants corresponding to the channel Fig. 2(f) [Fig. 2(g)]. The notation for the unitarity space is given in (3.2), and an analogous notation is used for the ordinary spin space, viz.,

$$\begin{aligned}&(\bar{\chi}\chi)_j{}^i=\bar{\chi}^i\chi_j-\tfrac{1}{2}\delta_j{}^i\langle\bar{\chi}\chi\rangle, &&\langle\bar{\chi}\chi\rangle=\bar{\chi}^k\chi_k,\\&(\bar{n}n)_j{}^i=\bar{n}^in_j-\tfrac{1}{2}\delta_j{}^i\langle\bar{n}n\rangle, &&\langle\bar{n}n\rangle=\bar{n}^kn_k, \quad (3.5)\\&(\bar{d}n)_j{}^i=\epsilon_{jk}\bar{d}^{ikl}n_l, &&(\bar{n}d)_j{}^i=\epsilon^{ik}\bar{n}^ld_{jkl}.\end{aligned}$$

In order to compare (3.4) with the static limit of (3.1), we must utilize the following identities:

$$\begin{aligned}\tfrac{1}{2}\langle\bar{n}\sigma_kn\rangle\langle\bar{\chi}\sigma_k\chi\rangle&=(\bar{n}n)_j{}^i(\bar{\chi}\chi)_i{}^j,\\(1/\sqrt{2})\langle\bar{d}_kn\rangle\langle\bar{\chi}\sigma_k\chi\rangle&=(\bar{d}n)_j{}^i(\bar{\chi}\chi)_i{}^j, \quad (3.6)\\(1/\sqrt{2})\langle\bar{n}d_k\rangle\langle\bar{\chi}\sigma_k\chi\rangle&=(\bar{n}d)_j{}^i(\bar{\chi}\chi)_i{}^j.\end{aligned}$$

Here d_k denotes the static limit of the Rarita-Schwinger spin-vector function. Now the comparison can be effected and we find the following relations:

$$\pm C_K{}^S+2iMC_{1,K}{}^V+C_{2,K}{}^V\{C_0{}^{Bq}\text{ or }C_0{}^{B\bar{q}}\}\times(\tfrac{1}{6},0,\tfrac{1}{6})\quad\text{for}\quad K=(F,D,S), \quad (3.7a)$$

$$\pm(-2MC_{1,K}{}^T+C_{2,K}{}^T)-C_{1,K}{}^A=\{C_0{}^{Bq}\text{ or }C_0{}^{B\bar{q}}\}\times(\tfrac{1}{9},\tfrac{1}{6},1/18)\quad\text{for}\quad K=(F,D,S), \quad (3.7b)$$

$$\pm C_2{}^T-2MC_2{}^A=0, \quad (3.7c)$$

$$\pm(-4iMC_1{}^T+C_3{}^T)-iC_1{}^A=\tfrac{1}{3}(C_0{}^{Bq}\text{ or }C_0{}^{B\bar{q}}). \quad (3.7d)$$

The upper (lower) sign corresponds to $C_0{}^{Bq}$ ($C_0{}^{B\bar{q}}$) and to the channel Fig. 2(f) [Fig. 2(g)]. The mass of the baryon is denoted by M.

A second set of relations arises from our requirement that in the nonstatic channels the vertex function must vanish in the static limit. This gives

$$C_{2,K}{}^V\pm C_{2,K}{}^T=0, \quad (3.8a)$$

$$(C_{1,K}{}^A-2iMC_{2,K}{}^A)\mp C_K{}^P=0, \quad (3.8b)$$

$$-C_2{}^T\mp 2iMC_4{}^T=0, \quad (3.8c)$$

$$C_3{}^T\pm 2MC_5{}^T=0. \quad (3.8d)$$

The upper (lower) signs arise from the channel Fig. 2(b) [Fig. 2(c)].

$$\begin{aligned}12iMC_{1,F}{}^V&=-9C_{1,F}{}^A=-6C_{1,D}{}^A=12iMC_{1,S}{}^V\\&=-18C_{1,S}{}^A=-18iMC_{2,F}{}^A\\&=-12iMC_{2,D}{}^A=C_+{}^{BM}, \quad (3.9a)\end{aligned}$$

$$\begin{aligned}6C_S{}^S=6C_F{}^S&=-18MC_{1,F}{}^T=-12MC_{1,D}{}^T\\&=-36MC_{1,S}{}^T=C_-{}^{BM},\\C_{1,D}{}^V=C_{2,K}{}^V&=C_{2,K}{}^T=C_K{}^P=C_D{}^S=0;\\-3iC_1{}^A&=C_+{}^{BM},\\-12iMC_1{}^T&=C_-{}^{BM}, \quad (3.9b)\\C_2{}^T=C_3{}^T&=C_4{}^T=C_5{}^T=C_2{}^A=0.\end{aligned}$$

Thus, all constants can be expressed in terms of the two parameters

$$C_\pm{}^{BM}\equiv\tfrac{1}{2}(C_0{}^{Bq}\pm C_0{}^{B\bar{q}}). \quad (3.9c)$$

Now we are prepared to apply our substitution law [cf. Eqs. (2.4) and (2.3)], and in so doing we finally obtain the effective Yukawa vertex function of the meson-baryon system:

$$\begin{aligned}A(p,p';Q)=\Big\{&\sum_{K=F,S}G_{1,K}{}^V\Big(\frac{1}{2iM}\Big)P_\mu(\bar{N}(p')N(p))_KiV_\mu(Q)+\sum_{K=F,D,S}G_{2,K}{}^V\Big(\frac{1}{2M}\Big)\epsilon_{\mu\nu\lambda\delta}Q_\nu P_\lambda(\bar{N}(p')\gamma_5\gamma_\delta N(p))_KiV_\mu(Q)\\&+\sum_{K=F,D,S}iG_K{}^PQ_\mu\Big[\bar{N}(p')\gamma_5\gamma_\mu\Big(1-\frac{Q^2}{4M^2}\Big)N(p)\Big]_KP(Q)\Big\}\\&+\Big\{G^AQ_\mu(\bar{D}_\mu(p')N(p))P(Q)+G^T\Big(\frac{1}{2M}\Big)\epsilon_{\mu\nu\kappa\lambda}P_\kappa Q_\mu(\bar{D}_\lambda(p')N(p))V_\nu(Q)+\text{H.c.}\Big\}+\cdots. \quad (3.10)\end{aligned}$$

Here the coupling constants obey the relations

$$6G_{1,F}{}^{V}=-9mG_F{}^P=-6mG_D{}^P=6G_{1,S}{}^V$$
$$=-18mG_S{}^P=-3mG^A=C_+{}^{BM}/K\,, \quad (3.11a)$$

$$9mG_{2,F}{}^V=6mG_{2,D}{}^V=18mG_{2,S}{}^V$$
$$=-6mG^T=C_-{}^{BM}/K\,, \quad (3.11b)$$

which follow from (3.9).

B. Comparison with Experiments

We first discuss the general structure of our effective vertex function (3.10) and (3.11).

(i) It is very interesting that in our scheme the vector mesons couple to octet baryons through a Sachs-type interaction, rather than through the usual Dirac-Pauli– (or vector-tensor–) type interaction. If we consider phenomenological effects of the form factor, the two different types are of course equivalent; however, they lead to different specific predictions if the coefficients G are restricted to be constants, as is our procedure. In the framework of the vector-meson dominance model, the two different types of vector-meson–octet-baryon couplings lead, of course, to the proportionality of the Sachs-type and vector-tensor–type electromagnetic form factors of the octet baryons.

(ii) The electric-type coupling of the vector mesons to the octet baryons is pure F coupling.[19] We recall that in any successful vector-meson dominance theory this is a necessity, because of charge universality (non-renormalization). On the other hand, we have a mixed D and F type for the magnetic coupling of vector mesons and octet baryons, with $D/F=\frac{3}{2}$.

Using the vector-meson dominance model, it can be easily shown[20] that the features summarized under (i) and (ii) lead to the familiar relations

$$\mu_p/\mu_n=-\tfrac{3}{2}\,,$$
$$G_M{}^p(Q^2)/\mu_p=G_M{}^n(Q^2)/\mu_n=G_E{}^p(Q^2)\equiv G(Q^2)\,, \quad (3.12)$$
$$G_E{}^n(Q^2)=0$$

between the proton and neutron magnetic moments and the proton and neutron magnetic and electric form factors. These relations are well borne out by experimental data. It is important to point out that (3.12) remains valid even if the mass splitting of the vector mesons is allowed for. This is so because, as will be discussed below under the paragraph marked (iv) and in Sec. V B, the φ meson does not couple[9,21,22] to the nucleon and because the ρ^0 and ω masses are practically equal.

In passing we note that (3.12) can be also derived[23] in other theories like $SU(6)_W$ or $M(12)$, *but only if it is assumed* that the electromagnetic current has some *higher than* $SU(3)$ transformation property. In our framework, we need only the *usual* $SU(3)$ assumption and the result is derived *dynamically*, resorting to the vector-meson model.

(iii) Our pseudoscalar meson-octet baryon interaction has both F and D couplings, with the familiar ratio $D/F=\frac{3}{2}$. This numerical value has been reconfirmed by a recent analysis[24] of data. It is also interesting to observe that the interaction involves an intrinsic form factor $(1-Q^2/4M^2)$. Of course, for the low-Q^2 region, where the extensive analysis of data was done, this factor does not lead to observable consequences.

(iv) As we shall discuss in Sec. V B, we use an extended nonet assumption. Consequently, the ratio of the unitary singlet-type (S) coupling constants for both the 0^- and 1^- nonets with the baryons relative to the corresponding F- and D-type unitary octet coupling constants is fixed, as seen from (3.11). From these relations we can derive that the φ meson with the ideal configuration[25] $V_3{}^3$ does not couple to the nucleon. As already pointed out, this result has an important relevance for the validity of (3.12).

After the general observations made above under (i) through (iv), we now make some numerical comparisons with experimental data on coupling constants.

From Eq. (3.11a) we have, in particular,

$$G_{1,F}{}^V=-\tfrac{3}{5}m(G_F{}^P+G_D{}^P)=-\tfrac{1}{2}mG^A. \quad (3.13a)$$

In terms of experimentally available quantities, $G_{1,F}{}^V$ can be represented by the $\rho\to 2\pi$ coupling constant $f_{\rho\pi\pi}$ if one describes the electromagnetic structure of the proton and of the pion by the vector-meson model[26] (cf. Fig. 4). The quantity $(G_F{}^P+G_D{}^P)$ is of course

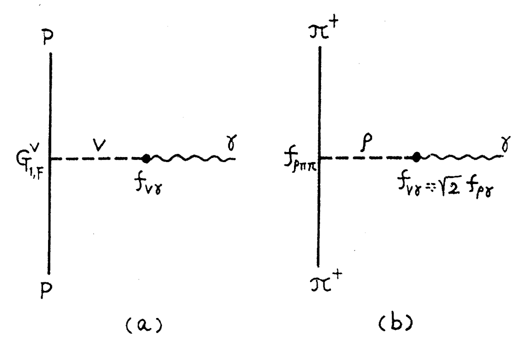

FIG. 4. Electromagnetic structure of proton and of pion.

[19] In the first term of (3.10), the summation *does not* include $K=D$.

[20] We mention here that there is a well-known difficulty in the vector-meson-dominance model when one wants to explain the experimentally observed absolute shape of $G(Q^2)$. A solution of this problem was proposed by S. Ishida, K. Konno, and H. Shimodaira, Nuovo Cimento **46A**, 189 (1966); and Progr. Theoret. Phys. (Kyoto) **36**, 1243 (1966). See also Y. Kinoshita, Y. Kobayashi, S. Machida, and M. Namiki, *ibid.* **36**, 107 (1966).

[21] H. Sugawara and F. von Hippel, Phys. Rev. **145**, 1331 (1966).

[22] S. Ishida, K. Konno, and H. Shimodaira, Ref. 20.

[23] See, for example, K. J. Barnes, P. Carruthers, and F. von Hippel, Phys. Rev. Letters **14**, 82 (1965); K. J. Barnes, Phys. Rev. **150**, 1331 (1966); or B. Sakita and K. C. Wali, *ibid.* **139**, B1355 (1965). See also the review article, Ref. 4.

[24] J. K. Kim, Phys. Rev. Letters **19**, 1074 (1967); **19**, 1079 (1967).

[25] G. Zweig, CERN Report, 1964 (unpublished); S. Ishida, Progr. Theoret. Phys. **32**, 922 (1964). See also Ref. 8.

[26] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).

directly related to the pion-nucleon coupling constant $f_{\pi N}$ where $(1/4\pi)f_{\pi N}^2=0.082$. The constant G^A is determined[27] from the decay width of N_{33}^*. With these considerations, (3.13a) can be expressed as

$$[\tfrac{5}{6}(m_\pi/m)f_{\rho\pi\pi}]^2=f_{\pi N}^2=\tfrac{1}{2}((5/6)m_\pi G^A)^2, \quad (3.13b)$$

where m_π is the pion mass. If we use the root-mean-square mass[28] of the 36-plet to represent m, i.e., set

$$m=\bar{m}=\left[\frac{1}{36}\sum_{i=1}^{36} m_i^2\right]^{1/2}=780 \text{ MeV}, \quad (3.14)$$

and if we adopt the experimental decay-width data[29] $\Gamma(\rho\to 2\pi)=157$ MeV, $\Gamma(N_{33}^*\to N\pi)=120$ MeV, then (3.13b) gives numerically

$$0.85=1.03=1.57.$$

The agreement is quite good.

Next we wish to check the coupling-constant relations (3.11b) against experimental data. In so doing, we again adopt the vector-meson model as a basis. Then we see that the equality between the first three quantities in (3.11b) leads to the familiar $\mu_p/\mu_n=-\frac{3}{2}$ magnetic-moment ratio, and to the decoupling of the φ meson and nucleon. These points have been already discussed above. The relation for G^T, which is also contained in (3.11b), leads to the usual $SU(6)$ prediction for the transition magnetic moment between the nucleon and the N_{33}^*. This prediction has been reasonably confirmed by experiment.[30] Our relation also predicts that the N^*-N transition form factor has the same form as that of the nucleon. For $Q^2<0.3(\text{GeV}/c)^2$, this has been also confirmed experimentally.[30] Thus, the relations (3.11b) are well supported by data.

IV. TRILINEAR MESON INTERACTION

In this section we shall apply our method to the second interesting example of a nonstatic vertex, viz., that between three meson 36-plets.

A. Derivation of the Vertex Function

By application of the fundamental substitution law to one of the participating mesons, the problem becomes very similar to the previous examples except that we must pay special attention to statistics, since we are treating a system of identical particles. The possible channels are again those of Fig. 2, where now the line 1 refers to a meson M, and the line 2 refers to the constituent quark q of the other meson. Assuming charge-conjugation invariance, and keeping only those terms which are permitted by statistics and anticipating our subsequent application of our substitution rule and of postulate 4 we can write, similarly as in the previous example, the general vertex function as

$$\begin{aligned}A(p,p';k,k')=&[C_1^S(P(p')P(p))_+ + C_2^S(V_\lambda(p')V_\lambda(p))_+]\bar{t}(k')t(k)\\&+[C_1^V(p-p')_\mu(P(p')P(p))_- + C_2^V(p-p')_\mu(V_\lambda(p')V_\lambda(p))_-]\bar{t}(k')\gamma_\mu t(k)\\&+[C_1^T\epsilon_{\mu\nu\kappa\lambda}p_\kappa(P(p')V_\lambda(p))_+ + C_2^T\epsilon_{\mu\nu\kappa\lambda}p_\kappa'(V_\lambda(p')P(p))_+ + C_3^T(V_\mu(p')V_\nu(p))_-]\bar{t}(k')\sigma_{\mu\nu}t(k)\\&\quad+[C_1^A\epsilon_{\mu\nu\kappa\lambda}(p-p')_\nu(V_\kappa(p')V_\lambda(p))_+ + C_2^A(P(p')V_\mu(p))_- + C_3^A(V_\mu(p')P(p))_-]\bar{t}(k')\gamma_5\gamma_\mu t(k). \quad (4.1)\end{aligned}$$

Here the abbreviation

$$(AB)_{+\beta}{}^\alpha\equiv A_\gamma{}^\alpha B_\beta{}^\gamma + A_\beta{}^\gamma B_\gamma{}^\alpha$$

is employed. We took p_μ and p_μ' for the two independent momenta rather than P_μ and Q_μ, so that the symmetry character is better displayed. As before, we regard the C coefficients as constants and try to find relations between them.

In the static channel the vertex function must have in the static limit the $SU(6)$-invariant form[10]

$$\begin{aligned}2\{&C_S^{Mq(M\bar{q})}\bar{t}^A t_B(M_C{}^B M_A{}^C + M_A{}^C M_C{}^B) + C_A^{Mq(M\bar{q})}\bar{t}^A t_B(M_C{}^B M_A{}^C - M_A{}^C M_C{}^B)\}\\&=C_S^{Mq(M\bar{q})}\bar{t}^\alpha t_\beta\{[(PP)_{+\alpha}{}^\beta + (V_kV_k)_{+\alpha}{}^\beta]\langle\bar{\chi}\chi\rangle + [(PV_k)_{+\alpha}{}^\beta + (V_kP)_{+\alpha}{}^\beta]\langle\bar{\chi}\sigma_k\chi\rangle - i\epsilon_{klm}(V_kV_l)_{-\alpha}{}^\beta\langle\bar{\chi}\sigma_m\chi\rangle\}\\&\quad+C_A^{Mq(M\bar{q})}\bar{t}^\alpha t_\beta\{[(PP)_{-\alpha}{}^\beta + (V_kV_k)_{-\alpha}{}^\beta]\langle\bar{\chi}\chi\rangle + [(PV_k)_{-\alpha}{}^\beta + (V_kP)_{-\alpha}{}^\beta]\langle\bar{\chi}\sigma_k\chi\rangle - i\epsilon_{klm}(V_kV_l)_{+\alpha}{}^\beta\langle\bar{\chi}\sigma_m\chi\rangle\}. \quad (4.2)\end{aligned}$$

Here $C_{S,A}^{Mq}$ ($C_{S,A}^{M\bar{q}}$) are constants corresponding to the channel Fig. 2(f) [Fig. 2(g)], and we have used the representation

$$M_B{}^A=P_\beta{}^\alpha\delta_j{}^i + V_{k,\beta}{}^\alpha(\sigma_k)_j{}^i \quad (4.3)$$

for the meson 36-plet.[10]

Taking now the static limit of (4.1) and comparing with (4.2) we get

$$\begin{aligned}\pm C_1^S=\pm C_2^S=\pm(-2imC_1^T)=\pm 2imC_2^T\\=\pm iC_3^T=C_S^{Mq} \text{ or } C_S^{M\bar{q}},\end{aligned}$$

$$\begin{aligned}2imC_1^V=2imC_2^V=-2imC_1^A=-iC_2^A\\=iC_3^A=C_A^{Mq} \text{ or } C_A^{M\bar{q}}, \quad (4.4)\end{aligned}$$

[27] M. Gourdin and Ph. Salin, Nuovo Cimento **27**, 193 (1963).

[28] S. Ishida and P. Roman, Phys. Rev. **159**, 1365 (1967).

[29] Throughout this paper, experimental data are taken from A. H. Rosenfeld, A. B. Galtieri, W. J. Podolsky, L. Price, P. Soding, C. G. Wohl, M. Ross, and W. J. Willis, Rev. Mod. Phys. **39**, 1 (1967). For $\Gamma(\rho\to 2\pi)$ we used the average of the ρ^+, ρ^-, ρ^0 data.

[30] W. W. Ash, K. Berkelman, C. A. Lichtenstein, A. Ramanauskas, and R. H. Siemann, Phys. Letters **24B**, 165 (1967).

where the upper (lower) sign corresponds to C^{Mq} ($C^{M\bar{q}}$) and to channel Fig. 2(f) [Fig. 2(g)].

The requirement of vanishing static limit in the nonstatic channels leads to the further relations

$$\pm C_1{}^S = \pm C_2{}^S = \pm C_1{}^T = \pm C_2{}^T = \pm C_3{}^T = 0, \quad (4.5)$$

where the upper (lower) sign arises from channel Fig. 2(d) [Fig. 2(e)].

From (4.4) and (4.5) we obtain

$$2imC_1{}^V = 2imC_2{}^V = -2imC_1{}^A = -iC_2{}^A = -iC_3{}^A = C_{A+}{}^{MM}, \quad (4.6)$$
$$C_1{}^S = C_2{}^S = C_1{}^T = C_2{}^T = C_3{}^T = 0.$$

All coupling constants are now expressed in terms of the single parameter

$$C_{A+}{}^{MM} \equiv \tfrac{1}{2}(C_0{}^{Mq} + C_0{}^{M\bar{q}}). \quad (4.6')$$

The analogous quantity $C_{A-}{}^{MM} \equiv \frac{1}{2}(C_0{}^{Mq} - C_0{}^{M\bar{q}})$ vanishes because, on account of a meson and its antimeson belonging to the same multiplet, the channels of Figs. 2(f) and 2(g) are identical, so that $C_A{}^{Mq} = C_A{}^{M\bar{q}}$. The second set of equalities in (4.6) can also be derived more generally from our requirement on nonstatic channels which forbids the appearance of the symmetric term on the left-hand side of Eq. (4.2), i.e., which leads to $C_S{}^{Mq} = C_S{}^{M\bar{q}} = 0$.

Finally, if we apply the substitution rule (2.4) and (2.3), we obtain the trilinear meson vertex function

$$A(p,p';Q) = G_{PPV}i(p-p')_\mu (P(p')P(p))_- V_\mu(Q) + G_{VVV}i(p-p')_\mu (V_\lambda(p')V_\lambda(p))_- V_\mu(Q) + G_{VVP}i\epsilon_{\mu\nu\kappa\lambda}Q_\mu(p-p')_\nu (V_\kappa(p')V_\lambda(p))_+ P(Q), \quad (4.7)$$

where, in consequence of (4.6), the coupling-constant relations

$$G_{PPV} = G_{VVV} = -mG_{VVP} = C_{A+}{}^{MM}/2imK \quad (4.8)$$

hold true.

B. Comparison with Experiments

We first note that our vertex function (4.7) and (4.8) contains only one free parameter and that the ratio of the unitary singlet-type coupling constant to the unitary octet-type coupling constant is fixed. This is again a consequence of the fact that we used the extended nonet assumption. Consequently, our theory will reproduce Okubo's results[8] on vector-meson decays, which are well borne out by experiments.

We also note that, as is seen from the first two terms of (4.7) and from $G_{PPV} = G_{VVV}$ [cf. (4.8)], the universality (nonrenormalization) of the electric charge for pseudoscalar and vector mesons is guaranteed in the vector-dominance model.

In order to make a numerical check on (4.8), we consider now, in particular, the processes $\rho \to 2\pi$ and $\omega \to \rho + \pi$. Extracting the relevant part of the vertex function, we get from (4.7)

$$A(p,p';Q) = if_{\rho\pi\pi}\epsilon_{ijk}(p-p')_\mu \pi_i(p')\pi_j(p)\rho_{k,\mu}(Q) + if_{\omega\rho\pi}\epsilon_{\mu\nu\kappa\lambda}Q_\nu(p-p')_\mu \omega_\kappa(p')\rho_{i,\lambda}(p)\pi_i(Q), \quad (4.9)$$

where we assumed the ideal configuration[25]

$$(1/\sqrt{2})(V_1{}^1 + V_2{}^2)$$

for the ω. Because of (4.8), we have the relation

$$(2/m)f_{\rho\pi\pi} = f_{\omega\rho\pi}. \quad (4.10)$$

We note here that essentially the same relation has been derived[4] also from $SU(6)_W$ and $M(12)$. For experimental comparison, we take $f_{\rho\pi\pi}$ from $\Gamma(\rho \to 2\pi) = 157$ MeV, as before,[29] which gives

$$f_{\rho\pi\pi}{}^2/4\pi = 3.02. \quad (4.11)$$

The $f_{\omega\rho\pi}$ coupling constant can be determined from the Gell-Mann–Sharp–Wagner[26] model. Using the experimental value[29] $\Gamma(\omega \to 3\pi) = 11$ MeV and combining it with (4.11), we get

$$m_\pi{}^2 f_{\omega\rho\pi}{}^2/4\pi = 0.37. \quad (4.12)$$

Finally, we use for the central mass m the value given by (3.14). If these data are substituted into the squared Eq. (4.10), i.e., into the relation

$$4(f_{\rho\pi\pi}{}^2/4\pi)(m_\pi/m)^2 = (f_{\omega\rho\pi}{}^2/4\pi)m_\pi{}^2,$$

then we get a very good agreement: Both sides give 0.37.

V. SOME RELATED PROBLEMS AND DISCUSSION

A. Static Vertex

We conclude this paper with a few brief comments. So far we have concentrated on nonstatic vertices. But also for static vertices (where we do not even need the somewhat speculative assumption of postulate 3 on fundamental substitution), our scheme provides a more clearcut method than the standard approach. We illustrate this point on the example of the vertex with two baryon and two meson lines.

In our framework we treat the static-channel process $(B+M \to B+M)$ and the nonstatic-channel process $(B+\bar{B} \to M+M)$ in a unified manner, whereas in the customary approach they are treated separately. From our scheme it is evident that the usual $SU(6)$ analysis of meson-baryon scattering[31] is valid only in the static region, whereas in the standard treatments no attention is paid to the region of the variables. Our viewpoint is supported by the results of experimental analysis,[32] according to which the $SU(6)$-symmetry predictions are better satisfied near threshold (and for the very-high-energy region) than they are elsewhere. Furthermore

[31] See, for example, V. Barger and M. H. Rubin, Phys. Rev. Letters 14, 713 (1965).
[32] T. Binford, D. Cline, and M. Olsson, Phys. Rev. Letters 14, 715 (1965).

the standard analysis[33] of B-$\bar{B}$ annihilation *at rest* into $M+M$ is performed by *assuming* the transformation property of the amplitude, because $SU(6)$ symmetry actually forbids that process. Our scheme, on the other hand, leads to the same results regarding $B+\bar{B}\rightarrow M+M$ at rest, *without* assuming anything about $SU(6)$ violation.

At this point, it may be worthwhile to make a comment concerning the so-called kinematic spurion method. In this method, special transformation properties of the spurion must be assumed. In our framework, however, we do not have to say anything about symmetry *breaking*. We talk only about symmetry properties that are strictly observed. If one insists on using spurion terminology, he may say that for *static* problems our theory *predicts* the transformation property of the spurion [35-plet of $SU(6)$].

B. Extended Nonet Assumption

Throughout this paper we tacitly used the following: *Assumption:* The $SU(6)$ singlet and 35-plet must be always treated as a single, 36-dimensional multiplet.

If we do not have this assumption, there exists another term[34] with a trace on the constituent quarks ($\bar{t}^A t_A$) in the $SU(6)$-invariant forms (2.8), (3.4), and (4.2). However, this affects only our conclusion for the electric coupling of the unitary singlet vector meson.

Our above theorem may be regarded as the $SU(6)$ analog of Okubo's nonet assumption[8] in $SU(3)$ symmetry. But we note that the standard nonet assumption has a rather arbitrary character and that it is incomplete because it gives different results for the baryon-meson vertex, depending[21] upon whether one uses the octet representation $N_\beta{}^\alpha$ or the quark representation $B_{\alpha\beta\gamma}=\epsilon_{\alpha\beta\delta}N_\gamma{}^\delta$ of the baryons. There is no similar problem in our case.

Iizuka[9] has proposed recently an interesting rule for hadron interactions, called "selection principle," which includes the nonet assumption. Our above assumption reproduces the implications of Iizuka's selection principle for the mesons and baryons.

C. Necessity of Postulate 4

In Secs. III and IV we used the somewhat obscure postulate 4, which was enunciated at the end of Sec. II. If we do not accept this postulate, results concerning static problems will of course not change. However, without postulate 4, many desirable results, such as electromagnetic form-factor relations (3.12) for the nucleon (with the obvious exception of the first, static relation) cannot be derived. These comments follow clearly from the analysis of Sec. II.

D. Concluding Remarks

The present study was based solely on the four postulates which were formulated in Sec. II. The physical reason for these postulates is certainly not clear. However, we saw that they provide a good framework for a remarkably successful and consistent treatment of higher symmetry. We may therefore consider our postulates as clues for a future theory. Such a future theory will presumably clarify the properties of the quark which, from the viewpoint of the present theory, has rather strange properties.

In our theory properties in the static limit play an important role. We may therefore suspect that the expected theory of the future could be closely connected with the extended-particle models, in which presumably only quantities with nonvanishing static limit play an essential role.[35]

Finally, we could comment on the relationship between our work and the investigations based on the $SU(6)_W$ or the $M(12)$ symmetry. Many of our relations obtained by our postulates are the same as or similar to those obtained by $SU(6)_W$ or $M(12)$. We do not have a simple explanation for this occurrence. One possible reason is that all these theories, as well as ours, are Lorentz covariant. It also appears that our substitution law plays a crucial role. On the other hand, it should be stressed that in our work we do not need the complex mathematical language as is needed in the $SU(6)_W$ or $M(12)$ framework. All we need to establish our results is standard $SU(6)$ and a few additional *physical* postulates.

[33] F. J. Dyson and H. N. Xuong, Phys. Rev. Letters **14**, 654 (1965); M. Konuma and E. Remiddi, *ibid.* **14**, 1082 (1965).

[34] We are obliged to Professor Z. Maki (Kyoto) and Professor J. Iizuka (Nagoya) for critical comments on this point.

[35] In this connection, we call attention to the work by O. Hara, T. Goto, S. Y. Tsai, and H. Yabuki, Progr. Theoret. Phys. (Kyoto) **39**, 203 (1968).