Bootstrap Calculation of Vector-Meson–Baryon Coupling Constants*†

G. L. KANE
Physics Department, University of Michigan, Ann Arbor, Michigan

AND

W. F. PALMER‡
Physics Department, The John Hopkins University, Baltimore, Maryland

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We perform a bootstrap calculation designed to study the vector-meson–baryon coupling constants. The vector mesons appear as baryon-antibaryon bound states, due to vector- and pseudoscalar-meson exchanges, so that the self-consistency between the couplings at the exchanges and the couplings that emerge at the residues allows the $VB\bar{B}$ coupling constants (including d/f ratios) to be calculated. The problem is studied both in the static model [no $SU(3)$ self-consistent solutions exist] and in the relativistic case with pseudoscalar-meson exhange contributing an inhomogeneous driving term, where approximately self-consistent solutions do exist.

I. INTRODUCTION

OVER the past years, a number of bootstrap calculations have been performed. Originally, many theorists had hoped that masses, coupling strengths, perhaps quantum numbers, and possibly even the existence of the hadrons would emerge from a bootstrap "program"; it is clear that there has been little progress in this direction. On the other hand, a number of bootstrap calculations with somewhat more modest aims have been partially successful in calculating coupling constants and coupling-constant ratios and have shown at least that the strong interactions are not inconsistent with the bootstrap dynamics, if not entirely determined by them.

We have performed a calculation that fits in this last category. We have studied the vector mesons as bound states of baryon-antibaryon pairs, mainly in hopes of determining their coupling strengths to the baryons, particularly the d/f ratios. Aside from their intrinsic interest within the bootstrap program, these couplings are interesting for two other reasons. First, it is difficult to obtain experimental values for essentially all of them, even the simplest coming only from complicated analyses involving, for example, models of electromagnetic form factors, so that a meaningful set of theoretical values would be useful. Second, there is no self-consistent static-model solution for these couplings. The forces are attractive, and either using relativistic kinematics (so that the solutions depend on mass ratios) or adding inhomogeneous driving terms (such as pseudoscalar exchange) is sufficient to give (approximately) self-consistent solutions. One might have expected that the elegance of the usual theory with pure F-type charge coupling would reflect itself in a static-model bootstrap solution of this sort. The absence of the static-model solution is particularly interesting because our full solutions are indeed generally consistent with pure F-type charge coupling.

Conceptually, our calculation is simple. We consider the $\bar{B}B$ elastic scattering amplitude in the neighborhood of the vector-meson pole. We assume that the main contributions to this amplitude come from the vector-meson exchange itself and from pseudoscalar-meson exchange. The positions of the vector-meson poles in the $\bar{B}B$ amplitude and the residues there are thus given in terms of themselves and the pseudoscalar parameters, and they can be determined.

A number of other calculations relevant to these couplings have been performed. Most recently, Ball and Parkinson[1] have studied the ρ as a bound state or resonance in a multichannel context and concluded that about 40% of the ρ is $N\bar{N}$; if the ρ is considered in any single-channel context, therefore, $N\bar{N}$ is probably the dominant channel. Carruthers and Krisch[2] have considered the amplitude for $\bar{B}B \to V \to PP$, with a vector-meson pole coupled to $\bar{B}B$ on one side and to two pseudoscalars on the other. The driving forces are then PB elastic scattering and can be taken as known, so that one can obtain the $V\bar{B}B$ couplings. They thus obtain them immediately in terms of the PB amplitudes and the $V \to PP$ width, whereas we have a self-consistency condition to satisfy to determine our results. Along still different lines, Ball, Scotti, and Wong[3] have studied NN scattering, using experimental data to determine the parameters involved in the vector- and pseudoscalar-meson exchanges. Then they used crossing to determine the $N\bar{N}$ amplitudes and studied the meson poles that resulted. Finally, Arnold[4] has studied the $N\bar{N}$ meson bootstrap, exchanging s-wave mesons and emphasizing the Regge cutoff of the high-energy behavior.

In Sec. II, we present the details of the calculation of

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† Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy at The Johns Hopkins University.
‡ Present address: Argonne National Laboratory, Argonne, Ill.

[1] J. Ball and M. Parkinson, Phys. Rev. **162**, 1509 (1967).
[2] P. Carrothers and J. P. Krisch (to be published).
[3] J. S. Ball, A. Scotti, and D. Y. Wong, Phys. Rev. **142**, 1000 (1966).
[4] R. C. Arnold, Nuovo Cimento **37**, 589 (1965).

the pole terms and crossing matrices. Section III is devoted to the various static-model calculations that are relevant, and Sec. IV to the discussion of the full calculation, including a summary of our results [Eqs. (4.12)]. Section V surveys the effects of other channels coupled to $B\bar{B}$. Two Appendices are included to make the text more readable.

II. KINEMATICS, SPACE-TIME SYMMETRIES, AND INTERNAL COORDINATES

A. Kinematical Notation and Choice of Amplitude

We use (p,λ) to denote baryon or antibaryon 4-momentum and helicity, with a metric such that $p^2=\bar{p}^2-E^2$. (See Fig. 1.) Subscripts refer to isospin or $SU(3)$ indices. The S-matrix element for the transition $\bar{B}B\leftrightarrow\bar{B}B$ can be written

$$S_{fi}=\delta_{fi}+i(2\pi)^4\frac{\delta^4(p_1+p_2-p_3-p_4)}{(\pi_i 2E_i)^{1/2}}\mathfrak{F}_{fi}, \qquad (2.1)$$

where (f,i) label final and initial states, normalized to a δ function. The polarized differential cross section in the c.m. system is given by

$$d\sigma/d\Omega=|\mathfrak{F}|^2/(8\pi)^2 s, \qquad (2.2)$$

where $\mathfrak{F}$ is a Lorentz scalar function of the scalar invariants

$$\begin{aligned} s&=-(p_1+p_2)^2=W^2=4E^2=4p^2+4M^2,\\ t&=-(p_1-p_3)^2=-2p^2(1-\cos\theta),\\ u&=-(p_1-p_4)^2=-2p^2(1+\cos\theta),\\ s&+t+u=4M^2. \end{aligned} \qquad (2.3)$$

Here W, E, p, and θ are, respectively, the total c.m. energy, the c.m. energy and 3-momentum of a single baryon, and the c.m. scattering angle. (Note that in these conventions the s and t channels describe $\bar{B}B$ scattering, whereas the u channel describes BB scattering.) The following relations hold between $\mathfrak{F}$ and the amplitudes τ and $\langle\lambda_3\lambda_4|\phi|\lambda_1\lambda_2\rangle$ defined by Goldberger, Grisaru, MacDowell, and Wong[5]:

$$\begin{aligned} &\mathfrak{F}=4M^2\tau=16\pi E\langle\lambda_3\lambda_4|\phi|\lambda_1\lambda_2\rangle,\\ &\frac{d\sigma}{d\Omega}=\frac{1}{(8\pi)^2 s}|\mathfrak{F}|^2=|\langle\lambda_3\lambda_4|\phi|\lambda_1\lambda_2\rangle|^2. \end{aligned} \qquad (2.4)$$

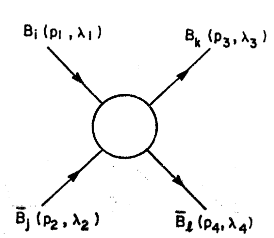

FIG. 1. $\bar{B}B$ kinematics. Latin subscripts refer to $SU(3)$ coordinates.

[5] M. L. Goldberger, M. T. Grisaru, S. W. MacDowell, and D. Y. Wong, Phys. Rev. **120**, 2250 (1960).

B. Partial-Wave Decomposition

Following the method of Jacob and Wick,[6] we construct the partial-wave series and its inverse

$$\langle\lambda_3\lambda_4|\mathfrak{F}|\lambda_1\lambda_2\rangle=\sum_J(2J+1)\langle\lambda_3\lambda_4|\mathfrak{F}^J|\lambda_1\lambda_2\rangle d_{\lambda\lambda'}{}^J(\mu),$$
$$\langle\lambda_3\lambda_4|\mathfrak{F}^J|\lambda_1\lambda_2\rangle=\tfrac{1}{2}\int_{-1}^{1}d_{\lambda\lambda'}{}^J(\mu)\langle\lambda_3\lambda_4|\mathfrak{F}|\lambda_1\lambda_2\rangle, \qquad (2.5)$$

with the definitions $\lambda=\lambda_1-\lambda_2$, $\lambda'=\lambda_3-\lambda_4$, and $\mu=\cos\theta$.

The elastic unitarity condition ($\lambda_1=\lambda_3$, $\lambda_2=\lambda_4$) on the $\mathfrak{F}^J$ amplitude is given by

$$\mathrm{Im}\mathfrak{F}^J(s)=(p/16\pi E)|\mathfrak{F}^J(s)|^2,\quad s>4M^2. \qquad (2.6)$$

We shall use the amplitude $T^J=\mathfrak{F}^J/16\pi$, with respect to which the elastic unitarity condition takes its conventional form

$$\mathrm{Im}T^J(s)=(p/E)|T^J(s)|^2. \qquad (2.7)$$

C. Symmetries and Angular Momentum

Parity and time-reversal invariance of the T matrix imply, respectively,

$$\langle\lambda_3\lambda_4|\mathfrak{F}^J|\lambda_1\lambda_2\rangle=\langle-\lambda_3-\lambda_4|\mathfrak{F}^J|-\lambda_1-\lambda_2\rangle, \qquad (2.8)$$
$$\langle\lambda_3\lambda_4|\mathfrak{F}^J|\lambda_1\lambda_2\rangle=\langle\lambda_1\lambda_2|\mathfrak{F}^J|\lambda_3\lambda_4\rangle. \qquad (2.9)$$

For identical-particle elastic scattering (e.g., $\bar{p}p\to\bar{p}p$) or for $SU(2)$ or $SU(3)$ eigenamplitudes, we can use charge-conjugation invariance and Fermi statistics to show that

$$\langle\lambda_3\lambda_4|\mathfrak{F}^J|\lambda_1\lambda_2\rangle=\langle\lambda_4\lambda_3|\mathfrak{F}^J|\lambda_2\lambda_1\rangle. \qquad (2.10)$$

We are left with five independent helicity transitions[5] among the possible 16 ($\langle\pm\pm|\mathfrak{F}^J|\pm\pm\rangle$):

$$\begin{aligned} \mathfrak{F}_1{}^J&\equiv\langle++|\mathfrak{F}^J|++\rangle,\\ \mathfrak{F}_2{}^J&\equiv\langle++|\mathfrak{F}^J|--\rangle,\\ \mathfrak{F}_3{}^J&\equiv\langle+-|\mathfrak{F}^J|+-\rangle,\\ \mathfrak{F}_4{}^J&\equiv\langle+-|\mathfrak{F}^J|-+\rangle,\\ \mathfrak{F}_5{}^J&\equiv\langle++|\mathfrak{F}^J|+-\rangle. \end{aligned} \qquad (2.11)$$

Linear combinations of these amplitudes represent the orbital transitions

$$\begin{aligned} &\text{singlet}\quad J=l\leftrightarrow J=l,\\ &\text{triplet}\quad J=l\leftrightarrow J=l,\\ &\qquad\qquad J=l+1\leftrightarrow J=l+1,\\ &\qquad\qquad J=l-1\leftrightarrow J=l+1,\\ &\qquad\qquad J=l-1\leftrightarrow J=l-1. \end{aligned} \qquad (2.12)$$

The $\bar{B}B$ system has parity $(-)^{l+1}$. We are interested in $J^P=1^-$ transitions, which restricts us to the coupled

[6] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

triplet $J=l\pm 1$ amplitudes, with $l=0$ (S wave) or $l=2$ (D wave). In terms of helicity states $|J\lambda_1\lambda_2\rangle$, we have[6]

$$P|J\lambda_1\lambda_2\rangle=(-)^J|J-\lambda_1-\lambda_2\rangle, \tag{2.13}$$

which allows the $J^P=1^-$ states

$$\begin{aligned}(\sqrt{\tfrac{1}{2}})\{|++\rangle+|--\rangle\}&\equiv|h_1\rangle,\\(\sqrt{\tfrac{1}{2}})\{|+-\rangle+|-+\rangle\}&\equiv|h_2\rangle.\end{aligned} \tag{2.14}$$

Both are triplet states. Neither is an eigenstate of orbital angular momentum.

The transformation between orbital angular momentum $J^P=1^-$ states ($|S\rangle$ and $|D\rangle$ for $l=0$ and $l=2$, respectively) and the helicity states $|h_i\rangle$ is given by the $SU(2)$ Clebsch-Gordan (CG) coefficient

$$\langle JJ_zls|JJ_zs_{1z}s_{2z}\rangle=\left(\frac{2l+1}{2J+1}\right)^{1/2}\langle JJ_z|lsl_zs_z\rangle \times\langle ss_z|s_1s_2s_{1z}s_{2z}\rangle,$$

where, in this case, $J=1$, $l=0$ or 2, $l_z=0$, $J_z=\lambda_1-\lambda_2$, $s_{1z}=\lambda_1$, and $s_{2z}=-\lambda_2$. That is,

$$\begin{aligned}|S\rangle&=(\sqrt{\tfrac{1}{3}})|h_1\rangle+(\sqrt{\tfrac{2}{3}})|h_2\rangle,\\|D\rangle&=-(\sqrt{\tfrac{2}{3}})|h_1\rangle+(\sqrt{\tfrac{1}{3}})|h_2\rangle.\end{aligned} \tag{2.15}$$

Writing $\mathfrak{F}_{ll'}$ for orbital transitions and $\mathfrak{F}_{ij}{}^J=\langle h_i|\mathfrak{F}^J|h_j\rangle$ for helicity transitions, one has the result

$$\begin{bmatrix}\mathfrak{F}_{SS} & \mathfrak{F}_{SD}\\ \mathfrak{F}_{DS} & \mathfrak{F}_{DD}\end{bmatrix}=\frac{1}{3}\begin{bmatrix}\mathfrak{F}_{11}+2\mathfrak{F}_{22}+2\sqrt{2}\mathfrak{F}_{12} & -\sqrt{2}\mathfrak{F}_{11}-\mathfrak{F}_{12}+\sqrt{2}\mathfrak{F}_{22}\\ -\sqrt{2}\mathfrak{F}_{11}-\mathfrak{F}_{12}+\sqrt{2}\mathfrak{F}_{22} & 2\mathfrak{F}_{11}-2\sqrt{2}\mathfrak{F}_{12}+\mathfrak{F}_{22}\end{bmatrix}. \tag{2.16}$$

D. $SU(3)$ and $\bar{B}B$ System

We use the Gell-Mann states[7] B_i, V_i and the Tarjanne[8] F, D matrices. (See Appendix A, where commutation relations, trace properties, and symmetry properties are tabulated.) Normalized singlet and octet states are given by

$$\begin{aligned}|0\rangle&=(1/\sqrt{8})\delta_{ij}|\bar{B}_iB_j\rangle,\\|8_s{}^k\rangle&=(\sqrt{3/20})D_{ij}{}^k|\bar{B}_iB_j\rangle,\\|8_a{}^k\rangle&=(i/\sqrt{12})F_{ij}{}^k|\bar{B}_iB_j\rangle.\end{aligned} \tag{2.17}$$

With the phases of (2.17) the octet-meson annihilation matrix has the form

$$T\sim\begin{pmatrix}(20/3)d^2 & (-4\sqrt{5})fd\\ (-4\sqrt{5})fd & 12f^2\end{pmatrix}\frac{1}{m_V{}^2-s}. \tag{2.18}$$

The invariant couplings to unitary singlet and unitary octet vector mesons are

$$\mathfrak{L}/i(4\pi)^{1/2}=g_8G_{ij}{}^k\bar{B}_iB_jV_k+g_0\bar{B}_iB_iV_0. \tag{2.19}$$

The CG coefficient $G_{ij}{}^k$ is Hermitian and allows for an arbitrary mixture of symmetric and antisymmetric coupling:

$$\begin{aligned}G_{ij}{}^k(f)&\equiv-ifF_{ij}{}^k+dD_{ij}{}^k\\&=2f\mathfrak{F}^k+2d\mathfrak{D}^k\\&=2[(1-\alpha)\mathfrak{F}^k+\alpha\mathfrak{D}^k],\end{aligned} \tag{2.20}$$

where $\mathfrak{F}$ and $\mathfrak{D}$ are the usual F and D matrices of Gell-Mann, and α is the Gell-Mann mixing parameter $(1-\alpha=f)$.

When the Lagrangian (2.19) is decomposed into couplings among isospin multiplets, the $\rho\bar{N}N$ term is

$$\mathfrak{L}/i(4\pi)^{1/2}=g_8\bar{N}_1\tau_{ij}{}^kN_j\rho_k+\cdots, \tag{2.21}$$

and g_8 is identical to the conventionally quoted (unrationalized) ρNN coupling constant.

We now tabulate some useful $SU(3)$ crossing coefficients. Consider the diagrams of Fig. 2. When we calculate such terms in $SU(3)$ eigenamplitudes, the result can be split into a CG part and a dynamical part. For illustration, consider the octet exchange of Fig. 3(b) in the symmetric octet amplitude:

$$\begin{aligned}\langle 8^s\bar{B}B|\mathfrak{F}|8^s\bar{B}B\rangle&=[(3/160)D_{lm}{}^kD_{ij}{}^kG_{il}{}^n(G_{jm}{}^n)^*]\\&\quad\times4\pi g_8{}^2\bar{u}_3\gamma_\mu u_1\bar{v}_2\gamma_\mu v_4/(m^2-t)\\&=[6f^2-2d^2]\\&\quad\times4\pi g_8{}^2\bar{u}_3\gamma_\mu u_1\bar{v}_2\gamma_\mu v_4/(m^2-t)\\&=[\text{CG part}]\times[\text{dynamical part}]\\&\equiv\langle 8^s\|8^s\rangle\times\langle\lambda_3\lambda_4\|\mathfrak{F}\|\lambda_1\lambda_2\rangle.\end{aligned} \tag{2.22}$$

The CG content of various relevant exchange and annihilation terms is tabulated in Table I. They are calculated in Appendix A. [These coefficients are merely elements of the $SU(3)$ 8×8 crossing matrix, aside from various normalization factors.]

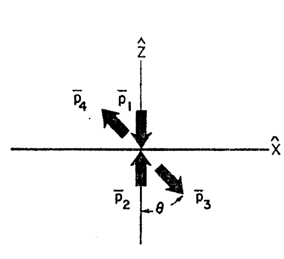

FIG. 2. Kinematics in the c.m. system.

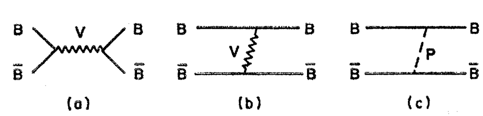

FIG. 3. Direct-channel pole term and Born exchange amplitudes.

[7] M. Gell-Mann and Y. Ne'eman, *The Eightfold Way* (W. A. Benjamin, Inc., New York, 1964).

[8] Pekka Tarjanne, Ann. Acad. Sci. Fennicae Ser. A VI, No. 105, 1 (1962).

Inspection of Table I yields the octet-pole matrix (2.18) which has the form $T_{ij}=T_iT_j$; that is, the residues factor, T_{ij}, has only one nonvanishing eigenvalue, and only one of the eigenphase shifts resonates, implying a definite mixture of symmetric and antisymmetric coupling, as we expect from (2.19). Complications arising from the existence of two independent dynamical couplings are discussed in a following section.

E. Dynamical Couplings and $SU(3)$ Coordinates Combined

The following phenomenological Lagrangians, when applied in second-order perturbation theory, serve to define the relationship between the coupling constants and the residues at the pole corresponding to the particle exchanged or produced in intermediate states:

(a) pseudoscalar octet coupling:

$$\mathcal{L}/i(4\pi)^{1/2}=g_{N\pi}G_{ij}{}^k(f_P)\bar{B}_i\gamma_5B_jP_k=g_{N\pi}\bar{N}_i\tau_{ij}{}^k\gamma_5N_j\pi_k+\text{other terms};\quad(2.23)$$

(b) vector-meson octet and singlet couplings—Dirac (vector) interaction:

$$\mathcal{L}/i(4\pi)^{1/2}=g_{V8}G_{ij}{}^k(f_V)\bar{B}_i\gamma_\mu B_jV_k{}^\mu+g_{V0}\bar{B}_i\gamma_\mu B_iV_0{}^\mu;\quad(2.24)$$

(c) vector-meson octet and singlet couplings—Pauli (tensor) interaction:

$$\mathcal{L}/i(4\pi)^{1/2}=g_{T8}G_{ij}{}^k(f_T)\bar{B}_i(\sigma_{\mu\nu}/2M)B_j\partial_\mu V_k{}^\nu+g_{T0}\bar{B}_i(\sigma_{\mu\nu}/2M)B_i\partial_\mu V_0{}^\nu.\quad(2.25)$$

F. More about Couplings (a)–(c)

The single pseudoscalar coupling corresponds to the single dynamical combination of $\bar{B}B$ pairs ($^1S_0$) which can form a $J^P=0^-$ state. The two vector-meson couplings (actually linear combinations of the ones written down) correspond to the two dynamical combinations of $\bar{B}B$ pairs ($^3S_1$,$^3D_1$) which can form a $J^P=1^-$ state.

The pseudoscalar coupling constant is well known from the application of forward dispersion relations to experimental data. The mixing parameter f_P is the subject of an $SU(6)$ prediction ($f_P=0.4$) and is an output of the Martin and Wali $SU(3)$ extension of the Chew-Low model ($f_P\approx0.25$).

The nucleon ρ-meson (Dirac) coupling constant is not well known experimentally, but if one accepts the Sakurai universal coupling of the ρ meson to the isospin current, we have

$$\mathcal{L}/i(4\pi)^{1/2}\sim G[\bar{N}\tfrac{1}{2}\tau\gamma_\mu N+\pi\times\partial_\mu\pi+\cdots]\cdot\rho_\mu,$$
$$g_{N\rho}{}^2=(\tfrac{1}{2}G)^2=\tfrac{1}{4}g_{\rho\pi}{}^2\approx\tfrac{3}{4},$$

where $g_{\rho\pi}{}^2\sim3$ is based on a ρ width of $\Gamma\sim150$ MeV. Phenomenological analyses suggest a somewhat stronger coupling; for example, Ball, Scotti, and Wong[3] predict

TABLE I. Some useful $SU(3)$ crossing coefficients.

$SU(3)$ representation	Direct pole	Octet exchange	Singlet exchange
1	$8g_{V0}{}^2$	$[12f^2+(20/3)d^2]g_{V8}{}^2$	$g_{V0}{}^2$
$8_s\leftrightarrow8_s$	$(20/3)d^2g_{V8}{}^2$	$[6f^2-2d^2]g_{V8}{}^2$	$g_{V0}{}^2$
$8_a\leftrightarrow8_a$	$12f^2g_{V8}{}^2$	$[6f^2+(10/3)d^2]g_{V8}{}^2$	$g_{V0}{}^2$
$8_s\leftrightarrow8_a$	$-(4\sqrt{5})fdg_{V8}{}^2$	$(4\sqrt{5})fdg_{V8}{}^2$	0
10	$\cdots$	$-(8/3)d^2g_{V8}{}^2$	$g_{V0}{}^2$
$\overline{10}$	$\cdots$	$-(8/3)d^2g_{V8}{}^2$	$g_{V0}{}^2$
27	$\cdots$	$[-4f^2+\frac{4}{3}d^2]g_{V8}{}^2$	$g_{V0}{}^2$

$g_{N\rho}{}^2$(Dirac)$\simeq2.0$, based on a phase-shift analysis of NN data.

The vector-coupling $SU(3)$ mixing parameter f_V is also not well known experimentally, but there are theoretical reasons for favoring $f_V=1$. If one infers the couplings from a gauge principle, then one expects that the $SU(3)$ current, a part of which is $F_{ij}{}^k\bar{B}_i\gamma_\mu B_j$, must be coupled to the octet gauge field $V_k{}^\mu$ and we have the $f_V=1$ prediction. One can also demand photon–neutral ρ-meson universality; then one must *not* have a $\rho^0\Sigma^0\bar{\Lambda}$ Dirac coupling, or else the "photonlike" ρ^0 would couple to a neutral current. Such a coupling vanishes only if $f_V=1$.

The ratio $g_{N\rho}$(Pauli)$/g_{N\rho}$(Dirac)$\equiv g_{T8}/g_{V8}=3.7$ is predicted by an electromagnetic form-factor analysis of the nucleons if it is assumed that the isovector part is dominated by the ρ meson. Note that the ratio of S/D wave at the vector-meson pole is controlled by the g_T/g_V ratio [at least in the case of $SU(2)$-invariant $\bar{N}N$ couplings; $SU(3)$ complications are discussed below]. For pure S-wave ρ mesons coupling to nucleons, we have

$$g_T/g_V=m_V/2M_N\approx2.4$$

("S-wave dominance"). The remaining coupling constants are relatively unknown.

G. Reduced Amplitudes

In Appendix B, we write down the single-meson-exchange and annihilation "reduced" amplitude $\langle\lambda_3\lambda_4\|\mathfrak{F}\|\lambda_1\lambda_2\rangle$, defined above, and their partial-wave projection in $J=1$ defined in (2.5). For the partial-wave amplitude, the rotation to an orbital basis defined by (2.16) has also been performed. Note the relative sign difference from what might be expected between the exchange and annihilation terms; that is, the reduced Dirac coupling exchange term is $4\pi g_V{}^2\bar{u}_3\gamma_\mu u_1\bar{v}_2\gamma_\mu v_4\times(m_V{}^2-t)^{-1}$, whereas the annihilation term is $-4\pi g_V{}^2\times\bar{u}_3\gamma_\mu v_4\bar{v}_2\gamma_\mu u_1(m_V{}^2-s)^{-1}$. The reduced amplitude of Appendix B, when combined with the crossing coefficients of Appendix A, yields the full amplitude indicated in (2.22). There are some subtleties associated with the cross-coupled vector-tensor terms in the $SU(3)$ octet channels, which we illustrate by calculating the vector-octet-exchange amplitude $\langle 8^s\lambda_3\lambda_4|\mathfrak{F}|8^s\lambda_1\lambda_2\rangle$:

$$\begin{aligned}\langle 8^s\lambda_3\lambda_4|\mathfrak{F}|8^s\lambda_1\lambda_2\rangle=&\tfrac{1}{8}\times(3/20)D_{ij}{}^kD_{lm}{}^k\{4\pi g_{V8}{}^2G_{il}{}^n(f_V)G_{mj}{}^n(f_V)\bar{u}_3\gamma_\mu u_1\bar{v}_2\gamma_\mu v_4/(m_{V8}{}^2-t)\\&+[4\pi g_{V8}g_{T8}/2M(m_{V8}{}^2-t)][G_{il}{}^n(f_V)G_{mj}{}^n(f_T)\bar{u}_3\sigma_{\mu\nu}q_\nu u_1\bar{v}_2\gamma_\mu v_4+G_{il}{}^n(f_T)G_{mj}{}^n(f_V)\bar{u}_3\gamma_\mu u_1\bar{v}_2\sigma_{\mu\nu}q_\nu{}^2v_4]\\&+4\pi g_{T8}{}^2G_{il}{}^n(f_T)G_{mj}{}^n(f_T)\bar{u}_3\sigma_{\mu\nu}q_\nu u_1\bar{v}_2\sigma_{\mu\lambda}q_\lambda v_4/4M^2(m_{V8}{}^2-t)\}\\=&\tfrac{1}{8}(3/20)\operatorname{tr}[G^n(f_V)D^kG^n(f_V)D^k]\langle\lambda_3\lambda_4\|\mathfrak{F}_{VV}\|\lambda_1\lambda_2\rangle+\tfrac{1}{8}(3/20)\operatorname{tr}[G^n(f_V)D^kG^n(f_T)D^k]\\&\times\langle\lambda_3\lambda_4\|\mathfrak{F}_{VT}\|\lambda_1\lambda_2\rangle+\tfrac{1}{8}(3/20)\operatorname{tr}[G^n(f_T)D^kG^n(f_T)D^k]\langle\lambda_3\lambda_4\|\mathfrak{F}_{TT}\|\lambda_1\lambda_2\rangle.\end{aligned}\tag{2.26}$$

The subscripts VV, TT, and VT refer to vector coupling, tensor coupling, and mixed coupling. Use has been made of the fact that $\operatorname{tr}[G^n(f_V)D^kG^n(f_T)D^k]=\operatorname{tr}[G^n(f_T)D^kG^n(f_V)D^k]$. The procedure is similar for the other amplitudes comprising the 2×2 octet-channel scattering matrix, and we have the octet–vector-meson exchange contributions ($i=1$, symmetric amplitude; $i=2$, antisymmetric)

$$\begin{aligned}\langle 8_i\lambda_3\lambda_4|\mathfrak{F}|8_j\lambda_1\lambda_2\rangle=&\begin{pmatrix}6f_V{}^2-2d_V{}^2 & 4(\sqrt{5})f_Vd_V\\ 4(\sqrt{5})f_Vd_V & 6f_V{}^2+(10/3)d_V{}^2\end{pmatrix}g_{V8}{}^2\langle\lambda_3\lambda_4\|\mathfrak{F}_{VV}\|\lambda_1\lambda_2\rangle\\&+\begin{pmatrix}6f_Vf_T-2d_Vd_T & 4(\sqrt{5})(f_Vd_T+f_Td_V)/2\\ 4(\sqrt{5})(f_Vd_T+f_Td_V)/2 & 6f_Vf_T+(10/3)d_Vd_T\end{pmatrix}g_{V8}g_{T8}\langle\lambda_3\lambda_4\|\mathfrak{F}_{VT}\|\lambda_1\lambda_2\rangle\\&+\begin{pmatrix}6f_T{}^2-2d_T{}^2 & 4(\sqrt{5})f_Td_T\\ 4(\sqrt{5})f_Td_T & 6f_T{}^2+(10/3)d_T{}^2\end{pmatrix}g_{T8}{}^2\langle\lambda_3\lambda_4\|\mathfrak{F}_{TT}\|\lambda_1\lambda_2\rangle.\end{aligned}\tag{2.27}$$

For singlet–vector-meson exchange we have the simpler result

$$\langle 8_i\lambda_3\lambda_4|\mathfrak{F}|8_j\lambda_1\lambda_2\rangle=\begin{pmatrix}1&0\\0&1\end{pmatrix}\{g_{V0}{}^2\langle\lambda_3\lambda_4\|\mathfrak{F}_{VV}\|\lambda_1\lambda_2\rangle+g_{V0}g_{T0}\langle\lambda_3\lambda_4\|\mathfrak{F}_{VT}\|\lambda_1\lambda_2\rangle\}+g_{T0}{}^2\langle\lambda_3\lambda_4\|\mathfrak{F}_{TT}\|\lambda_1\lambda_2\rangle.\tag{2.28}$$

It is easy to see how pseudoscalar-exchange terms are included (replace f_V by f_P, $g_{V8}{}^2$ by $g_{N\pi}{}^2$, and $\mathfrak{F}_{VV}$ by $\mathfrak{F}_\pi$). Finally, by using the partial-wave projections and rotation to orbital basis of Appendix B, the complete octet Born amplitudes may be put in the 4×4 matrix form $\langle 8_il|\mathfrak{F}^{J=1}|8_jl'\rangle$, with $l=0$ or 2, $i=$symmetric or antisymmetric octet. Note that there are 10 independent elements and that the matrix does not in general decompose into a Kronecker product (which would imply only six independent elements). That is, $\langle 8^sS|\mathfrak{F}|8^aD\rangle\neq\langle 8^sD|\mathfrak{F}|8^aS\rangle$, except under special conditions.[9]

The singlet amplitudes, which are 2×2 in spin space but one-dimensional in the $SU(3)$ coordinates, are constructed in an analogous manner.

The vector-meson pole matrix, corresponding to the (reduced) amplitudes ($J=1$) of Fig. 3(a), in the orbital-angular-momentum basis, is given by

$$\langle l\|\mathfrak{F}_p{}^1\|l'\rangle=\begin{pmatrix}R_{SS}&R_{SD}\\R_{DS}&R_{DD}\end{pmatrix}\frac{8\pi M^2}{m_V{}^2-s},\tag{2.29}$$

with

$$\begin{aligned}R_{SS}&=(4/9)[g_V(1+2\gamma)+g_T\gamma(2+\gamma)]^2,\\R_{DD}&=(8/9)(1-\gamma)^2(g_V-g_T\gamma)^2,\\R_{SD}&=(4\sqrt{2}/9)(\gamma-1)(g_V-g_T\gamma)\\&\qquad\times[g_V(1+2\gamma)+g_T\gamma(2+\gamma)]=R_{DS},\\\gamma&\equiv E/M=m_V/2M.\end{aligned}\tag{2.30}$$

The pole matrix for $SU(2)$ or $SU(3)$ eigenamplitudes (with the exception of the octet amplitudes) is a simple multiple of (2.30) with appropriate identification of g_V and g_T. Clearly, when $g_V/g_T=\gamma=m_V/2M$, the vector mesons are pure S wave at the pole; with $g_V/g_T=-\gamma(2+\gamma)/(1+2\gamma)$, the vector mesons are pure D wave at the pole. The residue matrix R_{ij} has the form $R_{ij}=G_iG_j$, and the remarks at the end of Sec. II D apply, with S/D mixture replacing F/D mixture.

In the $SU(3)$ octet case, however, there is an interesting complication. The vector-meson octet pole matrix is

$$\langle 8_il|\mathfrak{F}_p{}^J|8_jl'\rangle=\langle 8_il|R|8_jl'\rangle 8\pi M^2/(m_V{}^2-S),\tag{2.31}$$

[9] H. Ruegg, Nuovo Cimento 41, 576 (1966).

with

$$
\begin{aligned}
&(8^sS|R|8^sS)=(4/9)[d_Vg_V(1+2\gamma)+d_Tg_T\gamma(2+\gamma)]^2(20/3)\,,\\
&(8^sD|R|8^sD)=(8/9)(1-\gamma)^2(d_Vg_V-d_Tg_T\gamma)^2(20/3)\,,\\
&(8^sD|R|8^sS)=(20/3)(4\sqrt{2}/9)(\gamma-1)(d_Vg_V-d_Tg_T\gamma)[d_Vg_V(1+2\gamma)+d_Tg_T\gamma(2+\gamma)]=(8^sS|R|8^sD)\,,\\
&(8^aS|R|8^aS)=(4/9)[f_Vg_V(1+2\gamma)+f_Tg_T\gamma(2+\gamma)]^2(12)\,,\\
&(8^aD|R|8^aD)=(8/9)(1-\gamma)^2(f_Vg_V-f_Tg_T\gamma)^2(12)\,,\\
&(8^aD|R|8^aS)=12(4\sqrt{2}/9)(\gamma-1)(f_Vg_V-f_Tg_T\gamma)[f_Vg_V(1+2\gamma)+f_Tg_T\gamma(2+\gamma)]=(8^aS|R|8^aD)\,,\\
&(8^sS|R|8^aS)=-(4\sqrt{5})(4/9)[d_Vg_V(\gamma+2\gamma)+d_Tg_T\gamma(2+\gamma)][f_Vg_V(1+2\gamma)+f_Tg_T\gamma(2+\gamma)]\,,\\
&(8^sD|R|8^aD)=-4(\sqrt{5})(8/9)(1-\gamma)^2(d_Vg_V-d_Tg_T\gamma)(f_Vg_V-f_Tg_T\gamma)\,,\\
&(8^sD|R|8^aS)=-4(\sqrt{5})(4\sqrt{2}/9)(\gamma-1)(d_Vg_V-d_Tg_T\gamma)[f_Vg_V(1+2\gamma)+f_Tg_T\gamma(2+\gamma)]\,,\\
&(8^sS|R|8^aD)=-4(\sqrt{5})(4\sqrt{2}/9)(\gamma-1)(f_Vg_V-f_Tg_T\gamma)[d_Vg_V(1+2\gamma)+d_Tg_T\gamma(2+\gamma)]\,.
\end{aligned}
\tag{2.32}
$$

Note that $\langle 8^sS|R|8^aD\rangle\neq\langle 8^sD|R|8^aS\rangle$, except in the special case $f_V=f_T$, when the residue matrix decomposes into the Kronecker product

$$\langle 8_il|R|8_jl'\rangle=\begin{pmatrix}(20/3)d^2 & -4(\sqrt{5})fd\\ -4(\sqrt{5})fd & 12f^2\end{pmatrix}\otimes\langle l\|\mathfrak{F}_p{}^1\|l'\rangle\,, \tag{2.33}$$

where $\langle l\|\mathfrak{F}_p{}^1\|l'\rangle$ is defined in (2.30). Each element of the product can be diagonalized by independent similarity transforms [one in the space of $SU(3)$ coordinates, one in orbital-angular-momentum space], and the full residue matrix has a single nonvanishing eigenvalue whose eigenvector corresponds to a definite mixture of symmetric and antisymmetric representations as well as a definite mixture of S and D waves; that is, the eigenvector has the form

$$\{\alpha|8^a\rangle+\beta|8^s\rangle\}\otimes\{\alpha'|S\text{ wave}\rangle+\beta'|D\text{ wave}\rangle\}\,.$$

In the general case ($f_V\neq f_T$), one can not specify a single ratio of antisymmetric and symmetric coupling which applies both to S and D waves. The Kronecker decomposition is not possible, and the eigenvector which diagonalizes (2.32) has the form

$$\alpha_1|8^aS\rangle+\alpha_2|8^aD\rangle+\alpha_3|8^sS\rangle+\alpha_4|8^sD\rangle\,,$$

with $\alpha_1/\alpha_3\neq\alpha_2/\alpha_4$.

III. STATIC-MODEL SOLUTIONS

The "static"-model solution in the context of ND^{-1} dispersion calculations has come to refer to any set of assumptions which result in a single-pole approximation to the left-hand "exchange" cut, reducing the ND^{-1} equations (when a linear approximation is made to the D function) to algebraic relations between coupling constants which may or may not have real solutions. The set of assumptions usually includes elastic unitarity appropriate to low-energy scattering with bound or resonant states close to threshold—the region in which a potential theory is apt to be valid. A notable example of such a model is the Chew-Low reciprocal bootstrap of the N and N^* in $N\pi$ scattering (where the nucleon recoil is neglected—hence the term "static"). In this approximation, the crossing matrix—a matrix of products of CG coefficients expressing the effect in the s channel of exchange amplitudes in the t or u channel—essentially determines the solution. The existence of self-consistent N and N^* states in the Chew-Low model, for example, is equivalent to the statement, as will be shown below for a special case, that the relevant crossing matrix has a unit eigenvalue whose eigenvector has all positive components, to within a common multiplicative phase.

In the full $\bar{B}B$ scattering problem, which includes both Dirac and Pauli couplings for the vector meson as well as pseudoscalar exchange, the static model cannot be formulated in its usual form, and simple results, dependent only on the crossing matrices, do not exist. Nonetheless, as Wong[10] has shown, the truncated problem of S-wave $\bar{N}N$ scattering (with Dirac coupling only to the ρ and ω mesons) does possess a solution of the type described above and perhaps provides at least an indication of what to expect in the larger problem.

With an S-wave elastic $\bar{N}N$ isospin eigenamplitude $T^{(I)}(s)$ with $I=0$, 1 normalized so that

$$T^{(I)}(s)=e^{i\delta^{I}(s)}\sin\delta^{(I)}(s)\;\rho^{-1}(s)\,, \tag{3.1}$$
$$\rho(s)=p/E\,,$$

the elastic unitarity condition (δ real for $s>$physical threshold) reads

$$\operatorname{Im}[T^{(I)}(s)]^{-1}=-p/E,\quad s>4M^2,$$

with M the nucleon mass and $s=4p^2+4M^2=4E^2$ (p and E, respectively, are the c.m. momentum and energy of a *single* nucleon).

With the Lagrangian

$$\mathfrak{L}=i(4\pi)^{1/2}g_{\omega N}\bar{N}\gamma_\mu NV_\omega{}^\mu+i(4\pi)^{1/2}g_{\rho N}\bar{N}\boldsymbol{\tau}\gamma_\mu N\cdot\mathbf{V}_\rho{}^\mu, \tag{3.2}$$

where $V_\omega{}^\mu$ and $V_\rho{}^\mu$ represents the isotopic-scalar and

[10] D. Y. Wong (unpublished).

isotopic-vector Hermitian vector-meson fields, the exchange contributions are given, in S wave and with the indicated isospin, by

$$\begin{pmatrix} B^{(0)}(s) \\ B^{(1)}(s) \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} g_\omega^2 G(s,\mu^2) \\ g_\rho^2 G(s,\mu^2) \end{pmatrix}, \tag{3.3}$$

where $G(s,\mu^2)$, the dynamical content of the diagram, is calculated in Sec. II and Appendix B and is given by

$$G(s,\mu^2) = (1/18p^2)[(4p^2+5M^2+4ME)Q_0+12p^2Q_1 +2(p^2-2ME+2M^2)Q_2], \tag{3.4}$$

where μ is the meson mass and $Q_i = Q_i(1+\mu^2/2p^2)$ is the Legendre polynomial of the second kind. We then have

$$B^{(I)} = \sum_{I'} X_{II'} g_{I'}^2 G(s,\mu_{I'}^2), \tag{3.5}$$

with $X_{II'}$ the so-called "crossing" matrix.

Next, assume that μ^2 and M^2 are large compared with p^2, and expand $G(s,\mu^2)$ in powers of $(1+\mu^2/2p^2)^{-1}$; the desired pole approximation to the Born amplitude is then

$$G(s,\mu_I^2) \approx \frac{2M^2}{s-4M^2+2\mu_I^2} \equiv \frac{2M^2}{s+s_I}. \tag{3.6}$$

The approximation is a low-energy assumption, appropriate to bound states near threshold, i.e., to massive vector mesons with $\mu \sim 2M$. For vector mesons of this mass, the residue matrix is pure S-wave (this is shown in a succeeding section), and the exchange contributions in the D-wave and S-D transitions are strongly damped due to the assumption $p^2/M^2 \ll 1$.

Substituting (3.3) in the usual equation for N and using the approximation (3.6) yield

$$\begin{aligned} N^{(I)}(s) &= \frac{1}{\pi}\int_L \frac{\operatorname{Im}B^I(s')D^I(s')ds'}{s'-s} \\ &= \frac{1}{\pi}\int_L \sum_{I'} \operatorname{Im}\left(X_{II'}\frac{2M^2 g_{I'}^2}{s'+s_{I'}}\right)\frac{D^I(s')ds'}{s'-s} \\ &= \sum_{I'} \frac{X_{II'}2M^2 g_{I'}^2 D^I(-s_{I'})}{s+s_{I'}}, \\ T^{(I)}(s) &= N^I(s)/D^I(s), \\ T^{(I)}(s) &= \frac{1}{D^I(s)}\sum_{I'}\frac{X_{II'}2M^2 g_{I'}^2 D^I(-s_{I'})}{s+s_{I'}}. \end{aligned} \tag{3.7}$$

Guided by the $N\pi$ static theory, approximate the D functions by linear functions of s in the bound-state region. Because of the assumption that the ρ and ω are bound states, $D^{(0)}(\mu_0^2)$ and $D^{(1)}(\mu_1^2)$ must be set equal to zero, yielding

$$\begin{aligned} D^{(0)}(s) &\approx C_0(\mu_0^2-s), \\ D^{(1)}(s) &\approx C_1(\mu_1^2-s). \end{aligned} \tag{3.8}$$

As $s \to \mu_I^2$, the amplitudes are given by

$$T^{(I)}(s) \approx \sum_{I'} \frac{X_{II'}2M^2 g_{I'}^2}{s+s_{I'}} \frac{C_I(\mu_I^2+s_{I'})}{C_I(\mu_I^2-s)} \to \sum_{I'} \frac{X_{II'}2M^2 g_{I'}^2}{\mu_I^2-s}. \tag{3.9}$$

To be self-consistent, these pole terms must be equal to those calculated from the Lagrangian (3.2):

$$T_p^{(I)} = 2g_I^2 \bar{G}(s)/(\mu_I^2-s), \tag{3.10}$$

with $\bar{G}$ given by

$$\bar{G}(s) = (4M^2/18)(1+2E/M)^2 \approx 2M^2, \tag{3.11}$$

with the same approximation made above $(E\sim M)$. Combining (3.9) and (3.10), the self-consistency condition is

$$g_I^2 = \sum_{I'} \bar{X}_{II'} g_{I'}^2, \quad \bar{X}_{II'} = \begin{pmatrix} \frac{1}{2} & \frac{3}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}. \tag{3.12}$$

($\bar{X}$ is the conventionally quoted crossing matrix, corresponding to a normalization in which there is unit CG coefficient at the vertex arising from the various possible exchanges within an isospin multiplet.)

The self-consistency condition (3.12) states that the coupling constants squared must be an eigenvector of the crossing matrix belonging to the eigenvalue unity. Additionally, for the coupling constants to be all real, the ratio of any two components of the eigenvector must be positive. Such a solution indeed exists for the $\bar{N}N$ crossing matrix in (3.12), which implies

$$g_{\omega NN}^2/g_{\rho NN}^2 = 3. \tag{3.13}$$

Before discussing this result, let us see in what sense static-model solutions of this type persist in the Pagels approximation,[11,12] in which the left-hand cut is not replaced by a pole and in which the linear approximation to the D function is unnecessary. The self-consistency condition in each of the independent isospin channels now reads

$$B^I(a,\mu_I^2)(a-\mu_I^2) = 4M^2 g_I^2. \tag{3.14}$$

Writing

$$B^I(a,\mu_I^2) = 2\sum_{I'} \bar{X}_{II'} G(a,\mu_{I'}^2) g_{I'}^2 \tag{3.14$'$}$$

results in

$$2\sum_{I'} \bar{X}_{II'} g_{I'}^2 G(a,\mu_{I'}^2)(a-\mu_{I'}) = 4M^2 g_I^2. \tag{3.15}$$

[11] Heinz Pagels, Phys. Rev. **140**, B1599 (1965).
[12] See our discussion of the N/D solution below.

If the additional assumption is now made that the ω and ρ are *mass-degenerate*, then

$$\frac{G(a,\mu_I^2)}{2M^2}(a-\mu^2)\sum_{I'}\bar{X}_{II'}g_{I'}{}^2=g_I{}^2, \qquad (3.16)$$

which must be true for all I. The statement here is that a self-consistent solution exists if the inverse of $[G(a,\mu^2)/2M^2](a-\mu^2)=\lambda^{-1}$ is an eigenvalue of the crossing matrix; that is,

$$\bar{X}\mathbf{g}^2=\lambda\mathbf{g}^2, \qquad (3.17)$$

where λ need not be unity, and where the scale of coupling strengths is now fixed by the vanishing of the D function. λ must still be positive (which excludes the other, negative eigenvalue of $\bar{X}$) in order for the coupling strengths given by (3.14) to be real, and the requirement for all positive eigenvector components (to within the phase) still persists to ensure that all the coupling constants are real.

Since, in this case, the only positive eigenvalue is unity, (3.17) implies

$$(a-\mu^2)G(a,\mu^2)=2M^2,$$

which is satisfied (with $a\sim 6M^2$) for $\mu\sim 2M$.

Wong notes that $g^2\omega N/g^2\rho N=3$ corresponds to a pure F-type coupling in $SU(3)$.

Note, however, that the Wong result predicts pure F-type coupling only if the ω is regarded as the ω_8; if, on the other hand, one regards the ω as the physical ω and accepts the results of the mixing theory, then

$$\begin{pmatrix}\omega_1\\ \omega_8\end{pmatrix}=\begin{pmatrix}\alpha & -\beta\\ \beta & \alpha\end{pmatrix}\begin{pmatrix}\omega\\ \varphi\end{pmatrix}, \quad \alpha^2+\beta^2=1. \qquad (3.18)$$

Adding an $SU(3)$ singlet coupling to the Lagrangian

$$\mathcal{L}/i(4\pi)^{1/2}=g_0\omega_1{}^\mu\bar{B}_i\gamma_\mu B_i, \qquad (3.19)$$

it is clear that the Wong result is equivalent to

$$g_\omega{}^2/g_\rho{}^2=[\tfrac{1}{3}\sqrt{3}(4f-1)\beta g_{\rho N}+\alpha g_0]^2 g_{\rho N}{}^{-2}=3. \qquad (3.20)$$

The pure F type-coupling prediction is demanded, therefore, only in the limit of zero mixing angle ($\alpha=0$).

Further doubt is cast on the validity of this result by the companion static-model bootstraps $\bar{\Xi}\Xi$ exchanging ρ and ω, and $\bar{\Sigma}\Sigma$ exchanging ρ and ω, and a hypothetical $I=2$ vector meson. The method outlined in the equations leading up to (3.12) works equally well for these particles. The interaction is the same; only the CG coefficients, that is, the crossing matrices, change. For $\bar{\Xi}\Xi$ interaction, even the crossing matrix is the same, yielding the result

$$g_{\Xi\omega}{}^2/g_{\Xi\rho}{}^2=3=(1+2f)^2/3(1-2f)^2. \qquad (3.21)$$

The equation has two roots, $f=1$ and $f=\frac{1}{4}$. [At this point, let us note that the analogous equation for the Wong result

$$g_{N\omega}{}^2/g_{N\rho}{}^2=3=\tfrac{1}{3}(1-4f)^2 \qquad (3.22)$$

also has two roots, $f=1$ and $f=-\frac{1}{2}$.]

The $\bar{\Sigma}\Sigma$ crossing matrix is given by

$$\begin{bmatrix}1/3 & 1 & 5/3\\ 1/3 & 1/2 & -5/6\\ 1/3 & -1/2 & 1/6\end{bmatrix} \qquad (3.23)$$

and also has an eigenvector with eigenvalue unity corresponding to the static-model solution

$$g_{\Sigma\omega}{}^2:g_{\Sigma\rho}{}^2:g_{\Sigma(I=2)}{}^2=\tfrac{3}{2}:1:0. \qquad (3.24)$$

In terms of the $SU(3)$ parameters, this implies

$$\frac{g_{\Sigma\omega}{}^2}{g_{\Sigma\rho}{}^2}=\frac{(1-f)^2}{2f^2}=\tfrac{3}{2}\rightarrow f=(1\pm 3)^{-1}. \qquad (3.25)$$

The results (3.21), (3.22), and (3.25) taken together are clearly inconsistent—no one value of f satisfies all of them if in each case the ω is regarded as the $Y=0$, $I=0$ member of the octet. A similar situation arises, as Martin and Wali[13] have discussed, when the Chew-Low static theory is applied to $\Xi\pi$ scattering. Both the dynamics of the interaction and the isospin structure are the same as in the $N\pi$ interaction; one expects, therefore, a resonance in the $I=\frac{3}{2}$ $\Xi\pi$ state and not in the $I=\frac{1}{2}$ state, an expectation contrary to experiment. Martin and Wali go on to treat all the baryons and all the pseudoscalar mesons together, taking the relative couplings from $SU(3)$ and obtaining the d/f ratio for the pseudoscalar couplings as an output.

In view of the contradictory results of the static model, and motivated by the success of the Martin-Wali approach, this study, in addition to relaxing the static assumption, will similarly extend the particle multiplets scattered and exchanged to all the $J^P=\frac{1}{2}^+$ baryons and all the vector and pseudoscalar mesons.

Next, let us study static-model solutions analogous to those above, but with $SU(3)$ as the underlying symmetry, exchanging singlet and octet vector mesons in the cross channel and seeking singlet and octet vector-meson poles in the direct channel.

Referring to the crossing coefficients of Table I, the self-consistency condition in the singlet channel is simply

$$8g_{V0}{}^2=[12f^2+(20/3)d^2]g_{V8}{}^2+g_{V0}{}^2. \qquad (3.26)$$

For the cross-coupled octet channels the single-channel static-model equations may be trivially extended to matrix form. Using the results of Table I

[13] A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963).

and the pole approximation,

$$B^{(8)}(s)=\begin{pmatrix}6f^2-2d^2 & 4(\sqrt{5})fd\\ 4(\sqrt{5})fd & 6f^2+(10/3)d^2\end{pmatrix}\frac{2M^2g_{V8}^2}{s+s_8}+\begin{pmatrix}1&0\\0&1\end{pmatrix}\frac{2M^2g_{V0}^2}{s+s_8}$$

$$\equiv\frac{2M^2g_{V8}^2}{s+s_8}X_8+\frac{2M^2g_{V0}^2}{s+s_0}X_0. \qquad (3.27)$$

Then we have

$$N^{(8)}(s)=\frac{2M^2g_{V8}^2X_8D^{(8)}(-s_8)}{s+s_8}+\frac{2M^2g_{V0}^2X_0D^{(8)}(-s_0)}{s+s_0}, \qquad (3.28)$$

where $N^{(8)}$ and $D^{(8)}$ are now 2×2 matrices. Making the linear approximation to $D^{(8)}$,

$$D^{(8)}(s)\approx C(\mu_8^2-s),$$

where C is a matrix of constants. Then

$$T^{(8)}(s)\approx\frac{2M^2g_{V8}^2X_8C(\mu_8^2+s_8)C^{-1}}{(s+s_8)(\mu_8^2-s)}+\frac{2M^2g_{V0}^2X_0C(\mu_8^2+s_0)C^{-1}}{(s+s_0)(\mu_8^2-s)}, \qquad (3.29)$$

and near $s=\mu_s^2$,

$$T^{(8)}(s)\approx(X_8g_{V8}^2+X_0g_{V0}^2)2M^2/(\mu_8^2-s), \qquad (3.30)$$

which to be self-consistent must be equal to the S-wave pole terms (2.30) and (2.31):

$$T_{\text{pole}}^{(8)}=\begin{pmatrix}(20/3)d^2 & -4(\sqrt{5})fd\\ -4(\sqrt{5})fd & 12f^2\end{pmatrix}\frac{2M^2}{\mu_8^2-s}, \qquad (3.31)$$

yielding the three self-consistency conditions

$$\begin{gathered}(20/3)d^2g_{V8}^2=(6f^2-2d^2)g_{V8}^2+g_{V0}^2,\\ 12f^2g_{V8}^2=[6f^2+(10/3)d^2]g_{V8}^2+g_{V0}^2, \qquad (3.32)\\ -4(\sqrt{5})fdg_{V8}^2=4(\sqrt{5})fdg_{V8}^2.\end{gathered}$$

No set of real coupling constants satisfies the four conditions (3.26) and (3.32) in the three unknowns, g_{V0} g_{V8}, and f/d. While the approximations neglect the D wave, the Pauli coupling, and the effects of pseudoscalar exchange, and suffer as well from all the other (essentially low-energy) assumptions implicit in a static model, the failure of this solution is an unhappy turn of events, not because the assumptions are plausible, but because one might have hoped for a static solution to underlie and suggest the fully relativistic results. The "forces" are, of course, generally attractive (and hence favor the formation of poles). The structure of the self-consistency conditions (3.26) and (3.32) is such that the addition of inhomogeneous terms on the right sides may make real solutions possible. This may precisely be the effect of pseudoscalar-exchange terms. Possibly the low-energy assumptions and the suppression of one of the two spin-orbit degrees of freedom (the D wave) have conspired to destroy any resemblance between the model and the physics.

We note that C in (3.29) is implicitly a singular matrix, since we seek solutions corresponding to a simple pole of the scattering matrix. In general, we have $T=ND^{-1}=N\tilde{D}/\det D\sim N\tilde{D}/(\mu_8^2-s)$. The requirement of a simple pole, where only one of the eigenphases resonates, is

$$\lim_{\mu_8^2\to s}\det[(\mu_8^2-s)T]=\det N\det\tilde{D}=0.$$

Since $\det N$ is in general nonzero, $\det\tilde{D}(\mu_8^2)=0$. However, $D=\det D\tilde{D}^{-1}=C(\mu_8-s)$, which implies that $C\sim\tilde{D}^1$ is singular. Since C^{-1} occurs only in the combination $C^{-1}C$, Eqs. (3.29) are all well defined. It is easily checked that solutions to (3.32), if they exist, correspond to ordinary simple poles. Such solutions can be exhibited merely by changing elements of the crossing matrix so that (3.32) has a solution. C=constant matrix is not the most general linear form for D, and solutions may exist[14] to the nonlinear system of equations when we allow the form $D\approx C'-Is$, where C' is a matrix of constants and I is the unit matrix. We have incorporated fully relativistic dynamics before embarking on such a computer search.

IV. FULLY RELATIVISTIC SOLUTIONS

We attempt to solve the singlet and octet $J^P=1^-$ ND^{-1} equations, including S and D waves, pseudoscalar octet exchange, and the Pauli coupling, without making any low-energy assumptions. In doing this, we pass from the algebraic elegance of static-model results into the unhappy world of 4×4 matrices, Legendre polynomials of the second kind, and machine computation. The input parameters, upon which the input Born-approximation matrix depends, are

g_{V0}: singlet, Dirac coupling,
g_{T0}: singlet, Pauli coupling,
g_{V8}: octet, Dirac coupling,
g_{T8}: octet, Pauli coupling,
f_V: octet mixing parameter, Dirac coupling,
f_T: octet mixing parameter, Pauli coupling,
m_{V0}^2: vector singlet mass,
m_{V8}^2: vector octet mass,
m_P^2: pseudoscalar octet mass,
$g_{N\pi}$: pseuodscalar coupling,
f_P: pseudoscalar mixing parameter.

[14] We would like to thank Dr. M. Whippman for emphasizing this.

The following simplifying assumptions were made to reduce the number of free parameters:

$$\begin{aligned} m_V^2 &= m_{V8}^2 \approx 0.55M^2, \quad m_P^2 = 0.13M^2, \\ g_{N\pi}^2 &\approx 14, \qquad\qquad\qquad f_P = 0.4. \end{aligned} \tag{4.1}$$

M is the mean mass of the baryon octet. The remaining parameters were varied as follows:

$$\begin{aligned} &0.1 < g_{V0}^2 < 10, \\ &0.1 < g_{V8}^2 < 10, \\ &\left.\begin{aligned} -10 < g_{T0}/g_{V0} < 10 \\ -10 < g_{T8}/g_{V8} < 10 \end{aligned}\right\}, \quad \text{but } |g_{T0}/g_{V0}| \approx |g_{T8}/g_{V8}|; \\ &-10 < f_V < 10, \\ &-10 < f_T < 10. \end{aligned} \tag{4.2}$$

The Pagels approximation[11] was used to solve the ND^{-1} equations. In this approximation, the kinematical integral

$$F(z) = \frac{z}{\pi}\int_{4M^2}^{\infty} \frac{dx\rho(x)}{x^2(x-z)} \approx \frac{C_0}{z-a_0} \tag{4.3}$$

is approximated by a pole, as indicated. $\rho(x)$ is the phase-space factor appearing in the unitarity condition

$$\begin{aligned} \operatorname{Im}T^{-1}(s) &= -\rho(s), \quad s > 4M^2 \\ \rho(s) &= p/E. \end{aligned} \tag{4.4}$$

C_0 and a_0 were chosen for best fit in the bound-state region $0 < s < 4M^2$, with the result $C_0 = \pi^{-1}$, $a_0 = 6.0$. The Pagels solution to the matrix ND^{-1} equations is symmetric, satisfies

$$\operatorname{Im}T(s) = \operatorname{Im}B(s) \tag{4.5}$$

on the left cut, and is independent of the subtraction point chosen in D. Unsubtracted dispersion relations for T are assumed, but $B(s)$ is expected to describe the true discontinuity only in the region close to the beginning of the left-hand cut.

It is well known that approximate solutions to N/D equations are often very poor. By using a solution with the above properties, we have avoided the usual difficulties such as a strong dependence on the subtraction point or a nonsymmetric solution. Whether we have introduced other difficulties is not clear; they would show up, for example, in a strong dependence of the solution on the number of poles used in Eq. (4.3) to approximate $F(z)$.

If the behavior of the Born terms in the region where we use them is similar to the behavior of the actual left-hand cut of our amplitudes there, our solution may be qualitatively meaningful. Note especially that we never integrate over the Born terms (whose high-energy behavior is presumably not similar to that of the full amplitude).

The solutions which we have constructed have the following unfortunate feature: Since we have not factored powers of p from the D-wave and S-D-wave amplitudes, the solutions do not preserve the threshold behavior of the Born approximation amplitudes. (The ND^{-1} method "mixes" the threshold behaviors of the B matrix.) If the factoring process is attempted, we have

$$\bar{T}_{ll'} = T_{ll'}/p^l p^{l'}, \tag{4.6}$$

and all amplitudes behave like constants at threshold; there is no mixing problem, but the unitarity condition becomes

$$\operatorname{Im}T^{-1}(s) = -\frac{p}{E}\begin{pmatrix} 1 & 0 \\ 0 & p^4 \end{pmatrix}. \tag{4.7}$$

Since our primary interest was in low-lying bound states, far from the threshold region we have not followed this procedure, which severely complicates the matrix algebra.

We remark that the well-known left-hand kinematical singularity (of the type $\sqrt{s}$) in the F_5 amplitude causes no difficulty in the Pagels approximation, which preserves the given Born discontinuities of the left-hand cut, both dynamical and kinematical.

Our computational scheme was as follows: With masses and coupling constants as input, a program calculated the Born-approximation matrices and their derivatives and performed the matrix algebra of the Pagels approximation, resulting in a D matrix and a matrix R_0 corresponding to the output residue matrix when $\det D$ passed through a zero. Simultaneously, the input residue matrix R_I was calculated from the coupling constants and masses, and solutions were sought for which $R_0 = R_I$.

Let us first make some general comments on the existence, sensitivity, and uniqueness of the approximate solutions:

(*1*) *Existence.* Bound-state poles, that is, zeros of the determinant of the matrix D function, were plentiful for the input ranges mentioned above. Imposing the additional requirement that the zero appear in both the singlet and octet channels, for the same input, severely limited the input which could produce bound states. It was found possible, and relatively simple, to vary the input couplings so that the zero of the D function corresponded to the input vector-meson mass of the exchange diagram. The "elastic forces" of the $\bar{B}B$ system are, therefore, in this approximation fully able to produce low-lying bound states near the physical mass of the vector mesons; this is not in agreement with the $\bar{N}N$ calculation of Ball, Scotti, and Wong,[3] but it is in agreement with the Arnold[4] $\bar{N}N$ calculation (where exchange forces are Regge poles and the anomolous magnetic-moment coupling is neglected).

(*2*) *Sensitivity and uniqueness.* Let us say that for a given set of input parameters a bound state resulted.

Then it was found that relatively small variations of the input parameters could drastically shift the bound-state mass and even send it into the resonant region. (Our approximation certainly fails in this region, but we expect this qualitative behavior to persist nonetheless.) The positions of the poles vary sensitively with the input. Moreover, they are not unique; widely different sets of input could be adjusted to produce bound states which were self-consistent, at least in the sense that input and output masses were the same. The significant point is that in varying the parameters between different regions of parameter space, both of which were able to support bound states, one passed through regions where there were no bound states.

So far, we have discussed the straightforward procedure of seeking zeros of the determinant of the D matrix. This approach is similar to that of a potential theory in which we have used exchange diagrams to estimate the $\bar{B}B$ "potential." Clearly, many possible forms of this potential are expected to result in bound states, and we do not expect the form of the potential, from this requirement alone, to be unique.

In view of this ambiguity, we are led naturally to invoke the bootstrap condition ($R_0=R_I$) at the residues in the hope that this restriction will remove the unfortunate freedom implicit in a potential approach and sharpen out coupling-constant predictions. We therefore sought solutions in which the match between R_0 and R_I was optional. In a sense, this requirement was too restrictive: We cannot display any solutions which are fully self-consistent at the residues. Moreover, we found the output residue matrices to vary so sensitively with input, that it was difficult, with the present method of search and residue comparison, to resolve the ambiguities on the basis of self-consistency alone, although this is still, in principle, possible. Our final predictions for the coupling constants (perhaps they should be called suggestions) are based largely, therefore, on the requirement that bound states appear in both the octet and singlet channels, for similar input.

Let us illustrate residue consistency with two examples. With octet-channel input,

$$\begin{aligned} &g_{V0}^2=g_{V8}^2=4.0\,, \qquad g_{T8}/g_{V8}=g_{T0}/g_{V0}=5.0\,,\\ &f_V=1.1\,, \qquad f_T=0.4\,,\\ &m_{V8}^2=m_{V0}^2=0.55\,,\\ &g_{N\pi}^2=26\,, \quad f_P=0.4\,, \quad m_P^2=0.13\,, \end{aligned} \tag{4.8}$$

the output vector-meson mass is $m_{V8}^2=0.50$, so that the "solution" is self-consistent in mass. The input and output residue matrices are

$$R_I=\begin{pmatrix} -36 & -16 & 72 & -6.0\\ -16 & -7.0 & 30 & -2.7\\ 72 & 30 & -144 & 12\\ -6.0 & -2.7 & 12 & -1.0 \end{pmatrix},$$
$$R_0=\begin{pmatrix} -104 & 10 & 65 & 16\\ 10 & 11 & 7 & 8\\ 65 & 7 & -72 & 8\\ 16 & 8 & 8 & 14 \end{pmatrix}, \tag{4.9}$$

with $|1\rangle$=symmetric, S-wave; $|2\rangle$=symmetric, D-wave; $|3\rangle$=antisymmetric, S-wave; and $|4\rangle$=antisymmetric, D-wave.

The agreement is not very good. If we vary the two f/d ratios about the solution above, we obtain the det$D$ plots of Figs. 4(a) and 4(b).

Let us now display an "approximate" solution in a quite different region of parameter space, which we found upon a somewhat random variation of parameters. This solution will perhaps explain our reluctance to use the word "prediction."

With octet-channel input,

$$\begin{aligned} &g_{V0}^2=2.0\,, \qquad g_{V8}^2=3.0\,, \quad f_V=1.5\,, \quad f_T=0\,,\\ &g_{T0}^2=g_{T8}^2=0\,, \qquad g_{N\pi}^2=0\,,\\ &m_{V0}^2=m_{V8}^2=3.0\,, \end{aligned} \tag{4.10}$$

the output mass is $m_{V8}^2=2.95$. The input and output

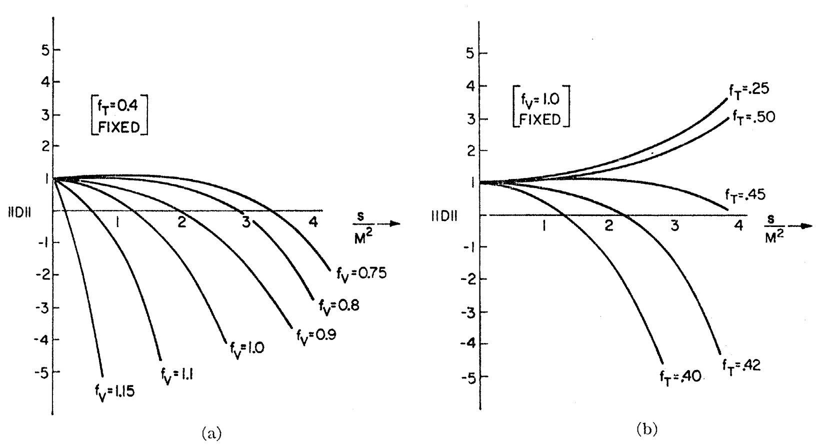

FIG. 4. (a) Determinant of the octet-channel D function $f_T=0.4$. See text for remaining parameters. (b) Determinant of the octet-channel D function $f_V=1.0$. See text for remaining parameters.

residue matrices are

$$R_I=\begin{bmatrix} -8.3 & 0.57 & -33 & 2.3 \\ 0.57 & -0.04 & 2.3 & -0.16 \\ 33 & 2.3 & -134 & 9.3 \\ 2.3 & -0.16 & 9.3 & -0.64 \end{bmatrix},$$

$$R_0=\begin{bmatrix} -0.60 & 0.94 & -1.7 & 0.73 \\ 0.94 & -0.45 & 0.73 & -0.52 \\ -1.7 & 0.73 & -0.27 & 0.79 \\ 0.73 & -0.52 & 0.79 & -0.34 \end{bmatrix}. \quad (4.11)$$

The agreement is almost as good as (that is, not much worse than) the previous solution with the "likely" couplings. It might be argued that we should discard this solution because the vector-meson mass is unphysical, but on the other hand it is also possible that in neglecting the inelastic contributions (which are "attractive," as we have seen) we have had to compensate by using erroneously large couplings. From this point of view, the second solution is the more reasonable one.

The singlet, 2×2 channels exhibit, qualitatively, the same behavior, with one important exception: The tensor coupling had to be reduced to produce bound states which appeared in both octet and singlet channels, which is a useful restriction.

Finally, we state the region of parameter space which we regard as our best estimate for the vector-meson coupling constants, based on the criterion that bound states appear at reasonable masses in both octet and singlet channels, for the same input:

$$\begin{aligned} &2<g_{V0}^2<4, && 1<g_{T0}/g_{V0}<4, \\ &2<g_{V8}^2<8, && 1<g_{T8}/g_{V8}<3.0, \\ &0.75<f_V<1.25, && 0.25<f_T<0.5. \end{aligned} \quad (4.12)$$

V. INELASTICITY EFFECTS

Properly speaking, we cannot regard the vector mesons as "bound states of $\bar{B}B$ pairs," whatever the preceding calculations and approximations suggest. It is most useful to speak of a given particle as a bound state or resonance of a *specific* particle pair when its coupling to *all other* states having the same quantum numbers as the given particle is negligible. In view of the apparent universality of the vector-meson couplings, this language is certainly not warranted. We should speak, rather, of the vector mesons as a singularity in the $J^P=1^-$ scattering matrix extended to include all the possible states that couple to the vector mesons, a few of which are given below for illustration:

$$\begin{bmatrix} \langle\bar{B}B|T|\bar{B}B\rangle & \langle\bar{B}B|T|PP\rangle & \langle\bar{B}B|T|VV\rangle & \langle\bar{B}B|T|VP\rangle \\ \cdots & \langle PP|T|PP\rangle & \langle PP|T|VV\rangle & \langle PP|T|VP\rangle \\ \cdots & \cdots & \langle VV|T|VV\rangle & \langle VV|T|VP\rangle \\ \cdots & \cdots & \cdots & \langle VP|T|VP\rangle \end{bmatrix} \quad (5.1)$$

(V = vector meson; P = pseudoscalar meson).

We have considered only the upper left-hand corner of this matrix, decoupled from the remainder by the assumption of elastic unitarity. Let us make this more explicit. The unitarity condition on the elastic amplitude

$$\mathrm{Im}T_{\bar{B}B,\bar{B}B}=\sum_n T_{\bar{B}B,n}{}^*\rho_n T_{n,\bar{B}B} \quad (5.2)$$

is exact, provided that all possible intermediate states $|n\rangle$ are included in the sum. [The diagonal matrix $\rho(s)$ is a set of phase-space factors and θ functions allowing for the onset of physical thresholds.] We indicate diagrammatically in Fig. 5(a) the discontinuities demanded of the elastic partial-wave amplitude by the unitarity condition (5.2) for a selected set of intermediate states. [We have not indicated the left-hand cuts due to t-channel singularities. These cuts actually overlap the unitarity cuts and would demand special treatment in an ND^{-1} approach. We have also assumed, if only to simplify the diagram, that the vector-meson pole lies below the PP threshold. In the real world of broken $SU(3)$ symmetry, some of the members of the 1^- octet are bound states of the pseudoscalar mesons, while others are resonances, and it is unclear which of the two conditions persists in the limit of exact $SU(3)$ symmetry.]

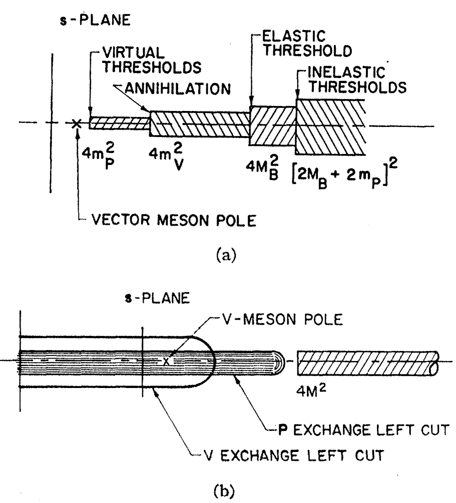

FIG. 5. (a) Right-hand (unitarity) cuts of the $\bar{B}B$ elastic-scattering partial-wave amplitudes. (b) Right-hand and left-hand cuts of the $\bar{B}B$ elastic-scattering partial-wave amplitudes in the one-meson-exchange, elastic unitarity approximation.

In our approximation, we have kept only $\bar{B}B$ states in the sum, with the resulting cut structure indicated in Fig. 5(b).

Let us attempt, from a general point of view, to parametrize the effects of the neglected channels. For this purpose, consider an $n\times n$ T matrix partitioned as follows:

$$T=T_{ij}\,,\quad i,j=1,\cdots,n;\quad T_0=T_{ij}\,,\quad i,j=1,\cdots,m<n;$$

$$T=\begin{array}{c} \begin{array}{cc} 1 & m \end{array} \\ \left[\begin{array}{c|c} T_0 & \\ \hline & \end{array}\right] \end{array}\begin{array}{l} 1 \\ m. \\ n \end{array} \tag{5.3}$$

The unitarity condition for the $n\times n$ matrix is

$$\mathrm{Im}T=T^{\dagger}\rho T\,,\quad \rho_{ij}=\theta(s-s_j)\delta_{ij}\rho_i(s)\,. \tag{5.4}$$

Then we have $\mathrm{Im}T_0=T_0^{\dagger}\rho_0T_0+\Delta$, where

$$\Delta_{ij}=\sum_{k=m+1;\,l=m+1}^{n} T_{ik}^{\dagger}\theta_{kl}T_{lj}\,,\quad i,j\leq m<n\,. \tag{5.5}$$

The unitarity condition for the truncated matrix T_0 is

$$\mathrm{Im}T_0^{-1}=-1\theta_0-(T_0^{T}T_0)^{-1}\Delta\,, \tag{5.6}$$

where it is well to remember that Δ contains θ functions appropriate to the excluded channels. The unitarity condition written in the form (5.6) is useful because the corrections due to the neglected states are separated from the elastic term $-1[\theta_0]$.

If we now write $N_0D_0^{-1}$ equations for T_0, we have for the D_0 matrix

$$D_0=1-\frac{s-s_0}{\pi}\int_R\frac{[1(\theta_0)+(T_0^{\dagger}T_0)^{-1}\Delta]N_0ds'}{(s'-s)(s'-s_0)}\,, \tag{5.7}$$

where *n-channel unitarity* has been satisfied for the $m\times m$ submatrix. The decoupling approximation is $(T_0^{\dagger}T_0)^{-1}\Delta\approx0$, or, more precisely,

$$\int_R\frac{\theta_0N_0ds'}{(s'-s)(s'-s_0)}\gg\int_R\frac{(T_0^{\dagger}T_0)^{-1}\Delta N_0ds'}{(s'-s)(s'-s_0)}\,. \tag{5.8}$$

In terms of familiar things, let T_0 be a single channel and let all the thresholds be equal. Then

$$(T_0^{\dagger}T_0)^{-1}\Delta=\frac{\sum_K|T_{0K}|^2}{|T_0|^2}=\frac{\sigma_{\mathrm{in}}}{\sigma_{\mathrm{el}}} \tag{5.9}$$

and

$$D_0=1-\frac{s-s_0}{\pi}\int_R\left(1+\frac{\sigma_{\mathrm{in}}}{\sigma_{\mathrm{el}}}\right)\frac{N_0ds'}{(s'-s)(s'-s_0)}\,. \tag{5.10}$$

When σ_{in} reaches its unitary bound, we have $\sigma_{\mathrm{in}}=\sigma_{\mathrm{el}}$, and the error in neglecting the inelasticity amounts to replacing 1 by 2, effectively doubling the squared coupling necessary to produce a given bound state. The unitarity bound on σ_{in} is, however, rather useless here, because the ratio $\sigma_{\mathrm{in}}/\sigma_{\mathrm{el}}$ is *not* bounded by unitarity. We can only say that the presence of inelasticity increases the effective attraction. Estimates of the coupling constants which neglect the inelasticity will be reasonably accurate only if σ_{in} is moderate in the range of integration which contributes most to the D integral. This statement can be sharpened if the inelastic thresholds lie far above the elastic threshold, in which case the correction terms are damped because of the lower limit of integration. We are interested, however, in the converse situation, that is, when the inelastic thresholds lie far below the elastic threshold. Let us, for example, identify the elastic channel as $\bar{B}B$ elastic scattering and the inelastic state as $|PP\rangle$. Then we have

$$D_{\bar{B}B,\bar{B}B}=1-\frac{s-s_0}{\pi}\int_R \times\left[\theta(s'-4m_B^2)+\frac{|T_{\bar{B}B,PP}|^2\theta(s'-4m_p^2)}{|T_{\bar{B}B,\bar{B}B}|^2}\right]\times\frac{N_{\bar{B}B,\bar{B}B}ds'}{(s'-s)(s'-s_0)}\,, \tag{5.11}$$

where we can no longer identify the ratio of amplitudes (squared) with the cross-section ratio because, for a range of the integration, $|T_{\bar{B}B,\bar{B}B}|^2$ is below the physical threshold. At the extreme left of the range of integration, the corrections $|T_{\bar{B}B,PP}/T_{\bar{B}B,\bar{B}B}|^2$ are likely to be dominated by the vector-meson pole terms, especially if the vector mesons are considered *resonances* of the PP system. In this range of integration we have approximately

$$|T_{\bar{B}B,PP}/T_{\bar{B}B,\bar{B}B}|^2\simeq g_{\rho\pi}^2/g_{\rho N}^2\,. \tag{5.12}$$

Only in the limit $g_{\rho N}\gg g_{\rho\pi}$ may we neglect the PP inelastic threshold. A similar argument may be made for the other two-meson states. This is perhaps a surprising result, because experimentally two-meson states are rare in $\bar{p}p$ (proton-antiproton) annihilation. The experimental result, however, tells us only what we can neglect *above* the physical threshold. Only a calculation will tell us if we can neglect them below the $\bar{B}B$ threshold. We are therefore led to insist that these states be included in any attempt to improve the calculation. (We should also include, of course, the many meson states, as well as states such as $\bar{B}BPP$, which lie above the $\bar{B}B$ threshold.)

The ratio of calculations suggested and planned to those actually performed in probably very small. Moreover, in a large calculation of the type being suggested, it is likely to be very difficult to understand the meaning of the results, however self-consistent they may or may not be. The problem will be complicated by the presence of various spins, by the $SU(3)$ degrees of freedom, by the presence of widely differing thresholds, and by a pathological cut structure. There are, however, several alternatives to this approach, in which we might seek very limited and perhaps initially unphysical results in exchange for increased insight into the various complicating features of the problem:

(a) We can study the effect of the spin degrees of freedom by considering a model in which all particles are scalars in the internal symmetry, *with degenerate masses* (for example, $M_B \approx m_P \approx m_V$). This model removes the $SU(3)$ degrees of freedom and the pathological cut structure. There may be interesting dynamical symmetries due to the assumption of mass degeneracy, and the model may simplify the study of the BBV and BBP interactions by removing some of the technical problems mentioned above.

(b) We can study the effect of widely different thresholds, together with the complicated cut structure, by considering a scattering matrix of scalar two-particle states with widely different thresholds and by investigating the behavior of the solutions as a function of the mass difference of the thresholds. This study may teach us more about the effects of low-lying inelastic thresholds and may result in useful machinery for handling the problem of overlapping cuts.

(c) The effects of $SU(3)$ might then be studied by incorporating the $SU(3)$ degrees of freedom in model (a). Once these main features are understood, a large calculation of the scattering matrix might be feasible. In addition, the couplings of this calculation—the VVV, PVV, and PPV meson couplings—would enter in both input and output residue matrices. Eventually, these practical computational difficulties will be overcome. It will then be seen whether a unique and fully self-consistent bootstrap solution exists for the $J^P=1^-$ system.

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APPENDIX A: $SU(3)$ NOTATION AND OCTET CROSSING COEFFICIENTS

A. B_i, V_i, P_i Octet Multiplets and Their Relations to Charge States

$$\sqrt{2}\Sigma^+ = B_1 - iB_2\,, \quad \sqrt{2}\bar{\Sigma}^+ = \bar{B}_1 + i\bar{B}_2\,,$$
$$\sqrt{2}\Sigma^- = B_1 + iB_2\,, \quad \sqrt{2}\bar{\Sigma}^- = \bar{B}_1 - i\bar{B}_2\,,$$
$$\Sigma^0 = B_3\,, \quad \bar{\Sigma}^0 = \bar{B}_3\,,$$
$$\sqrt{2}p = B_4 - iB_5\,, \quad \sqrt{2}\bar{p} = \bar{B}_4 - i\bar{B}_5\,,$$
$$\sqrt{2}\Xi^- = B_4 + iB_5\,, \quad \sqrt{2}\bar{\Xi}^- = \bar{B}_4 - i\bar{B}_5\,,$$
$$\sqrt{2}n = B_6 - iB_7\,, \quad \sqrt{2}n = \bar{B}_6 + i\bar{B}_7\,,$$
$$\sqrt{2}\Xi^0 = B_6 + iB_7\,, \quad \sqrt{2}\bar{\Xi}^0 = \bar{B}_6 - i\bar{B}_7\,,$$
$$\Lambda = B_8\,, \quad \bar{\Lambda} = \bar{B}_8\,,$$
$$\sqrt{2}\pi^+ = P_1 - iP_2\,, \quad \sqrt{2}\rho^+ = V_1 - iV_2\,, \text{ etc.},$$
$$\sqrt{2}\pi^- = P_1 + iP_2\,,$$
$$\pi^0 = P_3\,,$$
$$\sqrt{2}K^+ = P_4 - iP_5\,,$$
$$\sqrt{2}K^- = P_4 + iP_5\,,$$
$$\sqrt{2}K^0 = P_6 - iP_7\,,$$
$$\sqrt{2}K^0 = P_6 + iP_7\,,$$
$$\chi = P_8\,.$$

Note that the $i=2,5,7$ members have opposite charge parity from the $i=1,3,4,6,8$ members; that is,

$$C^{-1}P_iC = \eta(P)(-)^{\nu_i}P_i = (-)^{\nu_i}P_i\,,$$
$$C^{-1}V_iC = \eta(V)(-)^{\nu_i}V_i = (-)^{\nu_i+1}V_i\,,$$

where $\nu_i=0$, $i=1,3,4,6,8$ and $\nu_i=1$, $i=2,5,7$. η is the charge-conjugation number of the neutral member $[\eta(P)=+1,\ n(V)=-1]$.

B. Tarjanne F and D Matrices

The real, respectively antisymmetric and symmetric F and D are multiples of the Gell-Mann F and D matrices as noted in Eq. (2.25). They have the following properties:

$$(-)^{\nu_j+\nu_k}F_j{}^i = (-)^{\nu_i+1}F_{jk}{}^i\,, \qquad \mathrm{tr}F^i = 0\,,$$
$$(-)^{\nu_i+\nu_k}D_{jk}{}^i = (-)^{\nu_i}D_{jk}{}^i\,, \qquad \mathrm{tr}D^i = 0\,,$$
$$[F^i,F^j] = -F_{jk}{}^iF^k\,, \qquad [F^i,D^j] = -F_{jk}{}^iD^k\,,$$
$$[D^i,D^j] = F_{jk}{}^iF^k - (8/3)[ij]\,,$$
$$\{F^i,F^j\} = -3D_{jk}{}^iD^k + 4\{ij\} - 4\delta_{ij}\,,$$
$$\{D^i,D^j\} = -D_{ij}{}^iD^k + \tfrac{4}{3}\{ij\} + \tfrac{4}{3}\delta_{ij}\,,$$
$$\{F^i,D^j\} = D_{jk}{}^iF^k\,,$$

where $[ij] \equiv (ij)-(ji)$, $\{ij\} \equiv (ij)+(ji)$, and $(ij)_{\alpha\beta} \equiv \delta_{i\alpha}\delta_{j\beta}$.

The trace properties follow from the commutation relations

$$\mathrm{tr}F^iF^j = -12\delta_{ij}\,,$$
$$\mathrm{tr}D^iD^j = (20/3)\delta_{ij}\,,$$
$$\mathrm{tr}F^iD^j = 0\,,$$
$$\mathrm{tr}D^kD^nD^kD^n = -320/3\,, \qquad \mathrm{tr}D^nF^kD^nF^k = -320\,,$$
$$\mathrm{tr}F^kF^nF^kF^n = 8(72)\,, \qquad \mathrm{tr}D^nD^kF^nF^k = 320\,,$$
$$\mathrm{tr}F^kF^nF^kD^n = 0\,, \qquad \mathrm{tr}D^kD^nD^kF^n = 0\,.$$

C. Crossing Coefficients

Proceeding as in Sec. II, we have, for the direct terms of Fig. 3(a):

Vector-Meson Singlet Annihilation

$$\langle 1\|1\rangle = \tfrac{1}{8}\delta_{ij}\delta_{lm}\delta_{ij}\delta_{lm}g_{V0}{}^2 = 8g_{V0}{}^2;$$

Vector-Meson Octet Annihilation

$$\langle 8_s\|8_s\rangle=(3/160)D_{ij}{}^kD_{lm}{}^kG_{ml}{}^nG_{ij}{}^ng_{V8}{}^2$$
$$=(3/160)g_{V8}{}^2d^2\,\mathrm{tr}D^nD^k\,\mathrm{tr}D^kD^n=(20/3)d^2g_{V8}{}^2,$$
$$\langle 8_a\|8_a\rangle=(1/96)g_{V8}{}^2f^2\,\mathrm{tr}F^kF^n\,\mathrm{tr}F^kF^n=12f^2g_{V8}{}^2,$$
$$\langle 8_s\|8_a\rangle=(1/8\sqrt{80})g_{V8}{}^2fd\,\mathrm{tr}D^kD^n\,\mathrm{tr}F^kF^n$$
$$=-4(\sqrt{5})fdg_{V8}{}^2=\langle 8_a\|8_s\rangle.$$

For the exchange terms of Fig. 3(b), we have:

Vector-Meson Singlet Exchange

$$\langle 1\|1\rangle=\tfrac{1}{8}\delta_{ij}\delta_{lm}\delta_{il}\delta_{jm}g_{V0}{}^2=g_{V0}{}^2,$$
$$\langle 8_s\|8_s\rangle=(3/160)g_{V0}{}^2\,\mathrm{tr}D^kD^k=g_{V0}{}^2,$$
$$\langle 8_a\|8_a\rangle=(1/96)(-\mathrm{tr}F^kF^k)g_{V0}{}^2=g_{V0}{}^2,$$
$$\langle 8_s\|8_a\rangle=\sim\mathrm{tr}F^kD^k=0;$$

Vector-Meson Octet Exchange

$$\langle 1\|1\rangle=\tfrac{1}{8}g_{V8}{}^2[-f^2\,\mathrm{tr}F^nF^n+d^2\,\mathrm{tr}D^nD^n]$$
$$=g_{V8}{}^2[12f^2+(20/3)d^2],$$
$$\langle 8_s\|8_s\rangle=(3/160)g_{V8}{}^2[-f^2\,\mathrm{tr}D^kF^nD^kF^n$$
$$+\mathrm{tr}D^kD^nD^kD^n]=g_{V8}{}^2[6f^2-2d^2],$$
$$\langle 8_a\|8_a\rangle=(1/96)g_{V8}{}^2[f^2\,\mathrm{tr}F^kF^nF^kF^n-d^2\,\mathrm{tr}F^kD^nF^kD^n]$$
$$=g_{V8}{}^2[6f^2+(10/3)d^2],$$
$$\langle 8_s\|8_a\rangle=(i/8\sqrt{80})g_{V8}{}^2[-ifd\,\mathrm{tr}D^kF^nF^kD^n$$
$$-ifd\,\mathrm{tr}D^kF^nF^kD^n]=4(\sqrt{5})fdg_{V8}{}^2=\langle 8_a\|8_s\rangle.$$

Note that the last element is the sum of two equal terms. For another channel the terms can cancel, which is the case in $BP\leftrightarrow BP$ with B exchange (see Ref. 13).

For the 10, $\overline{10}$, and 27 representations, it is easier to construct charge eigenstates, exploiting the simple hypercharge and isospin content:

$$|\bar{\Sigma}^-\Sigma^+\rangle=|27\rangle,$$
$$\tfrac{1}{2}\sqrt{2}\{|p\bar{\Xi}^0\rangle+|n\bar{\Xi}^-\rangle\}=|\overline{10}\rangle,$$
$$1\{|\bar{p}\Xi^-\rangle+|\bar{p}\Xi^0\rangle\}=|\overline{10}\rangle.$$

For example, we have

$$\langle 27\|27\rangle=[-g_{\Sigma\rho}{}^2+g_{\Sigma\omega}{}^2]=[-4f^2+\tfrac{4}{3}d^2]g_{V8}{}^2,$$

where the Lagrangian (2.35) has been used to relate $g_{\Sigma\rho}$ and $g_{\Sigma\omega}$ to g_{V8} and the mixing parameter. The octet-exchange contributions to the $\overline{10}$ and 10 representations follow similarly.

APPENDIX B: SINGLE-MESON ANNIHILATION AND EXCHANGE AMPLITUDES

A. Vector-Meson Annihilation

See Fig. 3(a). The notation $\langle\lambda_3\lambda_4\|\mathfrak{F}\|\lambda_1\lambda_2\rangle$ is defined in Sec. II. The results are stated in terms of the rotation matrices

$$d_{00}{}^1=\mu\equiv\cos\theta,\quad d_{-11}{}^1=\tfrac{1}{2}(1-\mu),$$
$$d_{11}{}^1=\tfrac{1}{2}(1-\mu),\quad d_{10}{}^1=-\sin\theta/\sqrt{2},$$

$$\langle ++\|\mathfrak{F}\|++\rangle=\frac{d_{00}{}^1(\mu)16\pi M^2}{m_V{}^2-s}[g_V+g_T(1+p^2/M^2)]^2,$$
$$\langle ++\|\mathfrak{F}\|++\rangle=\langle ++\|\mathfrak{F}\|++\rangle,$$
$$\langle +-\|\mathfrak{F}\|+-\rangle=\frac{d_{11}{}^1(\mu)16\pi}{m_V{}^2-s}2E^2(g_V+g_T)^2,\tag{B1}$$
$$\langle +-\|\mathfrak{F}\|-+\rangle=\frac{d_{-11}{}^1(\mu)16\pi}{m_V{}^2-s}2E^2(g_V+g_T)^2,$$
$$\langle ++\|\mathfrak{F}\|+-\rangle=\frac{d_{10}{}^1(\mu)16\pi}{m_V{}^2-s}\sqrt{2}EM(g_V+g_T)\times[g_V+g_T(1+p^2/M^2)].$$

We now partial-wave project in accordance with (2.5) and rotate to the orbital-angular-momentum representation in accordance with (2.16). The result is

$$\langle l\|\mathfrak{F}^{J=1}\|l'\rangle=\frac{8\pi M^2}{m_V{}^2-s}\begin{pmatrix}R_{SS} & R_{SD}\\ R_{DS} & R_{DD}\end{pmatrix},\tag{B2}$$

with $R_{ll'}$ defined in (2.30).

B. Vector-Meson Exchange

$$\langle ++\|\mathfrak{F}\|++\rangle=\frac{16\pi g_V{}^2[2p^2+\frac{1}{2}M^2(1+\mu)]}{m_V{}^2-t}+\frac{16\pi g_T{}^2(\frac{1}{4}p^2)(3-4\mu+\mu^2)}{m_V{}^2-t}-\frac{16\pi g_Vg_Tt}{m_V{}^2-t},$$

$$\langle ++\|\mathfrak{F}\|--\rangle=16\pi g_V{}^2\frac{\frac{1}{2}M^2(\mu-1)}{m_V{}^2-t}+\frac{16\pi g_T{}^2}{m_V{}^2-t}\frac{p^2}{4M^2}[-3p^2-M^2+2p^2\mu+(p^2+M^2)\mu^2]-\frac{16\pi g_Vg_T}{m_V{}^2-t}(\tfrac{1}{2}t),$$

$$\langle +-\|\mathfrak{F}\|+-\rangle=\frac{16\pi g_V{}^2}{m_V{}^2-t}(1+\mu)(p^2+\tfrac{1}{2}M^2)+\frac{16\pi g_T{}^2}{m_V{}^2-t}(\tfrac{1}{4}p^2)(\mu-1)(\mu+1),\tag{B3}$$

$$\langle +-\|\mathfrak{F}\|-+\rangle=\frac{16\pi g_V{}^2}{m_V{}^2-t}\left(\frac{1-\mu}{2}\right)M^2+\frac{16\pi g_T{}^2}{m_V{}^2-t}\frac{p^2}{4M^2}(3p^2+M^2+p^2\mu+M^2\mu)(1-\mu)+\frac{16\pi g_Vg_Tp^2(\mu-1)}{m_V{}^2-t},$$

$$\frac{\langle ++|\mathfrak{F}|+-\rangle}{\sin\theta}=\frac{16\pi g_V{}^2}{m_V{}^2-t}(-EM)+16\pi g_T{}^2\frac{Ep^2}{4M}\frac{(1-\mu)}{m_V{}^2-t}+\frac{16\pi g_Vg_T}{m_V{}^2-t}\frac{Ep^2}{M}.$$

The partial-wave projections in the orbital basis are the following:

(a) Dirac coupling:

$$\begin{aligned}\langle S\|\mathfrak{F}^1\|S\rangle&=(8\pi g_V^2/9p^2)[(4p^2+5M^2+4ME)Q_0+12p^2Q_1+2(p^2-2ME+2M^2)Q_2],\\ \langle D\|\mathfrak{F}^1\|D\rangle&=(8\pi g_V^2/9p^2)[2(p^2+2M^2-2ME)Q_0+15p^2Q_1+(p^2+5M^2+4ME)Q_2],\\ \langle S\|\mathfrak{F}^1\|D\rangle&=(8\pi g_V^2\sqrt{2}/9p^2)[(2p^2+M^2-ME)Q_0-3p^2Q_1+(p^2-M^2+ME)Q_2];\end{aligned}\tag{B4}$$

(b) Pauli coupling:

$$\begin{aligned}\langle S\|\mathfrak{F}^1\|S\rangle&=(8\pi g_T^2/36M^2)[12p^2-4M^2-8ME]Q_0+(8\pi g_T^2/60M^2)[12M^2-34p^2+8EM]Q_1\\ &\quad+(8\pi g_T^2/36M^2)[-8M^2+6p^2+8EM]Q_2+(8\pi g_T^2/60M^2)[8M^2+4p^2-8EM]Q_3,\\ \langle D\|\mathfrak{F}^1\|D\rangle&=(8\pi g_T^2/36M^2)[-8M^2+9p^2+8EM]Q_0+(8\pi g_T^2/60M^2)[30M^2-35p^2-8EM]Q_1\\ &\quad+(8\pi g_T^2/36M^2)[-16M^2+9p^2-8EM]Q_2+(8\pi g_T^2/60M^2)(10M^2+5p^2+8EM)Q_3,\\ \langle S\|\mathfrak{F}^1\|D\rangle&=(8\pi g_T^2/36M^2)[4M^2+3p^2+2ME]\sqrt{2}Q_0+(8\pi g_T^2/60M^2)[-18M^2+p^2-2ME]\sqrt{2}Q_1\\ &\quad+(8\pi g_T^2/36M^2)[8M^2-3p^2-2ME]\sqrt{2}Q_2+(8\pi g_T^2/60M^2)[-2M^2-p^2+2ME]\sqrt{2}Q_3;\end{aligned}\tag{B5}$$

(c) Cross-coupling terms:

$$\begin{aligned}\langle S\|\mathfrak{F}^1\|S\rangle&=(8\pi g_Vg_T/9M^2)[(-7M^2-8EM)Q_0+15M^2Q_1+8M(E-M)Q_2],\\ \langle D\|\mathfrak{F}^1\|D\rangle&=(8\pi g_Vg_T/9M^2)[8M(E-M)Q_0+21M^2Q_1+(-13M^2-8EM)Q_2],\\ \langle S\|\mathfrak{F}^1\|D\rangle&=(8\pi g_Vg_T/9M^2)[(M^2+2EM)Q_0-6M^2Q_1+(5M^2-2EM)Q_2].\end{aligned}\tag{B6}$$

In all of the above, we have

$$Q_i\equiv Q_i(1+m_V^2/2p^2),$$

where the Q_i are Legendre polynomials of the second kind.

C. Pseudoscalar Exchange

See Fig. 3(c).

$$\begin{aligned}\langle++\|\mathfrak{F}\|++\rangle&=\langle+-\|\mathfrak{F}\|+-\rangle=\langle++\|\mathfrak{F}\|+-\rangle=0,\\ \langle++\|\mathfrak{F}\|--\rangle&=16\pi g_{N\pi}^2/(m_\pi^2-t)(-\tfrac{1}{4}t),\\ \langle+-\|\mathfrak{F}\|-+\rangle&=16\pi g_{N\pi}^2/(m_\pi^2-t)(-\tfrac{1}{4}t).\end{aligned}$$

The partial-wave projections are

$$\begin{aligned}\langle S\|\mathfrak{F}^1\|S\rangle&=(8\pi g_{N\pi}^2/6)(Q_0-Q_1),\\ \langle D\|\mathfrak{F}^1\|D\rangle&=(8\pi g_{N\pi}^2/6)(Q_1-Q_2),\\ \langle S\|\mathfrak{F}^1\|D\rangle&=(8\pi g_{N\pi}^2\sqrt{2}/6)(Q_0-2Q_1+Q_2),\end{aligned}$$

where

$$Q_i=Q_i(1+m_\pi^2/2p^2).$$