PHYSICAL REVIEW VOLUME 172, NUMBER 5 25 AUGUST 1968

Broken $SU(3)$ Symmetry, Sum Rules, and the Magnetic Moments of the Baryons

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(Received 15 February 1968)

Motivated by the increasing accuracy with which the magnetic moments of strange baryons are being measured, we present a calculation of these moments in broken $SU(3)$ symmetry without appeal to a highly detailed theory of symmetry breaking. As primary ingredients in the calculation, we assume only the $SU(2)$ transformation properties of the electromagnetic current, the octet transformation property of the symmetry-breaking medium-strong interaction, and the validity of the Drell-Hearn-Gerasimov sum rule for the anomalous moments. Saturating this sum rule with the dominant states, the decuplet and a singlet, but making no $SU(3)$ assumptions on the couplings of these states or their masses, we find that all eight baryon anomalous magnetic moments and the $\Sigma^0\Lambda$ transition moment can be expressed in terms of κ_n, κ_Λ, and κ_{Σ^+} through relations that are valid in broken symmetry with an expected accuracy of about 10%. Further assuming the absence of the **27** in the current, we have, in addition, $\kappa_{\Sigma^+}=\kappa_n-4\kappa_\Lambda$. We also find that our saturation assumption implies the absence of a unitary singlet piece in the electromagnetic current in the symmetry limit, so that the existing particle spectrum supports the hypothesis of a pure octet transformation property for the electromagnetic current. We also examine the Drell-Hearn-Gerasimov sum rule for high-spin systems, and this forward-direction sum rule would appear to require towers of states for its saturation. Forward-direction sum rules for the nonflip Compton amplitude, based on the absence of fixed or moving poles in the J plane with $I=2$, are also considered and found to be consistent with the results obtained from the magnetic-moment sum rules.

I. INTRODUCTION

THE primary purpose of this paper is to report a calculation of the influence of the $SU(3)$ symmetry-breaking forces on the magnetic moments of the baryons, which in the exact symmetry limit and with octet transformation properties for the photon, are predicted by the Coleman-Glashow formulas.[1] We are motivated to undertake this investigation by the improvement in the experimental accuracy with which the magnetic moments of the strange baryons Λ, Σ^+, and Ξ^- are being measured, improvements afforded by technical advances in detection systems used in conjunction with high-flux magnetic fields. It is possible that in the next several years, the magnetic moments of these baryons will be known to something like 10% accuracy, along with a possible measurement of the Ξ^0 moment. These experiments will provide definite tests of the $SU(3)$ transformation properties of the electromagnetic current. It is already evident that these experiments will be of such an accuracy as to provide information on $SU(3)$ symmetry breaking in electromagnetic interactions. Consequently, it becomes essential to know whether an experimental number which deviates, say, 30% from the value predicted in the symmetry limit is a verification or a failure of the symmetry. To answer such questions, one must have a dynamical theory of symmetry breaking.

At the present time, there is no well-established theory of symmetry breaking.[2] Although a great deal is known about the manifestations of the $SU(3)$ symmetry-breaking interaction, particularly in the mass spectrum, almost nothing is known about the dynamics presumably responsible for producing the symmetry breakdown. Our approach to this problem suggests that a great deal can be learned about symmetry violations without appeal to a *detailed* theory of the dynamics of symmetry breaking or a highly specific model. The central idea is the following: We assume that $SU(2)$ and the results of first-order *broken* $SU(3)$ [3] are valid for some set of physical parameters, such as magnetic moments. Secondly, we assume the validity of an exact sum rule for these parameters and we saturate this sum rule with a definite set of states whose couplings are also restricted by $SU(2)$ and also by first-order broken symmetry. At no stage do we assume the exact, full symmetry on the couplings or masses. The conjunction of the group-theoretical assumption plus the saturation assumption then leads to new restrictions on the physical parameters which must be valid in broken symmetry, and which are not implicit in either assumption when considered separately. It is clearly through the dynamical approximation inherent in the saturation of a truncated sum rule that one obtains the additional information.

First, we note the consequences of exact $SU(3)$ symmetry. If we assume that the photon transforms in the manner of U-spin scalar[4] under $SU(3)$, it follows that the magnetic moments of a U-spin multiplets are

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[1] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).
[2] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964); S. L. Glashow, *ibid.* **130**, 2132 (1963); R. E. Cutkosky and Pekka Tarjanne, *ibid.* **132**, 1354 (1963); R. F. Dashen and S. C. Frautschi, *ibid.* **143**, 1171 (1966); Y. Ne'eman, *ibid.* **134**, B1355 (1964).
[3] How to implement first-order violations of $SU(3)$ symmetry will be made clear in Sec. II. For reasons far from clear, the inclusion of just first-order symmetry breaking, as in various mass formulas where it can be tested, is remarkably accurate.
[4] The electromagnetic current could, under this assumption, accommodate a unitary singlet piece.

simply related. We have for the baryons

$$\begin{aligned}\mu_{\Sigma^+}&=\mu_p\,,\\ \mu_{\Xi^-}&=\mu_{\Sigma^-}\,,\\ \mu_{\Xi^0}&=\mu_n=\tfrac{3}{2}\mu_\Lambda-\tfrac{1}{2}\mu_{\Sigma^0}\,,\\ \mu_{\Sigma^0}-\mu_\Lambda&=(2/\sqrt{3})\mu_{\Sigma^0\Lambda}\,.\end{aligned}\tag{1.1}$$

If one makes the stronger assumption, still in the framework of the exact $SU(3)$ symmetry, that the hadronic electromagnetic current J_λ transforms in the manner of a member of an octet, i.e., like $\mathfrak{F}_\lambda{}^3+\mathfrak{F}_\lambda{}^8/\sqrt{3}$ in the usual notation,[5] so that there is no unitary singlet piece, then one specifies all the magnetic moments of the 8 baryons and the $\Sigma^0\to\Lambda+\gamma$ transition moment in terms of the proton and neutron magnetic moments, and obtains the Coleman-Glashow formulas[1]

$$\begin{aligned}\mu_{\Sigma^+}&=\mu_p\,,\\ 2\mu_\Lambda=\mu_{\Xi^0}=-2\mu_{\Sigma^0}&=-(2/\sqrt{3})\mu_{\Sigma^0\Lambda}=\mu_n\,,\\ \mu_{\Xi^-}=\mu_{\Sigma^-}&=-(\mu_p+\mu_n)\,.\end{aligned}\tag{1.2}$$

These formulas suffer from the deficiency common to all exact-symmetry results, that their interpretation in the real world of broken $SU(3)$ symmetry is ambiguous. For example, they do not specify the appropriate units in which the magnetic moments are to be measured. We now discuss the case of broken $SU(3)$ symmetry.

Only assuming $SU(2)$ symmetry leads to the relation[6] on the anomalous moments

$$\kappa_{\Sigma^+}+\kappa_{\Sigma^-}=2\kappa_{\Sigma^0}\,.\tag{1.3}$$

Assuming, further, the octet transformation property of the medium-strong interaction, we have[7]

$$2\sqrt{3}\kappa_{\Sigma^0\Lambda}=3\kappa_\Lambda+\kappa_{\Sigma^0}-2(\kappa_{\Xi^0}+\kappa_n)\,.\tag{1.4}$$

These broken-symmetry relations are not very interesting from the standpoint of experimental verification.

To go beyond these relations, we assume the validity of the Drell-Hearn-Gerasimov sum rule (DHGSR) for anomalous moments.[8,9] Saturating this sum rule with the dominant states (the decuplet and a singlet),[10,11] and assuming only $SU(2)$ and first-order broken-symmetry results for the radiative transitions $\mathbf{10}\to 8+\gamma$,[12] we find that the DHGSR will couple symmetry breaking in the anomalous moments with symmetry breaking in the radiative transitions, giving rise to the following consistency conditions:

$$\begin{aligned}\kappa_p+\kappa_n&=0\,,\\ \kappa_{\Xi^-}&=2\kappa_\Lambda+\kappa_{\Sigma^+}\,,\\ \kappa_{\Xi^0}&=4\kappa_\Lambda-\kappa_n\,,\\ \kappa_{\Sigma^-}+\kappa_{\Xi^-}&=0\,,\\ (2/\sqrt{3})\kappa_{\Sigma^0\Lambda}&=2\kappa_{\Sigma^0}=\kappa_{\Sigma^+}+\kappa_{\Sigma^-}\,.\end{aligned}\tag{1.5}$$

These conditions on the anomalous moments in *natural units*[13] are all that we can hope to obtain within the framework of our assumptions and are our main result. They fall short of, but are consistent with, the predictions of full $SU(3)$ symmetry [Eq. (1.2)]. However, they provide unambiguous relations which are capable of experimental test and should be valid to within 10%. Through Eq. (1.5) all the nine moments of the baryon octet can then be related to the n, Λ, and Σ^+ magnetic moments.

A significant consequence of this approach is that if one assumes $SU(3)$ symmetry, and assumes only that the electromagnetic current transforms like a U-spin singlet, in such a way that it might accommodate a unitary singlet piece, and if one saturates the DHGSR with a **10** and *any* number of singlets, then we find that the matrix elements of the singlet piece of the electromagnetic current must vanish, i.e., it is pure octet.

There is another remark worth making about our specific application. If we assume the complete suppression of the 27-dimensional piece of the electromagnetic current[14] and combine this additional information with our other results, then we find that all magnetic moments can be expressed in terms of the neutron and Λ magnetic moments. For example, we have $\kappa_{\Sigma^+}=\kappa_n-4\kappa_\Lambda$. The suppression of the **27**, which here enters as an additional group-theoretical assumption, may have a physical basis: If states exist filling this representation, they must either have a relatively high mass or couple weakly, or both, so that they have little influence in hadron dynamics in the 1–2-BeV range. We emphasize, however, that the assumption of complete **27** suppression in the electromagnetic current has, to our way of thinking, less of a rationale at present than the other assumptions that we have made. If we go still further and assume that the 10-dimensional representation is absent in the symmetry breaking, only then do we obtain the full set of Coleman-Glashow formulas [Eq. (1.2)] plus the vanishing of the isoscalar nucleon moments. However, we now may make the important observation that these formulas are unambiguously interpreted as conditions on magnetic moments measured in natural magnetons.

[5] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[6] R. E. Marshak, S. Okubo, and E. C. G. Sudarshan, Phys. Rev. **106**, 599 (1957).
[7] S. Okubo, Phys. Letters **4**, 14 (1963).
[8] S. D. Drell and A. C. Hearn, Phys. Rev. Letters **16**, 908 (1966).
[9] S. B. Gerasimov, Yadern. Fiz. **2**, 598 (1965) [English transl.: Soviet J. Nucl. Phys. **2**, 430 (1966)]; L. I. Lapidus and Chou Kuang-Chao, Zh. Eksperim. i Teor. Fiz. **39**, 1286 (1960) [English transl.: Soviet Phys.—JETP **12**, 898 (1961)].
[10] A. Pais, Phys. Rev. Letters **18**, 17 (1967); M. A. B. Bég and A. Pais, Phys. Rev. **160**, 1479 (1967).
[11] V. A. Matveev, L. D. Soloviev, B. V. Struminski, A. N. Tavkhelidze, and V. P. Shelest, Dubna report, 1967 (unpublished).
[12] There are no restrictions on the two couplings $1\to 8+\gamma$ in broken symmetry.
[13] By natural magneton we mean that the magnetic moment of a baryon is measured in units $e/2M$, where M is the mass of the baryon.
[14] S. Iwao, Nucl. Phys. **68**, 632 (1965).

This is precisely the conjecture of Bég and Pais,[15] and is presently in excellent accord with the measured moments. We refer the reader to the table in their paper[15] for numerical values.

We wish to comment briefly on the numerous previous attempts to estimate the influence of symmetry breaking on the magnetic moments. Some results, such as those obtained from a quark model[16] or chiral dynamics,[17] assume explicit dynamical models which, while they lead to definite predictions, utilize abstractions and approximations within the context of the model which resist precise physical (or experimental) interpretation. It is thus difficult to judge the validity of these results outside the framework of the highly specific models in which they are obtained. The primary virtue of these models lies in their heuristic value. Also relevant is the work of Pagels[18] and of Mathur and Pandit.[19] Both of these early calculations suffer from a common deficiency: They use sum rules which relate the anomalous moments to strong-interaction coupling constants which retain their $SU(3)$-symmetric values in the calculation, and the symmetry breaking is introduced only in the kinematics. Thus symmetry breaking is taken into account only in part of the amplitude and it is not known what error this neglect introduces. There is no reason to expect it to be small. Moreover, these calculations depend on the F/D ratio for the meson-baryon coupling, and although this ratio has a precise meaning only in the $SU(3)$ limit (and may be even fixed by a consistency condition), there is no reason to expect this to be the case in broken symmetry. As pointed out in Ref. 18, variations in this F/D ratio can considerably alter the values of the magnetic moments. This sensitivity to F/D is also a characteristic of the calculation of Ref. 19, so that it is difficult to attribute any reliability to the final results. The approach advocated in the present paper does not have this defect, because the exact symmetry is not assumed for the couplings at any step.

In Sec. II, we give the details of the calculation on the magnetic moments. In Sec. III, we also briefly examined the DHGSR for arbitrary spin targets. This strictly forward sum rule would seem to imply either the existence of towers of particles with arbitrarily high spin or "null" solutions. We also predict the magnetic moment of $\Omega^- \simeq -3e/2M_{\Omega^-}$. In Sec. IV, we examine some superconvergence sum rules (for the forward Thompson amplitude) based on the suppression of $I=2$ Regge poles and fixed poles at high energy.[20] Finally, in Sec. V, we review very briefly the experimental status and prospects for the measurement of strange-particle magnetic moments.

[15] M. A. B. Bég and A. Pais, Phys. Rev. **137**, B1514 (1965). In nuclear magnetons the predicted value is $\mu_\Lambda = -0.8$ and the experiments have $\mu_\Lambda = -0.73 \pm 0.16$.

[16] H. R. Rubinstein, F. Scheck, and R. H. Socolow, Phys. Rev. **154**, 1608 (1967).

[17] J. Schwinger, Phys. Rev. Letters **18**, 923 (1967).

[18] H. Pagels, Phys. Rev. **140**, B999 (1965).

[19] V. S. Mathur and L. K. Pandit, Phys. Rev. **147**, 965 (1966).

[20] H. Harari, Phys. Rev. Letters **18**, 319 (1967); H. Pagels, *ibid.* **18**, 316 (1967).

II. CALCULATION OF MAGNETIC MOMENTS IN BROKEN $SU(3)$

Here we present in detail the calculation of the magnetic moments in accord with the program discussed in the Introduction.

A. Definitions

The magnetic moments of the baryon $\frac{1}{2}^+$ octet are specified in terms of the matrix elements of the electromagnetic-current operator between baryon states of momentum p and p':

$$\langle B(p)|J_\mu(0)|B(p')\rangle = \left(\frac{M_B^2}{p_0'p_0}\right)^{1/2} \bar{U}(p) \times\left[F_1(q^2)e\gamma_\mu + F_2(q^2)\kappa_B\frac{e}{2M_B}i\sigma_{\mu\nu}q^\nu\right]U(p'), \quad (2.1)$$

where $p^2 = p'^2 = M_B^2$, $q = p' - p$, and we have normalized the Dirac and Pauli form factors $F_1(0) = F_2(0) = 1$. We shall be interested only in application for physical photons, $q^2 = 0$. Here κ_B is the anomalous moment of the corresponding baryon measured in natural magnetons $e/2M_B$, so that $\mu_B = (Q + \kappa_B)\, e/2M_B$ is the total magnetic moment. Q is the charge of the particle in units of e. To obtain the magnetic moment in terms of nuclear magnetons, one multiplies the value in natural magnetons by M_p/M_B.

The off-diagonal matrix element for $\Sigma^0 \to \Lambda + \gamma$ is defined according to[21]

$$\langle \Lambda(p)|J_\mu(0)|\Sigma^0(p')\rangle = \left(\frac{M_\Lambda M_{\Sigma^0}}{p_0'p_0}\right)^{1/2} \bar{U}_\Lambda(p) \times\left\{F_2(q^2)\kappa_{\Sigma^0\Lambda}\left(\frac{e}{M_{\Sigma^0}+M_\Lambda}\right)i\sigma_{\mu\nu}q^\nu + F_3(q^2)[q^2\gamma_\mu - (M_\Sigma - M_\Lambda)q_\mu]\right\}U_\Sigma(p'), \quad (2.2)$$

where $p^2 = M_\Lambda^2$, $p'^2 = M_{\Sigma^0}^2$, and the natural magneton for the transition moment here is defined as $e/(M_{\Sigma^0} + M_\Lambda)$. The parameter of physical interest is the lifetime τ_{Σ^0} for the Σ^0 in the electromagnetic transition $\Sigma^0 \to \Lambda + \gamma$:

$$\frac{1}{\tau_{\Sigma^0}} = \tfrac{1}{2}\alpha\left(\frac{\kappa_{\Sigma\Lambda}}{M_{\Sigma^0}}\right)^2\left(1 - \frac{M_\Lambda}{M_{\Sigma^0}}\right)(M_{\Sigma^0} - M_\Lambda)^3. \quad (2.3)$$

[21] J. Dreitlein and H. Primakoff, Phys. Rev. **125**, 1671 (1962).

We shall also have recourse to the use of matrix elements for transitions $\Delta(\frac{3}{2})^+ \to B(\frac{1}{2})^+ + \gamma$ which, assuming dominance of $M1$ transitions, has the form

$$\langle B(p)|J_\mu(0)|\Delta(p')\rangle = \frac{1}{\sqrt{2}}\left(\frac{M_B M_\Delta}{p_0 p_0'}\right)^{1/2} \bar{U}(p) \times \epsilon_{\mu\alpha\beta\lambda} p^\alpha p'^\beta W^\lambda(p') \frac{3e}{2M_\Delta}\mu_B^* \quad (2.4)$$

for $q^2=0$ and $p'^2 = M_\Delta^2$, $p^2 = M_B^2$. The radiative width is then specified by

$$\Gamma_\gamma = (\alpha k^{*3}/2M_B M_\Delta)\mu_B^{*2}, \quad (2.5)$$

where $k^* = (M_\Delta^2 - M_B^2)/2M_\Delta$. The transition moment is here given in units of $e/2M_B$.

B. Symmetry-Breaking Parameters

Next, we shall consider the general characterization of symmetry breaking in the nine magnetic moments (including the transition moment) of the baryon octet. Here we follow the treatment of de Swart,[22] expressing the anomalous-moment operator κ (or total moment operator $Q+\kappa$) as

$$\kappa = \sum_{\mu, I} T_{I,0,0}{}^{(\mu)}, \quad (2.6)$$

where the sum on tensor operators is extended over the allowable irreducible representations of $SU(3)$. For our application we consider the matrix elements of κ between states of the baryon **8**. We here denote by

$$a_{\mu_\gamma} = (8\|T_{0,0,0}{}^{(\mu)}\|8)_\gamma$$

those matrix elements exhibiting the breakdown of U-spin conservation for $\mu = 8_1$, 8_2, 27 and the singlet contribution for $\mu = 1$, and by

$$b_{\mu_\gamma} = (8\|T_{1,0,0}{}^{(\mu)}\|8)_\gamma,$$
$$c_{\mu_\gamma} = (8\|T_{2,0,0}{}^{(\mu)}\|8)_\gamma,$$

those coefficients denoting the breakdown of isospin conservation corresponding to $\Delta I=1$ and $\Delta I=2$ transitions, respectively. The anomalous moments are then expressed in terms of the nine coefficients[23] denoting the transformation property of these matrix elements of the electromagnetic current:

$$\begin{aligned}
\kappa_p &= (15^{1/2}/90)(9b_{8_1} - 4b_{27}) + (1/2\sqrt{3})b_{8_2} - (1/\sqrt{15}) \times (b_{10} - b_{10^*}) + a_1 - (5^{1/2}/10)a_{8_1} + \tfrac{1}{2}a_{8_2} + (1/3\sqrt{5})a_{27},\\
\kappa_n &= -(15^{1/2}/90)(9b_{8_1} - 4b_{27}) - (1/2\sqrt{3})b_{8_2} + (1/\sqrt{15})(b_{10} - b_{10^*}) + a_1 - (5^{1/2}/10)a_{8_1} + \tfrac{1}{2}a_{8_2} + (1/3\sqrt{5})a_{27},\\
\kappa_{\Sigma^0} &= (4/9)c_{27} + a_1 + (5^{1/2}/5)a_{8_1} - (1/9\sqrt{5})a_{27},\\
\kappa_{\Sigma^+} &= -(3/9)c_{27} + (1/\sqrt{3})b_{8_2} + (1/\sqrt{15})(b_{10} - b_{10^*}) + a_1 + (5^{1/2}/5)a_{8_1} - (1/9\sqrt{5})a_{27},\\
\kappa_{\Sigma^-} &= -(2/9)c_{27} - (1/\sqrt{3})b_{8_2} - (1/\sqrt{15})(b_{10} - b_{10^*}) + a_1 + (5^{1/2}/5)a_{8_1} - (1/9\sqrt{5})a_{27}, \quad (2.7)\\
\kappa_\Lambda &= a_1 - (5^{1/2}/5)a_{8_1} - (1/\sqrt{5})a_{27},\\
\kappa_{\Sigma\Lambda} &= (2/\sqrt{3})(15^{1/2}/90)(9b_{8_1} - 6b_{27}),\\
\kappa_{\Xi^0} &= -(15^{1/2}/90)(9b_{8_1} - 4b_{27}) + (1/2\sqrt{3})b_{8_2} - (1/\sqrt{15})(b_{10} - b_{10^*}) + a_1 - (5^{1/2}/10)a_{8_1} - \tfrac{1}{2}a_{8_2} + (1/3\sqrt{5})a_{27},\\
\kappa_{\Xi^-} &= (15^{1/2}/90)(9b_{8_1} - 4b_{27}) - (1/2\sqrt{3})b_{8_2} + (1/\sqrt{15})(b_{10} - b_{10^*}) + a_1 - (5^{1/2}/10)a_{8_1} - \tfrac{1}{2}a_{8_2} + (1/3\sqrt{5})a_{27}.
\end{aligned}$$

A priori, there are no conditions on the coefficients which characterized the magnetic moments; however, by appealing to theoretical principles which have been experimentally supported when applied to other amplitudes, some of these pieces may be small or may vanish. To see how the Coleman-Glashow formulas [Eqs. (1.2)] emerge, we shall consider a series of assumptions on the transformation properties of the electromagnetic current operator, starting from the weakest assumption and ending with the strongest.

First, one may assume that there is no $\Delta I=2$ piece in the electromagnetic current corresponding to $c_{27}=0$, so that the anomalous moments in an $SU(2)$ multiplet are characterized by two parameters. For $I \geqslant 1$ this gives a nontrivial constraint, which in our application results in

$$\kappa_{\Sigma^+} + \kappa_{\Sigma^-} = 2\kappa_{\Sigma^0}. \quad (2.8)$$

To go further, we must make explicit assumptions about $SU(3)$-breaking of the strong interaction upon the matrix elements of the electromagnetic current. We assume that the symmetry breaking will transform like the $I=0$, $I_3=0$ member of an octet, so that we may think in terms of a spurion transforming like the hypercharge inserted into the interaction. To get additional information from this assumption, we must also assume that the spurion acts only once, that is, we shall consider only first-order violations in the symmetry breaking. This corresponds to assuming the absence of the $(U=2, U_3=0)$ member of the **27** in the symmetry breaking or, in our notation, $b_{27} = (10/\sqrt{3})a_{27}$.

[22] J. J. de Swart, Rev. Mod. Phys. 35, 916 (1963).
[23] We have assumed time-reversal invariance, so that only the combination $10^* - 10$ appears.

Then we obtain the Okubo formula[7]

$$2\sqrt{3}\kappa_{\Sigma\Lambda}=3\kappa_{\Lambda}+\kappa_{\Sigma^0}-2\kappa_n-2\kappa_{\Xi^0}. \qquad (2.9)$$

These are the only conditions on the magnetic moments which one can presume to be true to the first order in broken symmetry. One point worth noting: Suppose that, instead of writing the anomalous moments in units of natural magnetons on the left-hand side of Eq. (2.9), we decided to scale them differently by replacing $\kappa_B \rightarrow M_B\kappa_B$, since we have no reason to prefer one choice over the other. This simply redefines the symmetry parameters on the right-hand side. The point is that formulas like Eqs. (2.8) and (2.9) are indifferent to this scale choice to the first order in symmetry-breaking interaction. For example, Eq. (2.9) becomes under this scale change

$$\begin{aligned}0&=\sqrt{3}\kappa_{\Sigma\Lambda}(M_\Sigma+M_\Lambda)-3\kappa_\Lambda M_\Lambda-\kappa_{\Sigma^0}M_\Sigma+2\kappa_n M_n+2\kappa_{\Xi^0}M_{\Xi^0}\\&=[2\sqrt{3}\kappa_{\Sigma\Lambda}-3\kappa_\Lambda-\kappa_{\Sigma^0}+2\kappa_n+2\kappa_{\Xi^0}]\bar{M}\\&\quad+\kappa_n[2(M_n+M_\Xi)-(3M_\Lambda+M_\Sigma)]\\&\quad+\text{high order in symmetry breaking},\end{aligned}$$

where $\bar{M}$ is some $SU(3)$-symmetric mass, and the second term vanishes because of the Gell-Mann–Okubo (GMO) formula. So the Okubo formula [Eq. (2.9)] remains unchanged to the first order. This indifference to the scale is a general characteristic of broken-symmetry results derived on the basis of the assumptions given above.

If we go to the full $SU(3)$ level, but presume only that the current transforms like a U-spin singlet, this corresponds to setting $c_{27}=a_{27}=b_{27}=b_{10}-b_{10^*}=\sqrt{3}a_{8_{1,2}}-b_{8_{1,2}}=0$, so that the magnetic moments are parametrized in terms of three numbers. Then we have the results in Eq. (1.1). Finally, assuming that the current has only octet transformation properties, we eliminate the unitary singlet piece $a_1=0$ and the full Coleman-Glashow formulas [Eq. (1.2)] follow.

This is the situation according to group theory. In what follows, we assume the validity of Eq. (2.8), which is probably very accurate, and Eq. (2.9), which neglects second- and higher-order violations of $SU(3)$, and is probably accurate to within 10% and perhaps much more (if the GMO mass formula is any indicator).

C. Drell-Hearn-Gerasimov Sum Rule

We shall assume the exact validity of the DHGSR[8,9] for forward Compton scattering to the first order in the electromagnetic coupling. In the case of the proton, the sum rule reads

$$\left(\frac{\kappa_p}{M_p}\right)^2=\frac{1}{2\pi^2\alpha}\int_0^\infty\frac{d\nu}{\nu}[\sigma_P(\nu)-\sigma_A(\nu)]. \qquad (2.10)$$

It relates the anomalous moment κ_p of the proton to an integral over the total production cross section for photons of energy ν, with spin parallel and antiparallel to the target-proton spin. The input into the derivation of this sum rule is the low-energy theorem for forward Compton scattering, analyticity, and a no-subtraction hypothesis. The most questionable of these is the subtraction hypothesis; however, this has received theoretical support from the Regge picture of high-energy scattering with the expected absence of fixed poles in this amplitude.[24] If one analyzes the contributions to the integral for proton Compton scattering ($\nu<1$ BeV) on the basis of both the theoretical assumptions and the experimental data on photoproduction from hydrogen, one finds that the contributions arise as follows[8]: At the threshold for photopion production the combination $\sigma_P-\sigma_A$ is given almost entirely by electric-dipole radiation E_{0^+}, and this contributes only to σ_A and hence contributes a negative amount to the integral according to $\sigma_P-\sigma_A\simeq-8\pi(q/p)|E_{0^+}|^2$. The largest contribution to the integral is by far the $N^*(1238)$ resonance, which contributes predominatly to σ_P, with $\sigma_P:\sigma_A=3:1$, and is roughly one order of magnitude larger than the threshold piece. The second resonance $N^{**}(1525)$ contributes a small positive quantity to the integral which almost exactly cancels the threshold contribution, leaving only the N^* as the dominant contribution. In the narrow-resonance approximation, the N^* contributes according to

$$\sigma_P(\nu)-\sigma_A(\nu)=(2\pi^2/k^{*2})\Gamma_\gamma\delta(E-M^*),$$

where $k^*=(M^{*2}-M_p^2)/2M^*$ is the momentum of the proton in the barycentric system for the decay $N^{*+}\rightarrow p+\gamma$; E is the total barycentric energy; $\nu=(E^2-M_p^2)/2M_p$; and

$$\Gamma_\gamma=\frac{M_pk^{*3}}{2\pi M^*}\left(\mu_p^*\frac{e}{2M_p}\right)^2 \qquad (2.11)$$

is the radiative width. Here μ_p^* is the same as the magnetic-dipole coupling defined by Dalitz and Sutherland[25] and is measured in units of $e/2M_p$ as indicated. We use the experimental value

$$\mu_p^*=(3.38\pm0.06)e/2M_p \qquad (2.12)$$

corresponding to $\Gamma_\gamma=0.65\pm0.02$ MeV. In this simple approximation, the DHGSR equation (2.10) then reads

$$\kappa_p^2=(M_p/2M^*)\mu_p^{*2}. \qquad (2.13)$$

Using the experimental value [Eq. (2.12)] for μ_p^*, we obtain $\kappa_p=2.08$ from Eq. (2.13), as compared with $\kappa_p^{\text{expt}}=1.79$. Hence the majority of the anomalous moment can be attributed to the $\gamma+p\leftrightarrow N^{*+}$ transition. If we do a better job by including the influence of the full energy-dependent width [$\Gamma(E=M^*)=120$

[24] A. H. Mueller and T. L. Trueman, Phys. Rev. **160**, 1306 (1967).

[25] R. H. Dalitz and D. G. Sutherland, Phys. Rev. **146**, 1180 (1966).

MeV], then the integral is decreased by $\sim 10\%$, improving the value of the anomalous moment to $\kappa_p \simeq 1.86$. These simple estimates support our conviction that the DHGSR is saturated by keeping only the decuplet and a singlet.

In what follows, we shall continue to use the narrow-resonance approximation in the application of the sum rule to the other baryons. In fact, for all the members of the **10**, with the exception of the N^*, this is an acceptable approximation, as we see from the observed full widths. Since we are interested in $SU(3)$-symmetry-breaking effects in the spectrum $\sigma_P - \sigma_A$, there will be an influence on the magnetic moments due to symmetry breaking in the full widths. However, this will be relatively small (a few percent) compared with the symmetry breaking in the radiative partial width.

D. Application to Strange-Baryon Magnetic Moments

Encouraged by the result for the proton, we now apply the DHGSR to Compton scattering from the baryon octet and also to the processes $\gamma + \Sigma^0 \to \gamma + \Lambda$. A set of nine sum rules for the 8 magnetic moments and the transition moment can then be written. We shall saturate these nine sum rules with the decuplet and a single unitary singlet which is required to have spin $\geqslant \frac{3}{2}$ and might be the $Y_0^*(1520)$, although this identification is not essential.[26] The unitary singlet is required in the processes $\gamma + \Lambda \to \gamma + \Lambda$, $\gamma + \Sigma^0 \to \gamma + \Sigma^0$, and $\gamma + \Sigma^0 \to \gamma + \Lambda$ in order to prevent a "null" solution in the symmetry limit.[10] In the narrow-resonance approximation (but making no assumptions on the symmetry of coupling, etc., at this stage), these nine sum rules read, in direct generalization of Eq. (2.13),

$$\kappa_p^2 = \tfrac{1}{2}(M_N/M_{N^*})\mu_p^{*2}, \quad \kappa_n^2 = \tfrac{1}{2}(M_N/M_{N^*})\mu_n^{*2}, \quad (2.14)$$

$$\kappa_{\Sigma^+}^2 = \tfrac{1}{2}(M_\Sigma/M_{Y_1^*})\mu_{\Sigma^+}^{*2}, \quad (2.15)$$

$$\kappa_{\Sigma^-}^2 = \tfrac{1}{2}(M_\Sigma/M_{Y_1^*})\mu_{\Sigma^-}^{*2}, \quad \kappa_{\Xi^-}^2 = \tfrac{1}{2}(M_\Xi/M_{\Xi^*})\mu_{\Xi^-}^{*2}, \quad (2.16)$$

$$\kappa_{\Xi^0}^2 = \tfrac{1}{2}(M_\Xi/M_{\Xi^*})\mu_{\Xi^0}^{*2}, \quad (2.17)$$

$$\kappa_\Lambda^2 + \kappa_{\Lambda\Sigma}^2 = \tfrac{1}{2}(M_\Sigma/M_{Y_1^*})\mu_\Lambda^{*2} + X_\Lambda^2, \quad (2.18a)$$

$$\kappa_{\Sigma^0}^2 + \kappa_{\Lambda\Sigma}^2 = \tfrac{1}{2}(M_\Sigma/M_{Y_1^*})\mu_{\Sigma^0}^{*2} + X_{\Sigma^0}^2, \quad (2.18b)$$

$$\kappa_{\Lambda\Sigma}(\kappa_\Lambda + \kappa_{\Sigma^0}) = \tfrac{1}{2}(M_\Sigma/M_{Y_1^*})\mu_\Lambda^*\mu_{\Sigma^0}^* + X_\Lambda X_{\Sigma^0}. \quad (2.18c)$$

In writing the rules [Eq. (2.18)], we have set $M_{\Sigma^0} = M_\Lambda$, which simplifies our analysis, and empirically is seen to introduce only errors of the order $(M_{\Sigma^0} - M_\Lambda)/M_\Lambda \sim 6\%$. That combinations of anomalous moments appear on the left side of the DHGSR follows from the observation that the Λ and Σ^0 states are both unexcited and contribute to the low-energy theorem. Here X_Λ and X_{Σ^0} represent the contributions of the singlet to these sum rules; in $SU(3)$ symmetry they are simply related by $\sqrt{3}X_\Lambda = X_{\Sigma^0}$, but we do not make that assumption here. What is essential is the factorization property $(X_\Lambda X_{\Sigma^0})^2 = X_\Lambda^2 X_{\Sigma^0}^2$ in the spectrum. This follows from the assumption that only a single unitary singlet is involved.

This is all we can obtain from the sum rule. Next, we make our group-theoretical assumptions. Assuming $SU(2)$ symmetry on the couplings, we have

$$\kappa_{\Sigma^+} + \kappa_{\Sigma^-} = 2\kappa_{\Sigma^0}, \quad \mu_p^* - \mu_n^* = 0, \quad \mu_{\Sigma^+}^* + \mu_{\Sigma^-}^* = 2\mu_{\Sigma^0}^*. \quad (2.19)$$

From Eq. (2.14) we have the result $\kappa_p^2 = \kappa_n^2$, and since we can also consider Compton scattering with isovector and isoscalar photons separately, we can get the relative signs of the anomalous moments. We also use them in what follows, for this case implying

$$\kappa_p + \kappa_n = 0, \quad (2.20)$$

a well-known consequence of the DHGSR, and one which is experimentally accurate. To account for a nonvanishing isoscalar nucleon moment requires a look at the noise in the spectrum, since neither the E_{0^+} threshold contribution nor the second resonance (which is observed to be excited by only isovector photons) contributes to $\kappa_p + \kappa_n$.

The other $SU(2)$ relation [Eq. (2.19)], when combined with the sum rules (2.15), (2.16), and (2.18), implies

$$\kappa_{\Sigma^0}^2 = \tfrac{1}{2}(M_\Sigma/M_{Y^*})\mu_{\Sigma^0}^{*2}, \quad (2.21a)$$

$$\kappa_{\Sigma\Lambda}^2 = X_{\Sigma^0}^2. \quad (2.21b)$$

As seen even on this $SU(2)$ level from Eq. (2.21b), if there is no singlet contribution, then $\kappa_{\Lambda\Sigma} = 0$, which, when combined with $\kappa_p + \kappa_n = 0$ on the $SU(3)$ level, means that all anomalous moments must vanish (the null solution). For this reason, we require that the singlet and its spin must be $\geq \frac{3}{2}$ so as to impose a relative plus sign in Eq. (2.21).

This is the content of the assumption that the photon will transform like a vector plus a scalar under $SU(2)$. We now go on to assume that the medium-strong symmetry-breaking interaction will transform like an octet. This assumption has considerable support when applied to the mass spectrum. Here it imposes one condition on the anomalous moments[7]

$$2\sqrt{3}\kappa_{\Lambda\Sigma} = 3\kappa_\Lambda + \kappa_{\Sigma^0} - 2(\kappa_{\Xi^0} + \kappa_n) \quad (2.9)$$

and two conditions on the transition moments

$$\mu_{\Sigma^-}^* + \mu_{\Xi^-}^* = 0, \quad (2.22a)$$

$$\mu_{\Sigma^0}^* + \mu_{\Xi^0}^* - \mu_n^* = \sqrt{3}\mu_\Lambda^*. \quad (2.22b)$$

All of these relations (to the first order in the symmetry breaking) are indifferent to whether we use natural units or common units to scale the moments.

[26] Using the coupling defined by F. M. Renard and Y. Renard, Orsay Report, 1966 (unpublished), and the estimate $f_{Y_0^*\Lambda\gamma} \sim 4 \times 10^{-2}$, we find that the $Y_0^*(1405)$ contributes in magnitude only about 10% of the full value of κ_Λ^2, and hence we neglect it.

Using Eqs. (2.22a) and (2.16), we have as an additional constraint

$$\kappa_{\Sigma^-}+\kappa_{\Xi^-}=0\,, \qquad (2.23)$$

since

$$(M_{\Xi^*}M_\Sigma/M_{Y_1^*}M_\Xi)^{1/2}=1.00.$$

Here we have used the photoproduction sum rule of Fubini, Furlan, and Rossetti[27] to determine the relative sign of κ_{Σ^-}, κ_{Ξ^-} (but not the magnitude). In $SU(3)$ these anomalous moments should be quite small, since they are proportional to the isoscalar nucleon magnetic moments. In the presence of $SU(3)$-symmetry breaking, this need not be the case; however, we find the sum rule [Eq. (2.23)] to be obeyed.

We proceed to extract the information from Eqs. (2.9) and (2.22b). Using Eqs. (2.14), (2.17), and (2.9), we have $\mu_n^*=-(2M_{N^*}/M_N)^{1/2}\kappa_n$, $\mu_{\Xi^0}^*=(2M_{\Xi^*}/M_\Xi)^{1/2}\kappa_{\Xi^0}$, $\mu_{\Sigma^0}^*=(2M_{Y_1^*}/M_\Sigma)^{1/2}\kappa_{\Sigma^0}$ [we assume that $SU(3)$ symmetry gives the correct relative signs], which are now substituted in (2.22b) to yield

$$\sqrt{3}\mu_\Lambda^*=(2M_{N^*}/M_N)^{1/2}\kappa_n+(2M_{\Xi^*}/M_\Xi)^{1/2}\kappa_{\Xi^0}+(2M_{Y_1^*}/M_\Sigma)^{1/2}\kappa_{\Sigma^0}.$$

We greatly simplify the consequent algebraic expressions by noting the empirical relations $(M_{N^*}M_\Sigma/M_{Y_1^*}M_N)^{1/2}=1.06$ and $(M_{\Xi^*}M_\Sigma/M_{Y_1^*}M_\Xi)^{1/2}=1.00$, so that we may set these quantities to 1.0 within the range of our approximation, although in principle this is not necessary. Then we have $(M_\Sigma/2M_{Y_1^*})^{1/2}\sqrt{3}\mu_\Lambda^*=\kappa_n+\kappa_{\Xi^0}+\kappa_{\Sigma^0}$, and now, using the Okubo formula [Eq. (2.9)], we may express μ_Λ^* purely in terms of quantities in the $\Lambda\Sigma^0$ system:

$$\mu_\Lambda^*(M_\Sigma/2M_{Y^*})^{1/2}=\tfrac{1}{2}\sqrt{3}(\kappa_\Lambda+\kappa_{\Sigma^0})-\kappa_{\Lambda\Sigma^0}. \qquad (2.24)$$

Using the above relation, Eq. (2.21), and substituting in Eq. (2.18c), we have

$$\kappa_{\Lambda\Sigma}(\kappa_\Lambda+\kappa_{\Sigma^0})=\tfrac{1}{2}\sqrt{3}(\kappa_\Lambda+\kappa_{\Sigma^0})\kappa_{\Sigma^0}-\kappa_{\Lambda\Sigma^0}\kappa_{\Sigma^0}+\kappa_{\Lambda\Sigma}X_\Lambda\,, \qquad (2.25)$$

and substituting Eq. (2.24) into Eq. (2.18a), we also have

$$\kappa_\Lambda^2+\kappa_{\Lambda\Sigma}^2=(\tfrac{1}{2}\sqrt{3}(\kappa_\Lambda+\kappa_{\Sigma^0})-\kappa_{\Lambda\Sigma^0})^2+X_\Lambda^2\,, \qquad (2.26)$$

which are then two equations for the unknown quantity X_Λ. Eliminating X_Λ^2 between these two independent equations then gives us an additional constraint on the anomalous moments, which reads

$$0=(\kappa_\Lambda+\kappa_{\Sigma^0})(\kappa_{\Sigma^0}-\kappa_{\Lambda\Sigma^0}/\sqrt{3})(\kappa_{\Sigma^0}-\sqrt{3}\kappa_{\Lambda\Sigma^0})\times[\kappa_\Lambda+\kappa_{\Sigma^0}-(4/\sqrt{3})\kappa_{\Lambda\Sigma^0}]. \qquad (2.27)$$

Rejecting solutions which are not valid in the $SU(3)$ limit (in the spirit of our perturbation approach), we must have either one or both, of two possible additional constraints

$$\kappa_\Lambda+\kappa_{\Sigma^0}=0\,, \qquad (2.28a)$$

$$\kappa_{\Sigma^0}-\kappa_{\Lambda\Sigma^0}/\sqrt{3}=0. \qquad (2.28b)$$

Within the approximations that we have made, it is impossible to determine which algebraic constraint [Eq. (2.28)] is required. We shall assume that both these conditions are satisfied in the broken symmetry. This is, in fact, what one would expect if $SU(3)$-symmetry predictions relating quantities in only the $\Lambda\Sigma^0$ system were correct even in the presence of symmetry breaking. Again, if the empirical mass relation $M_\Lambda\simeq M_{\Sigma^0}$, which has no theoretical justification in the broken symmetry, is any indication of symmetry violations in the $\Lambda\Sigma^0$ system, then we would expect both the relations [Eq. (2.28)] to be valid even in the presence of symmetry breaking. Our approach to symmetry breaking requires only that at least one of these constraints be satisfied.

Our conclusion from these assumptions is that even in the presence of symmetry breaking, we have the sum rules

$$\begin{aligned}\kappa_p+\kappa_n&=0\,,\\ \kappa_{\Xi^-}&=2\kappa_\Lambda+\kappa_{\Sigma^+}\,,\\ \kappa_{\Xi^0}&=4\kappa_\Lambda-\kappa_n\,,\\ \kappa_{\Sigma^-}+\kappa_{\Xi^-}&=0\,,\\ (2/\sqrt{3})\kappa_{\Sigma^0\Lambda}&=2\kappa_{\Sigma^0}=\kappa_{\Sigma^+}+\kappa_{\Sigma^-}\,,\end{aligned} \qquad (2.29)$$

on the anomalous moments of the octet *measured in natural units*. Had we used a different set of units, these relations would have been modified by the presence of kinematic factors involving mass ratios and not the simple form given above. In terms of the parameters defined in Eq. (2.7) we have, besides $c_{27}=b_{27}-(10/\sqrt{3})a_{27}=0$,

$$\begin{aligned}a_1-\tfrac{1}{9}(\sqrt{5})a_{27}&=0\,,\\ (1/\sqrt{5})b_{8_1}&=b_{8_2}\,,\\ a_{8_2}-(1/\sqrt{5})a_{8_1}&=-(16/9\sqrt{5})a_{27}\,,\\ b_{8_1}-\sqrt{3}a_{8_1}&=(8/\sqrt{3})a_{27}.\end{aligned} \qquad (2.30)$$

These relations do not exclude the possibility of a nonvanishing unitary singlet piece a_1; however, we see that it is required to be of the same order of magnitude as the **27** contribution, $a_1=(\frac{1}{9}\sqrt{5})a_{27}$, and hence proportional to the symmetry breaking. Indeed, suppose that we had assumed $SU(3)$ symmetry and that the electromagnetic current transformed like a U-spin

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[27] S. Fubini, G. Furlan, and C. Rossetti, Nuovo Cimento **40**, 1171 (1965). This sum rule applied to the processes $\gamma+\Sigma^-\to\pi^0+\Sigma^-$ and $\gamma+\Xi^-\to\pi^0+\Xi^-$ implies with **10** saturation $\kappa_{\Sigma^-}=\mu_{\Sigma^-}^*g^*(Y_1^{*-}\Sigma^-\pi^0)/g(\Sigma^-\Sigma^-\pi^0)$ and $\kappa_{\Xi^-}=\mu_{\Xi^-}^*g^*(\Xi^{*-}\Xi^-\pi^0)/g(\Xi^-\Xi^-\pi^0)$. Assuming that the relative sign of coupling constants as determined in the $SU(3)$ limit is not altered in the broken symmetry, we have $g^*(Y_1^*\Sigma^-\pi^0)/g^*(\Xi^{*-}\Xi^-\pi^0)=-1$ and $g(\Sigma^-\Sigma^-\pi^0)/g(\Xi^-\Xi^-\pi^0)=-1$ ($F/F+D=\frac{1}{4}$ so as to be consistent with Coleman-Glashow relations for κ's). Consequently, using the broken-symmetry relation [Eq. (2.22a)] $\mu_{\Sigma^-}^*/\mu_{\Xi^-}^*=-1$ [$\mu_{\Sigma^-}^*=\mu_{\Xi^-}^*=0$ in $SU(3)$ limit], we get $\kappa_{\Sigma^-}/\kappa_{\Xi^-}=-1$. This result differs from the $SU(3)$ relation of $\kappa_{\Sigma^-}/\kappa_{\Xi^-}=+1$. But with our approximation of $\kappa_p+\kappa_n=0$, κ_{Σ^-} and κ_{Ξ^-} actually vanish in the symmetric limit. So there is no contradiction.

scalar (which can, of course, accommodate a large unitary singlet piece), so that we had only the U-spin results for the magnetic moments and transition moments; and further suppose that we had saturated the DHGSR with a **10** and *any* number of **1**'s. Then the singlet piece in the magnetic moments must vanish, so that we must have octet transformation properties for the current and the Coleman-Glashow formula.[28] If our saturation assumption is fairly reasonable, we conclude that *there cannot be a large singlet contribution comparable to the octet to the electromagnetic-current operator*, at least for magnetic-moment matrix elements.[29] Either this or our saturation approximation is drastically wrong. In any event, we find that the singlet piece, if present at all, is proportional to symmetry breaking.

The sum rule $\kappa_p+\kappa_n=0$ is valid to about 6% and we might expect comparable error in our other sum rules. Using the well-known values for the nucleon moments, and

$$\begin{aligned}\mu_\Lambda&=(-0.73\pm0.16)\,e/2M_p\,,\\ \mu_{\Sigma^+}&=(+2.6\pm0.6)\,e/2M_p\end{aligned}\tag{2.31}$$

(which represent world-wide averages), we find, using Eq. (2.29),

$$\begin{aligned}\mu_{\Xi^0}&=-(1.1\pm0.5)\,e/2M_p\,,\\ \mu_{\Xi^-}&=(0.3+0.7)\,e/2M_p\,,\end{aligned}\tag{2.32}$$

which hardly represent precise numbers. We would be disturbed, however, if the measured $\Xi^{-,0}$ moments lay outside the indicated range. Future experiments will reduce the errors in Eq. (2.31) to $\delta_\Lambda\sim\pm0.05\;e/2M_p$, $\delta_{\Sigma^+}\sim\pm0.3\;e/2M_p$, so that Eq. (2.32) can be improved.

E. Suppression of the 27

Our sum rules [Eq. (2.29)], as we see from Eq. (2.30), still admit the possibility of a symmetry-breaking contribution from the **27**. From the general absence of such representations in the observed spectrum of particles we might expect the matrix elements of this piece of the electromagnetic current to be small. Here we examine the consequences of this additional assumption of all components $I=0$, 1, 2 of the **27** vanishing.[14] This implies $a_{27}=b_{27}=c_{27}=0$, which imposes an additional rule on the magnetic moments besides Eqs. (2.8) and (2.9):

$$\kappa_{\Xi^-}+\kappa_{\Xi^0}=\kappa_{\Sigma^0}+3\kappa_\Lambda-(\kappa_p+\kappa_n)\,.\tag{2.33}$$

When combined with Eq. (2.29), this is equivalent to

$$\kappa_{\Sigma^+}+\kappa_{\Xi^0}=0\,.\tag{2.34}$$

The assumption of **27** suppression can also be applied to the transition moments $\mu_B{}^*(\Delta\to B+\gamma)$. That implies an additional relation

$$\mu_{\Sigma^+}{}^*+\mu_{\Xi^0}{}^*=0\,,\tag{2.35}$$

which also leads to Eq. (2.34) through the DHGSR, since again the mass factor $(M_{\Xi^*}M_\Sigma/M_{Y^*}M_\Xi)^{1/2}=1.00$. Thus, in this instance, the DHGSR does not impose any additional constraint on the magnetic moments.

The relation $\kappa_{\Sigma^+}+\kappa_{\Xi^0}=0$ following from complete **27** suppression does not have the property of invariance under a scale change on the units as do our other relations [Eqs. (2.8), (2.9), and (2.22)], as discussed in Sec. II B. Here, precisely what is meant by complete **27** suppression depends on our choice of units for the anomalous moments appearing on the left-hand side of Eq. (2.7). For this reason, we have less confidence in the application of the postulate of **27** suppression.

Using Eq. (2.34) and our previous result [Eq. (2.29)], we find that all nine moments can be expressed in terms of κ_n and κ_Λ:

$$\begin{gathered}\kappa_p=-\kappa_n\,,\quad \kappa_{\Sigma^+}=\kappa_n-4\kappa_\Lambda\,,\quad \kappa_{\Sigma^0}=-\kappa_\Lambda\,,\\ \kappa_{\Sigma^-}=2\kappa_\Lambda-\kappa_n\,,\quad \kappa_{\Xi^0}=4\kappa_\Lambda-\kappa_n\,,\quad \kappa_{\Xi^-}=\kappa_n-2\kappa_\Lambda\,,\\ \kappa_{\Sigma\Lambda}=-\sqrt{3}\kappa_\Lambda\,.\end{gathered}\tag{2.36}$$

We note that in $\kappa_{\Sigma^+}=\kappa_n-4\kappa_\Lambda$, present experimental measurements give $(+1.1\pm0.5)\,e/2M_p$ for the right-hand side and $(+1.8\pm0.6)\,e/2M_p$ for the left-hand side.

Only if we further assume the absence of the 10-dimensional representation in the symmetry breaking, $b_{10}-b_{10^*}=0$, do we obtain the full set of Coleman-Glashow formulas and the vanishing of the isoscalar nucleon moment. Here these formulas are interpreted as conditions on the magnetic moments measured in natural units in accord with the conjecture of Bég and Pais.[15] However, the absence of both **10** and **27** is, in our opinion, a strong assumption and one which is difficult to test experimentally in an independent way.

The calculation that we have presented here represents a first approximation to symmetry breaking in the magnetic moments. It can be improved and refined in a systematic way by including in the DHGSR, as a next approximation, (i) the threshold contribution to the E_{0^+} multipole which can be expected to be important in Ξ^- and Σ^- Compton scattering, since the **10** does not contribute in the symmetry limit, (ii) additional singlets in the processes $\gamma+\Lambda$, $\Sigma^0\to\gamma+\Lambda$, Σ^0, and (iii) the second resonance region (in particular, the octet). To obtain additional constraints on the spectrum so that the symmetry-breaking effects in these contributions can also be computed, one can use the DHGSR gen-

[28] This result came about in the following way. Using $SU(3)$ and assuming that the photon is a U-spin scalar, we have the results [Eq. (11)] for the magnetic moments. Now, $SU(2)$ gives us Eq. (2.21), $\kappa_{\Sigma^0}{}^2=(M_8/2M_{10})\mu_{\Sigma^0}{}^{*2}$, from the DHGSR, no matter how many 1's we saturate with. Since by U-spin $\mu_{\Sigma^0}{}^*=\frac{1}{2}\mu_n{}^*$, we have (again dropping mass factor ~1) $\kappa_{\Sigma^0}=-\frac{1}{2}\kappa_n$; and using this and the U-spin results [Eq. (1.1)], we find Eq. (1.2), the Coleman-Glashow formula, plus $\kappa_p+\kappa_n=0$. Hence the photon must have pure octet transformation properties.

[29] For further experimental implications of the absence of a unitary singlet piece in the electromagnetic current, see H. Harari, in Proceedings of the Symposium on the Present Status of $SU(3)$ Symmetry, Argonne National Laboratory, Argonne, Ill., 1967 (unpublished).

eralized to nonforward scattering. Then, using the broken-symmetry sum rules on these couplings, an algebraic system of constraints will be imposed on these additional pieces in the amplitude, and their contribution to the magnetic moments computed.

III. MAGNETIC MOMENTS OF HIGHER-SPIN PARTICLES

In this section, we briefly consider some of the implications of the DHGSR for particles of arbitrary spin.[9] For a target of spin J, magnetic moment μ, and charge-to-mass ratio Qe/M, we have for forward scattering the rule

$$J\left(\frac{\mu}{J}-\frac{Qe}{M}\right)^2=\frac{1}{\pi}\int_0^\infty \frac{d\nu}{\nu}\,(\sigma_P-\sigma_A)\,, \qquad (3.1)$$

where $\sigma_{P,A}$ are the cross sections for photons with spin parallel and antiparallel to the maximum target spin, $J_z=J$. Since the left-hand side of Eq. (3.1) is always positive, the same must be demanded of the right-hand side. There does not appear to be any evident reason for this feature. Although $\sigma_{P,A}>0$, the weighted difference in Eq. (3.1) could well be negative. So there is a problem as to how Eq. (3.1) might in general be satisfied even qualitatively for targets of arbitrary spin.

Suppose we consider saturating the integral with sharp resonant states of the same parity as the target. Moreover, let us suppose that the contributing resonances of spin J_R make radiative transitions to the target of spin J primarily via S and P waves, the higher waves being suppressed by angular-momentum barriers. Then, in order to get a contribution to σ_P, and thus a positive contribution to the integral, requires a state with $J_R \geqslant J$. What we are suggesting is that to satisfy Eq. (3.1), one needs particles with spin greater than the target, or an infinite tower of states, when we also consider the high-spin states as targets.

For example, suppose that the resonant states radiatively decayed only via S waves, so that we need not consider orbital angular momentum, and the spins just add up. Denoting the contribution to the integral of the transition to a resonant state of spin J by $[\mu^*(J)]^2$, we then have

$$J[\mu(J)/J-Qe/M]^2=[\mu^*(J+1)]^2-[\mu^*(J-1)]^2\,, \qquad (3.2)$$

where we have assumed mass degeneracy in this gross model. A solution to this infinite system of equations satisfying the boundary condition $\mu(0)=0$ is

$$\begin{aligned}\mu(J)&=\mu_0 J\,,\\ \mu^*(J)&=\tfrac{1}{2}(\mu_0-Qe/M)J\,,\end{aligned} \qquad (3.3)$$

just as an illustration of how towers saturate the sum rule. There is, of course, always the trivial "null" solution for which all particles have their normal moment $\mu(J)=(Qe/M)J$ and all transitions vanish: $\mu^*(J)=0$.

As a slightly more serious application of the sum rule [Eq. (3.1)] to higher-spin systems, we consider its implications for the magnetic moment of the remaining metastable baryon, the Ω^-, $J^P=\frac{3}{2}^+$. Since the cross sections for photons on the Ω^- open only into channels with strangeness $=-3$, in the absence of any nearby resonant state with this quantum number or large continuum (which is difficult to estimate) the right-hand side of Eq. (3.1) vanishes. The magnetic moment of the Ω^- would then be given by its normal value,

$$\mu_{\Omega^-}\approx -\tfrac{3}{2}e/M_{\Omega^-}=-1.68e/2M_N\,, \qquad (3.4)$$

which differs from quark-model predictions.[16] In the distant future, this number might be measured.

When we consider, in addition, internal quantum numbers, the saturation of Eq. (3.1) becomes more difficult to implement without introducing high-spin states. For example, with $SU(3)$ symmetry the magnetic moments of the decuplets are proportional to their charge, with the proportionality constant being given by Eq. (3.4). So the integral $(1/\pi)\int_0^\infty(\sigma_P-\sigma_A)d\nu/\nu$ vanishes for the decuplet. In particular, for the N^{*+} we can expect a sizeable contribution to σ_A from the transitions $\gamma+N^{*+}\rightarrow p$ which must be cancelled, presumably by a high-spin state $J\geqslant\frac{3}{2}$ contributing to σ_P. So sum rules like Eq. (3.1), when considered in conjunction with internal symmetries, provide considerable constraints on the particle spectrum, as has been often emphasized by others.[10]

IV. FORWARD-COMPTON-SCATTERING SUM RULES

Let us consider Compton scattering on hadronic systems, and assume a simple Regge picture for the high-energy behavior.[20] Then, if $\alpha_{I=2}(0)<0$ is the leading trajectory for combinations of amplitudes with $I=2$ in the crossed channel, and assuming that there are no fixed J-plane poles with $J=0$, $I=2$, or assuming that their residues are small,[25] we have, from the no-subtraction hypothesis that this implies and the Thomson theorem for low-energy scattering,

$$\frac{2}{M_\Sigma}=\frac{1}{2\pi^2\alpha}\int_0^\infty d\nu\,[2\sigma_{\Sigma^0}(\nu)-\sigma_{\Sigma^+}(\nu)-\sigma_{\Sigma^-}(\nu)] \qquad (4.1)$$

for the $\Sigma^{\pm,0}$ system. Here σ_Σ is the total cross section for the photon on the hadron. If we make the drastic assumption that the $N^*(1238)$ is stable (under the strong interactions), then we would have in this approximation

$$\frac{2}{M_{N^*}}=\frac{1}{2\pi^2\alpha}\int_0^\infty d\nu\,[2\sigma_{N^{*+}}(\nu)-\sigma_{N^{*++}}(\nu)-\sigma_{N^{*0}}(\nu)]\,. \qquad (4.2)$$

Assuming that these sum rules are true, we now consider their possible implication for the particle spectrum.

First consider Eq. (4.1) for the spectrum of photons on the Σ's. The Λ can contribute to σ_{Σ^0}; however, since its coupling to the Σ^0 is magnetic, its contribution to this cross section is proportional to the excitation energy and hence vanishes as $M_\Lambda \to M_{\Sigma^0}$. So we may neglect the Λ. The $Y_1^*(1385)$ of the decuplet will contribute to the cross sections. Assuming $SU(3)$ for the radiative couplings, one finds that $Y_1^*(1385)$ contributes according to $\sigma_{\Sigma^+}:\sigma_{\Sigma^0}:\sigma_{\Sigma^-}=4:1:0$ and hence with the wrong sign to Eq. (4.1). Thus we require a unitary singlet which will contribute only to σ_{Σ^0} such as to make the right-hand side of the sum rule positive. If we assume this singlet to be the $Y_0^*(1520)$, $J^P=\frac{3}{2}$ and estimate its coupling to $\Sigma^0\gamma$, using our previous sum rule [Eq. (2.21b)] and using $SU(3)$ to relate the transition moments of the Y_1^* to the known value for the nucleon equation (2.12), then we find the sum rule [Eq. (4.1)] to be satisfied to 5%. Hence our saturation assumptions of the DHGSR used in Sec. II are consistent with the no-subtraction hypothesis required for the sum rule [Eq. (4.1)].

Next, we turn to the question as to how the sum rule [Eq. (4.2)] on the N^* system might be satisfied. The nucleon can contribute to the integral equation (4.2), but to see how this contribution enters, we must go back to the definition of the cross section in terms of an absorptive amplitude $\text{Im}f(\nu)=(\nu/4\pi)\sigma(\nu)$ and then, using the crossing property $\text{Im}f(\nu)=-\text{Im}f(-\nu)$, extend the integral over negative frequencies. Then the direct-channel nucleon pole contributes for negative-frequency photons and the cross-channel pole for positive frequencies. Since the nucleon contributes to the absorptive parts according to $\sigma_{N^{*+}}:\sigma_{N^{*0}}:\sigma_{N^{*++}}=1:1:0$, it has the correct sign in the contribution to the integral. Using the $M1$ coupling given by Eq. (2.4) and the expression for the width in terms of the coupling in Eq. (2.11), we find for the width

$$\Gamma_\gamma=\alpha M_{N^*}(1-M_N^2/M_{N^*}^2)^2=1.8 \text{ MeV}, \qquad (4.3)$$

which is not in good agreement with the experimental value 0.65 MeV. Typical of all relations between electric (e) and magnetic (μ^*) multipole coupling, the excitation energy enters the relation, and hence is very sensitive to the small difference of large masses and must be viewed with caution. The introduction of resonances like $N^{**}(1525)$ will contribute dominantly to $\sigma_{N^{*+}}$ and hence decreases the estimate for Γ_γ given above. The introduction of additional high-spin resonances in Compton scattering from the N^*'s was already required in the saturation of the DHGSR for N^*, as discussed in Sec. III, so that it comes as no surprise that they are needed here as well.

There are also corrections to sum rules involving unstable hadrons as targets due to the finite-width corrections. This problem of estimating corrections due to scattering from continuum states is usually neglected in most treatments and it is not altogether clear how to estimate these corrections properly. First of all, such scattering processes do not have well-defined "in" and "out" states, so that the scattering amplitude is not well-defined in the usual way. What one must do, for example, for the $\gamma+N^*\to\gamma+N^*$ processes, is to examine the three-body to three-body amplitude $\gamma+N+\pi\to\gamma+N+\pi$ when the c.m. energy of the final and initial πN system in the $I=\frac{3}{2}$, $J=\frac{3}{2}$ state approaches resonance energy. At this resonant c.m. energy the forward scattering amplitude of the photon of low frequency ν and the (πN) system might be something like $f(\nu)\approx-(e^2/M_R)\nu^2/(\nu^2+\frac{1}{4}\Gamma^2)$. The ν^2 factor in the numerator arises because of the absence of $0\to0$ transitions in the $\nu\to0$ limit and the denominator reflects the Breit-Wigner form. For a stable state ($\Gamma=0$) we have the usual Thompson limit $f=-e^2/M_R$, but for $\Gamma\neq0$ and $\nu<\frac{1}{2}\Gamma$ we would have deviations from the usual low-energy behavior. It is of course necessary to examine the three-body amplitude in detail to establish these conjectures.

V. EXPERIMENTAL MEASUREMENTS OF MAGNETIC MOMENTS

We shall review just very briefly the prospects of the measurements of strange-baryon magnetic moments. The proton and neutron magnetic moments are, of course, known with great precision.

The Λ magnetic moment is known to $\sim$20%. The Rosenfeld compilation of five experiments gives $\mu_\Lambda=(-0.73\pm0.16)\ e/2M_p$.[30] This already rules out a large unitary singlet piece comparable to the octet in the current. As has been pointed out, if one uses natural units in the Coleman-Glashow formulas $\mu_\Lambda=\frac{1}{2}\mu_n$, the experimental value is in remarkable agreement.[15] Emulsion experiments planned at CERN may provide measurement of this moment, possibly with errors of the order of $\delta_\Lambda\sim\pm0.05\ e/2M_p$.

The measurement of the Σ^+ magnetic moment is also gradually being resolved; the present world-wide weighted average gives $\mu_{\Sigma^+}=(2.6\pm0.5)\ e/2M_p$.[31] Ex-

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[31] V. Cook, T. Ewart, G. Masek, R. Orr, and E. Platner, Phys. Rev. Letters **17**, 223 (1966); C. R. Sullivan, A. D. McInturff, D. Kotelchuck, and C. E. Roos, *ibid.* **18**, 1163 (1967); D. Kotelchuck, E. R. Goza, C. R. Sullivan, and C. E. Roos, *ibid.* **18**, 1166 (1967); T. S. Mast, L. K. Gershwin, M. Alston-Garnjost, R. O. Bangerter, A. Barbaro-Galtieri, J. J. Murray, F. T. Solmitz, and R. D. Tripp, *ibid.* **20**, 1312 (1968). The weighted average $\mu_{\Sigma^+}=(2.6\pm0.5)e/2M_p$ also includes the preliminary value of (3.5 ± 1.2) as reported in *Proceedings of the Heidelberg International Conference on Elementary Particles, Heidelberg, 1967*, edited by H. Filthuth (Interscience Publishers, Inc., New York, 1968), by the CERN-Bristol-Lausanne-Munich-Rome Collaboration.

periments planned for the near future may well push the error to less than 10%.

Beyond this, an experiment has been performed at Brookhaven and Berkeley to measure the Ξ^- moment with an anticipated uncertainty on the order of $\delta_{\Xi^-} \sim \pm 0.35\ e/2M_p$.

It is possible that the near future will yield rough measurements of the Ξ^0 and Σ^- magnetic moments. Because of the small asymmetry parameter in the decay $\Sigma^- \to n+\pi^-$, this moment is difficult to measure in the usual way (by observing the rotation of the plane of the $n\pi^-$ system in the presence of a magnetic field); instead, one must examine the polarization of the final-state neutron.

On the other hand, considerable improvement in the experimental resolution is required for a measurement of the lifetime $\tau(\Sigma^0 \to \Lambda+\gamma)$ which is related to the transition moment $\kappa_{\Sigma^0\Lambda}$ by Eq. (2.3). Here one looks for the decay mode of Σ^0 into Λ and a Dalitz pair $\Sigma^0 \to \Lambda+e^+ +e^-$. The rate of this decay is smaller by a factor $\alpha=1/137$ and there is more hope of its being measured.[32]

ACKNOWLEDGMENTS

We would like to thank our colleagues here at Rockefeller University for helpful discussions, and also thank Dr. T. Kycia of Brookhaven National Laboratory for information on the experimental data.

[32] For other possible methods of measuring $\tau(\Sigma^0 \to \Lambda+\gamma)$, see Dreitlein and Primakoff (Ref. 21), and H. S. Mani and Y. P. Yao, Phys. Rev. (to be published).

PHYSICAL REVIEW VOLUME 172, NUMBER 5 25 AUGUST 1968

Pion Electromagnetic Form Factor

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(Received 27 December 1967; revised manuscript received 4 March 1968)

Expressions for the spacelike form factor and its phase in the timelike region are given in terms of the magnitude of the timelike form factor. A sum rule for the upper bound of the pion radius is given.

PRELIMINARY experimental results of the pion electromagnetic form factor in the timelike region have recently been obtained.[1,2] The measurement of the form factor is limited only to the region near the ρ resonance. It is the purpose of this paper to suggest that measurements in other energy regions, especially near the two-pion threshold, are of considerable interest. This is so because, under certain assumptions about the zeros of the form factor, which can be checked experimentally, we can obtain the following relations:

(i) The spacelike pion form factor is given by a dispersion relation in terms of its magnitude in the timelike region. In particular, a dispersion relation for the upper bound of the rms radius of the pion with estimated errors can be derived.

(ii) The phase of the pion form factor above the two-pion threshold is given by a similar dispersion relation. For values of $s=(p_+ + p_-)^2 < 16\mu^2$, where p_+, p_- are the pion 4-momenta and μ is the pion mass, the phase of the pion form factor is that of pion-pion scattering in the P state, owing to the final-state theorem. If the inelastic effect can be neglected, this relation also holds for higher values of s, say, near the ρ resonance.

(iii) The formulation given here applies also to other form-factor problems such as those of the nucleon and K meson, but, unlike the pion, their timelike form factor cannot be measured to the lowest threshold.

The pion form factor $F(s)$ is assumed to be analytic in the cut s plane with a cut from $4\mu^2$ to ∞. Time-reversal invariance is assumed to be valid. Apart from the finite number of subtractions, we have

$$F(s)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{\sigma(s')ds'}{s'-s}\,, \tag{1}$$

with the condition $F(0)=1$.

The timelike region is defined for $s>0$ and the spacelike region for $s<0$. Let ϕ be the phase of $F(s)$. The following remarks are useful for subsequent

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[1] V. L. Auslander *et al.*, Phys. Letters **25B**, 433 (1967).

[2] J. E. Augustin *et al.* (to be published).

[3] We exclude in this discussion an exponential behavior of the form factor. This possibility has been discussed by T. T. Wu and C. N. Yang, Phys. Rev. **137**, B708 (1965). The case of N=half-integer and $F(4\mu^2)=0$ will be discussed elsewhere.