High-Energy Proton-Proton Scattering and Nucleon Structure: The Scalar Interaction in $O(4,2)$*

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A formula is derived from the simple scalar interaction in the relativistic $O(4,2)$ model of the baryons that accounts for the high-energy proton-proton scattering at fixed angle over a cross-section range of eight decades, with a single over-all constant as the parameter. The theory also explains why there is an apparent relation between the nucleon-nucleon scattering and the electromagnetic form factors. Further implications and developments of the theory are discussed.

THERE is now a considerable amount of accurate experimental data on large-momentum-transfer proton-proton scattering[1–5] that shows that the earlier phenomenological or semiphenomenological fits by many authors[6–9] only give the very qualitative form of the data over a limited range. Krisch[10] has recently given a universal empirical curve for the differential cross section with six parameters, but at the expense of neglecting the interference term which is not explained.

The purpose of this paper is to show that the high-energy nucleon-nucleon scattering at fixed angle can be well accounted for in the relativistic theory of composite structures that uses the representations of the dynamical group $O(4,2)$ for the baryons.[11] In this theory, the proton is the ground state of a composite structure having infinitely many higher excited states. It is the presence of these excited states that contains indirectly, via the "wave function" of the system, information on the mesonic structure around the nucleon. The theory has been successfully applied to explain the magnetic moments, form factors, and mass spectrum,[12] as well as partial decay rates[13] of hadrons.

In previous applications only the vertex function has been considered. In the present new application of the theory to scattering processes we adopt the following simple physical picture: The two nucleons approaching each other at fixed impact parameter simply transfer momentum to each other, and their "wave functions" are coupled via a scalar interaction. We therefore evaluate the analog of the scalar coupling term $g\{\psi^\dagger(p_2)\times\psi(p_1)\psi^\dagger(p_4)\psi(p_3)-\text{exchange}\}$ in $O(4,2)$; that is, the $\psi(p)$'s are now the boosted $O(4,2)$ states. The more general current-current interaction will be discussed in a future publication. More precisely, for two fermions the scalar-scalar interaction amplitude in $O(4,2)$ has the following form:

$$A=g_S\{\langle\bar{n}_3p_3|\mathcal{S}|\bar{n}_1p_1\rangle\langle\bar{n}_4p_4|\mathcal{S}|\bar{n}_2p_2\rangle-\text{exchange}\}\,,\quad(1)$$

where $\mathcal{S}$ is the interaction operator, $|\bar{n}p\rangle$ are the "tilted" and boosted (in that order) physical $O(4,2)$ states[11,12]

$$|\bar{n}p\rangle=(1/\mathfrak{N})e^{i\xi\cdot\mathbf{M}}e^{i\vartheta_nL_{45}}|n\rangle\,,\quad(2)$$

with ϑ_n = the tilting angle, $\boldsymbol{\xi}$ the parameters of the Lorentz transformation, $\tanh\xi=p/E$, and $M_i=L_{i5}$ the generators of the Lorentz transformation. In the center-of-mass frame,

$$p_1=m(\cosh\xi,0,0,\sinh\xi),\quad p_2=m(\cosh\xi,0,0,-\sinh\xi)\,,$$
$$p_3=m(\cosh\xi,\sin\theta\sinh\xi,0,\cos\theta\sinh\xi)\,,$$
$$p_4=m(\cosh\xi,-\sin\theta\sinh\xi,0,-\cos\theta\sinh\xi);\quad(3)$$

hence

$$s=4m^2\cosh^2\xi,\quad t=2m^2\sinh^2\xi(1-\cos\theta)\,,$$

so that

$$A=g_S\{\langle n_3|e^{-i\vartheta_3L_{45}}e^{-i\xi_3\cdot\mathbf{M}}\mathcal{S}e^{i\xi_1\cdot\mathbf{M}}e^{i\vartheta_1L_{45}}|n_1\rangle\times\langle n_4|e^{-i\vartheta_4L_{45}}e^{-i\xi_4\cdot\mathbf{M}}\mathcal{S}e^{i\xi_2\cdot\mathbf{M}}e^{i\vartheta_2L_{45}}|n_2\rangle-\text{exchange}\}\,,\quad(4)$$

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where

$$\xi_1=\xi(0,0,1),\quad \xi_2=\xi(0,\,0,\,-1),$$
$$\xi_3=\xi(\sin\theta,0,\cos\theta),\quad \xi_4(-\sin\theta,\,0,\,-\cos\theta).$$

If we now first take $\mathcal{S}$ to be simply the identity operator, we can evaluate the matrix elements in (4). For this purpose we first make an Euler angle transformation and take the spin factors out:

$$e^{-i\xi_3\cdot\mathbf{M}}e^{i\xi_1\cdot\mathbf{M}}=e^{-i\varphi_D L_{13}}e^{i\chi_D L_{35}}e^{-i\psi_D L_{13}}, \tag{5}$$

where

$$\sin\varphi_D=-\frac{\cos\frac{1}{2}\theta}{\cosh\frac{1}{2}\chi_D}[\cosh\xi(1-\cos\theta)+\cos\theta],$$

$$\sin\psi_D=\frac{\cos\frac{1}{2}\theta}{\cosh\frac{1}{2}\chi_D},\quad \sinh\tfrac{1}{2}\chi_D=\sin\tfrac{1}{2}\theta\,\sinh\xi.$$

We have then

$$F_{n_3n_1}(\xi,\theta)\equiv\langle n_3|e^{-i\vartheta_3L_{45}}e^{-i\varphi_DL_{13}}e^{i\chi_DL_{35}}e^{-i\psi_DL_{13}}e^{i\vartheta_1L_{45}}|n_1\rangle$$
$$=d_{m_3m}{}^{1/2}(-\varphi_D)F_{mm}(\vartheta,\chi_D)d_{mm_1}{}^{1/2}(-\psi_D). \tag{6}$$

For the ground state, $\vartheta_3=\vartheta_1=\vartheta$ and the remaining matrix element in (6) can be evaluated as in the previous form-factor calculations[11–13]:

$$F_{1/2,1/2}=F_{-1/2,-1/2}=\cosh\tfrac{1}{2}\chi_D/\cosh^4(\tfrac{1}{2}\beta_D),$$
$$\sinh\tfrac{1}{2}\beta_D=\cosh\vartheta\,\sinh\tfrac{1}{2}\chi_D. \tag{7}$$

Similarly, we evaluate the second factor in (4) and the exchange term with the corresponding angles χ_E, φ_E, ψ_E and obtain

$$(d\sigma/d\Omega)_{\text{unpol}}=(g_S{}^2/s)\{|F^2(\vartheta,\chi_D)|^2+|F^2(\vartheta,\chi_E)|^2$$
$$-\tfrac{1}{2}\cos(\varphi_E+\psi_E-\varphi_D-\psi_D)[F^2(\vartheta,\chi_D)F^2(\vartheta,\chi_E)^*$$
$$+F^2(\vartheta,\chi_D)^*F^2(\vartheta,\chi_E)]\}. \tag{8}$$

We then express all the angles and the matrix elements F in terms of the invariants s, t, and u. The final result is

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}}=\frac{g_S{}^2}{s}\left\{\frac{(1-t/4m^2)^2}{(1-at)^8}+\frac{(1-u/4m^2)^2}{(1-au)^8}-\frac{s/4m^2-ut/(4m^2)^2}{(1-at)^4(1-au)^4}\right\}, \tag{9}$$

where $a=\cosh^2\vartheta/4m^2$, and the value of a as determined[11] from the experimental form factor is $a=1.4$.

Figure 1 shows the differential cross section $d\sigma/dt=[4\pi/(s-4m^2)]d\sigma/d\Omega$ plotted against $-t$ for two angles, $\theta=90°$ and 75°, together with the experimental points. A single analytic curve thus fits the experiment quite well, and we see no reason to invoke a layer (onion) structure for the proton with a number of different empirical radii.[14]

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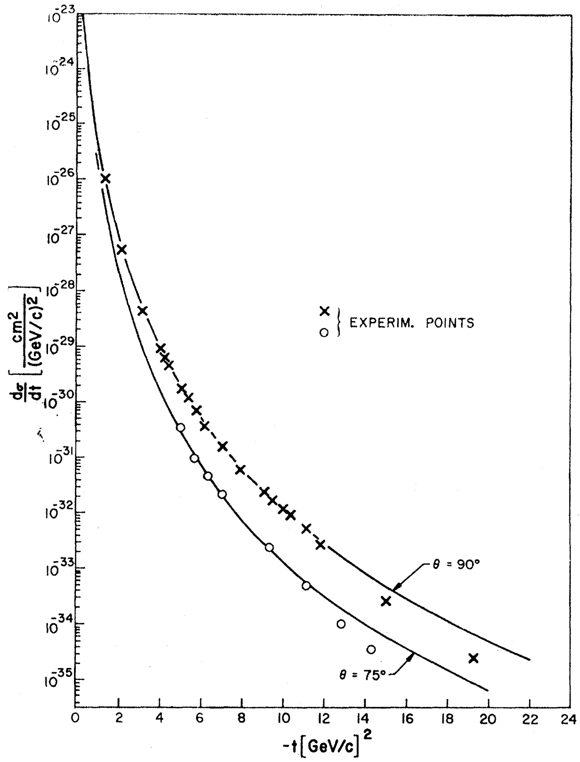

FIG. 1. Proton-proton differential elastic cross section at fixed angle as calculated from Eq. (9) together with experimental points (Refs. 1–5) for two angles [normalized at $t=-9.11$ and at $t=-7.03$ (GeV/c)^2, respectively].

It is clear from our derivation that the differential cross section depends on the fourth power of the form factor for a scalar vertex:

$$\frac{d\sigma}{dt}=\frac{4\pi g_S{}^2}{s(s-4m^2)}\{G_S{}^4(t)\cosh^4(\tfrac{1}{2}\chi_D)+G_S{}^4(u)\cosh^4(\tfrac{1}{2}\chi_E)+\text{cross term}\}. \tag{10}$$

Now it is a characteristic of the $O(4,2)$ theory that the scalar form factor, and the magnetic and the charge form factors, although quite different in strength, have, to a good approximation, the same functional dependence of the form

$$G_S(t)=1/(1-at)^2,\quad G_M{}^P(t)\cong G_E{}^P(t)\cong 1/(1-at)^2. \tag{11}$$

Thus, the observation that the proton-proton differential cross section is related to the electromagnetic form factors[15]—which would be only accidental from the usual point of view, because one is a strong and the other an electromagnetic interaction—is now explained in terms of the composite structure. Note also that the

[15] The relation between $d\sigma/dt$ and the electromagnetic form factor have been given in the quark model by L. Van Hove, in *Symmetry Principles and Fundamental Particles* (W. H. Freeman and Co., San Francisco, 1967), p. 173. See also T. T. Wu and C. N. Yang, Phys. Rev. **137B**, 708 (1965).

neutron has almost no electric form factor, yet n-p scattering is about the same as p-p scattering.

We should like to emphasize that the present simple theory is valid only at fixed angle. That means that the coupling constant g_S in Eq. (9) is still a function of the angle. We have more elaborate formulas than (9) involving propagators in the t and u channels, or involving more general scalar, pseudoscalar, and vector interactions, with which one can give a more detailed fit of the data. These considerations, however, always bring new parameters into the theory and do not change in an essential way the salient feature of the nucleon-nucleon scattering at high energies given by (9). Our objective was to point out this feature in a simple way. A further important point is the immediate applicability of the present theory to inelastic processes of the form

$$N+N\rightarrow N'+N'',$$

where N', N'' are arbitrary nucleon resonances. These amplitudes can be evaluated from Eq. (6) by simply inserting the appropriate higher-spin $O(4,2)$ state for $|n\rangle\equiv|njm\pm\rangle$ and evaluating the matrix elements. Note that all known excited states of baryons can be accommodated in a single representation of $O(4,2)$.[16]

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Determination of Vacuum Regge Poles and Cuts*

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The sum of the π^+p and π^-p forward elastic scattering amplitudes at high energies is studied from the standpoint of complex angular momentum. Experimental total cross sections and real parts, as well as the Igi sum rule, are used. It is found that excellent fits can be obtained using either two Regge poles (the P and the P') or a pole and a cut (the P and the PP). Two-pole fits indicate that the Pomeranchukon intercept lies in the interval $0.92\leq\alpha_P(0)\leq1.00$. Prospects for improving these bounds with data at higher energies are investigated.

I. INTRODUCTION

An investigation is made of the singularity structure in the angular-momentum plane of amplitudes for which t-channel exchanges are restricted to vacuum quantum numbers. The discussion is further limited to the point $t=0$ (corresponding to forward elastic scattering). The present work is motivated by a number of recent theoretical studies,[1-4] each of which has questioned the common assumption that the leading singularity is a simple pole (called the Pomeranchukon) with intercept $\alpha_P(0)=1$. This assumption, as well as the importance of contributions arising from angular-momentum branch points, is studied phenomenologically. Recent experimental measurements of pion-nucleon total cross sections and forward real parts at high energy by Foley *et al.*,[5] to significantly greater accuracy than was previously available, are also important stimuli to this work.

The theoretical facts and proposals that bear on the problem are discussed in Sec. II. The assumptions and hypotheses to be tested in the subsequent phenomenology are indicated. It is necessary to be rather restrictive, as only a few parameters can be meaningfully determined from the data. In Sec. III we discuss the choice of experimental data to be used, the degree of our confidence in them, and the significance of any conclusions that numerical analysis might provide. The numerical methods and results are presented in Sec. IV. Excellent fits are obtained with a rather wide range of expressions. However, we do obtain a lower limit for the intercept of the leading singularity, which should be reasonably model-independent. Finally, in Sec. V we summarize the conclusions based on the present experimental and theoretical situation and indicate the prospects for resolving the remaining ambiguities with data at higher energies than are presently accessible.

II. THEORETICAL BACKGROUND

As it is the purpose of this work to study the properties of amplitudes having vacuum quantum numbers in the t channel, it is important to consider expressions which do not also have contributions from other quan-

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