Kim's results are self-consistent to within experimental errors. To rigorously satisfy our family of sum rules Eq. (1), however, the above values of $g_{p\Lambda K}^2$ and $g_{p\Sigma^0K}^2$ should be slightly pushed down and brought up, respectively (in the same direction as all previous results,[4,16] including those obtained from photoproduction analysis).[17] This can still preserve SU_3 invariance, with an f value smaller than 0.41.

[16] G. H. Davies *et al.*, Nucl. Phys. **B3**, 616 (1967), and references therein.

[17] See, for example, T. K. Kuo, Phys. Rev. **129**, 2264 (1963); **130**, 1539 (1963); S. Hatsukade and H. J. Schnitzer, *ibid.* **132**, 1301 (1963); Fayyazuddin, *ibid* **134**, B182 (1964).

[18] Y. C. Liu and S. Okubo, Phys. Rev. **168**, 1712 (1968).

The same conclusion applies to the crossing-even amplitude $C^{(+)}(\omega)$.[18] This time a subtraction constant is needed. From the ordinary dispersion relation itself (where data in Ref. 1 are taken) we evaluate this number as $\frac{1}{2}\pi C^{(+)}(0)=10.13M_K^{-1}$. The three terms on the left-hand side for $C^{(+)}(\omega)$ [analogous to that of Eq. (1) for $C^{(-)}(\omega)$] cancel almost completely.

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Regge-Pole Analysis of Charge-Exchange and Hypercharge-Exchange Reactions in Pseudoscalar-Meson–Baryon Scattering*

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Within the context of the vector-tensor Regge-pole exchange model, the data on charge-exchange reactions ($\pi^-p\to\pi^0n$, $\pi^-p\to\eta n$, $K^-p\to\bar{K}^0n$) and hypercharge-exchange reactions [$\pi^-p\to K^0\Lambda(\Sigma^0)$, $\pi^+p\to K^+\Sigma^+$, $K^-p\to\pi^0\Lambda$, $K^-p\to\pi^-\Sigma^+$] are systematically analyzed. A statistical fit is presented which provides a good quantitative description of these data. In particular, the measured differential cross sections are adequately fitted over the momentum transfer range $0\leq|t|\leq2.6$ (GeV/c)^2. The magnitude and sign changes of the polarization data are explained by the model with the exception of the $\pi^-p\to\pi^0n$ polarization at high energies. Predictions of immediate experimental interest include (i) near-maximal polarization in $K^-p\to\bar{K}^0n$ near $t=0$, (ii) a large polarization independent of energy for all reactions involving exchange of both a vector and tensor trajectory, (iii) differential cross sections for $K^-p\to\eta\Lambda(\Sigma^0)$, (iv) structure in differential cross sections near $t\sim-2.5$ (GeV/c)^2, and (v) polarization for $\pi^-p\to\pi^0n$ at intermediate energies. Assuming $SU(3)$ symmetry for the factorized residues, the f/d ratios were determined for the vector and tensor nonet $J^P=\frac{1}{2}^+$ baryon octet couplings.

1. INTRODUCTION

THERE now exists a wealth of experimental information on meson-baryon scattering regarding the energy and angular dependence of the cross section and the baryon polarization for elastic charge-exchange and hypercharge-exchange reactions.[1] Some general qualitative features of the available data are: (i) The total scattering cross sections seem to approach constant values at high energies while the partial cross sections for most inelastic reactions appear to fall off with a power-law energy dependence $(E_{\rm lab})^{-n}$ where $n>0$; (ii) most of the differential cross sections show sharp peaks in the forward direction, $\cos\theta_{\rm c.m.}\simeq+1$ ($\theta_{\rm c.m.}$ is defined as the angle between the directions of the incident and outgoing mesons in the center-of-mass system); (iii) in a variety of reactions a secondary maximum occurs following the forward peak.

The forward peaks suggest that the scattering occurs through a peripheral mechanism, i.e., through the exchange of a boson. Calculations based on exchange of an elementary meson of spin J have failed to describe the energy and angle dependence of cross sections for reactions in which $J\geq1$.[2] In the Regge-pole model, however, the exchanged object has angular momentum which is a function of the invariant momentum transfer t, and as a result the energy dependence of the cross section is a power law consistent with the experimental

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[1] The general trends of the data can be seen in the excellent compilation by D. R. O. Morrison, Phys. Letters **22**, 528 (1966), in which references to the original sources are given. See also D. R. O. Morrison, Centre d'Etudes Recherche National Report No. CERN/TC/Physics 66-20 (unpublished).

[2] See, for example, J. D. Jackson, J. Donahue, K. Gottfried, R. Keyser, and B. Svensson, Phys. Rev. **139**, B428 (1965).

data. Thus a peripheral model based on the exchange of a Regge pole may be expected to provide a useful framework for the description of scattering processes.

Extensive studies[3–12] have indicated that indeed such a model can prove useful in the description of strong-interaction processes. These analyses are encouraging since a large body of experimental data can be systematized and correlated by means of a few parameters.

However, these models involve parameters which as yet cannot be calculated theoretically, and consequently must be determined from the experimental data. Thus we are led to apply the model to situations requiring a small number of parameters. Pseudoscalar-meson–spin-$\frac{1}{2}$-baryon scattering provides such a situation. Only two independent spin amplitudes are required. Furthermore, by restricting the analysis to quasi-elastic reactions, selection rules limit the number of trajectories exchanged to one or two. Additional simplifications follow from assuming $SU(3)$ symmetry for the factorized residues.[13]

The studies reported here indicate that a peripheral model based on the exchange of boson Regge poles and on $SU(3)$ symmetry can provide a consistent quantitative description of a large body of experimental data on reactions due to the inelastic scattering of pseudoscalar mesons by spin-$\frac{1}{2}$ baryons. In Sec. 2 the formalism of the model is presented. The applications to charge-exchange reactions are made in Sec. 3 and to hypercharge-exchange reactions in Sec. 4. Section 5 is devoted to summary and discussions.

2. FORMALISM

We consider a pseudoscalar meson M_1 of mass μ_1 scattering on a $J^P=\frac{1}{2}^+$ baryon B_1 of mass m_1, producing another pseudoscalar meson M_2 of mass μ_2 and another $\frac{1}{2}^+$ baryon B_2 of mass m_2,

$$M_1+B_1 \to M_2+B_2. \tag{2.1}$$

In the spirit of the peripheral model we shall assume that the scattering (2.1) near the forward direction at high energies is adequately described by a coherent sum of Regge amplitudes, each representing the exchange of a boson Regge trajectory. The formulation of the model consists in making the Sommerfeld-Watson transformation on the partial-wave expansion of the helicity amplitudes of the t-channel reaction

$$\bar{B}_2+B_1 \to M_2+\bar{M}_1, \tag{2.2}$$

and extracting the asymptotic energy behavior with the help of the substitution rule. This procedure has been carried out by many authors[14–17] and we shall briefly mention a few steps leading to the assumptions that will be made.

Let $F_{++}{}^t$ and $F_{+-}{}^t$ denote the helicity-nonflip and helicity-flip amplitudes, respectively, for the reaction (2.2). In terms of the conventional Mandelstam invariant amplitudes A and B, they are

$$F_{++}{}^t=-\left\{-A(t-4\bar{M}^2)^{1/2}+\frac{B\bar{M}}{(t-4\bar{M}^2)^{1/2}}\left[2s+t-\sum m^2+\frac{(m_1-m_2)}{2\bar{M}}(\mu_1{}^2-\mu_2{}^2)\right]\right\}, \tag{2.3}$$

$$F_{+-}{}^t=-[+B\phi^{1/2}/(t-4\bar{M}^2)^{1/2}],$$
$$F_{--}{}^t=F_{++}{}^t,\quad F_{-+}{}^t=-F_{+-}{}^t, \tag{2.4}$$

where $\bar{M}=\frac{1}{2}(m_1+m_2)$, $\sum m^2=m_1{}^2+m_2{}^2+\mu_1{}^2+\mu_2{}^2$,

$$\phi=st(\sum m^2-s-t)-t(\mu_1{}^2-m_1{}^2)(\mu_2{}^2-m_2{}^2)-s(m_1{}^2-m_2{}^2)(\mu_1{}^2-\mu_2{}^2)-(\mu_1{}^2m_2{}^2-\mu_2{}^2m_1{}^2)(\mu_1{}^2+m_2{}^2-\mu_2{}^2-m_1{}^2),$$

and s, t, u, are the Mandelstam variables. In order to avoid the branch-point singularities at $t=4\bar{M}^2$, we shall define the following functions:

$$G_{++}=-(t-4\bar{M}^2)^{1/2}F_{++}{}^t, \tag{2.5}$$

$$G_{+-}=+B. \tag{2.6}$$

The G's have analytic properties implied by Mandelstam representation while $F_{+\pm}{}^t$ have additional kinematical singularities.[18] The formulas for the differential cross section and polarization (written in natural units $\hbar=c=1$) are

$$\frac{d\sigma}{dt}=\frac{1}{64\pi s p_1{}^2}\frac{1}{4\bar{M}^2-t}[|G_{++}|^2+|\phi^{1/2}G_{+-}|^2], \tag{2.7}$$

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[18] L. C. Wang, Phys. Rev. **142**, 1187 (1965); L. Durand, III, and C. B. Chiu (unpublished).

$$\mathbf{P}\frac{d\sigma}{dt}=+\hat{n}\frac{(1-z^2)^{1/2}}{16\pi\sqrt{s}}\frac{p_2}{p_1}\frac{1}{(4\bar{M}^2-t)}\operatorname{Im}(G_{++}{}^*G_{+-})\,, \quad (2.8)$$

where

$$\hat{n}\equiv\frac{\hat{p}_1\times\hat{p}_2}{|\hat{p}_1\times\hat{p}_2|} \quad \text{(Basel convention)}.$$

Here $\mathbf{p}_1$ and $\mathbf{p}_2$ are the three-momenta of the baryons B_1 and B_2 in the s-channel reaction (2.1) and z is the cosine of the scattering angle, $\hat{p}_1\cdot\hat{p}_2$.

Applying the Sommerfeld-Watson transformation to the partial-wave expansions of the G's, we can write the Regge amplitudes at asymptotic values of z_t as follows:

$$G_{++}\simeq\sum\beta_{++}(\alpha,t)\frac{1\pm e^{-i\pi\alpha}}{\sin\pi\alpha}\frac{\Gamma(\alpha+\frac{3}{2})}{\Gamma(\alpha+1)}\left(\frac{-2z_tp_tk_t}{s_{++}}\right)^{\alpha}, \quad (2.9)$$

$$G_{+-}\simeq\sum\beta_{+-}(\alpha,t)\frac{1\pm e^{-i\pi\alpha}}{\sin\pi\alpha}\frac{\Gamma(\alpha+\frac{3}{2})}{\Gamma(\alpha+1)}\left(\frac{-2z_tp_tk_t}{s_{+-}}\right)^{\alpha-1}. \quad (2.10)$$

Here z_t is the t-channel scattering angle taken between the "uncrossed lines," and $p_t(k_t)$ is the magnitude of the three-momentum of the baryon (meson) in the t channel. The summations of (2.9) and (2.10) are over the contributing Regge poles. We have removed the threshold dependence $(p_tk_t/s_{+\pm})^J$ from the residue functions where the constants $s_{+\pm}$ are energy scale parameters. The reduced residues $\beta_{+\pm}$ and the trajectory α are real functions of t below the t-channel threshold $(\mu_1+\mu_2)^2$.

We first note that in view of the Mandelstam symmetry[19] the functions $[\beta_{+\pm}\Gamma(\alpha+\frac{3}{2})]$ do not have poles at $\alpha=-\frac{3}{2}$, $-\frac{5}{2}$, etc. The factor $[\Gamma(\alpha+1)\sin\pi\alpha]^{-1}$ is analytic on the left-half of the real axis in the J plane except for a simple pole at $\alpha=0$. For the analysis of experimental data using trajectories which do not reach negative values smaller than -1, it is convenient to replace $[\Gamma(\alpha+1)\sin\pi\alpha]^{-1}$ by $(\alpha+1)/\sin\pi\alpha$, which still has the pole at $\alpha=0$. Notice that the poles at the real positive integers, which have the meaning of physical bound states, are still preserved by our replacement. The pole at $\alpha=0$, however, is cancelled by the signature zero for the case of odd-trajectory exchange.

In the case of even-trajectory exchange, we introduce a "ghost-killing factor" α in the amplitudes, thus obtaining

$$G_{++}=\sum\beta_{++}'(\alpha,t)\frac{1\pm e^{-i\pi\alpha}}{\sin\pi\alpha}\begin{Bmatrix}\alpha\\1\end{Bmatrix}(\alpha+1)R_{++}, \quad (2.11)$$

$$G_{+-}=\sum\beta_{+-}'(\alpha,t)\frac{1\pm e^{-i\pi\alpha}}{\sin\pi\alpha}\begin{Bmatrix}\alpha\\1\end{Bmatrix}(\alpha+1)R_{+-}, \quad (2.12)$$

where

$$R_{+\pm}\equiv(-2z_tp_tk_t/s_{+\pm})^{(\alpha-1/2)\pm1/2} \quad (2.13)$$

and the upper (lower) entry in the bracket refers to even (odd) signature amplitude. To proceed further, we must make specific assumptions about the residue functions $\beta_{+\pm}'$.

In an analysis of the π^-p charge exchange reaction using Regge ρ exchange, Arbab and Chiu[7] have assumed $\beta_{+-}'\sim\alpha$ and interpreted the observed minimum at $t\sim-0.6$ (GeV/c)^2 in the differential cross section as arising from the vanishing of G_{+-} at $\alpha=0$ (the sense-nonsense point[20]). Frautschi[21] has suggested that the observed dips in the elastic scattering reactions at high energies may have a similar interpretation if the amplitudes corresponding to even-trajectory exchanges also vanish at the sense-nonsense point.[22] We shall assume that the amplitude vanishes at the sense-nonsense transition point (irrespective of the signature of the exchanged trajectory); in other words, the trajectory chooses "sense."

On the other hand, the factor $(\alpha+1)$ in Eqs. (2.11) and (2.12) produces a zero (assuming β has no pole at $\alpha=-1$) in the amplitudes associated with the exchange of an even trajectory but not in the amplitudes associated with an odd trajectory. We shall assume, without any attempt at justification, that the amplitudes vanish at the nonsense-nonsense point $\alpha=-1$ irrespective of the signature. For this purpose we should have $\beta_{+\pm}\sim(\alpha+1)$ for the odd-trajectory exchange. This assumption, as we shall see later especially in the case of $\pi^-p\to\pi^0n$, helps us to achieve the rapid decline of the differential cross-section curves for $t\lesssim-1$ (GeV/c)^2.

Inserting the various factors implied by the above two assumptions, we can write (2.11) and (2.12) finally as

$$G_{++}=\sum\gamma_{++}(t)\frac{1\pm e^{-i\pi\alpha}}{\sin\pi\alpha}g_\pm(\alpha)(\alpha+1)R_{++}, \quad (2.14)$$

$$G_{+-}=\sum\gamma_{+-}(t)\frac{1\pm e^{-i\pi\alpha}}{\sin\pi\alpha}\alpha g_\pm(\alpha)(\alpha+1)R_{+-}, \quad (2.15)$$

where

$$g_\pm(\alpha)\equiv(\alpha+\tfrac{1}{2})\pm(-\tfrac{1}{2}). \quad (2.16)$$

These are the forms of the amplitudes that will be used in this paper. The functions $\gamma_{+\pm}$ will be assumed to be constant or at most linear functions of t (crossover factors). As for the energy dependent factor R, we shall

[19] S. Mandelstam, Ann. Phys. (N. Y.) **19**, 251 (1959).

[20] M. Gell-Mann, in *Proceedings of the International Conference on High-Energy Physics, Geneva, 1962*, edited by J. Prentki (CERN Scientific Information Service, Geneva, 1962).

[21] S. Frautschi, Phys. Rev. Letters **17**, 722 (1966).

[22] G. F. Chew, Phys. Rev. Letters **16**, 30 (1966).

write[23,24]

$$R_{+\pm}=\left[\frac{s-\frac{1}{2}(m_1^2+m_2^2+\mu_1^2+\mu_2^2)+\frac{1}{2}t}{(m_1+m_2)E_\pm}\right]^{(\alpha-1/2)\pm1/2}, \quad (2.17)$$

where $E_\pm$ are two arbitrary parameters. For πN elastic scattering at $t=0$, the expression in the square brackets of Eq. (2.17) reduces to the customary form $[E_{\text{lab}}/E_\pm]$.

Interference Model

At intermediate energies (laboratory momenta of the incident mesons in the range 2–5 GeV/c) the formation of resonant states in meson-baryon scattering can be an important process. The resonances in the s-channel reaction (2.1) will be termed as direct channel resonances. In extrapolating the Regge-pole analyses to intermediate energy ranges, we shall allow for the possibility of resonance formation. This is done by assuming that the total amplitude for a given process can be represented by a sum of two terms: (i) the resonant amplitude, which is a sum of Breit-Wigner forms and (ii) the Regge-pole amplitude. Such an interference model was originally considered by Barger and Cline[12] to explain the structure in $\pi^\pm p$ elastic scattering in the backward direction.

The statement of the interference model is that the helicity amplitudes for meson-baryon scattering are given by

$$F_{+\pm}=F_{+\pm}^{\text{Regge}}+F_{+\pm}^{\text{Res}}. \quad (2.18)$$

The general structure of the Regge-pole amplitude $F_{+\pm}^{\text{Regge}}$ to be used is that of (2.14) and (2.15). To obtain the explicit forms of the resonance amplitudes $F_{+\pm}^{\text{Res}}$, we start with the scattering amplitude for meson-baryon scattering:

$$F_{fi}=(\xi_f|f+i\boldsymbol{\sigma}\cdot\hat{n}g|\xi_i), \quad (2.19)$$

where

$$\hat{n}=\frac{\hat{p}_1\times\hat{p}_2}{|\hat{p}_1\times\hat{p}_2|}.$$

[23] In the literature, the high-energy behavior of the Regge amplitude is exhibited by writing for the expression in the square bracket of Eq. (2.17) the form $[s/s_0]$ or $[E_{\text{lab}}/E_0]$. At asymptotic energies the above two forms are equivalent if we identify $s_0\simeq 2mE_0$. However, for intermediate energies (~2 or 3 GeV) and for unequal-mass kinematics, the manner in which the energy variable enters the amplitude is very important. Since one is considering the expansion of z_t in the Regge amplitude, the expression (2.17) seems to be more exact. The factor (m_1+m_2) multiplying E_0 is inserted, so that we recover the expression E_{lab}/E_0 for πN kinematics. Also by retaining the term $\frac{1}{2}t$ in Eq. (2.17), we can achieve better fits to the generally small values of the experimental cross sections observed at large negative t.

[24] We recall that for the case of unequal-mass scattering the Regge asymptotic behavior at $t=0$ is maintained through the introduction of an infinite number of daughter trajectories. This problem is studied in detail by D. Z. Freedman and J. M. Wang [Phys. Rev. **153**, 1596 (1967)], who show that successive $t=0$ intercepts of the daughters are spaced by one unit and the successive daughters have opposite signature. Not much is known about the daughter poles as a function of t and it is difficult to take them into account in phenomenological analyses [see R. E. Cutkosky and B. B. Deo, Phys. Rev. Letters **19**, 1256 (1967)]. We shall assume that the contributions from the daughter poles can be neglected.

The partial-wave expansions of f and g are

$$f=\sum_l[(l+1)f_{l+}+lf_{l-}]P_l(z),$$
$$g=\sum_l[f_{l+}-f_{l-}]\frac{d}{dz}P_l(z)(1-z^2)^{1/2}, \quad (2.20)$$

where $f_{l\pm}$ are the partial-wave amplitudes for $J=l\pm\frac{1}{2}$. Breit-Wigner forms for the partial-wave amplitudes imply

$$f=\sum_{\text{Res}}c(J+\tfrac{1}{2})P_l(z)\left(\frac{1}{p_1p_2}\right)^{1/2}\frac{(x\Gamma'/\Gamma)^{1/2}}{\epsilon-i}, \quad (2.21)$$

$$g=\sum_{\text{Res}}c(1-z^2)^{1/2}(-)^{J-l-1/2}\frac{d}{dz}P_l(z)\times\left(\frac{1}{p_1p_2}\right)^{1/2}\frac{(x\Gamma'/\Gamma)^{1/2}}{\epsilon-i}, \quad (2.22)$$

$$\epsilon\equiv(s_R-s)/\Gamma\sqrt{s_R},\quad x\equiv\Gamma_{\text{el}}/\Gamma.$$

Γ is the total width at half-maximum of the resonance peak, Γ_{el} and Γ' are the partial widths of the resonance for decay into the initial and final channels; s_R is the mass squared of the resonance; c is the appropriate Clebsch-Gordan coefficient coming from $SU(2)$ or $SU(3)$ symmetry. The summations on the right of (2.21) and (2.22) are over the various resonances.

Little is known about the momentum dependence of the elasticity x or total width Γ for an inelastic resonance. The well-known centrifugal barrier factors from potential scattering are not reliable when dealing with a resonance whose mass is large compared to the threshold of the final states. Consequently, we make the simplest possible assumption that x and Γ are constants independent of momentum.

We can write A and B in terms of f and g as

$$A=\frac{4\pi}{[(E_1+m_1)(E_2+m_2)]^{1/2}}\times\left\{(\sqrt{s}+\bar{M})f+\frac{g}{(1-z^2)^{1/2}}\left[(\sqrt{s}+\bar{M})z+\frac{(\sqrt{s}-\bar{M})(E_1+m_1)(E_2+m_2)}{p_1p_2}\right]\right\}, \quad (2.23)$$

$$B=\frac{4\pi}{[(E_1+m_1)(E_2+m_2)]^{1/2}}\left\{f+\frac{g}{(1-z^2)^{1/2}}\times\left[z-\frac{(E_1+m_1)(E_2+m_2)}{p_1p_2}\right]\right\}. \quad (2.24)$$

Substituting for f and g from Eqs. (2.21) and (2.22) in the above formulas for A and B, we obtain the t-channel helicity amplitudes from Eqs. (2.3) and (2.4).

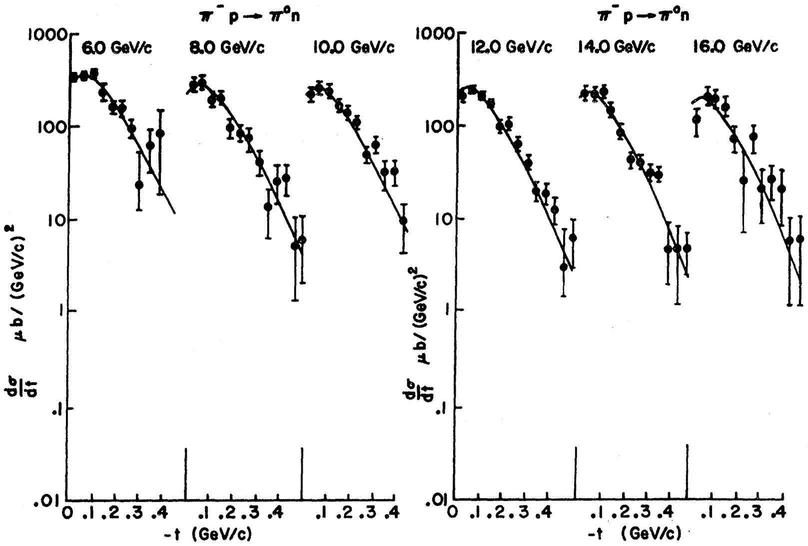

FIG. 1. The data on $\pi^- p \to \pi^0 n$ from Mannelli *et al.*, Ref. 27, compared to our solution.

3. CHARGE-EXCHANGE REACTIONS

A. Reaction $\pi^- p \to \pi^0 n$

The analysis of the $\pi^- p$ charge-exchange reaction

$$\pi^- p \to \pi^0 n \tag{3.1}$$

is important for two reasons. At present, this is the only inelastic meson-baryon scattering reaction that has been studied experimentally up to energies $\sim$18 GeV. Secondly, from the standpoint of Regge-pole phenomenology, this reaction is ideal in that only one known trajectory has the appropriate quantum numbers ($I=1$, $G=+$, $Y=0$) to be exchanged. Among the presently known boson resonances[25] only the vector meson $\rho(m=760$ MeV, $J^P=1^-)$ has the above quantum numbers.

From the expressions (2.14)-(2.16) we now write down the helicity amplitudes for the reaction (3.1) as

$$G_{++}(\rho)=C_+\gamma_{++}{}^V(\alpha_\rho+1)^2(1+t/t_\rho) \times(i+\tan\tfrac{1}{2}\pi\alpha_\rho)R_{++}{}^\rho, \tag{3.2}$$

$$G_{+-}(\rho)=C_-\gamma_{+-}{}^V(\alpha_\rho+1)^2\alpha_\rho(i+\tan\tfrac{1}{2}\pi\alpha_\rho)R_{+-}{}^\rho, \tag{3.3}$$

where

$$R_{+\pm}{}^\rho\equiv\left(\frac{s-\frac{1}{2}(m_1{}^2+m_2{}^2+\mu_1{}^2+\mu_2{}^2-t)}{(m_1+m_2)E_\pm{}^V}\right)^{(\alpha_\rho-1/2)\pm 1/2}. \tag{3.4}$$

The superscript V denotes the exchange of an odd trajectory associated with vector meson. The factor $(1+t/t_\rho)$ is the "crossover" factor[26] in the nonflip amplitude $G_{++}(\rho)$. The Clebsch-Gordan coefficient corresponding to the coupling of the ρ exchange to the meson vertex is $\sqrt{2}$ and to the baryon vertex is -1. By the factorization theorem, we have

$$C_+=C_-=-\sqrt{2}. \tag{3.5}$$

We shall parametrize the $\gamma_{+\pm}{}^V$ as constants independent of t. For α_ρ we choose the quadratic form with positive curvative ($\alpha_{1,2}\geq 0$)

$$\alpha_\rho=\alpha_0+\alpha_1 t+\alpha_2 t^2. \tag{3.6}$$

Thus our expression for the cross section contains eight adjustable parameters: $\gamma_{++}{}^V$, $\gamma_{+-}{}^V$, t_ρ, $E_+{}^V$, $E_-{}^V$, α_0, α_1, and α_2.

The experimental data on $(d\sigma/dt)$ of the reaction (3.1) at the laboratory incident pion momenta 6, 8, 10, 12, 14, and 16 GeV/c for $0\lesssim -t\lesssim 0.5$ (GeV/c)^2 were obtained by Mannelli *et al.*[27] The data at 4.83, 5.85, 9.8, 13.3, and 18.2 GeV/c for $0\lesssim -t\lesssim 2.5$ (GeV/c)^2 are those of Sonderegger *et al.*[28] and Sterling *et al.*[29] The errors in these experiments are mainly statistical. The authors of Ref. 27 quote an over-all normalization error of $\pm 8\%$ at each value of the momentum, while the authors of Refs. 28 and 29 estimate their normalization error as $\pm 10\%$ ($\pm 15\%$ at the highest two momenta).

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[26] There are various possible explanations of the crossover effect in the context of Regge-pole phenomenology as discussed originally by Phillips and Rarita (Ref. 3). We follow here the simple explanation that $G_{++}(\rho)$ has a zero near the forward direction and that the contributions of the P (Pomeranchuk) and P' trajectories to G_{+-} are small compared to that of ρ. This argument is supported by the absence of the initial turnover in the experimental $\pi^\pm p$ elastic cross sections. Of course, if a second trajectory exists with the quantum numbers as ρ (e.g., ρ'), then the situation is considerably more complicated.

[27] I. Mannelli, A. Bigi, R. Carrara, M. Wahlig, and L. Sodickson, Phys. Rev. Letters **14**, 408 (1965).

[28] P. Sonderegger, J. Kirz, O. Guisan, P. Falk-Vairant, C. Bruneton, P. Borgeaud, A. V. Stirling, C. Caverzasio, J. Guillaud, M. Yvert, and B. Amblard, Phys. Rev. Letters **14**, 763 (1965).

[29] A. Stirling, P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, P. Borgeaud, M. Yvert, J. Guillaud, C. Caverzasio, and B. Amblard, Phys. Letters **20**, 75 (1966).

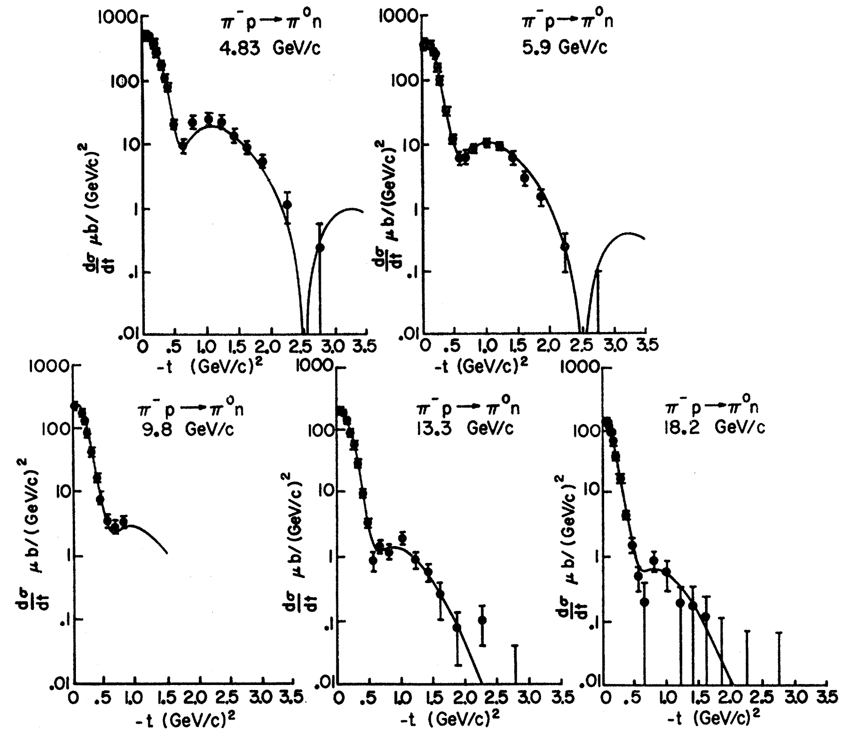

FIG. 2. The data on $\pi^- p \to \pi^0 n$ from Sonderegger *et al.*, Ref. 28, compared to our solution.

We have used a conventional χ^2 minimization procedure to determine the eight parameters from the above data for the $\pi^- p \to \pi^0 n$ differential cross section. Our definition of χ^2 is as follows: if $\sigma_{in} \pm \Delta\sigma_{in}$ is the measured value of the cross section at $t = t_i$ and $p_{\rm lab} = p_n$, and ϵ_n is the quoted normalization error in the experiment, we define

$$\chi^2 = \sum_i \sum_n \left[\frac{\sigma_{in}^c - (1+g_n)\sigma_{in}}{(1+g_n)\Delta\sigma_{in}}\right]^2 + \sum_n \left[\frac{g_n}{\epsilon_n}\right]^2. \quad (3.7)$$

Here σ_{in}^c is the calculated value of the cross section and the normalization-error parameter g_n, along with the parameters in the Regge amplitudes, is selected to yield a minimum value for the χ^2.

We obtained a χ^2 at a minimum of 198 for 160 data points. This value of χ^2 should only be taken as indicative of a reasonable fit to the data because of the possibility of unknown systematic errors (point-to-point) and the uncertainties in the measurement of t. Thus a textbook interpretation of χ^2 (optimum χ^2 = number of data points − number of parameters) should not be expected when we are fitting a steep curve to data points which may be horizontally displaced. For example, an increase of 10% in the errors reduces χ^2 to 152.

Our best values of the Regge ρ parameters are[30]:

$$\begin{aligned} &\gamma_{++}^V = 13.55\ \text{GeV}, \quad \gamma_{+-}^V = 37.4\ \text{GeV}^{-2}, \\ &E_+^V = 2.12\ \text{GeV}, \quad E_-^V = 1.81\ \text{GeV}, \\ &\alpha_0 = 0.564, \quad \alpha_1 = 1.03\ (\text{GeV}/c)^{-2}, \\ &\alpha_2 = 0.16 (\text{GeV}/c)^{-4}, \\ &t_\rho = 0.31\ (\text{GeV}/c)^2. \end{aligned} \quad (3.8)$$

Previous analyses of certain segments of this data on $\pi^- p$-charge-exchange reaction have been carried out by various authors.[3,4,7,8,11] It is not appropriate here to present a detailed evaluation of the relative merits of these works. However, we should like to mention some points by which our analysis differs from all or most of the above authors. All these authors have restricted their analyses to values of $-t \lesssim 1$ (GeV/c)^2. In addition, the above analyses have not considered either one or several of the following features: (i) the Reggeization of the kinematical singularity-free helicity amplitudes, (ii) the insertion of a zero in $G_{++}(\rho)$ to account for the crossover effect, (iii) allowing $E_\pm^V$ as arbitrary parameters to be determined from the data, (iv) allowing for the possible curvature of the trajectory, and (v) treatment of statistical and normalization errors. Our analysis incorporates all these features besides fitting all available data at $-t > 1$ (GeV/c)^2.

Our best fit is presented in Figs. 1 and 2 together with the experimental data. The initial convexity in the calculated curve near $t \simeq 0$ is due to the fact that the term $|\phi^{1/2} G_{+-}|^2$ is large compared to $|G_{++}|^2$ near the

[30] The parameters g_n [see Eq. (3.7)] representing a shift in the scale of the differential cross section were at each energy less than the quoted errors ϵ_n.

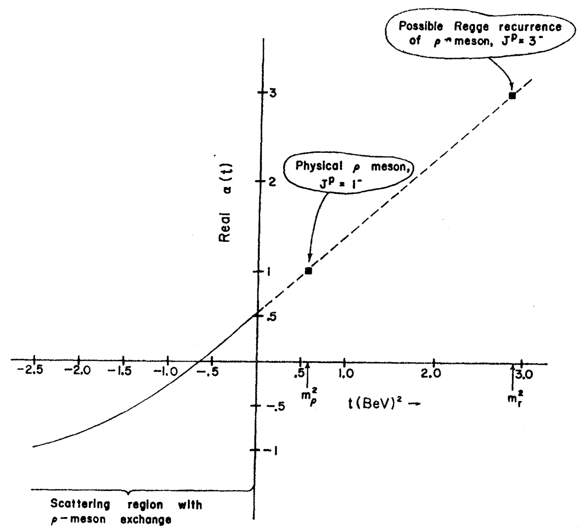

FIG. 3. The Regge trajectory of the ρ mesons as determined in this analysis.

forward direction. The secondary minimum, or "dip," at $t\simeq -0.6$ is a consequence of the vanishing of α (see Fig. 3), which implies the vanishing of G_{+-} [Eq. (3.3)]. The precipitous dip at $t\simeq -2.5$ is a result of the factor $(\alpha+1)^2$ in both Eqs. (3.2) and (3.3) so that the cross section vanishes when $\alpha(t\simeq -2.5)=-1$ (see Fig. 3). The presently available data are consistent with the presence of such a "second dip." It will be interesting to see whether more precise data bear out this characteristic feature of our residues, viz., the vanishing of the amplitudes at $\alpha_\rho=-1$ for which $t=-2.5$ (GeV/c)^2.

The signs of $\gamma_{+\pm}{}^V$ cannot be determined from our fits to the data on the differential cross section. The experimental results on the difference of total cross sections $\sigma_t(\pi^\pm p)$ fixes the sign of $\gamma_{++}{}^V$ to be positive. This result follows from the charge-independence relation:

$$\sqrt{2}A(\pi^- p\rightarrow \pi^0 n)=A(\pi^+ p\rightarrow \pi^+ p)-A(\pi^- p\rightarrow \pi^- p).$$

Thus, the difference of total cross sections

$$\Delta(\pi^- p)\equiv\sigma_t(\pi^- p)-\sigma_t(\pi^+ p)$$

can be related by the optical theorem to $G_{++}(\rho)$;

$$\Delta(\pi^- p)=(1/[2m_1{}^2 p_{\rm lab}])\,{\rm Im}[G_{++}(s,t=0)/c_+]. \quad (3.9)$$

Experimentally, it is known that the left-hand side of Eq. (3.9) is positive at high energy and hence we conclude $\gamma_{++}{}^V>0$.

Through formula (3.9), we can relate the values of the forward differential cross section to the experimental values of $\Delta(\pi^- p)$:

$$\left(\frac{d\sigma(\pi^- p\rightarrow \pi^0 n)}{dt}\right)_{t=0}=\frac{\sec^2\frac{1}{2}\pi\alpha_\rho(0)}{32\pi}[\Delta(\pi^- p)]^2, \quad (3.10)$$

where we have here neglected the mass differences between p and n and between π^- and π^0. In this way the Regge ρ exchange has been tested[6] and Eq. (3.10) is found to be consistent with the available data. The value of $\alpha_\rho(0)$ obtained in Ref. 6 agrees with ours within errors.

The sign of $\gamma_{+-}{}^V$ is indicated to be positive for the following reason. In the framework of the interference model (Sec. 2), we analyzed the intermediate energy data[31] using the formulas (2.21) and (2.22), where the $SU(2)$ Clebsch-Gordan coefficients are

$$c=\begin{pmatrix}+\sqrt{\frac{2}{3}}\\ -\sqrt{\frac{2}{3}}\end{pmatrix}$$

for

$$I=\begin{pmatrix}\frac{3}{2}\\ \frac{1}{2}\end{pmatrix},\quad Y=1 \text{ baryon resonances.} \quad (3.11)$$

The resonance parameters are taken from the work of Barger and Cline.[12] We find, as shown in Fig. 4, that the fit is unacceptable when we choose $\gamma_{+-}{}^V<0$.

An independent justification for the crossover factor in $G_{++}{}^\rho$ comes from a recent analysis by Baacke and Yvert,[31] who by considering the interference model concluded that $G_{++}{}^\rho(s,t=-0.6)$ has opposite sign to $G_{++}{}^\rho(s,t=0)$.[32] The essential point is that $G_{+-}{}^\rho$ vanishes identically at $\alpha(t\simeq -0.6)=0$ and hence the sign of $G_{++}{}^\rho$ is determinable through the interference of the direct-channel resonant amplitude.

If we now extrapolate the model in to the range $1\leq p_{\rm lab}\leq 5$ (GeV/c), we find that the average cross

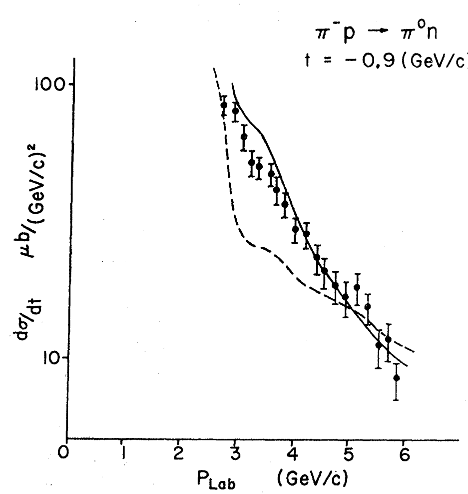

FIG. 4. The fit of the interference model described in this paper to the data of Baacke and Yvert, Ref. 31. Solid (dashed) line represents calculation with positive (negative) sign of $\gamma_{+-}{}^V$.

[31] J. Baacke and M. Yvert, Nuovo Cimento **51**, 761 (1967).
[32] In this connection, we have reexamined the model considered by A. Yokasawa [Phys. Rev. **159**, 1431 (1967)] at 2.07 GeV/c. When the exact energy dependence as given in Eqs. (2.17), (2.24), and (2.25) is inserted, we find a satisfactory fit *with* the amplitude containing crossover zero.

section is predicted correctly but there are numerous fluctuations usually interpreted as the effects of direct or s-channel resonances. We have applied the interference model of Sec. 2 to the low-energy ($\lesssim 4$ GeV) $\pi^- p$ charge-exchange data and found reasonable agreement. A more accurate test of the model, especially the relative phase of Regge and resonance amplitudes, is in describing the polarization distributions. Unfortunately, no data exist[33] at this time with which to compare the polarization distributions (Fig. 5) predicted[34] by our model. Of particular interest is the curve for $t=-0.3$ $(\text{GeV}/c)^2$ in Fig. 5, which illustrates the sensitivity of the polarization to the location of the various resonances and becomes vanishingly small for $p_{\text{lab}} \gtrsim 4$ GeV/c, reflecting the imposed crossover zero in the nonflip amplitude.

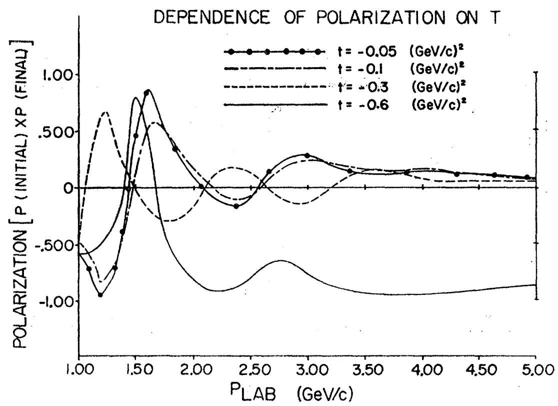

FIG. 5. Polarization at fixed momentum transfer as a function of incident pion momentum for $\pi^- p \to \pi^0 n$ in the resonance region.

B. Reaction $\pi^- p \to \eta n$

The exchange of the tensor trajectory associated with the A_2 meson (mass$=1.3$ GeV, $J^P=2^+$, $I^G=1^-$, $Y=0$) can be studied by analyzing the high-energy data on the reaction

$$\pi^- p \to \eta n. \tag{3.12}$$

The A_2 meson which has $J^P=2^+$ should lie on a Regge trajectory of even signature.[25] In the literature, the trajectory on which the A_2 meson lies is sometimes referred to as the R trajectory.

The Regge amplitudes for the exchange of an even-signature trajectory are [see Eqs. (2.14)–(2.16)]

$$G_{++}(A)=C_+\gamma_{++}{}^T(1+t/t_A)\alpha_A(\alpha_A+1) \times(i-\cot\tfrac{1}{2}\pi\alpha_A)R_{++}{}^A, \tag{3.13}$$

$$G_{+-}(A)=C_-\gamma_{+-}{}^T\alpha_A{}^2(\alpha_A+1)(i-\cot\tfrac{1}{2}\pi\alpha_A)R_{+-}{}^A,$$

where

$$R_{+\pm}{}^A\equiv\{(s-\tfrac{1}{2}\sum m^2+\tfrac{1}{2}t)/[(m_1+m_2)E_\pm{}^T]\}^{(\alpha_A-1/2)\pm1/2}.$$

The superscript T denotes even trajectory exchange. In the helicity-nonflip amplitude G_{++} we inserted a crossover factor as in the case of $\pi^- p$ charge exchange. Because it is not clear whether or not the crossover factor is needed in the reaction (3.12), we have analyzed the data with and without such a factor. The Clebsch-Gordan coefficients C_+ and C_- are

$$C_+=C_-=\sqrt{\tfrac{2}{3}}. \tag{3.14}$$

The experimental data[35] were obtained by observing the 2γ decay mode of the η. To obtain the differential cross section from these experimental data, we have used the branching ratio[25]

$$\text{Rate}(\eta\to2\gamma)/\text{Rate}(\eta\to\text{all decay modes})=0.314. \tag{3.15}$$

We have considered both segments of the data of Ref. 35 (classified by the experimental geometry) and the quoted normalization errors of 10% at each momentum (±20% at the highest momentum).

Unfortunately, the high-energy data ($p_{\text{lab}}\gtrsim 5$ GeV/c) on this reaction are neither as accurate nor as extensive as those on the $\pi^- p$-charge-exchange reaction. For this reason we have analyzed the reaction (3.12) in conjunction with the $K^- p$-charge-exchange reaction.

C. Reaction $K^- p \to \bar{K}^0 n$

Reactions (3.1) and (3.12) are clean examples of meson-baryon inelastic scattering, in that only one Regge trajectory is presumed to dominate each reaction. The KN-charge-exchange reactions

$$K^- p \to \bar{K}^0 n, \tag{3.16}$$

$$K^+ n \to K^0 p, \tag{3.17}$$

are next in complexity. Both these reactions allow exchanges of bosons with natural parity $[(-)^J]$, $I=1$, and $Y=0$. The two boson Regge trajectories ρ and A_2 are the known trajectories with the required quantum numbers.

[33] We have no explanation for the nonzero polarization in $\pi^- p\to\pi^0 n$ observed at 5.9 and 11.2 GeV/c by P. Bonamy *et al.* [Phys. Letters **23**, 501 (1966)]. For the various attempts to explain this polarization within the context of the Regge model, see the contribution of L. Durand, III, to the Argonne symposium on Regge poles [Argonne National Laboratory Report, 1966 (unpublished)].

[34] Measurements of the $\pi^- p$-charge-exchange polarization in the range 2–3.5 GeV/c have recently been reported by D. D. Drobnis *et al.* [Phys. Rev. Letters **20**, 274 (1968)], which appear to disagree with our predictions. However, the recent phase-shift analyses indicate new low-spin resonances which may extend into the intermediate-energy region. [C. Lovelace, in Proceedings of the International Conference on High-Energy Physics, Heidelberg, 1967 (unpublished).] Furthermore, these analyses show that the elasticities of the currently established resonances are poorly determined. Our predictions of the low-energy polarization depend critically on the baryon spectrum and elasticities. Thus at this time we do not consider the apparent failure of our prediction to be significant. Other calculations based on the interference model have been reported by R. C. Lamb *et al.* [Phys. Rev. Letters **20**, 353 (1968)].

[35] O. Guisan, J. Kirz, P. Sonderegger, A. Stirling, P. Borgeaud, C. Bruneton, P. Falk-Vairant, B. Omblard, C. Caverzasio, J. Guillaud, and M. Yvert, Phys. Letters **18**, 200 (1965).

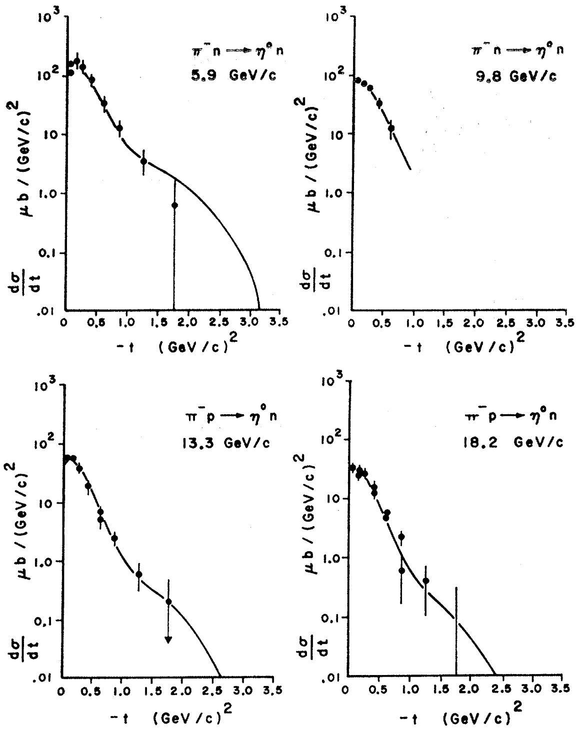

FIG. 6. The data on $\pi^- p \to \eta n$ from Guisan *et al.*, Ref. 35, compared to solution 1 of Table I.

Denoting by $g_{+\pm}$ the helicity amplitudes from which the Clebsch-Gordan coefficients $C_\pm$ are factored out, the Regge amplitudes for the reactions (3.16) and (3.17) are

$$G_{++}{}^{KN} = \pm g_{++}(\rho) + g_{++}(A), \qquad (3.18)$$
$$G_{+-}{}^{KN} = \pm g_{+-}(\rho) + g_{+-}(A).$$

The $\pm$ sign in front of the ρ amplitude refers to reactions (3.16) and (3.17), respectively.

The high-energy experimental data on $(d\sigma/dt)$ of the reaction (3.16) exist[36] at incident K^- laboratory momenta 5, 7.1, 9.5, and 12.3 GeV/c with a quoted 10% normalization error. To date, we have no high-energy experimental data on reaction (3.17).

In an effort to determine a best set of parameters regarding the A_2 exchange, we have analyzed reaction (3.12) in conjunction with (3.16). In this analysis the ρ-exchange parameters have been fixed at the values obtained from our analysis of $\pi^- p$-charge-exchange reaction. We have found four solutions for the A_2 exchange corresponding to the sign of $\gamma_{+-}{}^T$ being positive or negative and corresponding to t_A being finite or infinite. Our solutions in the simultaneous analysis of 102 data points for these reactions are presented in Table I.

TABLE I. Parameters of the A_2 trajectory exchange amplitudes for the four solutions discussed in the text. The solutions are distinguished by the presence of a crossover factor and the sign of $\gamma_{+-}{}^T$.

Solution	t_A	$\gamma_{++}{}^T$	$\gamma_{+-}{}^T$	$E_+{}^T$	$E_-{}^T$	α_A	χ^2
1	∞	37.4	−588.0	0.55	0.16	$0.41+0.41t$	89
2	3.1	46.6	−592.0	0.58	0.16	$0.39+0.37t$	104
3	3.1	46.6	592.0	0.58	0.152	$0.4+0.38t$	97
4	∞	26.4	169.6	0.35	0.018	$0.45+0.38t$	83

Our fits with solution 1 are presented in Figs. 6 and 7. Similar curves are obtained with the other solutions. We have also presented our prediction[37] of the differential cross section in $K^+n \to K^0 p$ in Fig. 8 using solution 1 and Eqs. (3.18).

Previous analyses on reactions (3.12) and (3.16) above have also been carried out by other authors.[3,11,37] Our analysis, in addition to incorporating the normalization errors, includes more of the available data. The shoulder at $t \sim -1$ (GeV/c)^2 in Fig. 6 is due to the vanishing of the $G_{+-}{}^A$ because of the imposed sense-nonsense zero. At present the data are too poor to be used to test this feature of our residues.

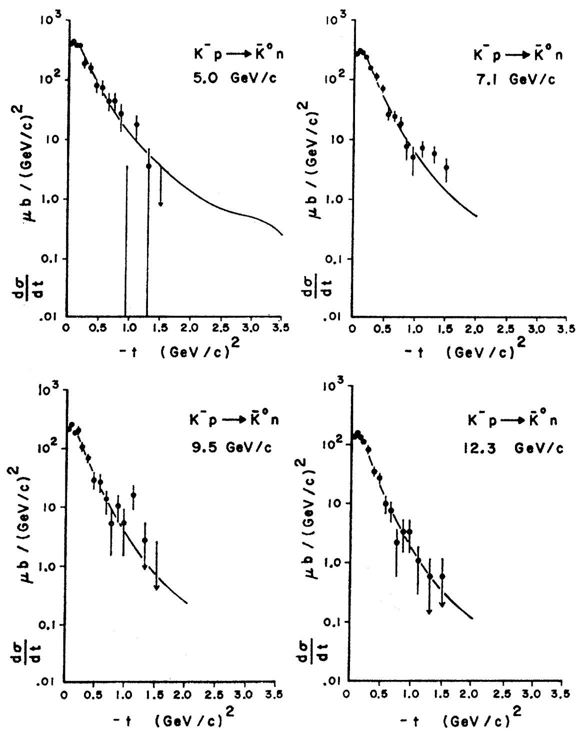

FIG. 7. The data on $K^- p \to \bar{K}^0 n$ from Astbury *et al.*, Ref. 36, compared to solution 1 of Table I.

[36] P. Astbury, G. Brautti, G. Finocchiaro, A. Michelini, K. Terwilliger, D. Websdale, C. West, P. Zanella, W. Beusch, W. Fischer, G. Gobbi, M. Pepin, and E. Polgar, Phys. Letters 23, 396 (1966).

[37] Our curve at 5 GeV/c is very similar to the predicted curve of W. Rarita and B. M. Schwarzschild [Phys. Rev. **162**, 1378 (1967)], who introduced a ρ' trajectory to explain simultaneously the high-energy polarization in $\pi^- p \to \pi^0 n$ and K^+n-charge exchange data at 2.3 GeV/c.

The sign of $\gamma_{++}{}^T$ can be shown[6] to be positive by examining the data on the combination

$$\sigma_t(K^+p)+\sigma_t(K^-p)-\sigma_t(K^+n)-\sigma_t(K^-n),$$

to which only the imaginary part of $G_{++}{}^A(s, t=0)$ contributes (optical theorem). Unfortunately, the sign of $\gamma_{+-}{}^T$ cannot be fixed by the interference model, as the partial widths of $I=\frac{1}{2}$, $Y=1$ resonances to decay into the ηn channel are not known. However, as we shall see the magnitude of the polarization in KN-charge-exchange can determine the sign of $\gamma_{+-}{}^T$.

It is apparent from Fig. 9 that our trajectory does not extrapolate to the A_2 pole. This perhaps is not surprising because the position of the A_2 pole ($m_A{}^2=1.7$ GeV, $J=2$) is far away from $t=0$, where $\alpha(0)=0.4$. Some authors in their analyses have constrained the A_2 trajectory to go through the pole. We feel that this constraint is unwarranted and probably misleading because the behavior of the A_2 trajectory from $t=9\ \mu\pi^2$ up to $t=m_A{}^2$ need not be smooth in the presence of a large number of inelastic thresholds in the 3π channel.

As the A_2 trajectory alone dominates the reaction $\pi^-p\rightarrow\eta n$, the polarization of the recoil neutron should be zero at high energies. At present no experimental information is available on this point.

As the KN-charge-exchange reactions proceed by the exchange of two Regge trajectories, we can calculate the polarization $\mathbf{P}$.[3,9] The predicted polarization[10] near the forward direction in the reaction $K^-p\rightarrow\bar{K}^0n$ turns out to be *very large* (nearly -1) for solutions 1 and 2 (where $\gamma_{+-}{}^T<0$), and $\sim(-0.5)$ for solutions 3 and 4 ($\gamma_{+-}{}^T>0$), as shown in Fig. 10. There are three interesting features of this polarization:

(i) The sign of the polarization is negative for all the four solutions in Table I.

(ii) The magnitude of the polarization is close to 100% when $\gamma_{+-}{}^T<0$ and to 50% when $\gamma_{+-}{}^T>0$.

(iii) The polarization near the forward direction ($t\neq 0$) is independent of the incident K^- energy.

More precisely, at $t=-0.075$ $(\text{GeV}/c)^2$ solutions 1, 2, 3, and 4 of Table I give $\mathbf{P}=(-0.95, -0.97, -0.57,$

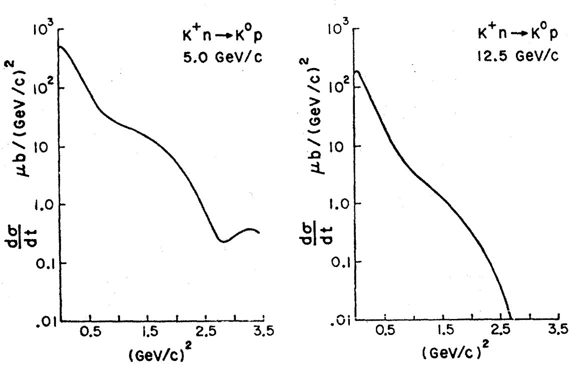

FIG. 8. The predicted differential cross section for the reaction $K^+n\rightarrow K^0p$.

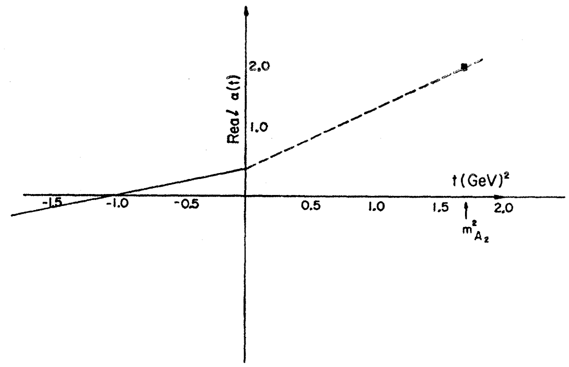

FIG. 9. The Regge trajectory of the A_2 meson as determined in this analysis.

and $-0.39)\hat{n}$, respectively. As the differential cross section near $t\sim 0$ is larger than at any other values of t, it is experimentally feasible to detect the predicted polarization. The constant polarization independent of energy is due to the fact that $\alpha_A\sim\alpha_\rho$ in the range $t=0$ to $(-0.5\ \text{GeV}/c)^2$ and

$$|\mathbf{P}|\sim s^{\alpha_A-\alpha_\rho}, \quad \text{for large } s. \tag{3.19}$$

It should be noted, however, that although $|\mathbf{P}|$ is large, experimentally one measures the polarized cross section $\mathbf{P}d\sigma/dt$, which decreases as s increases.

The reaction $K^+n\rightarrow K^0n$ is the "line-reversed" reaction of $K^-p\rightarrow\bar{K}^0n$. The vector trajectory (ρ) contributes to the former reaction with a negative sign as shown in Eqs. (3.18). For this reason the polarized cross section ($\mathbf{P}d\sigma/dt$) in $K^+n\rightarrow K^0p$ is equal in magnitude and opposite in sign to the polarized cross section in $K^-p\rightarrow\bar{K}^0n$.[9] This elegant prediction should be tested experimentally.

4. HYPERCHARGE-EXCHANGE REACTIONS

Among the hypercharge-exchange reactions in πN and $\bar{K}N$ scattering that have a pseudoscalar meson and

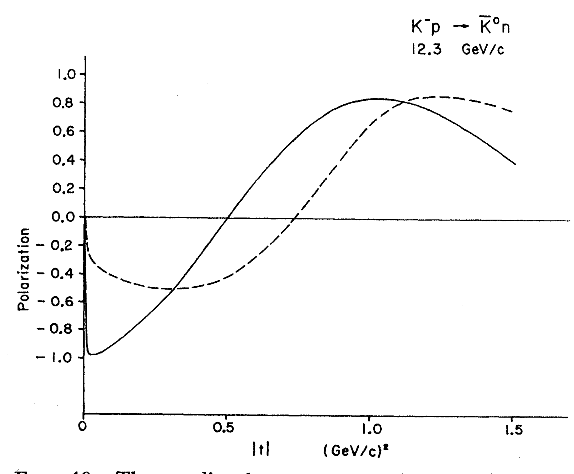

FIG. 10. The predicted neutron polarization distribution. The solid (dashed) line is the prediction for solution 2(3) of Table I.

a spin-$\frac{1}{2}$ baryon in the final state, we shall analyze only those that involve peripheral meson exchange. Furthermore, since little or no experimental data exist on the reactions

$$K^-p \to \pi^0\Sigma^0, \quad K^-p \to \eta^0\Lambda, \quad K^-p \to \eta^0\Sigma^0, \quad (4.1)$$

we do not analyze these reactions nor can we analyze reactions producing the X^0 or $\eta'(960)$ meson. Consequently, the reactions, we can consider are

Λ class:

$$\pi^-p \to K^0\Lambda^0, \quad (4.2a)$$

$$K^-p \to \pi^0\Lambda^0, \quad (4.2b)$$

and

Σ class:

$$\pi^+p \to K^+\Sigma^+, \quad (4.3a)$$

$$\pi^- p \to K^0\Sigma^0, \quad (4.3b)$$

$$K^-p \to \pi^-\Sigma^+. \quad (4.3c)$$

Henceforth, when we refer to the hypercharge-exchange reactions (HCEX) we mean specifically reactions (4.2) and (4.3).

In the context of the peripheral model a meson of strangeness $S\neq 0$ must be exchanged. The $K\bar{K}\pi$ vertex is not allowed by parity and angular-momentum conservation. Therefore, of the well-established resonances,[25] only the K^* (mass$=890$ MeV $J^P=1^-$, $I=\frac{1}{2}$ and $Y=-1$) and the K^{**} (mass 1420 MeV, $J^P=2^+$, $I=\frac{1}{2}$, $Y=-1$) can participate in the HCEX reactions. We denote the Regge trajectory on which the K^* (K^{**}) lies as the q (Q) trajectory.

Although the available data on the HCEX reactions are fragmentary, a few trends can be discerned. The differential cross sections show the characteristic forward peak of meson exchange and qualitatively are similar to the cross sections in charge exchange scattering. The partial cross sections appear[1] to decrease as a power law of the laboratory energy E_{lab}^{-n} where $n\simeq 2$. This is to be compared with $n\simeq 1.5$ for the charge-exchange reactions considered above.

Because of the forward peaking, the predominant contribution to the partial cross section is made by $d\sigma/dt$ at $t\simeq 0$. That is,

$$\sigma\sim\int_{\text{forward peak}}\frac{d\sigma}{dt}dt\sim s^{2\langle\alpha\rangle-2}, \quad (4.4)$$

where $\langle\alpha\rangle$ is an effective value for the leading Regge trajectory near $t=0$. Assuming the HCEX reactions can be described by Regge q exchange, we estimate

$$\langle\alpha_\rho\rangle-\langle\alpha_q\rangle\sim 0.25. \quad (4.5)$$

This crude estimate indicates the difference in the intercepts to be anticipated in the Regge trajectories belonging to same $SU(3)$ representation.

The trajectories α_q and α_Q can be determined from α_ρ and α_A, provided we know how to take into account the effect of $SU(3)$-symmetry breaking on Regge trajectories. In the limit of exact $SU(3)$ symmetry all the vector mesons have a degenerate mass and hence lie on a degenerate trajectory. In the physical situation, $SU(3)$ symmetry is broken and the masses of the particles are nondegenerate; the corresponding Regge trajectories are also then nondegenerate. A plausible way to take into account the broken $SU(3)$ symmetry, insofar as the Regge trajectories are concerned, is to postulate that

$$\alpha_j(t)=\alpha_i(t-\Delta_{ji}), \quad (4.6)$$

where $\Delta_{ji}=m_j^2-m_i^2$.

Here α_i and α_j are trajectories associated with the members i and j of the given $SU(3)$ multiplet. Substituting $t=m_j^2$, we recover the equality of the spins of the members i and j. We note that Eq. (4.6) is intended only for the real part of $\alpha(t)$, which is all that is relevant for our present purposes.

From the ρ trajectory determined above, we obtain, using Eq. (4.6), the q trajectory

$$\alpha_q(t)=0.35+0.96t+0.16t^2, \quad (4.7)$$

where we have used $\Delta_{K^*\rho}=0.215$ (GeV)^2. A similar calculation for the Q trajectory yields

$$\alpha_Q(t)=0.276+0.41t \quad (4.8)$$

where we used $\alpha_A=0.41+0.41t$ and $\Delta_{K^{**}A_2}=0.326$ (GeV)^2. Note that the intercept in Eq. (4.7) is consistent with our expectation in Eq. (4.5).

There is no HCEX reaction to which the Regge exchange of either q or Q alone contributes. However, the large number of reactions that can be analyzed in terms of simultaneous exchanges of q and Q partly compensates for the fact that these exchange contributions cannot be directly isolated (as was possible for the ρ and A_2 trajectories).

If $G_{+\pm}{}^\Lambda(q)$ and $G_{+\pm}{}^\Lambda(Q)$ denote the four helicity amplitudes describing the reaction (4.2a), then we can write the helicity amplitudes as follows:

reaction (4.2a):

$$G_{+\pm}{}^\Lambda(q)+G_{+\pm}{}^\Lambda(Q), \quad (4.9a)$$

reaction (4.2b):

$$-(1/\sqrt{2})G_{+\pm}{}^\Lambda(q)+(1/\sqrt{2})G_{+\pm}{}^\Lambda(Q). \quad (4.9b)$$

Thus the two Λ-class reactions can be analyzed in terms of the same four helicity amplitudes. The minus sign in front of the $G_{+\pm}{}^\Lambda(q)$ in (4.9b) is a consequence of charge-conjugation invariance for the trilinear meson vertex.

We proceed in a similar fashion for the Σ-class reactions. If $G_{+\pm}{}^\Sigma(q)$ and $G_{+\pm}{}^\Sigma(Q)$ are the four helicity amplitudes describing the reaction $\pi^+p\to K^+\Sigma^+$, then we have

reaction (4.3a):

$$G_{+\pm}{}^\Sigma(q)+G_{+\pm}{}^\Sigma(Q), \quad (4.10a)$$

reaction (4.3b):

$$(1/\sqrt{2})G_{+\pm}{}^{\Sigma}(q)+(1/\sqrt{2})G_{+\pm}{}^{\Sigma}(Q)\,, \qquad (4.10b)$$

reaction (4.3c):

$$-G_{+\pm}{}^{\Sigma}(q)+G_{+\pm}{}^{\Sigma}(Q)\,. \qquad (4.10c)$$

The factor $1/\sqrt{2}$ in (4.10b) is the Clebsch-Gordan coefficient relating to isospin symmetry and the minus sign before $G_{+\pm}{}^{\Sigma}(q)$ in (4.10c) again arises from C invariance at the boson vertex.

The helicity amplitudes in (4.9) and (4.10) are parametrized in accordance with Eqs. (2.14)–(2.16). Suppressing superscripts Λ and Σ on the G's and C's, we write

$$G_{++}(q)=C_+(q)\gamma_{++}{}^V(\alpha_q+1)^2(i+\tan\tfrac{1}{2}\pi\alpha_q)R_{++}{}^q\,, \quad (4.11)$$

$$G_{+-}(q)=C_-(q)\gamma_{+-}{}^V(\alpha_q+1)^2\alpha_q \times(i+\tan\tfrac{1}{2}\pi\alpha_q)R_{+-}{}^q\,, \quad (4.12)$$

$$G_{++}(Q)=C_{++}(Q)\gamma_{++}{}^T(1+t/t_Q)\alpha_Q(\alpha_Q+1) \times(i-\cot\tfrac{1}{2}\pi\alpha_Q)R_{++}{}^Q\,, \quad (4.13)$$

$$G_{+-}(Q)=C_{-}(Q)\gamma_{+-}{}^T\alpha_Q{}^2(\alpha_Q+1) \times(i-\cot\tfrac{1}{2}\pi\alpha_Q)R_{+-}{}^Q\,, \quad (4.14)$$

where

$$R_{+\pm}{}^{q,Q}=\left(\frac{s-\frac{1}{2}\sum m^2+\frac{1}{2}t}{(m_1+m_2)E_\pm{}^{q,Q}}\right)^{(\alpha_{q,Q}-1/2)\pm1/2}. \quad (4.15)$$

The introduction of a crossover factor[38] in Eq. (4.13) is suggested by the data as follows. The polarization resulting from the amplitudes (4.11)–(4.14) is

$$\mathbf{P}\propto[(\alpha_Q\tan\tfrac{1}{2}\pi\alpha_q+\alpha_Q\cot\tfrac{1}{2}\pi\alpha_Q)(\alpha_Q+1)(\alpha_q+1)^2] \times[\alpha_Q g_{++}{}^q g_{+-}{}^Q-\alpha_q(1+t/t_Q)g_{++}{}^Q g_{+-}{}^q]\hat{n}\,, \quad (4.16)$$

where the g's have fixed signs and are defined as

$$g_{++}{}^q=C_+(q)\gamma_{++}{}^V R_{++}{}^q, \text{ etc.} \qquad (4.17)$$

We note that when α_Q passes through zero and $|\alpha_q|<1$ the expression in the curly bracket does not change sign. The observed polarization in the reaction $\pi^+p\to K^+\Sigma^+$ changes sign near $t=t_z\sim-0.5$ (GeV/c)^2 and becomes very large in magnitude for $t<t_z$. According to our model (neglecting the direct-channel resonance effects for the moment) the zero in the polarization at t_z should result from a zero of the expression in the square bracket in (4.16). The first term in the square bracket changes sign at $\alpha_Q=0$. We might expect to obtain a large polarization for $t<t_z$ provided α_Q vanishes at $t\sim t_z$ and the second term in the square bracket reinforces the contribution from the first term for $t<t_z$. In other words, the second term should not be allowed to change sign in the vicinity of $\alpha_Q=0$, where the first term changes sign. As there is already a factor α_q present in the second term, the sign change due to the vanishing of α_q can be reversed if there is a crossover factor in $G_{++}(Q)$.[39]

The C's in Eqs. (4.11)–(4.14) are appropriate Clebsch-Gordan coefficients for the reaction under consideration. If we know the f/d ratios for the vector and tensor couplings to baryons, we can, in principle, determine the C's. As the f/d ratios are not known, and moreover as the sign on γ_{+-T} is ambiguous, we shall instead determine from experiment the quantities

$$D_+(q)\equiv C_+(q)\gamma_{++}{}^V,\quad D_-(q)\equiv C_-(q)\gamma_{+-}{}^V, \quad D_+(Q)\equiv C_+(Q)\gamma_{++}{}^T,\quad D_-(Q)\equiv C_-(Q)\gamma_{+-}{}^T. \qquad (4.18)$$

As a working hypothesis we shall take the q and Q energy-scale parameters equal to the corresponding values determined above for the ρ and A_2 exchanges:

$$E_\pm{}^q=E_\pm{}^V,\quad E_\pm{}^Q=E_\pm{}^T. \qquad (4.19)$$

Using the trajectories, assuming the forms (4.7) and (4.8) obtained from the broken $SU(3)$ symmetry, we have not been able to obtain adequate fits to the data on HCEX reactions. As the data on $\pi^-p\to\eta n$ are not very precise, the errors on the A_2 trajectory are large. For this reason, instead of assuming (4.8), we shall allow the intercept and the slope of α_Q to vary. However, we shall assume the form as given by (4.7) for α_q.

Thus our analysis consists in seeking the best possible values for the residue constants D (four D's for the Λ class and four D's for the Σ class), the crossover constant t_Q, and the two parameters in α_Q.

Experimental data on the reaction $\pi^-p\to K^0\Lambda$ at laboratory momenta 2.6, 3.15, and 4.0 GeV/c are taken from Dahl *et al.*[40] (with normalization errors of ±12, 16, and 20%). We have also used the data at 7.91 GeV/c (±15% normalization error) of Ehrlich *et al.*[41] The existing high-energy data at 4.1 and 5.5 GeV/c on the reaction $K^-p\to\pi^0\Lambda$ are taken from the preliminary work of a Wisconsin group,[42] with estimated normalization errors of ±12 and 10%.

Data on the reaction $\pi^+p\to K^+\Sigma^+$ exists[43] at $p_{\text{lab}}=3.23$ GeV/c with a quoted normalization error of ±12%. We have used the available data[40] on $\pi^-p\to K^0\Sigma^0$ at incident momenta 2.6, 3.15, and 4.0 GeV/c (with ±9, 25% normalization uncertainties). The preliminary data[44] at 4.1 and 5.5 GeV/c on the reaction $K^-p\to\pi^-\Sigma^+$ is incorporated in our analysis with an

[38] K. V. L. Sarma and D. D. Reeder, Nuovo Cimento 53A, 808 (1968).

[39] It might be tempting, in view of the π^-p-charge exchange analysis, to insert the crossover zero in $G_{++}(q)$ rather than in $G_{++}(Q)$. In that case, the slope of the Q trajectory controls the slope of the forward peaks and, as a consequence, we expect the calculated curves of $d\sigma/dt$ to be flatter than the experimental ones especially in the region $-0.5\leq t\leq 0$. In general, we have found it difficult to fit the steep forward peaks simultaneously with the polarization data by introducing a crossover factor in $G_{++}(q)$.

[40] O. I. Dahl, L. M. Hardy, R. I. Hess, J. Kirz, D. H. Miller, and J. A. Schwartz, Phys. Rev. 163, 1430 (1967).

[41] R. Ehrlich, W. Selove, and H. Yuta, Phys. Rev. 152, 1194 (1966).

[42] D. Hodge, University of Wisconsin (private communication).

[43] R. R. Kofler, R. Hartung, and D. D. Reeder, Phys. Rev. 163, 1479 (1967).

[44] J. Loos, University of Illinois (private communication).

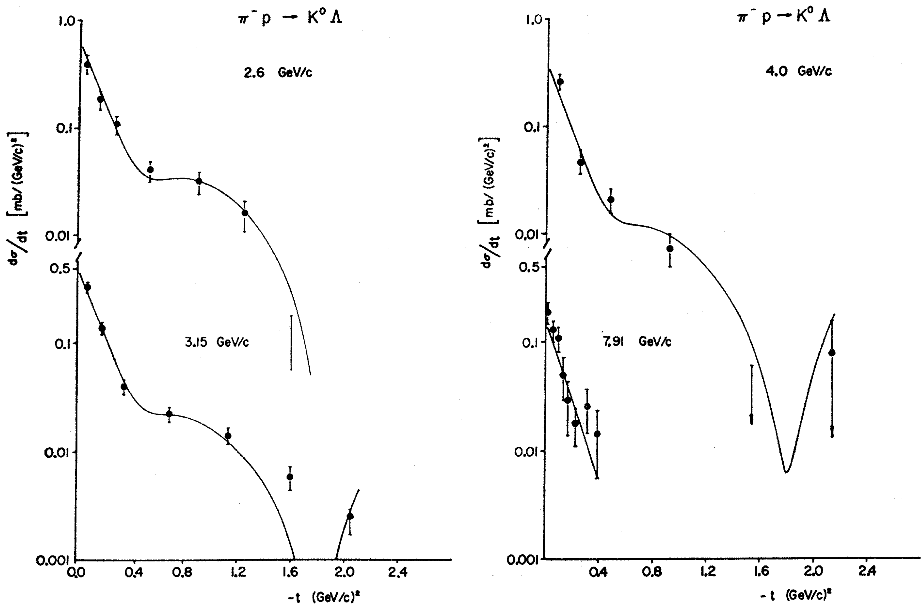

FIG. 11. The data on $\pi^- p \to K^0\Lambda$ of Kirz *et al.*, and Ehrlich *et al.*, Refs. 40 and 41, compared to the calculations of this model.

assumed normalization error of 10%. In analyzing the above data we have restricted ourselves to the data for which the *s*-channel scattering angle is less than 90°.

In the analysis of 40 data points belonging to the Λ class we have obtained a $\chi^2 = 74$. The χ^2 for 57 data of the Σ class is 111. We have not considered the effects of direct-channel resonances when analyzing the Λ-class reactions. However, we have studied the effects of N^* resonances on the Σ-class reactions, particularly for $\pi^+ p \to K^+\Sigma^+$.

As described above we have fixed the energy-scale parameters at the following values:

$$E_+^q = E_+^V = 2.12 \text{ GeV}, \quad E_-^q = E_-^V = 1.81 \text{ GeV}, \quad E_+^Q = E_+^T = 0.58 \text{ GeV}, \quad E_-^Q = E_-^T = 0.152 \text{ GeV}. \tag{4.20}$$

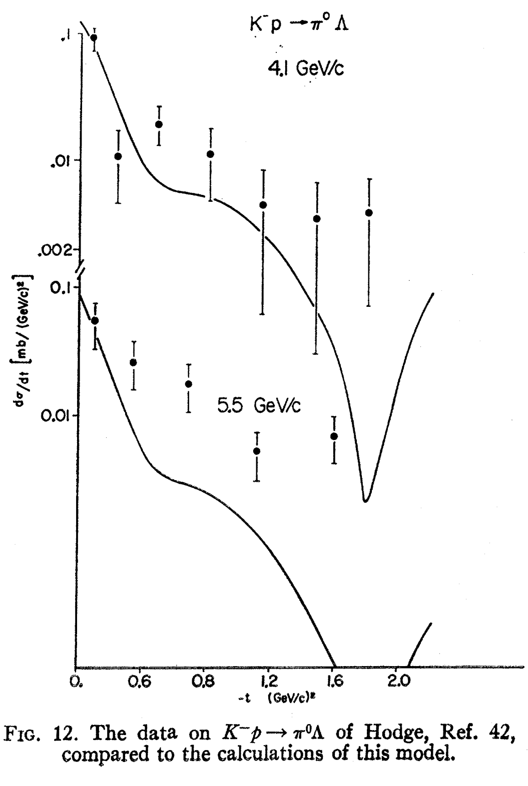

FIG. 12. The data on $K^- p \to \pi^0\Lambda$ of Hodge, Ref. 42, compared to the calculations of this model.

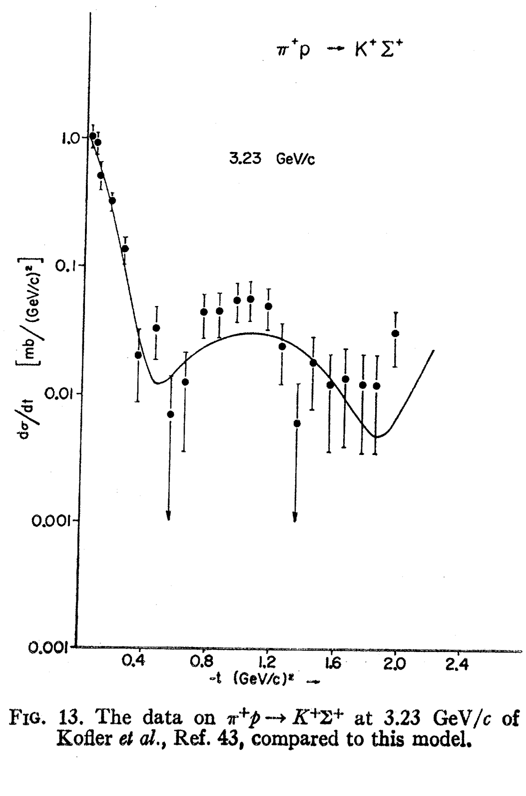

FIG. 13. The data on $\pi^+ p \to K^+\Sigma^+$ at 3.23 GeV/c of Kofler *et al.*, Ref. 43, compared to this model.

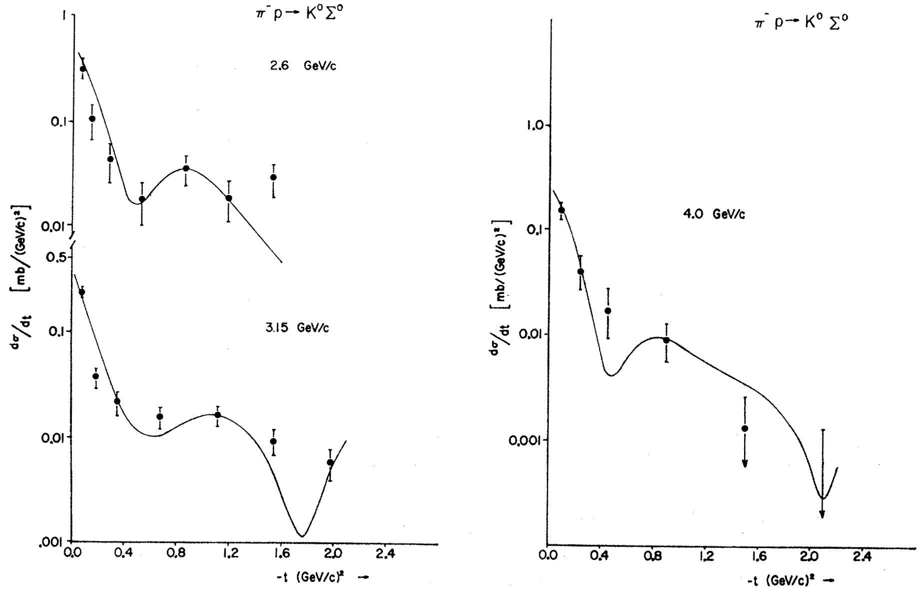

FIG. 14. The data on $\pi^- p \to K^0\Sigma^0$ of Kirz *et al.*, Ref. 40, compared to this model.

The K^* trajectory is assumed to be given by Eq. (4.7):

$$\alpha_q(t) = 0.35 + 0.96t + 0.16t^2. \tag{4.21}$$

The trajectory corresponding to $K^{**}(1420)$ is determined to be

$$\alpha_Q(t) = 0.24 + 0.69t \tag{4.22}$$

and the best value for the crossover constant t_Q is found to be

$$t_Q = 0.526 \text{ (GeV}/c)^2. \tag{4.23}$$

The residue constant D's [defined in Eq. (4.18)] are found to have the following values:

$$\begin{aligned} &D_+^\Lambda(q) = -24.93 \text{ GeV}, \; D_+^\Sigma(q) = -24.76 \text{ GeV}, \\ &D_-^\Lambda(q) = -0.44 \text{ (GeV)}^{-2}, \\ &\qquad D_-^\Sigma(q) = -16.8 \text{ (GeV)}^{-2}, \\ &D_+^\Lambda(Q) = 60.1 \text{ GeV}, \quad D_-^\Sigma(Q) = 96.7 \text{ GeV}, \\ &D_-^\Lambda(Q) = 974.9 \text{ (GeV)}^{-2}, \\ &\qquad D_-^\Sigma(Q) = 1514.5 \text{ (GeV)}^{-2}. \end{aligned} \tag{4.24}$$

We recall that the D^Λ's refer to the reaction (4.2a) and the D^Σ's refer to the reaction (4.3a).

Our best fits to the data with the above parameters are to be found in Figs. 11–15. In Fig. 16 a plot of the

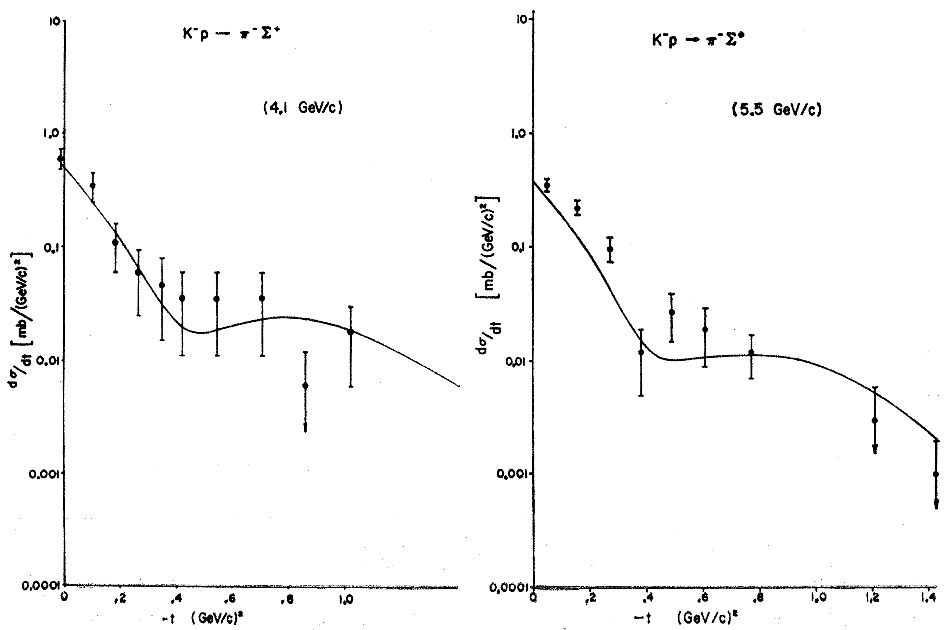

FIG. 15. The data on $K^- p \to \pi^-\Sigma^+$ of Loos, Ref. 44, compared to this model.

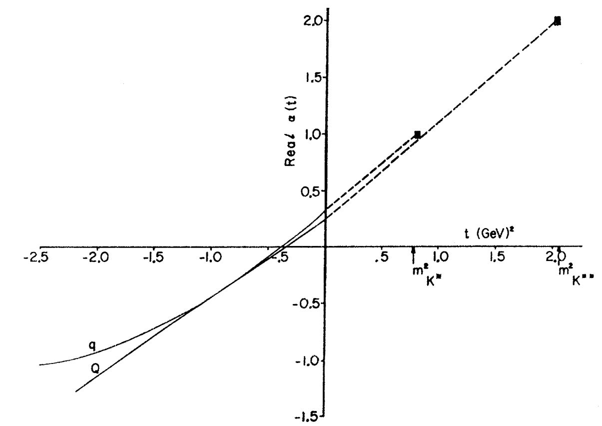

FIG. 16. The Regge trajectories of the vector $K^*(890)J^P=1^-$ and the tensor $K^*(1420)J^P=2^+$ as determined in this analysis.

two trajectories q and Q is given. Typical predictions of the polarization are displayed in Figs. 17 and 19. As we have not studied the effects of s-channel resonances on HCEX reactions we have not presented detailed polarization predictions at various energies. Indeed the neglect of direct-channel resonances may account in part for the large values of χ^2 that were obtained.

The characteristic features of our fits can be easily understood through the trajectory functions and the crossover factor. Noting that $\alpha_q(t=-0.4)\cong 0$, $\alpha_Q(t=-0.35)\cong 0$, and $(1+t/t_Q)\cong 0$ at $t=-0.5$, we expect a relative minimum in $(d\sigma/dt)$ in the region $t\sim -0.4$ $(\text{GeV}/c)^2$. The cross section at this minimum, or dip, then is dominated only by the helicity nonflip amplitude corresponding to q exchange, $G_{++}(q)$. The recent data of Mannelli *et al.*[45] at 6 and 11.2 GeV/c show that the marked change of slope of the forward peak at $t\sim -0.4$ $(\text{GeV}/c)^2$ in $\pi^-p\to K^0\Lambda^0(\Sigma^0)$ persists up to their highest energy, conforming to the expectations of our model. The common factor (α_Q+1) in $G_{+\pm}(Q)$ makes the tensor contribution vanish identically at $\alpha_Q=-1$, which results in another sharp falloff $t\sim -1.8$ $(\text{GeV}/c)^2$ in the curve $d\sigma/dt$ versus t. This "second dip" seems to be suggested by the present data although it should be mentioned that the subsequent rise of the cross section

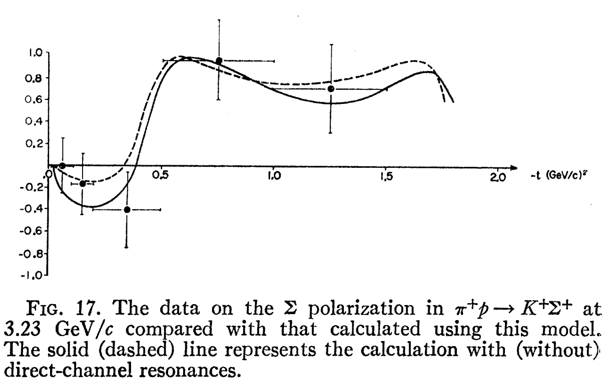

FIG. 17. The data on the Σ polarization in $\pi^+p\to K^+\Sigma^+$ at 3.23 GeV/c compared with that calculated using this model. The solid (dashed) line represents the calculation with (without) direct-channel resonances.

[45] I. Mannelli *et al.*, University of Pisa Report No. INFN/AE-67/9 (unpublished).

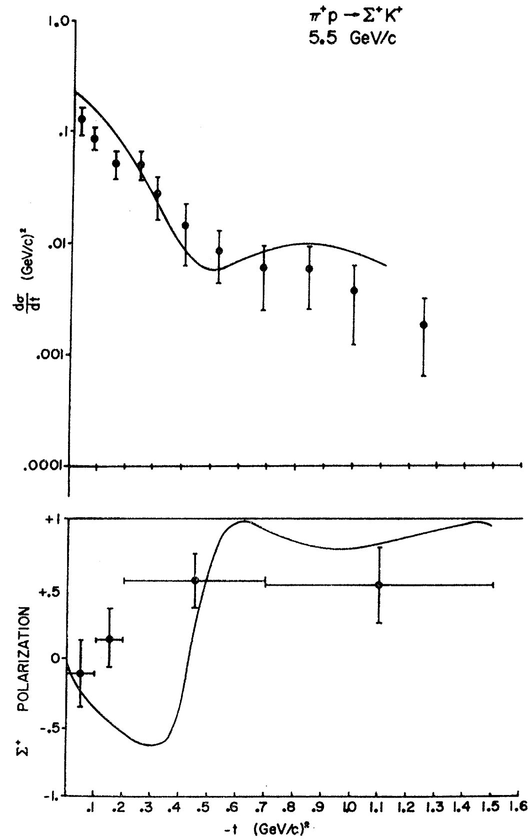

FIG. 18. The data of Ref. 46 on $d\sigma/dt$ and polarization for the reaction $\pi^+p\to K^+\Sigma^+$ at 5.5 GeV/c, compared to the prediction of this model. These data were not used in the determination of the parameters.

predicted by our model seems to be rather sharp. In general, our fits presented in Figs. 11–15 are in good agreement with the data, especially near the forward direction.

The behavior of the expected polarization has already been discussed in motivating the inclusion of a cross-

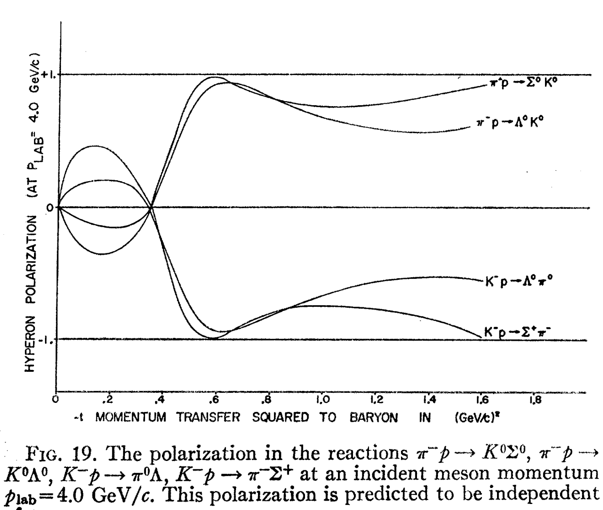

FIG. 19. The polarization in the reactions $\pi^-p\to K^0\Sigma^0$, $\pi^-p\to K^0\Lambda^0$, $K^-p\to\pi^0\Lambda$, $K^-p\to\pi^-\Sigma^+$ at an incident meson momentum $p_{\text{lab}}=4.0$ GeV/c. This polarization is predicted to be independent of p_{lab}.

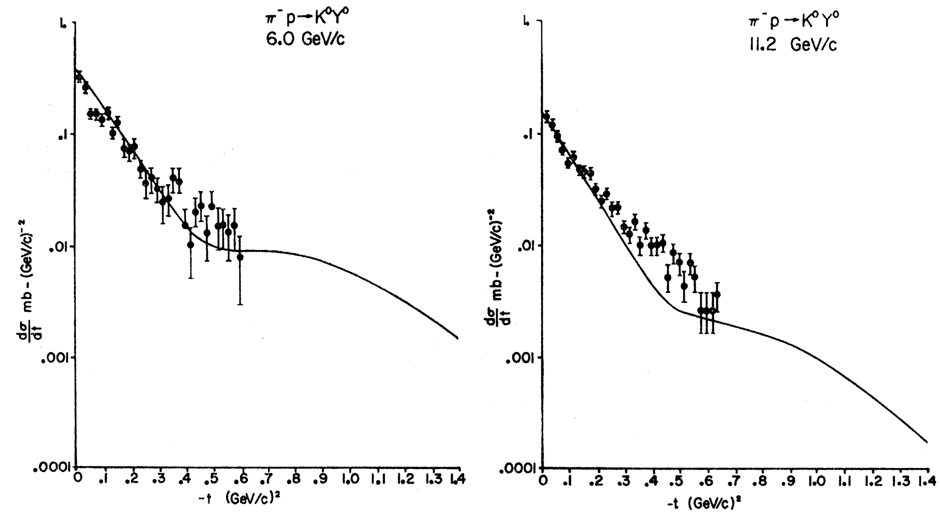

FIG. 20. The differential cross sections for the reaction $\pi^- p \rightarrow K^0\Lambda^0(\Sigma^0)$ predicted by the model compared to the data of Mannelli *et al.*, Ref. 45. There are no free parameters in the comparison.

over factor. The main feature of the polarization in our model is a change of sign around $t \sim -0.5$ (GeV/c)^2 becoming rapidly large in magnitude for $t < -0.5$ (GeV/c)^2. Our prediction of negative polarization in $\pi^- p \rightarrow K^0\Lambda$ is in agreement with the data[40] at 4.0 GeV/c. The observed positive polarization in this reaction at lower laboratory momenta may be the result of s-channel resonances. We have not studied this point in detail as the partial widths of $I=\frac{1}{2}N^*$ resonances to decay into ΛK^0 are poorly known. This preliminary indication[42] of a positive polarization in $K^- p \rightarrow \pi^0\Lambda$ is consistent with our prediction, which is a simple consequence of line reversal.

In Fig. 17 we have plotted the polarization data on reaction (4.3a) along with the prediction of the two-pole Regge model with and without the inclusion of direct-channel resonances. We have taken into account the direct-channel effects by assuming that

$$\Gamma_{\Sigma^+K^+}/\Gamma_{\text{el}} = 0.08, \quad \text{for} \quad \Delta_\delta(2450)$$
$$= 2.9, \quad \text{for} \quad \Delta_\delta(2840) \qquad (4.25)$$

and for the remaining inelastic resonances which can couple to the physical ΣK final states we have taken a common value 0.06 for the above ratio of partial widths. These numerical values are chosen to achieve a simultaneous fit to the differential cross section and the polarization in the reaction (4.3a) at 3.23 GeV/c. It is clear from Fig. 17 that the polarization is unaffected at large $|t|$ but modified slightly near the forward direction by the inclusion of direct-channel resonances. Consequently, as we move to higher energies where the effects of s-channel resonances can be neglected, we should expect to find agreement with the dashed curve in Fig. 17. Figure 18 shows a comparison of the recent data[46] on the differential cross section and the polarization for the reaction $\pi^+ p \rightarrow K^+\Sigma^+$ at 5.5 GeV/c to the predictions of the Regge model (neglecting direct-channel resonances). The agreement in Fig. 18 is very encouraging. Finally, we note that $\alpha_V \sim \alpha_Q$ (Fig. 6); therefore we expect the polarization in this region to be independent of the incident energy [see (3.19)], i.e., a large positive polarization in the region $|t| \sim 1$ (GeV/c)^2 is expected in the reaction $\pi^+ p \rightarrow K^+\Sigma^+$ at all high energies.

[46] W. Cooper, W. Manner, B. Musgrave, and L. Voyvodic, Phys. Rev. Letters **20**, 472 (1968).

The polarization distributions for other reactions for which no data exist are presented in Fig. 19. If this model is correct, then all these hypercharge-exchange reactions will show a polarization large in magnitude and independent of energy.

There are also additional data[45] on the reactions $\pi^- p \rightarrow K^0\Sigma^0(\Lambda^0)$, which were not available at the time of our fit. The predictions of the model are shown with these data in Fig. 20. The agreement is excellent. No normalization effects have been included. Experimentally, it was impossible to distinguish the two reactions and the cross sections calculated for the two reactions were simply summed.

We now deduce the f/d ratios from the fitted values of the D's in Eq. (4.24). Referring to the helicity-nonflip amplitude, we attach to the parameters f and d the subscript V (T) when dealing with the exchange of an odd (even) trajectory associated with the vector (tensor) meson. The corresponding parameters for the helicity-flip amplitude will be capitalized (F_V, D_T, etc.). In this notation, we have the following expressions for the Clebsch-Gordan coefficients C:

$$\begin{aligned} C_+^\Lambda(q) &= -(1/\sqrt{6})(1+2f_V), \\ C_-^\Lambda(q) &= -(1/\sqrt{6})(1+2F_V), \\ C_+^\Lambda(Q) &= -(1/\sqrt{6})(1+2f_T), \\ C_-^\Lambda(Q) &= -(1/\sqrt{6})(1+2F_T), \\ C_+^\Sigma(q) &= 1-2f_V, \\ C_-^\Sigma(q) &= 1-2F_V, \\ C_+^\Sigma(Q) &= 1-2f_T, \\ C_-^\Sigma(Q) &= 1-2F_T. \end{aligned} \qquad (4.26)$$

By taking the ratios of appropriate D's in Eq. (4.24) we can determine the corresponding f/d ratios to be

$$\begin{aligned} (f/d)_V &= -6.5, & (F/D)_V &= -0.36, \\ (f/d)_T &= -1.71, & (F/D)_T &= -1.81. \end{aligned} \qquad (4.27)$$

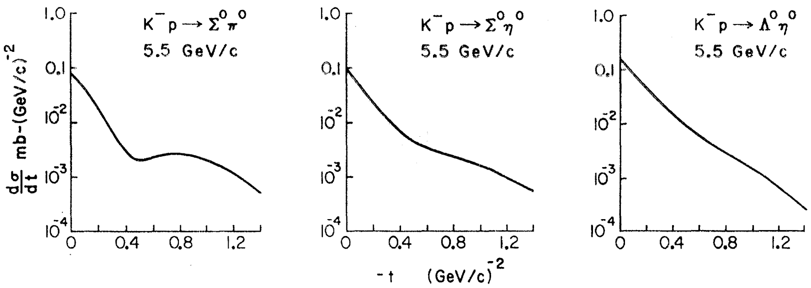

FIG. 21. The differential cross section for the reaction $K^-p \to \eta\Lambda^0(\Sigma^0)$ and $K^-p \to \pi^0\Sigma^0$ at 5.5 GeV/c as predicted by the model.

We emphasize that the values (4.27) are determined independently of our assumption on trajectories in broken $SU(3)$ symmetry. It is not clear theoretically whether one should evaluate the f/d ratios from the ratios of C's alone or from the ratios of C's along with certain factors containing α. Our procedure in determining (4.27) clearly does not suffer from this ambiguity. Furthermore, in considering the ratio of the D's the reduced residues γ's cancel so that the sign ambiguities do not effect our values (4.27). Finally, although the value of $(f/d)_V$ determined from our analysis in Eq. (4.27) is a factor of 3 larger then the corresponding value determined from the Regge-pole analysis of data on total cross sections $[(f/d)_V=-2,\ (f/d)_T=-2]$,[6] it may be significant that the signs on $(f/d)_V$ and $(f/d)_T$ in both analyses are the same.

On the basis of the f/d ratios deduced from our analysis, we have calculated the expected cross section in the reactions $K^-p \to \eta^0\Lambda^0(\Sigma^0)$. These predictions (Fig. 21) constitute an important test of the f/d ratios.

5. DISCUSSION

The purpose of this paper was to construct a phenomenological model for the charge-exchange and HCEX reactions based on Regge poles with residues related by $SU(3)$ symmetry.[47] Although we do not, as yet, have an understanding at a fundamental level, it is very encouraging that a variety of reactions lend themselves to a unified description in the framework of a $SU(3)$ Regge-pole model.

Instead of using exponential residues, we have chosen to exploit the arbitrariness in the choice of the energy-scale parameters; the bulk of the t dependence is thereby determined by the Regge trajectory. Our intent in pursuing the Regge pole model up to $t\sim-2$ (GeV/c)^2 was to study how the model fares as a function of t, as we move far away from the forward direction. In this respect, we found no significant departures from the Regge-pole model up to $t\approx-2$ (GeV/c)^2 and over a four-decade variation of the cross section (see Fig. 1). The available high-energy data for values of $t\lesssim-2$ (GeV/c)^2 are too imprecise to permit a test at these large values of momentum transfer. On the other hand, the Regge-pole models are also successful[48] in predicting the qualitative features of the cross section in $\pi^-p \to \pi^0 n$ at incident momenta as low as 2 or 3 GeV/c [$t\lesssim-0.2$ (GeV/c)^2] and quantitative fits result when account is taken of the resonant states present in the intermediate energy region by means of the interference model.

A simple interpretation of the dips can be made on the basis of the two assumptions: (i) The helicity-flip amplitudes, for both even- and odd-trajectory exchanges, vanish at $\alpha=0$ and (ii) the amplitudes for odd-trajectory exchange vanish at $\alpha=-1$. Here we have assumed that the contributions from the fixed poles[49] and cuts to the amplitude are negligible. This assumption is necessary in the absence of any theoretical model to estimate the contributions coming from the singularities other than Regge poles. Thus the analyses in this paper are made under the tacit assumption that the Mandelstam cuts and other moving poles (e.g., daughters) or fixed poles (e.g., Mandelstam-Wang poles) are secondary effects which can be ignored to a first approximation.

It is of interest to compare our vector-coupling f/d ratios with the ones deduced from the electromagnetic structure of the nucleon. While our value $f_V\sim1.18$ is not inconsistent with pure f coupling, our value of $(F/D)_V\sim-\frac{1}{3}$ is to be compared with the famous $SU(6)$ value, $+\frac{2}{3}$. [Note, however, that the $SU(6)$ value applies at the vector meson pole whereas our value was obtained from $t\leq0$ data.] A simple test of the values in (4.27) would be provided when more accurate data on HCEX reactions become available, particularly on the reactions involving η-meson final states. Finally, we would like to emphasize that accurate measurements of the polarization in HCEX reactions and in KN-charge-exchange reactions would be of considerable value in further testing the double-Regge-pole models and in permitting further refinement of the present analysis.

ACKNOWLEDGMENTS

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[47] Our model does not incorporate the "exchange degeneracy" hypothesis of A. Ahmadzadeh [Phys. Rev. Letters **16**, 952 (1966)]. Our vector and tensor trajectories are quite different. To our knowledge there is no prediction of the model for the residues at $-t>0$.

[48] K. V. L. Sarma (unpublished).

[49] S. Mandelstam and L. L. Wang, Phys. Rev. **160**, 1490 (1967).