Superconvergent Dispersion Relation and Low-Energy Kaon-Proton Interaction*

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A simple superconvergent dispersion relation for forward $K^{\pm}p$ scattering is used to test the self-consistency of a new result obtained by Kim on low-energy $\bar{K}p$ interactions. The answer is in the affirmative, within experimental accuracy, as far as the S wave is concerned. There is an indication that Kim's value of $g_{p\Lambda K}^2$ (13.5±2.4) should be lowered, while that of $g_{p\Sigma^0 K}^2$ (0.26±0.43) should be increased.

RECENTLY a new determination of $p\Lambda K$ and $p\Sigma K$ coupling constants has been obtained by Kim[1] using forward dispersion relations for $K^{\pm}p$ and $K^{\pm}n$ scattering.[2] The values $g_{p\Lambda K}^2=13.5\pm2.4$ and $g_{p\Sigma^0K}^2=0.26\pm0.43$ [3] are quite different from all previous results,[4] but are consistent with SU_3 symmetry. The crucial step consisted in the careful treatment in the unphysical region of the dispersion integral. Considerations on kinematics and unitarity for each partial wave led to the use of the multichannel effective-range theory[5] over the constant-scattering-lengths method.[6] Analytic continuation of the forward scattering amplitude, determined from low-energy $\bar{K}N$ interaction in the physical region, into the unphysical region then yielded good predictions for the position and width of $Y_0^*(1405)$. However, $Y_1^*(1385)$ was shown to be weakly coupled to the $\bar{K}N$ system, a fact not expected hitherto. It is, therefore, desirable to see if there is any other means to support all these experimental results.

It turns out that, using a technique similar to that developed in Ref. 7 in connection with the πN case, a simple continuous family of superconvergent dispersion relations for forward kaon-proton scattering can serve for this purpose. All we need are Kim and Goldhaber's experimental parameters[8] describing the $\bar{K}N$ and KN interactions below 300 MeV/c (and its analytic continuation) as well as the $K^{\pm}p$ total cross sections up to 20 BeV/c. Essentially, we are free of (1) $K^{\pm}n$ data,[9] and (2) Regge extrapolation beyond 20 BeV/c.

In analogy with Ref. 7, let us consider the function

$$F^{(-)}(\omega)=\frac{C^{(-)}(\omega)}{(\omega^2-\omega_0^2)^{\beta}(\omega^2-M_K^2)^{1-\beta}},$$

where ω is the laboratory energy of the incident kaon M_K its rest mass, ω_0 a constant energy lying in the unphysical region $(\omega_{\Lambda\pi}\leqslant\omega_0\leqslant M_K)$, $C^{(-)}(\omega)$ a crossing-odd forward kaon-proton scattering amplitude, normalized so that

$$\mathrm{Im}C^{(-)}(\omega)=\tfrac{1}{2}(\omega^2-M_K^2)^{1/2}(1/4\pi)[\sigma_{K^-p}(\omega)-\sigma_{K^+p}(\omega)],$$

and β is an arbitrary real number between 0 and 1 $(0<\beta<1)$. In addition to those standard singularities associated with $C^{(-)}(\omega)$ and $(\omega^2-M_K^2)^{1-\beta}$ explained in Ref. 7, here we have to introduce two more cuts for the factor $(\omega^2-\omega_0^2)^{\beta}$, running from $-\infty$ to $-\omega_0$ and from ω_0 to ∞. Then the crossing-odd function $F^{(-)}(\omega)$ is meromorphic in the upper-half complex-ω plane, and we can write the following family of superconvergent dispersion sum rules:

$$\int_{\omega_{\Lambda\pi}}^{\omega_0}d\omega\frac{\mathrm{Im}C^{(-)}(\omega)}{(\omega_0^2-\omega^2)^{\beta}(M_K^2-\omega^2)^{1-\beta}}+\int_{\omega_0}^{M_K}d\omega\frac{\cos\pi\beta\,\mathrm{Im}C^{(-)}(\omega)+\sin\pi\beta\,\mathrm{Re}C^{(-)}(\omega)}{(\omega^2-\omega_0^2)^{\beta}(M_K^2-\omega^2)^{1-\beta}}-\int_{M_K}^{\infty}d\omega\frac{\mathrm{Im}C^{(-)}(\omega)}{(\omega^2-\omega_0^2)^{\beta}(\omega^2-M_K^2)^{1-\beta}}$$

$$=-\tfrac{1}{2}\pi\sum_Y\frac{X(Y)}{(\omega_0^2-\omega_Y^2)^{\beta}(M_K^2-\omega_Y^2)^{1-\beta}}\quad(0<\beta<1)\qquad(1)$$

where

$$\omega_Y=(M_Y^2-M_p^2-M_K^2)/2M_p,$$
$$X(Y)=[(M_Y-M_p)^2-M_K^2]g_{pYK}^2/4M_p^2,$$

with $Y=\Lambda$ and Σ. In this derivation, use has been made of the fact $C^{(-)}(\omega\to\infty)\sim\omega^{\alpha}$, $\alpha<1$. (A specific value for

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[1] J. K. Kim, Phys. Rev. Letters **19**, 1079 (1967).

[2] P. T. Matthews and A. Salam, Phys. Rev. **110**, 565 (1958). See also J. D. Jackson, in *Dispersion Relations*, edited by G. R. Screaton (Oliver and Boyd, Edinburgh, Scotland, 1961), pp. 44-47.

[3] C. H. Chan and F. T. Meiere [Phys. Rev. Letters **20**, 568 (1968)] pointed out that, in order to use SU_3 symmetry, the correct residue function for the Y pole ($Y=\Lambda$ or Σ) should be $X(Y)=[(M_Y-M_p)^2-M_K^2]g_{pYK}^2/4M_p^2$ instead of the one used by Kim in Ref. 1. Thus one obtains $g_{p\Lambda K}^2=13.5\pm2.4$ and $g_{p\Sigma^0K}^2=0.26\pm0.43$ to replace the old values $g_{p\Lambda K}^2=16.0\pm2.5$ and $g_{p\Sigma^0K}^2=0.3\pm0.5$.

[4] M. Lusignoli *et al.*, Phys. Letters **21**, 229 (1966); Nuovo Cimento **45A**, 792 (1966); N. Zovko, Phys. Letters **23**, 143 (1966); H. P. C. Rood, Nuovo Cimento **50A**, 493 (1966); A. D. Martin and F. Poole, Phys. Letters **25B**, 343 (1967).

[5] M. Ross and G. Shaw, Ann. Phys. (N. Y.) **13**, 147 (1961).

[6] R. Dalitz and S. Tuan, Ann. Phys. (N. Y.) **10**, 307 (1960).

[7] Y. C. Liu and S. Okubo, Phys. Rev. Letters **19**, 190 (1967).

[8] J. K. Kim, Phys. Rev. Letters **19**, 1074 (1967); S. Goldhaber *et al.*, *ibid.* **9**, 135 (1962).

[9] As is evident in Ref. 1, the inclusion of $K^{\pm}n$ data gave a small value of $g_{p\Sigma^0K}^2$, and, consequently, a large value of $g_{p\Lambda K}^2$. Another independent method without using such data is therefore desirable.

TABLE I. Numerical results for Eq. (1) at $k_0=(M_K{}^2-\omega_0{}^2)^{1/2}=125$ MeV/c and 295 MeV/c in kaon natural units ($\hbar=c=M_K=1$). On the left-hand side the contribution from 20 BeV/c to ∞ will be -0.20 for all values of β, if the ordinary Regge pole model is assumed. On the right-hand side we take $g_{p\Lambda K}{}^2=13.5$ and $g_{p\Sigma^0K}{}^2=0.26$. The errors in the right-hand side can be easily calculated ($\approx 20\%$ for the Λ pole term and $\approx 200\%$ for the Σ pole term). It is evident that better agreement can be achieved if $g_{p\Lambda K}{}^2$ and $g_{p\Sigma^0K}{}^2$ are slightly pushed down and brought up, respectively.

k_0 (MeV/c)	β	Unphysical region, real part	Unphysical region, imaginary part	Physical region up to 20 BeV/c	Left-hand side up to 20 BeV/c	Λ pole term $g_{p\Lambda K}{}^2=13.5$	Σ pole term $g_{p\Sigma^0K}{}^2=0.26$	Right-hand side
125	0.0	−1.80	11.09	−7.68	1.61	1.30	0.02	1.32
	0.3	−2.29	8.24	−4.34	1.61	1.32	0.02	1.34
	0.5	−2.46	7.46	−3.38	1.62	1.35	0.02	1.37
	0.8	−2.57	6.96	−2.76	1.63	1.37	0.02	1.39
	1.0	−2.54	6.71	−2.52	1.65	1.39	0.02	1.41
295	0.0	−1.80	11.09	−7.68	1.61	1.30	0.02	1.32
	0.3	0.66	4.33	−3.32	1.67	1.48	0.02	1.50
	0.5	2.83	1.29	−2.39	1.73	1.63	0.03	1.66
	0.8	5.89	−2.25	−1.83	1.81	1.86	0.03	1.89
	1.0	7.43	−3.95	−1.63	1.85	2.04	0.03	2.07

α is unnecessary.) At $\beta=0^+$ and $\beta=1^-$, Eq. (1) coincides with the ordinary dispersion relation for $C^{(-)}(\omega)$ at $\omega=M_K$ and at $\omega=\omega_0$, respectively. When $\beta=\frac{1}{2}$, only the real part of $C^{(-)}(\omega)$ survives in the second integral of Eq. (1). This type of dispersion relation in which $\mathrm{Re}C^{(-)}(\omega)$ appears in a finite region of the integral has been called by Adler the broad-area subtraction dispersion relation.[10]

To put numbers in Eq. (1), we proceed as follows. For simplicity, we put $M_K=1$ and keep only Kim's S and P waves. This should be a good approximation for $\mathrm{Im}C^{(-)}(\omega)$ below 300 MeV/c and in the unphysical region, for in contrast to the πN case, the S wave dominates the low-energy $\bar{K}N$ (KN) interaction, and, as shown in Ref. 1, $Y_0{}^*(1405)$ predominates throughout the whole unphysical region. We do not have the same good approximation for $\mathrm{Re}C^{(-)}(\omega)$ in the whole unphysical region with the present experimental result.[11] If $\mathrm{Re}C^{(-)}(\omega)$ is well known everywhere, the best choice for ω_0 is at $\omega_{\Lambda\pi}$, because only one principal value has to be evaluated at $\omega=M_K$ when $\beta\rightarrow 0^+$, and the calculation of Eq. (1) is easier.[12] Since the analytic continuation of $\mathrm{Re}C^{(-)}(\omega)$ down to $\omega=\omega_{\Lambda\pi}$ is not meaningful at present, we have to keep ω_0 not too far away from M_K, where the S wave is still dominant and the approximation is still very good, at the expense of evaluating one more principal value when $\beta\rightarrow 1^-$. Above physical 300 MeV/c, accurate total cross sections for $K^\pm p$ can be found in the literature.[13] We note that, when $\omega=\omega_m \gg M_K$ (and therefore $\gg\omega_0$) the integral

$$\int_{\omega_m}^{\infty} d\omega \frac{\mathrm{Im}C^{(-)}(\omega)}{(\omega^2-\omega_0{}^2)^\beta(\omega^2-M_K{}^2)^{1-\beta}} \propto \int_{\omega_m}^{\infty} d\omega\, \omega^{\alpha-2}$$

becomes a constant with respect to β. In a family of sum rules like Eq. (1), a constant plays no role when β is varied. Thus we need not concern ourselves with the Regge extrapolation beyond the experimentally accessible energy.

In Table I[14] we show a typical numerical result for $k_0=(M_K{}^2-\omega_0{}^2)^{1/2}=125$ MeV/c. We do not include errors, which is equivalent to using the central value of all experimental quantities only. When k_0 is varied in the unphysical region, the sum rule (1) is more satisfactory for smaller k_0. When k_0 gets bigger, as expected from the poor analytic continuation of $\mathrm{Re}C^{(-)}(\omega)$, the discrepancy is large for both sides of Eq. (1) evaluated at $k_0=295$ MeV/c, as can be seen from the same Table. This prevents us from claiming a more reliable determination of $g_{p\Lambda K}{}^2$ and $g_{p\Sigma^0K}{}^2$ than Kim's, which should be possible if more accurate analysis on P and D waves is carried out.[15] As far as the S wave is concerned,

[10] S. L. Adler, Phys. Rev. **137**, B1022 (1965). We are grateful to N. Zovko for pointing out this reference to us.

[11] The real part is not measurable in the unphysical region. Its values obtained from the analytic continuation of the multichannel effective-range parametrization and from the evaluation of the ordinary dispersion relation should agree with each other, if all input parameters in these calculations make sense. Numerically, this is not true for large k_0 (see Table I), where the contribution from the P waves is not small compared to the S waves (in contrast to the imaginary part), and one does not know which one is correct (unless we have information on πY scattering). According to Kim, his analysis is reliable only for the S wave.

[12] One can always make an arbitrary amplitude superconvergent by dividing as many convergent factors as possible, see, e.g., L. A. Khalfin, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **6**, 606 (1967) [English transl.: Soviet Phys.—JETP Letters **6**, 107 (1967)]. The trouble is the difficulty associated with the numerical calculation at so many branch points.

[13] S. Goldhaber *et al.*, Ref. 8; R. L. Cool *et al.*, Phys. Rev. Letters **17**, 102 (1966); **19**, 259 (1967); W. F. Baker *et al.*, Phys. Rev. **129**, 2285 (1963); W. Galbraith *et al.*, *ibid.* **138**, B913 (1965); J. K. Kim, Ref. 8; Columbia University Report No. Nevis-149 1966 (unpublished); M. B. Watson *et al.*, Phys. Rev. **131**, 2248 (1963); G. Goldhaber and G. Giacomelli, CERN Report No. 67-24, 1967 (unpublished).

[14] Our result should coincide with Kim's at $\beta=0$, irrespective of k_0. Because of a different treatment for physical 300 MeV/c to 550 MeV/c, there is a small difference between the left-hand side and the right-hand side at this value of β, which, however, is within experimental error.

[15] In Table I, if the difference between the values of the left-hand side at $\beta=0$ and $\beta=1$ is large enough, one can confidently solve the simultaneous equations at various β, and obtain $g_{p\Lambda K}{}^2$ and $g_{p\Sigma^0K}{}^2$ solely from experimental $K^\pm p$ data. Unfortunately, this occurs only at large $k_0=(M_K{}^2-\omega_0{}^2)^{1/2}$, where a more accurate analysis of P and D waves is needed.

Kim's results are self-consistent to within experimental errors. To rigorously satisfy our family of sum rules Eq. (1), however, the above values of $g_{p\Lambda K}^2$ and $g_{p\Sigma^0K}^2$ should be slightly pushed down and brought up, respectively (in the same direction as all previous results,[4,16] including those obtained from photoproduction analysis).[17] This can still preserve SU_3 invariance, with an f value smaller than 0.41.

The same conclusion applies to the crossing-even amplitude $C^{(+)}(\omega)$.[18] This time a subtraction constant is needed. From the ordinary dispersion relation itself (where data in Ref. 1 are taken) we evaluate this number as $\frac{1}{2}\pi C^{(+)}(0)=10.13M_K^{-1}$. The three terms on the left-hand side for $C^{(+)}(\omega)$ [analogous to that of Eq. (1) for $C^{(-)}(\omega)$] cancel almost completely.

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[16] G. H. Davies *et al.*, Nucl. Phys. **B3**, 616 (1967), and references therein.

[17] See, for example, T. K. Kuo, Phys. Rev. **129**, 2264 (1963); **130**, 1539 (1963); S. Hatsukade and H. J. Schnitzer, *ibid.* **132**, 1301 (1963); Fayyazuddin, *ibid* **134**, B182 (1964).

[18] Y. C. Liu and S. Okubo, Phys. Rev. **168**, 1712 (1968).

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Regge-Pole Analysis of Charge-Exchange and Hypercharge-Exchange Reactions in Pseudoscalar-Meson–Baryon Scattering*

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Within the context of the vector-tensor Regge-pole exchange model, the data on charge-exchange reactions ($\pi^-p\to\pi^0n$, $\pi^-p\to\eta n$, $K^-p\to\bar{K}^0n$) and hypercharge-exchange reactions [$\pi^-p\to K^0\Lambda(\Sigma^0)$, $\pi^+p\to K^+\Sigma^+$, $K^-p\to\pi^0\Lambda$, $K^-p\to\pi^-\Sigma^+$] are systematically analyzed. A statistical fit is presented which provides a good quantitative description of these data. In particular, the measured differential cross sections are adequately fitted over the momentum transfer range $0\leq|t|\leq2.6$ (GeV/c)^2. The magnitude and sign changes of the polarization data are explained by the model with the exception of the $\pi^-p\to\pi^0n$ polarization at high energies. Predictions of immediate experimental interest include (i) near-maximal polarization in $K^-p\to\bar{K}^0n$ near $t=0$, (ii) a large polarization independent of energy for all reactions involving exchange of both a vector and tensor trajectory, (iii) differential cross sections for $K^-p\to\eta\Lambda(\Sigma^0)$, (iv) structure in differential cross sections near $t\sim-2.5$ (GeV/c)^2, and (v) polarization for $\pi^-p\to\pi^0n$ at intermediate energies. Assuming $SU(3)$ symmetry for the factorized residues, the f/d ratios were determined for the vector and tensor nonet $J^P=\frac{1}{2}^+$ baryon octet couplings.

1. INTRODUCTION

THERE now exists a wealth of experimental information on meson-baryon scattering regarding the energy and angular dependence of the cross section and the baryon polarization for elastic charge-exchange and hypercharge-exchange reactions.[1] Some general qualitative features of the available data are: (i) The total scattering cross sections seem to approach constant values at high energies while the partial cross sections for most inelastic reactions appear to fall off with a power-law energy dependence $(E_{\mathrm{lab}})^{-n}$ where $n>0$; (ii) most of the differential cross sections show sharp peaks in the forward direction, $\cos\theta_{\mathrm{c.m.}}\simeq+1$ ($\theta_{\mathrm{c.m.}}$ is defined as the angle between the directions of the incident and outgoing mesons in the center-of-mass system); (iii) in a variety of reactions a secondary maximum occurs following the forward peak.

The forward peaks suggest that the scattering occurs through a peripheral mechanism, i.e., through the exchange of a boson. Calculations based on exchange of an elementary meson of spin J have failed to describe the energy and angle dependence of cross sections for reactions in which $J\geq1$.[2] In the Regge-pole model, however, the exchanged object has angular momentum which is a function of the invariant momentum transfer t, and as a result the energy dependence of the cross section is a power law consistent with the experimental

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[1] The general trends of the data can be seen in the excellent compilation by D. R. O. Morrison, Phys. Letters **22**, 528 (1966), in which references to the original sources are given. See also D. R. O. Morrison, Centre d'Etudes Recherche National Report No. CERN/TC/Physics 66-20 (unpublished).

[2] See, for example, J. D. Jackson, J. Donahue, K. Gottfried, R. Keyser, and B. Svensson, Phys. Rev. **139**, B428 (1965).