K_{l3}-Decay Form Factors, Current Algebra, and Subtracted Dispersion Relations. II*

NORMAN H. FUCHS
Department of Physics, Purdue University, Lafayette, Indiana
(Received 15 April 1968)

Once-subtracted dispersion relations are used together with the algebra of currents and soft-pion techniques to investigate the form factors $f_\pm(t)$ which describe K_{l3} decays. Particular attention is paid to the influence of S-wave $K\pi$ states, and results are obtained which depend explicitly on the S-wave phase shift. Various parametrizations of this phase shift are made in an effort to clarify the interdependence of the $K\pi$ scattering amplitude and the K_{l3} form factors. It is found to be difficult to obtain a value for $\xi \equiv f_-(0)/f_+(0)$ which is not small and negative, while the momentum dependence of $f_-(t)$ may be quite rapid in some cases.

I. INTRODUCTION

In a previous report[1] we have described a new attempt to determine the K_{l3}-decay form factors in a manner which avoids several of the more questionable assumptions, such as the soft-kaon approximation that have been made in the literature. We assumed only once-subtracted dispersion relations for most amplitudes that we encounter and used simple pole approximations together with current algebra to eliminate arbitrary parameters or functions. Our final results for the form factors $f_\pm(t)$ were very close to being the same as the naive K^*-pole model predictions. This was presumably due to our neglect of the possible contribution of $J^P=0^+$ $K\pi$ states to the dispersion integrals. The purpose of the present work is to extend the calculation to include this contribution and determine its effect on the scale as well as the t dependence of the form factors.

Of course, the contribution of such S-wave states to $f_\pm(t)$ has been investigated before[2]; furthermore, there have been many applications of current-algebra techniques to the determination of $f_\pm(t)$.[3] However, we believe that by avoiding assumptions such as soft-kaon approximation or superconvergence,[4] and by not using arguments based on small $SU(3)$-symmetry breaking,[5] we have made a reliable computation. Indeed, we feel that the most interesting theoretical aspect of the problem is the determination of the effect of $SU(3)$-symmetry breaking in a situation where such breaking may very well be large.

In Sec. II we compute the contribution of the $K\pi$ S-wave intermediate states to the matrix element $T_{\mu\nu}$, which is given by

$$T_{\mu\nu}=\int d^4x\, e^{iqx}\theta(x_0)\langle 0|[A_\mu{}^3(x),V_\nu{}^{4+i5}(0)]|K^-(k)\rangle \quad (1)$$

and to the matrix element W_ν,

$$W_\nu=\int d^4x\, e^{iqx}\theta(x_0)\langle 0|[D_A{}^3(x),V_\nu{}^{4+i5}(0)]|K^-(k)\rangle. \quad (2)$$

All notation in this paper will be the same as that used in I, whenever there is an overlap. The results of Sec. II are expressions for $f_\pm(t)$ in terms of the $K\pi$ scattering phase shift for $l=0$ and isospin $\frac{1}{2}$. In Sec. III several parametrizations of this phase shift are studied and the effects of the variation of the parameters on the form factors is finally presented in Table I. A discussion of our results as well as a comparison with some recent work of others is given in Sec. IV.

II. INCLUSION OF $K\pi$ S-WAVE STATES

In order to investigate the effect of the S-wave $K\pi$ states we must determine the contribution of these states to the matrix element W_ν as well as to the matrix element $T_{\mu\nu}$. That is to say, we must modify the usual soft-pion technique in order to take these direct channel states into account. The situation here is analogous to that of the πN case where the nucleon Born term must be treated carefully even when the intermediate-state baryon does not have precisely the same mass as the external baryon. We are not proposing here that there is a $K\pi$ 0^+ resonance, but we will allow for that possibility.

The absorptive part of W_ν, which we denote by w_ν, receives a contribution from $K\pi$ S-wave intermediate states given by

$$w_\nu=-\frac{1}{2}\frac{f_\pi m_\pi^2}{m_\pi^2-q^2}\left[\frac{m_K^2-m_\pi^2}{t}f_+^*(t)+f_-^*(t)\right](k-q)_\nu \times e^{-i\delta}\sin\delta\,\theta[t-(m_K+m_\pi)^2], \quad (3)$$

where $\delta=\delta(t)$ is the phase shift for S-wave isospin-$\frac{1}{2}$ $K\pi$ scattering at center-of-mass energy squared equal to t,

---

* This work was supported in part by the U. S. Atomic Energy Commission, Contract No. AT(11-1)-1428.
[1] N. H. Fuchs, Phys. Rev. **170**, 1310 (1968), referred to as I.
[2] See, for example, S. W. MacDowell, Phys. Rev. **116**, 1047 (1959); J. Iizuka, Progr. Theoret. Phys. (Kyoto) **26**, 554 (1961); Y. Ito, *ibid.* **36**, 182 (1966); J. Bernstein and S. Weinberg, Phys. Rev. Letters **5**, 481 (1960); H. Chew, Phys. Rev. Letters **8**, 297, (1962).
[3] Current algebra and unsubtracted dispersion relations are used by V. S. Mathur, L. K. Pandit, and R. E. Marshak, Phys. Rev. Letters **16**, 947, 1135(E) (1966).
[4] H. T. Nieh, Phys. Rev. **164**, 1780 (1968).
[5] S. Matsuda and S. Oneda, Phys. Rev. **169**, 1172 (1968).

and the asterisk indicates complex conjugation. In obtaining this result, we have used partial conservation of axial-vector current (PCAC) to relate the matrix element $\langle \pi K|D_A{}^3(0)|K^-(k)\rangle$ to that for πK scattering. This is the reason for the appearance of the pion pole in q^2 as well as of the phase shift $\delta(t)$. The form factors $f_+{}^*$ and $f_-{}^*$ arise from the matrix element $\langle 0|V_\nu{}^{4+i5}(0)|K\pi\rangle$.

Expressing W_ν as before,

$$W_\nu=(k+q)_\nu G_+(q^2,t)+(k-q)_\nu G_-(q^2,t)\,, \tag{4}$$

with the absorptive parts of $G_\pm$ denoted by $g_\pm$, we find that only g_- receives a contribution from these $K\pi$ states. Furthermore, in I we have argued that G_- should satisfy an unsubtracted dispersion relation in t for fixed q^2. Therefore, our corrected $G_\pm$ are given by

$$G_+=\bar{G}_+\,,$$

$$G_-=\bar{G}_-+\frac{1}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{dt'}{t'-t}\left[\frac{m_K{}^2-m_\pi{}^2}{t'}f_+{}^*(t')+f_-{}^*(t')\right]\times e^{-i\delta(t')}\sin\delta(t')\frac{\frac{1}{2}f_\pi m_\pi{}^2}{m_\pi{}^2-q^2}, \tag{5}$$

where $\bar{G}_\pm$ are the values for $G_\pm$ which we found in I by neglecting the $K\pi$ 0^+ states.

We now consider the matrix element $T_{\mu\nu}$, with absorptive part $t_{\mu\nu}$. The contribution of $K\pi$ intermediate states to $t_{\mu\nu}$ involves an integral over the internal momenta (k_1,k_2 of the K and π, respectively) of the product of the matrix element $\langle 0|V_\nu{}^{4+i5}(0)|K\pi\rangle$ as above and the matrix element T_μ given by

$$T_\mu=\langle K\pi|A_\mu{}^3(0)|K^-(k)\rangle\,, \tag{6}$$

which has the form dictated by Lorentz invariance

$$T_\mu=Pk_\mu+Qq_\mu+R(k_1-k_2)_\mu\,,\quad k_1+k_2=k+q\,, \tag{7}$$

where P,Q,R are scalar functions of the invariants formed from the momenta k_1,k_2,k,q. Since $k^2=k_1{}^2=m_K{}^2$ and $k_2{}^2=m_\pi{}^2$, as these particles are on the mass shell, we may choose as independent variables q^2 and t, and either s or u, where

$$\begin{aligned}t&=(k_1+k_2)^2=(k-q)^2\,,\\ s&=(k_1-k)^2=(k_2+q)^2\,,\\ u&=(k_1+q)^2=(k_2-k)^2\,.\end{aligned} \tag{8}$$

In our case, however, the $K\pi$ state has zero total angular momentum. If we go to the center of mass of the $K\pi$ system and define

$$k_1=(E_1,\mathbf{p})\,,\quad k_2=(E_2,-\mathbf{p})\,, \tag{9}$$

then

$$T_i=k_i(P+Q)+2p_iR\,. \tag{10}$$

But T_i cannot depend on the direction of $\mathbf{p}$, since the $K\pi$ state is invariant under rotations. Therefore, we conclude that

$$R=0\,. \tag{11}$$

Now, recalling that $T_{\mu\nu}$ may be written as

$$iT_{\mu\nu}=Ag_{\mu\nu}+Bq_\mu q_\nu+Cq_\mu k_\nu+Dk_\mu q_\nu+Ek_\mu k_\nu\,, \tag{12}$$

where $A,\cdots,E$ are scalar functions of q^2 and t, and denoting their respective absorptive parts by $a,\cdots,e$, we find that the $K\pi$ S-wave states contribute as follows:

to $-b,c$:

$$\left[\frac{m_K{}^2-m_\pi{}^2}{t}f_+{}^*(t)+f_-{}^*(t)\right]Q\theta[t-(m_K+m_\pi)^2]\,,$$

to $-d,e$:

$$\left[\frac{m_K{}^2-m_\pi{}^2}{t}f_+{}^*(t)+f_-{}^*(t)\right]P\theta[t-(m_K+m_\pi)^2]\,,$$

to a: 0. (13)

We now want to determine the functions P and Q. If we multiply the matrix element T_μ by q^μ, we get something proportional to $\langle K\pi|D_A{}^3(0)|K^-\rangle$, which is the matrix element of the divergence of the axial-vector current. According to the usual pole-dominance form of the PCAC hypothesis, this matrix element satisfies an unsubtracted dispersion relation in q^2 for fixed $q\cdot k$ and $q\cdot(k_1-k_2)$; moreover, the dispersion relation is dominated for small q^2 by the pion pole in q^2. This reasoning then leads to the following expressions for P and Q:

$$\nu P=f_\pi T_{\pi K}(\nu)\,, \tag{14}$$

$$Q=\frac{f_\pi}{m_\pi{}^2-q^2}T_{\pi K}(\nu)\,, \tag{15}$$

where $\nu=k\cdot q$ and $T_{\pi K}(\nu)$ is the physical $K\pi$ S-wave scattering amplitude for pion lab energy ν. Notice that Eq. (14) implies that $T_{\pi K}(\nu=0)=0$, which is just Adler's consistency condition for πK scattering in this dispersion-theoretic context. If, in fact, $q^\mu T_\mu$ did not go to zero for large q^2, but rather approached a constant λ, then we would still obtain Eq. (15), but Eq. (14) would be modified to

$$\nu P=f_\pi[T_{\pi K}(\nu)-T_{\pi K}(\nu=0)] \tag{16}$$

and

$$f_\pi T_{\pi K}(\nu=0)=-\lambda\,. \tag{17}$$

We will choose $\lambda=0$ for the ensuing discussion. It will be clear how to proceed if $\lambda\neq0$.

We have previously argued that of the functions appearing in the expansion of $T_{\mu\nu}$, Eq. (12), $E(q^2,t)$ should satisfy an unsubtracted dispersion relation in t for fixed q^2, while the combination $C(q^2,t)+\frac{1}{2}E(q^2,t)$ should satisfy an unsubtracted dispersion relation in q^2

for fixed t. Using Eqs. (13), we then find

$$E=\bar{E}-\frac{1}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{dt'}{t'-t}\times\left[\frac{m_K^2-m_\pi^2}{t'}f_+^*(t')+f_-^*(t')\right]\times\frac{f_\pi e^{-i\delta(t')}\sin\delta(t')}{m_K^2+q^2-t'},$$

$$C+\tfrac{1}{2}E=\bar{C}+\tfrac{1}{2}\bar{E}, \tag{18}$$

where, in analogy with Eq. (5) for $G_\pm$, the bar indicates the function as computed in I without the contribution of the S-wave $K\pi$ states. Similar results obtain for the functions B and D, but these will not be needed here.

The basic equation for further discussion is the relation between the functions appearing in the expression for $T_{\mu\nu}$, Eq. (12), and those appearing in the expression for W_μ, Eq. (14):

$$q^2C+\nu E=-\tfrac{1}{2}f_K-(G_++G_-). \tag{19}$$

We proceed by first equating the residues of the left- and right-hand sides of Eq. (19) at the pion pole $q^2=m_\pi^2$; this gives

$$f_-(t)=\frac{f_{K^*}(m_K^2-m_\pi^2)}{m_{K^*}^2(m_{K^*}^2-t)}g_{K^*\pi K}-\frac{1}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{dt'}{t'-t}\left[\frac{m_K^2-m_\pi^2}{t'}f_+^*(t')+f_-^*(t')\right]\times e^{-i\delta(t')}\sin\delta(t'). \tag{20}$$

Next we evaluate Eq. (19) at $q^2=0$ for fixed t; this gives

$$f_+(t)=\frac{f_K}{f_\pi}+t\frac{f_{K^*}}{m_{K^*}^2}\frac{g_{K^*\pi K}}{m_{K^*}^2-t}-\frac{1}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{dt'}{m_K^2-t'}\left[\frac{m_K^2-m_\pi^2}{t'}f_+^*(t')+f_-^*(t')\right]\times e^{-i\delta(t')}\sin\delta(t'). \tag{21}$$

The only difference between this result for $f_+(t)$ and our previous result is the appearance of the third term in Eq. (21), which we notice is a constant independent of t. Thus the addition of S-wave intermediate states does not effect the t dependence of the form factors $f_+(t)$, but does affect that of the form factor $f_-(t)$, as one can see from Eq. (20).

These coupled integral equations may be solved as follows. We first define the linear combination $f_0(t)$ of $f_\pm(t)$ which corresponds to $J^P=0^+$ intermediate states:

$$f_0(t)=(m_K^2-m_\pi^2)f_+(t)+tf_-(t). \tag{22}$$

Then, from Eqs. (20) and (21), we deduce an integral equation for f_0:

$$f_0(t)=(m_K^2-m_\pi^2)\frac{f_K}{f_\pi}+\frac{m_\pi^2}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{dt'e^{-i\delta(t')}\sin\delta(t')\,f_0^*(t')}{t'(m_K^2-t')}-\frac{1}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{dt'}{t'-t}\frac{m_K^2-t}{m_K^2-t'}\times e^{-i\delta(t')}\sin\delta(t')\,f_0^*(t'), \tag{23}$$

which would be of the usual Omnès type if it were not for the second term on the right-hand side. This causes only minor difficulties, however, and we find the solution

$$f_0(t)=\frac{(m_K^2-m_\pi^2)(f_K/f_\pi)\Omega(t)}{1-(m_\pi^2/m_K^2)[\Omega(0)-1]}, \tag{24}$$

$$\Omega(t)=\exp\left[\frac{1}{\pi}\int_{(m_K+m_\pi)^2}^{\infty}\frac{dt'}{t'-t}\frac{m_K^2-t}{m_K^2-t'}\delta(t')\right]. \tag{25}$$

From $f_0(t)$ we may obtain $f_+(t)$ directly, since

$$f_+(0)=(m_K^2-m_\pi^2)^{-1}f_0(0) \tag{26}$$

and

$$f_+(t)=f_+(0)+t\frac{f_{K^*}}{m_{K^*}^2}\frac{g_{K^*\pi K}}{m_{K^*}^2-t} \tag{27}$$

from Eq. (21). Furthermore, once we have $f_+(t)$ we may find $f_-(t)$, since

$$f_-(t)=[f_0(t)-(m_K^2-m_\pi^2)f_+(t)]t^{-1}. \tag{28}$$

Therefore, we conclude that in the present context, the entire effect of the additional consideration of the $J=0^+$ $K\pi$ intermediate states is contained in the function $\Omega(t)$. Once we specify $\delta(t)$, the $K\pi$ S-wave isotopic spin-$\frac{1}{2}$ phase shift, we have fixed $f_0(t)$ and thus $f_\pm(t)$ as well. However, since one has little experimental information on this phase shift, we are forced at this point to make some estimates based on various reasonable assumptions. Notice that the integral defining $\Omega(t)$, Eq. (25), is determined by the values of $\delta(t)$ for small t, since there is a damping factor of $(t'-m_K^2)^{-1}$. We will, therefore, parametrize $\delta(t)$ according to some low-energy approximations and compute $\Omega(t)$. We also will investigate the effect of the possible existence of a $K\pi$ resonance on $\Omega(t)$.

III. PARAMETRIZATIONS OF $K\pi$ SCATTERING AMPLITUDE

In the scattering length (zero effective range) approximation for $K\pi$ scattering, the amplitude is just

$$e^{i\delta}\sin\delta=ka[1-ika]^{-1}, \tag{29}$$

where a is the scattering length. For simplicity, we will use for k the approximation

$$k=(m_K m_\pi)^{1/2}[t-(m_K+m_\pi)^2]^{1/2}(m_K+m_\pi)^{-1}. \quad (30)$$

This will enable us to do the integral over $\delta(t)$ and compute $\Omega(t)$ in closed form. If $a>0$, which corresponds to an attractive force insufficiently strong to bind, we find

$$\Omega(t)=\frac{1+k_0 a}{1-ika}, \quad (31)$$

where k is given above in Eq. (30) and

$$k_0=(m_K m_\pi)^{1/2}(2m_K m_\pi+m_\pi^2)^{1/2}(m_K+m_\pi)^{-1}. \quad (32)$$

That is, when $t=m_K^2$, $k=+ik_0$. On the other hand, if $a<0$, which corresponds to a repulsive force or one bound state, we find

$$\Omega(t)=\frac{B-t}{B-m_K^2}\,\frac{1-k_0|a|}{1+ik|a|}, \quad (33)$$

where k_0 and k are as above and the quantity B is

$$B=(m_K+m_\pi)^2[1-(m_K m_\pi a^2)^{-1}]. \quad (34)$$

If, in fact, there were a bound state, then we would have to go back and modify our equations to include this possibility, adding a pole at the bound-state mass. However, there is good reason to believe that such a particle κ does not exist. If its mass m_κ were less then m_K, then the K meson would decay into it,

$$K\rightarrow\kappa+2\gamma,$$

and, subsequently, the κ would decay weakly just as the K meson does. No such particle has been seen. Moreover, if $m_K<m_\kappa<m_K+m_\pi$, then the κ would decay according to

$$\kappa\rightarrow K+2\gamma.$$

Beam surveys have not given any evidence for the existence of a particle in this mass range, nor is there any indication for κ from missing-mass experiments of the type

$$\pi^- p\rightarrow\kappa+\Lambda.$$

Of course, if the κ is not coupled appreciably to hadrons, it would be difficult to produce; however, if this is the case, the residue of the κ pole would be small and our neglect of it would be justified. Therefore, we interpret $a<0$ as a repulsive interaction and ignore the possibility of a bound $K\pi$ state.

There is a theoretical estimate of the $K\pi$ scattering length a, based on a current algebra soft-pion calculation, which gives $a\approx+\frac{1}{2}m_K^{-1}$.[6] Of course, this calculation proceeds under the assumption that the $K\pi$ interaction is not strong near threshold and it therefore would presumably require modification if this were not the case.

Finally, we consider the possibility of the existence of a $K\pi$ resonance with $J^P=0^+$. Experimental evidence for such a resonance has come and gone in recent years, and at present there is no strong indication for its existence. On the other hand, a resonance with a broad width and a moderately large central mass would not contradict experiment. Such a resonance has been proposed for the $\pi\pi$ 0^+ system and is the subject of current controversy. If this $\pi\pi$ resonance existed, one would expect analogous behavior for the $K\pi$ system. In any case, we note that in the one-resonance approximation,

$$k\cot\delta=(M^2-t)/\gamma, \quad (35)$$

$$\Omega(t)=\frac{M^2-m_K^2+k_0\gamma}{M^2-t-ik\gamma}, \quad (36)$$

where M is the resonance mass and γ is its width.

In Table I we present the predictions of the present work, using some representative values of the parameters which characterize $\Omega(t)$, for the quantities $\Omega(0)$ and $f_+(0)$.

From the expression Eq. (28) for $f_-(t)$ we see that $f_-(0)$ depends not only on $\Omega'(0)$ but also on $f_+'(0)$:

$$f_-(0)=\left[\frac{\Omega'(0)}{\Omega(0)}f_+(0)-f_+'(0)\right](m_K^2-m_\pi^2). \quad (37)$$

But from Eq. (27) we see that

$$f_+'(0)=\frac{f_{K^*}g_{K^*\pi K}}{m_{K^*}^2}\,\frac{1}{m_{K^*}^2}. \quad (38)$$

Now the combination $f_{K^*}g_{K^*\pi K}/m_{K^*}^2$ is not experimentally known; however, it is expected theoretically to be approximately equal to $f_+(0)$. [This would be exact if $f_+(t)$ vanished for $t\rightarrow\infty$.] If we make this identification, then we obtain

$$\xi(0)=\frac{f_-(0)}{f_+(0)}=\frac{m_K^2-m_\pi^2}{m_{K^*}^2}\left[m_{K^*}^2\frac{\Omega'(0)}{\Omega(0)}-1\right], \quad (39)$$

$$\lambda_-=m_K^2\frac{f_-'(0)}{f_-(0)}=\frac{m_K^2}{m_{K^*}^2}\,\frac{[1-\frac{1}{2}m_{K^*}^2\Omega''(0)/\Omega(0)]}{[1-m_{K^*}^2\Omega'(0)/\Omega(0)]}. \quad (40)$$

TABLE I. Computed values for parameters describing the K_{l3} form factors under various assumptions for S-wave $K\pi$ scattering. For notation, see the text.

Input	$\Omega(0)$	$(f_\pi/f_K)f_+(0)$	$\xi(0)$	$(m_{K^*}^2/m_K^2)\lambda_-$
$a=1$	1	1	−0.29	1
$a=\frac{1}{2}m_K^{-1}$	0.92	0.91	−0.23	1.1
$a=m_K^{-1}$	0.87	0.86	−0.19	1.3
$a=-\frac{1}{2}m_K^{-1}$	1.09	1.10	−0.34	1
$a=-m_K^{-1}$	1.15	1.16	−0.35	1.5
$M=725$ MeV, $\gamma=0$	0.53	0.49	−0.14	0.8
$M=725$ MeV, $\gamma=m_K$	0.55	0.51	≈0	large
$M=2m_K$, $\gamma=0$	0.75	0.73	−0.06	1.7
$M=2m_K$, $\gamma=m_K$	0.74	0.72	−0.13	0.9
$M=2m_K$, $\gamma=2m_K$	0.73	0.71	−0.18	1
$M=3m_K$, $\gamma=0$	0.89	0.88	−0.19	1.3
$M=3m_K$, $\gamma=3m_K$	0.95	0.95	−0.24	1

[6] See, for example, S. Weinberg, Phys. Rev. Letters 17, 616 (1966).

Referring to Table I, we see that $\xi(0)$ is insensitive to the various parametrizations we have examined. It is difficult to obtain a value for $\xi(0)$ that is not small and negative. On the other hand, λ_- is extremely sensitive to variations in the parameters describing the resonant $K\pi$ interaction and insensitive to the other fits for the phase shift $\delta(t)$. In particular, for the case of a resonance at 725 MeV and a width of 500 MeV *the value as well as the sign* of λ_- are indeterminate. This comes about because there are cancellations which give a small value of $\xi(0)$ [i.e., small $f_-(0)$] while no such cancellations appear in the calculation of the derivative, $f'(0)$, which is proportional to $\xi\lambda_-$. This result of small ξ and (possibly) large λ_- is the same as that obtained by Lee[7] and Majumdar[8] for entirely different reasons. However, we are not forced to this conclusion, as we see from the above discussion.

IV. CONCLUDING REMARKS

In the context of the present investigation, with the assumptions we have made on subtractions in dispersion relations as well as on dominance of certain contributory states to absorptive parts, we conclude that the S-wave πK scattering states are expected to have an important effect on the K_{l3} form factors $f_\pm(t)$. This is true independent of the detailed form of the πK amplitude. Even for a relatively small positive scattering length, as has been predicted from current algebra and some reasonable extrapolation recipe, the value of $f_+(0)$ is reduced by about 9%. This makes a comparison of K-decay rates with Cabibbo theory difficult. For stronger interaction, giving rise to a resonance in the $K\pi$ system with $J^P=0^+$, the effect on the form factors is even more pronounced. In particular, for some resonance positions and widths it was shown that $f_-(t)$ will vary quite rapidly with t. On the other hand, although a wide range of parametrizations was investigated, including both positive and negative scattering lengths, resonances of 725 to 1500 MeV and resonance widths of zero to 1500 MeV, no set of parameters was found to give a value of ξ that was not small and negative.

Recently, Lee[7] has obtained expressions for $f_\pm(t)$ based on the notions of chiral dynamics and the field-current identity as applied to broken $SU(3)$. He finds a small ξ, and a large $|\lambda_-|$, and a small value for the product $\xi\lambda_-$. The theoretical bases of Lee's work and the present work are similar insofar as current algebra is built into both; however, the chiral dynamics approach is a purely low-energy technique, while the dispersion-theoretic approach we have used is of necessity dependent upon relations between high- and low-energy behavior. Direct comparison is therefore difficult. Results very close to those of Lee have been obtained recently by Majumdar,[8] using the algebra of currents and the Weinberg spectral-function sum rules. Majumdar considers the effect of a possible κ-meson contribution, and finds that it leads to a large negative value for ξ and a small positive value for λ_-. This is in contradiction with our results, which, of course, follow from quite different assumptions. It should be mentioned that both Lee and Majumdar work with amplitudes for which the kaon (as well as the pion) is off the mass shell. There is thus a large extrapolation to be made in returning to a physical amplitude. We have avoided making this extrapolation.

We have, however, made assumptions about subtractions in dispersion relations for which we can offer no justification other than simplicity. That is, weaker assumptions would lead to results depending on undetermined subtraction constants. In particular, we have assumed that the quantity $S(q^2,t)$, defined by

$$S=i\int d^4x\, e^{i(k-q)x}\theta(-x_0) \times\langle 0|[D_A{}^3(0),D_V{}^{4+i5}(x)]|K^-(k)\rangle \quad (41)$$

so that S is related to the other functions we have defined by

$$S=(m_K{}^2-q^2)G_+(q^2,t)+tG_-(q^2,t)+\tfrac{1}{2}f_Km_K{}^2, \quad (42)$$

satisfies once-subtracted dispersion relations in q^2 for fixed t and in t for fixed q^2. Using the results of the present work, we obtain

$$S=\tfrac{1}{2}f_K(m_K{}^2-m_\pi{}^2)\left[1-\frac{m_\pi{}^2}{m_\pi{}^2-q^2}\Omega(t)\right]. \quad (43)$$

Previously we had neglected $K\pi$ S-wave states, which means $\Omega(t)=1$, so that we had found

$$S=\frac{\tfrac{1}{2}(m_\pi{}^2-m_K{}^2)q^2}{m_\pi{}^2-q^2}. \quad (44)$$

Our present result, Eq. (43), is more reasonable in that S now is not independent of t. However, it is difficult to know what to expect for this function apart from the obvious property of vanishing in the $SU(3)$ limit.

Finally, we would like to point out that using our results we may infer properties of the low-energy $K\pi$ scattering amplitude from measurements of $f_\pm(t)$. Conversely, from measurements of the $K\pi$ S-wave phase shift (for example, by use of analyses similar to those of Johnson *et al.*[9]), we may obtain some constraints on the K_{l3} form factors.

ACKNOWLEDGMENTS

We would like to express our gratitude to A. Garfinkel, T. K. Kuo, and K. Raman for several fruitful discussions.

[7] B. W. Lee, Phys. Rev. Letters **20**, 617 (1968).
[8] D. P. Majumdar, Phys. Rev. Letters **20**, 971 (1968).
[9] P. B. Johnson *et al.*, Phys. Rev. **163**, 1497 (1967).