Scalar- and Tensor-Tadpole Contribution to K^0-K^+ Mass Difference

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A calculation of the scalar-meson "tadpole" gives the correct sign but too small a magnitude for $m_K - m_{K^+}$. As an alternative, a "tadpole" due to 2^+ mesons is calculated. Here the concept of relating the trace of tensor currents to scalar mesons is introduced which makes the dispersion integrals for the tensor particle 2γ vertex convergent. Then the tensor tadpoles give $m_{K^0}-m_{K^+}\approx 0.1$ MeV, which has the correct sign but is too small in magnitude compared to the experimental value of 4 MeV.

I. INTRODUCTION

THE problem of electromagnetic mass differences is complicated by the strong interaction and it seems that there may not be any single, simple approach which can explain most of the electromagnetic mass differences δm. It has been pointed out by Harari[1] that the $\Delta I>1$ parts of these δm may allow unsubtracted dispersion relations and this is related to the fact that the $\Delta I=2$ parts are more or less correctly obtained from several approaches.[2] However, the $\Delta I=1$, seems not only difficult to calculate, but also to require different approaches to different situations such as proton, neutron, and K^+,K^0. Thus, for example, a recent calculation[3] of the p-n mass difference based on dispersion relations rests heavily on the fact that there exists a Roper resonance not far from the nucleon mass and having the same quantum numbers as the nucleon. The situation for K^0-K^+, however, does not allow such an explanation, since there is no particle near the K mass having the same quantum numbers as K.

It has been suggested by Coleman and Glashow[4] that the tadpole mechanism may contribute a large part of the $\Delta I=1$ mass differences. In Sec. II, we calculate this explicitly for K^0-K^+ mass difference, using some recent information on s-wave $\pi\pi$ scattering. However, although the sign of this contribution comes out right, i.e., $m_{K^0}>m_{K^+}$, the magnitude comes out to be of the order of 0.1 MeV, which is at least an order of magnitude too small.

The idea of tadpoles has been given a somewhat different interpretation by Okubo,[5] that the Reggeized tadpoles may dominate the high-energy contribution in the Cottingham formula[6] and hence are important. In this context, we note that the tensor-meson propagators have pure spin-2 on the mass shell, but when away from the mass shell they develop spin-0 parts as well. Here we are assuming that the tensor-meson propagator is of the form

$$T_{\mu\alpha}{}^{\nu\beta}=g_{\mu\alpha}g^{\nu\beta}+g_\mu{}^\beta g_\alpha{}^\nu-\tfrac{2}{3}g_\mu{}^\nu g_\alpha{}^\beta-(1/m^2)(k_\mu k_\alpha g^{\nu\beta}+k^\nu k_\alpha g_\mu{}^\beta+k_\mu k^\beta g_\alpha{}^\nu+k^\nu k^\beta g_{\mu\alpha}-\tfrac{2}{3}k_\mu k^\nu g_\alpha{}^\beta-\tfrac{2}{3}k_\alpha k^\beta g_\mu{}^\nu)+(4/3m^4)k_\mu k^\nu k_\alpha k^\beta\,, \quad (1)$$

where m is the tensor-meson mass. This therefore means that the vacuum expectation value of the tensor fields, when off the mass shell, is not zero. This is similar to the idea of Regge poles where the spin of the "particle" changes with the mass.

The tadpole picture one has in mind is that a scalar or a tensor meson is given off from the particle, say K^+, in whose mass we are interested. The meson then disappears through the photon bubble. It is clear that the tadpole is related to the decay of the particle into 2γ. In Sec. III, we analyze the 2γ decay of tensor mesons in a gauge-invariant way, using dispersion relations. The intermediate states are saturated by two-pseudo-scalar-meson states. The perturbation calculation is divergent; however, it can be made convergent if we introduce form factors.

In order to determine the form factors, we introduce the concept of "partially traceless" tensor currents, the statement of which is

$$g^{\mu\nu}J_{\mu\nu}=c\phi_S\,, \quad (2)$$

where $J_{\mu\nu}$ is the source of the tensor meson and ϕ_S is the scalar-meson field. With this relation, one finds that the form factors are dominated by the scalar-meson poles.

The 2γ decay calculation for the f^0 yields the result that

$$\Gamma(f^0\to 2\gamma)/\Gamma(f^0\to 2\pi)\approx 2.5\times 10^{-4}\approx 5\alpha^2\,, \quad (3)$$

where α is the fine-structure constant. These calculations are then used for calculating the K^0-K^+ mass difference. The result is

$$\delta m=m_{K^0}-m_{K^+}\approx 0.1 \text{ MeV}\,, \quad (4)$$

which is too small compared with the experimental value of 4 MeV. In the model discussed, therefore, the tadpoles give an unimportant contribution to the K^0-K^+ mass difference, even though the sign of their contribution is correct.

[1] H. Harari, Phys. Rev. Letters **17**, 1303 (1966).
[2] T. Das, G. S. Guralnik, V. S. Mathur, F. E. Low, and J. E. Young, Phys. Rev. Letters **18**, 759 (1967); G. C. Wick and B. Zumino, Phys. Letters **25B**, 7 (1967); **25B**, 479 (1967).
[3] J. Cornwall and S. Patil, Phys. Rev. Letters **18**, 757 (1967).
[4] S. Coleman and S. Glashow, Phys. Rev. **134**, B671 (1964).
[5] S. Okubo, Phys. Rev. Letters **18**, 256 (1967).
[6] W. N. Cottingham, Ann. Phys. (N. Y.) **25**, 424 (1963).

II. SCALAR TADPOLES

Recently, a phase-shift analysis[7] for $\pi\pi$ scattering has yielded some information about $\pi\pi$ scattering, and the indications are that there is an $I=0$ s-wave resonance called the S meson around 800 MeV and having a width of about m_π. Earlier experiments[8] yielded a smaller mass of about 720 MeV and a width of about 50 MeV. Postulating that the S belongs mostly to an $SU(3)$ octet, one obtains a good candidate for the $I=1$ tadpole we used.

We shall first analyze the 2γ decay of S. The gauge-invariant matrix element for the 2γ decay of S is[9]

$$M=\left(\epsilon_1\cdot\epsilon_2-\frac{k_1\cdot\epsilon_2 k_2\cdot\epsilon_1}{k_1\cdot k_2}\right)G(t)\,, \tag{5}$$

where k_i,ϵ_i $(i=1,2)$ are the momenta and the polarization vectors of the photons. The amplitude $G(t)$ has been analyzed by Barger.[9] Using dispersion relations, he shows that if the intermediate states are saturated by the 2π state and the $2\pi\rightarrow 2\gamma$ amplitude is approximated by the gauge-invariant Born term, then

$$\Gamma(S\rightarrow 2\gamma)/\Gamma(S\rightarrow\pi^+\pi^-)\approx(\alpha/2\pi)^2\,, \tag{6}$$

where α is the fine-structure constant. Similarly, the value[9] of G at $t=0$ is

$$G(0)=-\alpha\left(\frac{2g}{\pi}\right)\int_{4\mu^2}^{\infty}dt\frac{\ln[(\frac{1}{4}t)^{1/2}+(\frac{1}{4}t-\mu^2)^{1/2}]}{t^2}\,, \tag{7}$$

where g is the $S\pi^+\pi^-$ coupling and μ is the pion mass. Numerical evaluation of (7) yields

$$G(0)\approx\alpha g/3\pi\,. \tag{8}$$

To calculate the $m_{K^0}-m_{K^+}$ due to the scalar tadpole, we use $SU(3)$ to obtain the coupling of the $I=1$ scalar meson to 2γ, assuming that the S meson belongs predominantly to an octet. Also the coupling of the $I=1$ scalar to K^+K^- is related to the coupling of S to $\pi^+\pi^-$ by $SU(3)$. One then gets

$$\delta m^2\approx\frac{1}{(2\pi)^4}\frac{i}{m_S^2}\frac{\alpha g^2}{\pi}(\tfrac{3}{2})^{1/2}\int\frac{d^4q_1}{q_1^2}F^2(q_1^2)\,, \tag{9}$$

where $F(q_1^2)$ is the photon form factor. For the form factor, we take a double-pole form, i.e., $m_\rho^4/(q_1^2-m_\rho^2)^2$ and $m_S\approx 800$ MeV, and the S-meson decay width $\approx m_\pi$, which determines g, and we get

$$\delta m\approx 0.05\text{ MeV}. \tag{10}$$

This is about two orders of magnitude too small, although the sign is correct. A single-pole form factor gives $\delta m\approx 0.15$ MeV which is still too small. This suggests that one should look for a different mechanism. With this in view, we will analyze the tensor-meson "tadpole." We first discuss the $f^0\gamma\gamma$ vertex.

III. DISPERSION RELATIONS OF THE $f^0\gamma\gamma$ VERTEX

We assume that the tensor current $J_{\mu\nu}$ which is the source of f^0 is conserved. Then the gauge-invariant matrix element for the $f^0\gamma\gamma$ vertex is of the form

$$\begin{aligned}M_{\mu\nu}=&A_1\left(g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right)\left(\epsilon_1\cdot\epsilon_2-\frac{\epsilon_1\cdot k_2\epsilon_2\cdot k_1}{k_1\cdot k_2}\right)\\&+A_2\left(\epsilon_{1\mu}\epsilon_{2\nu}-\frac{\epsilon_{2\nu}k_{1\mu}\epsilon_1\cdot k_2}{k_1\cdot k_2}-\frac{\epsilon_{1\mu}\epsilon_2\cdot k_1k_{2\nu}}{k_1\cdot k_2}\right.\\&\left.+k_{1\mu}k_{2\nu}\frac{\epsilon_1\cdot k_2\epsilon_2\cdot k_1}{(k_1\cdot k_2)^2}\right)+A_3\frac{q_\mu q_\nu}{q^2}\left(\epsilon_1\cdot\epsilon_2-\frac{\epsilon_1\cdot k_2\epsilon_2\cdot k_1}{k_1\cdot k_2}\right)\,,\end{aligned} \tag{11}$$

where k_i, ϵ_i $(i=1,2)$ are the momenta and the polarization vectors of the photons, $k=k_1+k_2$ and $q=k_1-k_2$. In these expressions, A_i are taken to be free from kinematic singularities and to satisfy the dispersion relations,

$$A_i=\frac{1}{\pi}\int\frac{\mathrm{Im}A_i}{t'-t}dt'\,. \tag{12}$$

For calculating $\mathrm{Im}A_i$, we assume that the intermediate states are saturated by the 2π states. The $f^0\pi\pi$ vertex is of the form

$$(p_1-p_2)_\mu(p_1-p_2)_\nu g\,, \tag{13}$$

where g is the $f^0\pi\pi$ coupling constant, and p_1 and p_2 are the pion momenta. For the $2\pi\rightarrow 2\gamma$ amplitude we take Born diagrams with a single π pole in the t and u channels along with the gauge term, so that

$$\begin{aligned}\mathrm{Im}M_{\mu\nu}{}^B=&\frac{\alpha g}{\pi}\int d^4p_1d^4p_2\delta(p_1^2-\mu^2)\delta(p_2^2-\mu^2)\eta(p_{10})\eta(p_{20})\\&\times\delta^4(p_1+p_2-k_1-k_2)(p_1-p_2)_\mu(p_1-p_2)_\nu\\&\times\left[\frac{\epsilon_1\cdot p_2\epsilon_2\cdot p_1}{q_1\cdot p_2}+\frac{\epsilon_2\cdot p_2\epsilon_1\cdot p_1}{q_2\cdot p_2}-\epsilon_1\cdot\epsilon_2\right]\,,\end{aligned} \tag{14}$$

where the superscript B stands for the Born approximation. Carrying out the integrations and projecting

[7] W. D. Walker *et al.*, Phys. Rev. Letters **18**, 630 (1967); E. Malamud and P. Schlein, *ibid.* **19**, 1056 (1967).

[8] V. Hagopian *et al.*, Phys. Rev. Letters **14**, 1077 (1965); M. Feldman *et al.*, *ibid.* **14**, 869 (1965).

[9] V. Barger, Nuovo Cimento **32**, 127 (1964).

out the amplitudes A_i, we get

$$\begin{aligned}
\mathrm{Im}A_1{}^B &= \alpha g\frac{|\mathbf{p}|^2}{2}\left[-\frac{1}{12a}+\frac{a}{4}\right.\\
&\qquad\left.+\left(-\frac{1}{8a^2}+\frac{1}{4}-\frac{a^2}{8}\right)\ln\left(\frac{1+a}{1-a}\right)\right],\\
\mathrm{Im}A_2{}^B &= -\alpha g\frac{|\mathbf{p}|^2}{a^2}\left[\frac{5}{12}a-\frac{a^3}{4}\right.\\
&\qquad\left.+\frac{(1-2a^2+a^4)}{8}\ln\left(\frac{1+a}{1-a}\right)\right],\\
\mathrm{Im}A_3{}^B &= -\alpha g\frac{|\mathbf{p}|^2}{2a^2}\left[\frac{5}{4}a^3-\frac{13}{12}a\right.\\
&\qquad\left.+\left(\frac{5}{8}a^4+\frac{3}{4}a^2-\frac{1}{8}\right)\ln\left(\frac{1+a}{1-a}\right)\right],
\end{aligned}\tag{15}$$

where $a^2=k^2/|\mathbf{p}|^2$, $|\mathbf{p}|^2=t-4\mu^2$, and $t=k^2$. However, with these Born expressions, the dispersion integrals in (12) are severely divergent. One can introduce form factors to make the integrals convergent, and with this intention we shall discuss the concept of "partially traceless" tensor currents.

IV. PARTIALLY TRACELESS TENSOR CURRENTS

In the theory of Regge poles, tensor particles and scalar particles may be manifestations of the same Regge trajectory, and hence related to each other. In field theories also, relations may exist between tensor and scalar particles. Following the spirit of partially conserved axial-vector currents, we postulate that the trace of a tensor current, which is the source of tensor particles, is proportional to the scalar-meson field. The consequence of this is that the form factor for the 2^+ meson is dominated by the scalar-meson pole. Specifically

$$g^{\mu\nu}J_{\mu\nu}=c\phi_S\,,\tag{16}$$

where ϕ_S is the scalar-meson field. The expectation value of (16) between 2π state and the vacuum gives

$$g^{\mu\nu}g(t)(p_1-p_2)_\mu(p_1-p_2)_\nu=\frac{cg_{S\pi\pi}}{t-m_S{}^2},\tag{17}$$

where $g(t)$ is the coupling of f^0 to $\pi^+\pi^-$, $g_{S\pi\pi}$ is the coupling of the scalar meson, and m_S is its mass. Taking the trace gives

$$g(t)(4\mu^2-t)=\frac{cg_{S\pi\pi}}{t-m_S{}^2}.\tag{18}$$

This relation, however, is expected to be strictly true only at $t\approx m_S{}^2$. Away from that value one should add a subtraction constant d, i.e.,

$$g(t)(4\mu^2-t)=\frac{cg_{S\pi\pi}}{t-m_S{}^2}+d\,.\tag{19}$$

The constant d is determined by noticing that $t=4\mu^2$ is not a singular point and therefore the factor $4\mu^2-t$ on the left should be compensated by a similar factor on the right. We then get

$$g(t)=\frac{cg_{S\pi\pi}}{(4\mu^2-m_S{}^2)(t-m_S{}^2)}.\tag{20}$$

Thus the form factor for the f^0 is given by

$$f(t)=\frac{m_f{}^2-m_S{}^2}{t-m_S{}^2}.\tag{21}$$

Because the S meson is unstable, the formula (21) should be slightly modified to read

$$f(t)=(m_f{}^2-m_S{}^2)/(t-m_S{}^2+i\Gamma_S)\,,\tag{22}$$

where Γ_S is m_S multiplied by the full width of the S meson.

We are now in a position to carry out the dispersion integrals using (15). The effect of the form factor on expressions of type (15) has already been analyzed by Barger,[9] and the result is that while g is multiplied by $f^*(t)$, the projection of the Born term for $2\pi\to 2\gamma$ should be multiplied by $f(t)/f(0)$. The intuitive meaning of this is that the projection of the Born term is very large near $t\approx 0$ and therefore dominates the amplitude in that region. The multiplication by $f(t)/f(0)$ then allows one to continue from 0 to t. Combining the two effects, the consequence of introducing the form factor is to multiply the expressions in (15) by $|f(t)|^2/f(0)$.

V. CALCULATION OF $\Gamma(f^0\to 2\gamma)$ AND $m_{K^0}-m_{K^+}$

The introduction of the form factors makes the dispersion integrals fairly easy. In the approximation of small Γ_S, we have

$$|f(t)|^2\approx(\pi/\Gamma_S)(m_f{}^2-m_S{}^2)^2\delta(t-m_S{}^2)\,.\tag{23}$$

Therefore,

$$A_i(t)\approx\frac{(m_f{}^2-m_S{}^2)^2}{\Gamma_S f(0)}\frac{\mathrm{Im}A_i{}^B(m_S{}^2)}{m_S{}^2-t},\tag{24}$$

where $\mathrm{Im}A_i{}^B$ are the Born terms given in (15). Taking $m_S\approx 800$ MeV and its full width $\approx m_\pi$, we get

$$\begin{aligned}
&A_1(m_f{}^2)\approx -A_3(m_f{}^2)\approx 15\alpha g\,,\\
&A_2(m_f{}^2)\approx -2A_1(m_f{}^2)\,,
\end{aligned}\tag{25}$$

where α is the fine-structure constant. Similarly, at $t=0$,

$$A_1(0)\approx -A_3(0)\approx -20\alpha g\,,\qquad A_2(0)\approx -2A_1(0)\,. \tag{26}$$

Using the numbers in (25), we find that

$$\Gamma(f^0\to 2\gamma)/\Gamma(f^0\to \pi^+\pi^-)\approx 2.5\times 10^{-4}. \tag{27}$$

This is a large enhancement over the expected value of $(\alpha/2\pi)^2$ if we did not have the scalar meson.

The analysis for $f^0\to\rho\gamma$ is more complicated, but it is likely that this decay is also enhanced by the scalar-meson pole. A simple ρ-dominance model for γ coupling suggests that the $f^0\to\rho\gamma$ decay could be as large as

$$\Gamma(f^0\to\rho\gamma)/\Gamma(f^0\to\pi^+\pi^-)\sim 0.05\,.$$

For the K^0-K^+ mass difference, we should consider the A_2-meson tadpole. For the $A_2\gamma\gamma$ coupling, we use $SU(3)$ and the expressions (26) for $f^0\gamma\gamma$ couplings. The coupling g itself is calculated from the $f^0\to\pi\pi$ decay width, which is about 100 MeV. This gives $g\approx(\frac{1}{2})^{1/2}$. Taking octet singlet mixing into account, and $SU(3)$ symmetry, the values of the coupling constants for $A_2\gamma\gamma$ are the same as those in (25) with $g\approx(\frac{1}{2})^{1/2}$. Also the coupling for $A_2K^+K^-$ as expressed in the form (13) is about $\frac{1}{4}$. However, for continuing this from $t=m_f^2$ to $t=0$, one should multiply it by $f(0)$. Then the K^0-K^+ mass difference comes out as

$$\delta m^2=\frac{1}{2(2\pi)^4}\frac{i}{m_f^2}\left(\frac{m_f^2-m_S^2}{m_S^2}\right)\times\int\frac{d^4q_1}{q_1^2}[F(q_1^2)]^2\left[\frac{2A_2(0)(q_1\cdot Q)(q_2\cdot Q)}{q_1\cdot q_2}+2A_3(0)\left(Q^2-\frac{3q\cdot Qq\cdot Q}{q^2}\right)\right], \tag{28}$$

where Q is the sum of the momenta of the incoming and outgoing K mesons, and $F(q_1^2)$ is the photon form factor. We can take the form factor to be dominated by the ρ-meson pole, i.e.,

$$F(q_1^2)=-m_\rho^2/(q_1^2-m_\rho^2)\,.$$

Then the mass difference $m_{K^0}-m_{K^+}$ comes out to be[10]

$$\delta m\approx 0.3\text{ MeV}.$$

However, if we take the quadratic form factor, as seems to be the experimental situation, i.e.,

$$F(q_1^2)=m_\rho^4/(q_1^2-m_\rho^2)^2\,,$$

then

$$m_{K^0}-m_{K^+}\approx 0.1\text{ MeV}.$$

This is once again considerably smaller than the experimental value of about 4 MeV. Thus tensor tadpoles also give negligible contribution ot K^0-K^+ mass difference.

VI. CONCLUSIONS

The discussion indicates that the simple-minded scalar-meson tadpole, when one uses the information from a recent analysis of the $\pi\pi$ phase shift,[7] gives the correct sign but too small a magnitude for $m_{K^0}-m_{K^+}$. As another possibility, we considered the tadpole due to a 2^+ meson which develops a spin-0 part when it is off-shell. A gauge-invariant calculation is made, using the concept of partially traceless tensor currents, which introduces form factors and hence allows a cutoff-free calculation. The mass difference is once again of the correct sign but too small in magnitude. This leads us to the conclusion that tadpoles, at least in the model discussed, do not contribute significantly to the K'-K^+ mass difference.

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[10] We have assumed that the kinematics is unchanged when the photon is off the mass shell, which is not strictly correct. However, with $i\epsilon$ added to the denominators, the integrals are well defined and finite. Alternatively, we could have written the dispersion relations (12) for $A_2/q_1\cdot q_2$ and A_3/q^2 instead of A_2 and A_3, in which case the integrals (28) are well defined. This does not alter any of the qualitative results.