Consequences of a $QQ\bar{Q}$ Model for Meson-Baryon Processes

S. Das Gupta, V. K. Gupta,* and A. N. Mitra
Department of Physics and Astrophysics, University of Delhi, Delhi, India
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The consequences of a $QQ\bar{Q}$ model for meson-baryon processes, proposed recently by one of the authors (ANM), are examined in relation to vector-meson production in association with both the octet and the decuplet of baryons. As shown in that paper, the basic quark-meson amplitudes which represent the processes $PQ\rightarrow PQ$ and $PQ\rightarrow VQ$ (where P and V denote pseudoscalar and vector mesons, respectively) can be classified in terms of two sets $A^{(\pm)}$, where $A^{(+)}$ represents the transition amplitude between positive-parity $QQ\bar{Q}$ states, and $A^{(-)}$ between negative-parity $QQ\bar{Q}$ states. The analysis of the data, which is made most conveniently in terms of density matrices for spin-1 and spin-$\frac{3}{2}$ final-state objects, shows that several of the experimental density matrices can be fitted in terms of either of the sets $A^{(+)}$ and $A^{(-)}$, though a dominance of $A^{(-)}$ seems to be favored by experiment. However, density matrices for certain processes like $PB\rightarrow VB$ are found to require both $A^{(+)}$ and $A^{(-)}$ for a proper fit to the experimental data. A sum rule of the form $\frac{4}{3}[\rho_{3,3}{}^D+\sqrt{3}\rho_{3,-1}{}^D]=\rho_{1,1}{}^V+\rho_{1,-1}{}^V$, which is derived for the density matrices using both the amplitudes $A^{(\pm)}$, is found to be identical with one obtained by other authors using the additivity assumption, thus extending the range of validity of this result beyond pure additivity. Using only $A^{(-)}$ amplitudes, certain results on the angular distribution of the density matrices for $PB\rightarrow VB^*$ processes, especially their zero-angle behavior, are found to agree rather well with experiment, the agreement being somewhat better than for $SU(6)_W$. From the experimental point of view, the $QQ\bar{Q}$ model also seems to work somewhat better than the additivity principle in respect of density matrices like $\rho_{1,0}$, inasmuch as this model predicts them to be nonzero in nonforward directions, while the additivity assumption makes them identically zero for all angles. The main conclusions of this investigation are as follows: (1) Experiment is consistent with the dynamical assumption of dominance of the $Q\bar{Q}$ force over the QQ force, which implies a higher priority for multiple-scattering effects within the $QQ\bar{Q}$ system than for the scattering of the meson by the quark constituents of the baryon. (2) Experiment is also consistent with the $SU(3)$ and spin independence of the quark forces. (3) While the precise mechanism of the quark forces cannot be studied in this model, it nevertheless suggests a general classification of the meson amplitudes in terms of two distinct types $A^{(+)}$ and $A^{(-)}$ of which the latter is favored most by experiment.

1. INTRODUCTION

While the additivity principle in the independent quark model[1,2] received a fairly extensive degree of confirmation from experiment in the earlier stages, more recent analyses have led to significant departures from this simple assumption. One of the important areas of disagreement with the predictions of this model lies in the baryon (B) and antibaryon ($\bar{B}$) annihilation contributions to the high-energy total cross sections.[3] Another source of violation of this model is in the possibility of multiple-scattering effects[4] between the various quark constituents. Since these effects are of a dynamical origin, it may be of interest to keep on record the predictions of certain other types of dynamical assumptions.

In this connection, a dynamical model of meson-baryon processes, based on a comparative evaluation of the relative tightness of the various quark constituents of a meson-baryon system, was recently developed by one of us,[5] as an extension of the "elementary meson model" for similar processes.[6] This $QQ\bar{Q}$ model, which takes account of the $Q\bar{Q}$ structures of the mesons within a meson-quark system, is designed to give a unified treatment of both the processes $PQ\rightarrow PQ$ and $PQ\rightarrow VQ$, where P and V denote pseudoscalar and vector mesons, respectively. The model, which does not regard the P mesons as elementary, nevertheless assumes the mesons as $Q\bar{Q}$ composites to be more tightly bound structures than baryons as $3Q$ composites. Dynamically, such a feature comes about in a natural way through the assumption of mainly two-body potentials (V) among quarks, such that $V_{Q\bar{Q}}\gg V_{QQ}$. Within the $QQ\bar{Q}$ model of a meson-quark system, the quark can "see through" the mesonic structure and hence distinguish between the $^1S_0$ and $^3S_1$ states characterizing the P and V mesons, respectively. On the other hand, in a meson-baryon process, the tighter structure of the meson enables it to "see through" the looser structure of the baryons, without the former

* Present address: Centre for Advanced Study in Physics, Department of Physics, University of Delhi, Delhi, India.

[1] E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu 2, 105 (1965) [English transl.: Soviet Phys.—JETP Letters 2, 65 (1965)].

[2] H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966).

[3] J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento 42, 711 (1966); Nucl. Phys. B1, 169 (1967).

[4] V. Franco, Phys. Rev. Letters 18, 1159 (1967); D. R. Harrington and A. Pagnamenta, *ibid.* 18, 1147 (1967).

[5] A. N. Mitra, Phys. Rev. 167, 1382 (1968). This paper is referred to as QMB in what follows.

[6] The "elementary meson" model has a rather long history. In connection with strong decays of hadrons, it was proposed by C. Becchi and G. Morpurgo [Phys. Rev. 149, 1284 (1966)] and also by A. N. Mitra and M. H. Ross [*ibid.* 158, 1630 (1967)]. For meson-baryon processes, the same model was suggested by G. C. Joshi, V. S. Bhasin, and A. N. Mitra [*ibid.* 156, 1572 (1967)] and independently by J. L. Friar and J. S. Trefil [CERN Report No. TH. 723, 1966 (unpublished)]. For photoproduction processes, a corresponding idea was proposed by S. Das Gupta and A. N. Mitra [Phys. Rev. 156, 1581 (1967)]. Somewhat similar ideas were proposed by H. Satz [Phys. Letters 25B, 27 (1967)], but this author also made use of the $Q\bar{Q}$ structures of the emitted mesons through suitable rearrangements of the quark constituents of the final hadron system.

having to "expose" itself before the latter. Thus the $QQ\bar{Q}$ model which, in short, is based on a "hierarchy of elementarity" (in the descending order of quarks, mesons, and baryons), can predict the amplitudes for meson-baryon processes by evaluating the matrix elements of the amplitudes $PQ \to PQ$ and $PQ \to VQ$, regarded as operators between appropriate initial and final baryonic states. The amplitude operators $PQ \to PQ$ and $PQ \to VQ$ were shown in QMB to depend on a certain number (12) of parameters in the most general case. The additional assumptions of spin and $SU(3)$-spin independence of the $Q\bar{Q}$ force, as well as its operation mainly in s waves, led to certain relations and inequalities among these parameters so as to reduce their effective number considerably.

The purpose of this paper is to make a fairly detailed comparison of the $QQ\bar{Q}$ model, as developed in QMB, with experiment, with a view to illuminate some qualitative dynamical features of this model. In this connection the density matrix clearly provides a more meaningful basis of comparison, as was emphasized (and carried out) by various authors.[7–9] For production of two high-spin ($S \geqslant 1$) particles, even some limited double-density matrices, involving the simultaneous spin correlation of both the emitted particles have been evaluated[10,11] and compared with experimental data. The main advantage of the density-matrix relations, as has been pointed out by Trefil *et al.*,[7,9] is that they involve amplitudes that must all be evaluated at a common s and t. A second advantage, pointed out by the same authors, is that in the density-matrix elements, effects like multiple scattering, annihilation, etc., should tend to cancel out. This last point is no less relevant to our $QQ\bar{Q}$ model which, though more restrictive than the pure additivity assumption, includes only one stage of the multiple-scattering effects (viz., the ones within the $QQ\bar{Q}$ system), while leaving out those of the meson by the quarks in the baryon.

As has been the more recent practice in quark-model analyses of high-energy processes, we shall try to present *not* the amplitude sum rules for various processes, but (1) differential cross-section sum rules, (2) sum rules for single-density matrices, and (3) the actual values for density matrices for cases of particular experimental interest. The processes to be analyzed will include (i) production of vector mesons in association with both the octet and decuplet of baryons, and (ii) production of decuplet and negative-parity singlet baryons in association with pseudoscalar mesons, for all of which reasonable experimental data are available. As was already pointed out in QMB, the $QQ\bar{Q}$ model, unlike the pure additivity assumption, is not able to correlate processes like $BB \to BB$ or $B\bar{B} \to B\bar{B}$ within the framework relevant to meson-baryon processes. While this fact necessarily restricts the possibility of obtaining the rich variety of sum rules characteristic of the additivity principle, we shall show that there are still enough of them (even within the meson-baryon framework) to facilitate a meaningful comparison with experiment.

In Sec. 2, we summarize the essential results of the $QQ\bar{Q}$ model as found in QMB, and in the same notation. The density matrices for vector mesons as well as the baryon decuplet are defined and their reality properties are discussed in relation to the pure additivity model. Section 3 lists the relations between the differential cross sections for various scattering and production processes within the **56** of baryons, and their (limited) comparison with experiment. For this last purpose, it is found more useful to consider these sum rules in the form of relations between total cross sections. In Sec. 4 are discussed the density matrices (i) for the processes $PB \to PB^*$, which are compared with the predictions of the additivity model, and (ii) for the process $\pi^- p \to K^0 Y_0{}^*{}_{3/2^-}$ (1520), which are compared with the results of the Stodolsky-Sakurai model. Sections 5 and 6 give fairly detailed analyses of the processes $PB \to VB$ and $PB \to VB^*$, respectively, with emphasis on density-matrix sum rules, their angular distribution and zero-angle behavior in relation to experiment. Section 7 gives a summary of essential results together with a comparison with related approaches.

2. ESSENTIAL RESULTS OF THE $QQ\bar{Q}$ MODEL

We summarize here, for convenience, the essential results of the $QQ\bar{Q}$ model obtained in QMB, for the processes $PQ \to PQ$ and $PQ \to VQ$. In the notation of QMB, the $SU(3)$ elastic terms of the meson-quark amplitude (including both scattering of P mesons and production of V mesons) are given by the expression

$$\Pi_\beta{}^\dagger \Pi_\alpha [\tilde{A}(\tfrac{2}{3}\delta_{\alpha\beta} + u_{\beta\alpha}{}^{(+)}) + \tfrac{1}{2}\tilde{B} u_{\beta\alpha}{}^{(-)}], \qquad (2.1)$$

where

$$u_{\beta\alpha}{}^{(\pm)} = (\pm i f_{\beta\alpha\gamma} + d_{\beta\alpha\gamma}) \lambda_\gamma{}^{(1,2)}, \qquad (2.2)$$

the superscripts (1,2) on λ_γ being associated with the ($\pm$) signs, respectively. The symbols $\Pi_\beta{}^\dagger$, Π_α ($\alpha, \beta = 1, \cdots, 8$) represent the creation and annihilation operators of an $SU(3)$ octet of mesons, and $\tilde{A}$ and $\tilde{B}$ are the following combinations [cf. Eqs. (4.26)–(4.30) of QMB] of the parameters for the processes $PQ \to PQ$ and $PQ \to VQ$:

$$\begin{aligned}\tilde{A} = {} & \tfrac{1}{2}A^{(+)}(3+\sqrt{3}V_0) + \tfrac{1}{2}\bar{A}^{(+)}(1-\sqrt{3}V_0) + \sqrt{3}a^{(+)}V_3 \\ & + \tfrac{1}{2}A^{(-)}(3P+\sqrt{3}V_1) + \tfrac{1}{2}\bar{A}^{(-)}(P-\sqrt{3}V_1) \\ & + \sqrt{3}a^{(-)}V_2, \end{aligned} \qquad (2.3)$$

$$\begin{aligned}\tilde{B} = {} & \tfrac{1}{2}B^{(+)}(3+\sqrt{3}V_0) - \tfrac{1}{2}\bar{B}^{(+)}(1-\sqrt{3}V_0) + \sqrt{3}b^{(+)}V_3 \\ & + \tfrac{1}{2}B^{(-)}(3P+\sqrt{3}V_1) - \tfrac{1}{2}\bar{B}^{(-)}(P-\sqrt{3}V_1) \\ & + \sqrt{3}b^{(-)}V_2, \end{aligned} \qquad (2.4)$$

[7] J. L. Friar and J. S. Trefil, Nuovo Cimento **49A**, 642 (1967).
[8] M. Jacob and C. Itzykson, Nuovo Cimento **48A**, 909 (1967).
[9] K. Kanantie and J. S. Trefil, Nucl. Phys. **B2**, 243 (1967).
[10] A. Bialas, A. Gula, and B. Muryan, Phys. Letters **24B**, 428 (1967).
[11] A. Bialas and K. Zalewski, Phys. Letters **26B**, 170 (1968).

$$A^{(\pm)}=2D^{(\pm)}+F^{(\pm)},\quad \bar{A}^{(\pm)}=\bar{D}^{(\pm)}+2\bar{F}^{(\pm)}, \tag{2.5}$$

$$B^{(\pm)}=D^{(\pm)}-F^{(\pm)},\quad \bar{B}^{(\pm)}=\bar{D}^{(\pm)}-\bar{F}^{(\pm)}, \tag{2.6}$$

$$a^{(\pm)}=2d^{(\pm)}+f^{(\pm)},\quad b^{(\pm)}=d^{(\pm)}-f^{(\pm)}, \tag{2.7}$$

$$P=k_\mu k_\nu'(\delta_{\mu\nu}+i\epsilon_{\mu\nu\lambda}\sigma_\lambda^{(2)}), \tag{2.8}$$

$$V_1=\hat{V}_\lambda k_\mu k_\nu'(\sigma_\mu^{(2)}\delta_{\nu\lambda}+\sigma_\nu^{(2)}\delta_{\mu\lambda}-i\epsilon_{\mu\nu\lambda}-\delta_{\mu\nu}\sigma_\lambda^{(2)}), \tag{2.9}$$

$$V_2=\hat{V}_\lambda k_\mu k_\nu'(\sigma_\mu^{(2)}\delta_{\nu\lambda}-\tfrac{1}{2}\sigma_\nu^{(2)}\delta_{\mu\lambda}+\tfrac{1}{2}i\epsilon_{\mu\nu\lambda}+\tfrac{1}{2}\delta_{\mu\nu}\sigma_\lambda^{(2)}), \tag{2.10}$$

$$V_3=\hat{V}_\lambda k_\mu' k_\nu'(\sigma_\mu^{(2)}\delta_{\nu\lambda}+\sigma_\nu^{(2)}\delta_{\mu\lambda}-\tfrac{2}{3}\sigma_\lambda^{(2)}\delta_{\mu\nu}), \tag{2.11}$$

$$V_0=\boldsymbol{\sigma}^{(2)}\cdot\mathbf{V}. \tag{2.12}$$

Here $\mathbf{k}$, $\mathbf{k}'$ are the respective momenta of the initial and final mesons in the c.m. frame of the meson-quark system. The amplitudes with superscripts $(\pm)$ represent transitions between two $(+)$ parity $QQ\bar{Q}$ states and two $(-)$ parity $QQ\bar{Q}$ states, respectively. Similarly the amplitude for the production of an $SU(3)$-singlet meson (Π_0) by an $SU(3)$-octet on a quark is[5]

$$(\sqrt{\tfrac{2}{3}})\Pi_0^\dagger\Pi_\alpha\lambda_\alpha^{(2)}\tilde{B},$$

where $\tilde{B}$ is the same expression as (2.4).

Under certain simplifying dynamical assumptions explained in QMB, viz., (i) spin and unitary-spin independence of the $Q\bar{Q}$ force, (ii) neglect of the QQ force compared with the $Q\bar{Q}$ force, and (iii) operation of the $Q\bar{Q}$ force mainly in the s wave, the following approximate relations and inequalities were noted in QMB:

$$D^{(\pm)}\approx\bar{D}^{(\pm)},\quad F^{(\pm)}\approx\bar{F}^{(\pm)}, \tag{2.13}$$

$$D^{(+)},\bar{D}^{(+)},d^{(+)}\gg F^{(+)},\bar{F}^{(+)},f^{(+)}, \tag{2.14}$$

$$D^{(-)},\bar{D}^{(-)},d^{(-)}\ll F^{(-)},\bar{F}^{(-)},f^{(-)}. \tag{2.15}$$

Equation (2.13) is merely the result of assumptions (i) and (ii), and is probably on a stronger footing than the inequalities (2.14) and (2.15), which depend on the additional assumption (iii).

As can be seen from Eqs. (2.3) and (2.4), the quantities that are directly expected to appear in the density-matrix formalism are

$$A_1^{(\pm)}=A^{(\pm)}-\bar{A}^{(\pm)},\quad A_2^{(\pm)}=3A^{(\pm)}+\bar{A}^{(\pm)}, \tag{2.16}$$

$$B_1^{(\pm)}=B^{(\pm)}+\bar{B}^{(\pm)},\quad B_2^{(\pm)}=3B^{(\pm)}-\bar{B}^{(\pm)}. \tag{2.17}$$

Now if use is made only of Eq. (2.13), the eight parameters $A_{1,2}^{(\pm)}$ and $B_{1,2}^{(\pm)}$ become effectively reduced to *four*, since one then obtains

$$B_2^{(\pm)}\approx B_1^{(\pm)}\approx 2A_1^{(\pm)}, \tag{2.18}$$

so that the independent parameters can be taken as $A_1^{(\pm)}$ and $A_2^{(\pm)}$, together of course with the other four quantities $a^{(\pm)}$ and $b^{(\pm)}$. If, in addition, one uses the inequalities (2.14) and (2.15), the total number of the independent parameters gets reduced from eight to four, since in that case

$$7A_1^{(+)}\approx A_2^{(+)},\quad 5A_1^{(-)}\approx -A_2^{(-)}, \tag{2.19}$$

$$a^{(+)}\approx 2b^{(+)},\quad a^{(-)}\approx -b^{(-)}. \tag{2.20}$$

This classification of the successive reduction of parameters should facilitate a step-wise comparison with experiment, to judge the validity of the three main dynamical assumptions of the $QQ\bar{Q}$ model.

The density-matrix elements for one of the particles (D) produced in a certain reaction,

$$A+B\to C+D,$$

can be written down according to the general definition

$$\rho_{m,m'}(D)=N\sum_{abc}\langle D_mC_c|A_aB_b\rangle\langle D_{m'}C_c|A_aB_b\rangle^*, \tag{2.21}$$

where a, b, c, and d $(=m,m')$ are the magnetic quantum numbers associated with the respective particles A, B, C, and D, and N is a normalization constant to ensure

$$\mathrm{Tr}\rho=1. \tag{2.22}$$

The matrix ρ of course satisfies the usual conditions

$$\rho=\rho^\dagger,\quad \rho_{m,m'}=(-1)^{m-m'}\rho_{-m,-m'}. \tag{2.23}$$

Correspondingly, the double-density matrix for both the final-state particles may be defined as

$$\rho_{m,m'}{}^{n,n'}(C,D)=N\sum_{ab}\langle D_mC_n|A_aB_b\rangle\times\langle D_{m'}C_{n'}|A_aB_b\rangle^*, \tag{2.24}$$

where the magnetic quantum numbers (m,m') and (n,n') are associated with D and C, respectively.

In general, some of the off-diagonal elements of ρ, such as $\rho_{1,0}$ are complex, though the condition (2.23) ensures reality of off-diagonal elements of the form $\rho_{m,-m}$ for integral m. Now, in the pure additivity model[7] all the elements of ρ are real. Indeed, elements of the form $\rho_{1,0}$ which should be proportional to $\sin\theta$ are exactly zero in the additivity model, and this feature led the authors of Ref. 7 to compare their predictions with experiment only near $\theta=0$. In the present $QQ\bar{Q}$ model, on the other hand, the elements of $\rho_{m,m'}$ [as can be seen from direct substitution of the amplitudes in Eq. (2.21)] are complex in general. However, if we consider only the amplitudes $(A_{1,2}^{(+)}, B_{1,2}^{(+)}, a^{(+)}$, and $b^{(+)})$ corresponding to the overlap of positive-parity wave functions in the initial *and* final states, and likewise only the amplitudes $(A_{1,2}^{(-)}, B_{1,2}^{(-)}, a^{(-)}$, and $b^{(-)})$, then all the elements of ρ turn out to be *real*. Even so, the elements of the form $\rho_{1,0}$ are *no longer zero*, unlike the prediction of pure additivity. This important feature of the $QQ\bar{Q}$ model, which allows the full effect of nonadditive corrections within the $QQ\bar{Q}$ system, should facilitate a comparison of such elements with experiment, even for finite values of θ.

3. SUM RULES FOR DIFFERENTIAL CROSS SECTIONS

In this section, we discuss relations between various differential meson-baryon cross sections obtainable in the $QQ\bar{Q}$ model from the corresponding relations between the amplitudes. For example, for processes involving a P or a V meson in association with the octet of baryons, there are only six independent combinations of the $QQ\bar{Q}$ parameters detailed in Sec. 2. Thus, for the processes $PB\to PB$, some of the sum rules are[12]

$$\frac{d\sigma}{d\Omega}(\pi^-p\to\pi^0n)+12\frac{d\sigma}{d\Omega}(\pi^-p\to\eta n)+8\frac{d\sigma}{d\Omega}(K^-p\to\pi^-\Sigma^+)$$
$$=12\frac{d\sigma}{d\Omega}(K^+p\to K^+p)+4\frac{d\sigma}{d\Omega}(K^-p\to\bar{K}^0n)\,,\quad(3.1)$$

$$3\frac{d\sigma}{d\Omega}(K^+p\to K^+p)=3\frac{d\sigma}{d\Omega}(\pi^-p\to X^0n)+2\frac{d\sigma}{d\Omega}(K^-p\to\pi^-\Sigma^+)\,,\quad(3.2)$$

$$2\frac{d\sigma}{d\Omega}(\pi^-p\to K^0\Lambda)=2\frac{d\sigma}{d\Omega}(\pi^+p\to K^+\Sigma^+)+\frac{d\sigma}{d\Omega}(K^-p\to\bar{K}^0n)\,,\quad(3.3)$$

$$9\frac{d\sigma}{d\Omega}(\pi^+p\to K^+\Sigma^+)+13\frac{d\sigma}{d\Omega}(K^-p\to\pi^-\Sigma^+)+11\frac{d\sigma}{d\Omega}(K^+p\to K^+p)+10\frac{d\sigma}{d\Omega}(\pi^-p\to\pi^-p)$$
$$=16\frac{d\sigma}{d\Omega}(\pi^+p\to\pi^+p)+8\frac{d\sigma}{d\Omega}(K^-p\to K^-p)+\frac{d\sigma}{d\Omega}(K^-p\to\bar{K}^0n)\,,\quad(3.4)$$

$$12\frac{d\sigma}{d\Omega}(\pi^+p\to\pi^+p)+33\frac{d\sigma}{d\Omega}(\pi^+p\to K^+\Sigma^+)+2\frac{d\sigma}{d\Omega}(\pi^-p\to K^0\Lambda)+24\frac{d\sigma}{d\Omega}(K^-p\to\eta\Lambda)$$
$$=18\frac{d\sigma}{d\Omega}(\pi^-p\to\pi^-p)+12\frac{d\sigma}{d\Omega}(K^-p\to\pi^0\Lambda)+9\frac{d\sigma}{d\Omega}(K^-p\to\pi^-\Sigma^+)\,,\quad(3.5)$$

$$4\frac{d\sigma}{d\Omega}(K^-p\to\pi^0\Lambda)=3\frac{d\sigma}{d\Omega}(K^+p\to K^+p)\,,\quad(3.6)$$

$$3\frac{d\sigma}{d\Omega}(K^-p\to X^0\Lambda)=2\frac{d\sigma}{d\Omega}(K^-p\to\pi^0\Lambda)\,.\quad(3.7)$$

Equation (3.3) is identical with Eq. (10) of Lipkin, Scheck, and Stern[13] (abbreviated hereafter as LSS) in terms of the total cross sections for the corresponding relations. It is, however, difficult to compare all the relations listed above with those given in LSS, because the pure additivity model predicts amplitudes not only for meson-baryon (MB) processes, but BB and $B\bar{B}$ processes as well, in terms of a *common* set of parameters representing the QQ or $Q\bar{Q}$ scattering amplitudes. The present relations are more specialized, in that they involve only those MB processes that are deducible within the framework of the $QQ\bar{Q}$ model without any reference to BB or $B\bar{B}$ processes.

In the forward direction, we have the additional relations

$$\frac{d\sigma}{d\Omega}(\pi^-p\to\pi^-p)=4\frac{d\sigma}{d\Omega}(K^-p\to\bar{K}^0n)\,,\quad(3.8)$$

$$\frac{d\sigma}{d\Omega}(K^+p\to K^+p)=4\frac{d\sigma}{d\Omega}(K^-p\to\pi^0\Sigma^0)\,,\quad(3.9)$$

$$\frac{d\sigma}{d\Omega}(K^-p\to\bar{K}^0n)=2\frac{d\sigma}{d\Omega}(\pi^-p\to K^0\Sigma^0)\,,\quad(3.10)$$

which simplify Eqs. (3.3) and (3.6), respectively, to

$$2\frac{d\sigma}{d\Omega}(\pi^-p\to K^0\Lambda)=3\frac{d\sigma}{d\Omega}(K^-p\to\bar{K}^0n)\,,\quad(3.11)$$

$$\frac{d\sigma}{d\Omega}(K^-p\to\pi^0\Lambda)=3\frac{d\sigma}{d\Omega}(K^-p\to\pi^0\Sigma^0)\,.\quad(3.12)$$

Equations (3.10)–(3.12) are together equivalent to Eqs. (9a)–(9c) of LSS.

Of so many relations, we can at present discuss only a few which have a bearing on current experimental knowledge. According to the prescription of Meshkov, Snow, and Yodh,[14] the sum rules for inelastic processes should be compared at the same "Q value" for each process to take approximate account of nondegeneracy of masses. Thus for example, Eq. (3.8) can be tested in terms of the data of Allitti *et al.*[15] and Dauber *et al.*,[16] which roughly satisfy the above requirement. The

[12] In all the relations appearing in this section, the differential cross section $d\sigma/d\Omega$ or the total cross section σ are corrected for the mass defect, so that $\sigma=(k/k')\sigma_{\rm expt}$, where $\sigma_{\rm expt}$ is the experimental cross section, k and k' are, respectively, the c.m. momenta of the initial and final mesons.

[13] H. J. Lipkin, F. Scheck, and H. Stern, Phys. Rev. **152**, 1375 (1966), referred to as LSS.

[14] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters **12**, 87 (1964).

[15] Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento **29**, 515 (1963).

[16] P. M. Dauber *et al.*, in *Proceedings of the Second Topical Conference on Resonant Particles, Athens, Ohio, 1965*, edited by B. A. Munin (Ohio University, Athens, Ohio, 1965), p. 380.

TABLE I. Comparison with experiment of the total cross sections for Eqs. (3.5) and (3.6) for processes of the type $PB \to PB$ and Eq. (3.13) for the processes $PB \to VB$, in terms of their left-hand and right-hand sides.

Equation number	Total cross section[a] (mb) Left-hand side	Right-hand side	Approximate Range of Q values (BeV)
(3.5)	108 ±7	141 ±7	1.3
(3.6)	8.4±0.8	29.1±1.5	0.6
(3.13)	26 ±3	28 ±5	0.8

[a] The data for these processes have been taken from Ref. 26 and (i) Saclay-Orsay-Bari-Bologna Collaboration, Nuovo Cimento **37**, 361 (1965); (ii) T. P. Wangler, A. R. Erwin, and W. D. Walker, Phys. Rev. **137**, B414 (1965); (iii) Birmingham-Glasgow-London (I.C.)-Oxford-Rutherford Collaboration, Phys. Rev. **152**, 1148 (1966); (iv) A. Bettini *et al.*, Phys. Letters **16**, 83 (1964); (v) William Grazinao and Stanley G. Wojcicki, Phys. Rev. **128**, 1868 (1962); (vi) R. Barloutaud *et al.*, Phys. Letters **12**, 352 (1964); (vii) D. H. Miller *et al.*, Phys. Rev. **140**, B360 (1965); (viii) S. S. Yamamoto *et al.*, Phys. Rev. **134**, B383 (1964).

figures[17] for the left-hand side and the right-hand side of (3.8) are, respectively, (2.87±0.15) mb/sr and (2.16±0.32) mb/sr, which are in rather good agreement within experimental errors.

As for many other relations, while the data are not comprehensive enough for comparison in a differential form, they are nevertheless sufficient for a discussion of the corresponding *total* cross sections.[18] For example, Eqs. (3.5) and (3.6) represent processes for which the data are available at nearly the same Q values. Table I gives a collection of the necessary experimental data for the $PB \to PB$ processes involved. The data indicate fair agreement for Eq. (3.5), but a striking disagreement for Eq. (3.6), the two sides of which differ by more than a factor of three. However, it has been pointed out in LSS that a comparison with experiment at low Q values[19] may not be very meaningful. We believe, on the other hand, that the very prediction of relations like (3.6), which appear even more difficult to satisfy at higher energies, should probably be recognized as an unfortunate feature of the quark model.

The same sum rules as given in Eqs. (3.1)-(3.7) are also valid for the processes $PB \to VB$, with a P meson replaced by the corresponding V meson of the same $SU(3)$ quantum numbers. However, one must now use the *physical* ω and φ,[20] instead of the corresponding $SU(3)$ singlet or octet states, which are probably not bad approximations for the pseudoscalars X^0 and η, respectively. We give below one such relation, which does not involve the φ and ω states and can also be put to experimental test:

$$16\frac{d\sigma}{d\Omega}(\pi^+p \to \rho^+p)+8\frac{d\sigma}{d\Omega}(K^-p \to K^{*-}p) +\frac{d\sigma}{d\Omega}(K^-p \to \bar{K}^{*0}n)=9\frac{d\sigma}{d\Omega}(\pi^+p \to K^{*+}\Sigma^+) +13\frac{d\sigma}{d\Omega}(K^-p \to \rho^-\Sigma^+)+11\frac{d\sigma}{d\Omega}(K^+p \to K^{*+}p) +10\frac{d\sigma}{d\Omega}(\pi^-p \to \rho^-p). \quad (3.13)$$

At this stage, such a relation can be tested in terms of the corresponding total cross sections. Again, Table I, which gives the experimental data necessary for a comparison of the left-hand sides and right-hand sides of this equation, shows a surprisingly good agreement.

One can also derive sum rules for processes of the type $PB \to PB^*$ and $PB \to VB^*$. For example, for $PB \to PB^*$ some of the sum rules are

$$\frac{d\sigma}{d\Omega}(\pi^+p \to K^+Y_1^{*+})=\frac{d\sigma}{d\Omega}(K^-p \to \bar{K}^0N^{*0}), \quad (3.14)$$

$$\frac{d\sigma}{d\Omega}(K^+p \to K^0N^{*++})=3\frac{d\sigma}{d\Omega}(K^-p \to \pi^-Y_1^{*+}), \quad (3.15)$$

$$\frac{d\sigma}{d\Omega}(\pi^+p \to \pi^0N^{*++})=3\frac{d\sigma}{d\Omega}(\pi^+p \to \eta N^{*++}), \quad (3.16)$$

$$\frac{d\sigma}{d\Omega}(K^+p \to K^0N^{*++})=12\frac{d\sigma}{d\Omega}(K^-p \to \pi^0Y_1^{*0}), \quad (3.17)$$

$$3\frac{d\sigma}{d\Omega}(\pi^-p \to \eta N^{*0})+\frac{d\sigma}{d\Omega}(\pi^-p \to \pi^0N^{*0}) =\frac{d\sigma}{d\Omega}(K^+p \to K^+N^{*+}) +\frac{d\sigma}{d\Omega}(K^-p \to K^-N^{*+}), \quad (3.18)$$

$$\frac{d\sigma}{d\Omega}(K^+p \to K^0N^{*++})=3\frac{d\sigma}{d\Omega}(\pi^+p \to X^0N^{*++}), \quad (3.19)$$

$$6\frac{d\sigma}{d\Omega}(K^-p \to \eta Y_1^{*0})+2\frac{d\sigma}{d\Omega}(K^-p \to \pi^0Y_1^{*0}) =2\frac{d\sigma}{d\Omega}(\pi^-p \to K^0Y_1^{*0}) +\frac{d\sigma}{d\Omega}(\pi^+p \to \pi^+N^{*+}). \quad (3.20)$$

[17] The data for the process $\pi^-p \to \pi^-p$ refer to a Q value of 0.79 BeV and an average angle given by $\langle\cos\theta\rangle \sim 0.8-0.9$ and those for $K^-p \to \bar{K}^0n$ correspond to a Q value of 0.78 BeV and $\langle\cos\theta\rangle \sim 0.95-1.0$.

[18] It may be noted that these total-cross-section relations, derived as they are from the corresponding differential cross sections, are somewhat stronger than those derivable from the forward-scattering amplitude, which (i) do not give the charge exchange scattering cross sections, and (ii) must be corrected for the momentum factor k involved in the relation

$$\sigma_{\rm tot}=(4\pi/k)\,{\rm Im}\,f(0).$$

[19] For Eq. (3.5) the Q value is around 1.2 BeV, while for Eq. (3.6) it is around 0.6 BeV.

[20] S. Okubo, Phys. Letters **5**, 165 (1963); S. L. Glashow, Phys. Rev. Letters **11**, 48 (1965).

TABLE II. Comparison with experiment of the total-cross-section sum rules (3.17) and (3.18) for the processes $PB \to PB^*$.

Equation	Process	Q value (BeV)	Total cross section, σ (μb)	Process	Q value (BeV)	Total cross sections, σ (μb)
(3.16)	$\pi^+p \to \pi^0 N^{*++}$ [a]	1.187	345±35	$\pi^+p \to \eta N^{*++}$ [b]	1.114	35; ($3\sigma=105$)
(3.17)	$K^+p \to K^0 N^{*++}$ [c]	0.874	930±120	$K^-p \to \pi^0 Y_1^{*0}$ [d]	1.265	68.4±11.4; ($12\sigma=820\pm140$)

[a] Reference a (i) of Table I. [b] Reference 33. [c] Reference a (vi) of Table I. [d] Reference a (iii) of Table I.

Equations (3.14) and (3.15) agree with those of LSS given in terms of total cross sections. All these differential cross sections *vanish identically* in the forward direction. Again, the same sum rules hold for $PB \to VB^*$ with the replacement of a P by a V meson of the same $SU(3)$ quantum numbers. However, it may be noted that for these V-meson production processes, the amplitudes do *not* vanish, in general, in the forward direction. In terms of physical ω and φ, of course, the relations would look somewhat different, but we do not reproduce them here since at this stage they can not be experimentally verified.

For the processes $PB \to PB^*$, Eqs. (3.16) and (3.17) for the total cross sections can be put to experimental test, the necessary figures being shown in Table **II**. The sum rule (3.17) seems to be well satisfied, but (3.18) is in total disagreement. This last discrepancy could at least partly be attributed to possible $SU(3)$ mixing effects for the physical η state.

Lastly, we discuss relations between processes of the type $PB \to PB$ and $PB \to PB^*$. Once again, relations connecting $PB \to VB$ and $PB \to VB^*$ will be similar, except for the effect of ω-φ mixing angle. Thus, for example, we have

$$\frac{d\sigma}{d\Omega}(K^-p \to \bar{K}^0 n) = 3\frac{d\sigma}{d\Omega}(\pi^+p \to K^+Y_1^{*+}) + 2\frac{d\sigma}{d\Omega}(\pi^-p \to \pi^0\Sigma^0)\,, \quad (3.21)$$

$$4\frac{d\sigma}{d\Omega}(K^-p \to \pi^0\Lambda) = 3\left[\frac{d\sigma}{d\Omega}(K^-p \to \pi^-\Sigma^+) + \frac{d\sigma}{d\Omega}(K^-p \to \pi^-Y_1^{*+})\right], \quad (3.22)$$

$$\frac{d\sigma}{d\Omega}(K^+p \to \bar{K}^0 N^{*++}) + 6\frac{d\sigma}{d\Omega}(\pi^-p \to \pi^-p) = 6\frac{d\sigma}{d\Omega}(\pi^+p \to \pi^+N^{*+}) + 2\frac{d\sigma}{d\Omega}(\pi^-p \to K^0\Lambda) + 21\frac{d\sigma}{d\Omega}(\pi^+p \to \pi^+\Sigma^+)\,. \quad (3.23)$$

Equation (3.22) agrees with Eq. (12) of LSS. There are, however, not enough data for an experimental test of these relations.

4. DENSITY MATRICES FOR $PB \to PB^*$ AND $PB \to PB^{(-)}$

In this section, we discuss the results for the density matrices for the production of the **56** baryon decuplet as well as negative-parity singlets in association with pseudoscalar mesons.

A. Process $PB \to PB^*$

This process is the simplest of all the cases under consideration, the amplitudes being expressible in terms of the quantities A^S and B^S defined in QMB, as

$$A^S \equiv \langle \chi^s | \tilde{A} | \chi'' \rangle\,, \quad (4.1)$$

$$B^S = \langle \chi^s | \tilde{B} | \chi'' \rangle\,, \quad (4.2)$$

where $\tilde{A}$ and $\tilde{B}$ are given by (2.3) and (2.4), respectively; χ^s is the (symmetric) spin-$\frac{3}{2}$ function of the quark constituents and (χ', χ'') are the two corresponding (mixed-symmetric) spin-$\frac{1}{2}$ functions.

The density matrices for the individual processes of this type are completely independent of these parameters, in agreement with the corresponding results of Ref. 7 as well as those of other authors.[21,22] An explicit evaluation of Eqs. (4.1) and (4.2) in terms of the results of Sec. 2 now shows that both A^S and B^S receive contributions only from the $(-)$ terms and none from the $(+)$ terms.[23] This fact therefore tells us that the $(-)$ terms are essential for a basic understanding of this unique process within the $QQ\bar{Q}$ model while the $(+)$ terms are more or less superfluous in this regard. Since this process is observed to be very copious, this fact brings out the more important role of the $(-)$ terms within the $QQ\bar{Q}$ formalism. The experimental status of this purely geometrical density matrix has been discussed by previous authors, and the present model does not give anything new in this regard, except to suggest that experiment discriminates in favor of the $(-)$ terms and against the $(+)$ terms.

B. Process $PB \to PB^{(-)}$

The evaluation of such amplitudes can be made through the use of the $(\mathbf{70},1^-)$ wave functions of $SU(6)$

[21] L. Stodolsky and J. J. Sakurai, Phys. Rev. Letters **11**, 90 (1963); L. Stodolsky, Phys. Rev. **134**, B1099 (1964).

[22] Y. Hara, Phys. Rev. **140**, B1170 (1965).

[23] This is of course expected from the general structure of $\tilde{A}$ and $\tilde{B}$ given by Eqs. (4.26) and (4.27) of QMB, in terms of $(+)$ and $(-)$ amplitudes, where the term $P = (\boldsymbol{\sigma}\cdot\mathbf{k})(\boldsymbol{\sigma}\cdot\mathbf{k}')$, which is always associated with $(-)$ amplitudes, provides the necessary spin flip for an octet to decuplet transition.

$\otimes O(3)$ exactly as in an earlier paper by Joshi, Bhasin, and Mitra.[6] The case of greatest physical interest is one involving the production of $Y_0{}^*{}_{3/2}$-(1520), which is presumably an $SU(3)$ singlet. This fact makes the density matrix depend on a very few parameters in close analogy with the cases $PB \to PB^*$, where it just happens to be a geometrical number. It is clear from the structure of the amplitudes that the $(-)$ terms which have the structure $\alpha^{(-)}\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k}\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k}'$ can contribute to such process along with the spin-independent parts $(\alpha^{(+)})$ of the $(+)$ terms. In exact analogy with the earlier subsection, we define the amplitude for $PB \to PY^{(-)}$ as

$$\langle m|\mu\rangle = \langle Y_{3/2,m}{}^{(-)}|\alpha^{(+)} + (\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k})(\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k}')\alpha^{(-)}|B_{1/2}{}^{(+)}\rangle\,, \quad (4.3)$$

where the "operator" amplitudes are defined in Sec. 2, and $Y^{(-)}$, $B^{(+)}$ are the respective wave functions of the final $Y_0{}^*{}_{3/2}$-(1520) and the initial baryon (proton). The latter functions are given by[24]

$$B_{1/2,\mu}{}^{(+)} = (\psi^s/\sqrt{2})(\chi_\mu{}'\phi' + \chi_\mu{}''\phi'')\,,$$
$$Y_{3/2,m}{}^{(-)} = [\psi'\sigma'' - \psi''\sigma']_m \chi_{3/2}{}^s \phi_\alpha\,,$$

where the notation for the right-hand sides is the same as explained in Ref. 24. Equation (4.3) immediately simplifies for the process $\pi^- p \to K^0 Y_0{}^{*(-)}$ to

$$\langle m|\mu\rangle \sim \langle(-\psi''\sigma')_m \chi_{3/2}{}^s|\alpha^{(+)} + (\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k})(\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k}')\alpha^{(-)}|\psi^s\chi_\mu{}'\rangle\,. \quad (4.4)$$

These amplitudes are easily seen to have the following structures. The "scalar part" of the operator, viz.,

$$[\alpha^{(+)} + \alpha^{(-)}\mathbf{k}\cdot\mathbf{k}']\,,$$

gives rise to various spherical components of the vector integral

$$\mathbf{I} = \int \psi''(\alpha^{(+)} + \alpha^{(-)}\mathbf{k}\cdot\mathbf{k}')\psi^S \quad (4.5)$$

in the different amplitudes. The vector part of the operator, viz.,

$$\alpha^{(-)}\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k}\times\mathbf{k}'\,,$$

on the other hand, gives rise to three types of integral harmonics corresponding, respectively, to the tensor, vector, and scalar products of the two vectors ψ'' and $\mathbf{q} = \mathbf{k}\times\mathbf{k}'$. In the language of multipole transitions, these individual contributions would be called M_2, E_1, and L_1 (longitudinal) amplitudes, respectively. Of these, the L_1 amplitude could also receive an additional contribution from (4.5), but E_1 and M_2 would remain unaffected by (4.5). It is now a straightforward matter to evaluate the density matrices for the process in terms of the individual contributions of the M_2, E_1, and L_1 amplitudes, according to the formula

$$\rho_{m,m'}{}^D = \textstyle\sum_\mu \langle m|\mu\rangle\langle m'|\mu\rangle^*\,.$$

[24] A. N. Mitra, Ann. Phys. (N.Y.) 43, 126 (1967).

For example, the E_1 contributions to the density matrices are the following:

$$\rho_{3,3}(E_1) = \tfrac{3}{8}\,,$$
$$\rho_{3,1}(E_1) = 0\,,$$
$$\rho_{3,-1}(E_1) = \sqrt{3}A_{-1}A_{-1}/8|A_{-1}|^2\,,$$

where

$$A_\mu = \int (\psi''\times q)_\mu \psi^S\,.$$

It is clear that $\rho_{3,-1}$ would vanish on taking an average over the azimuthal dependence, while the other two numbers are geometrical even without this approximation. As for the M_2 contributions, these are expressible in terms of the amplitudes

$$T_\mu = \sum_\nu \int C(1,\nu;1,\mu-\nu|2,\mu)\psi_\nu{}''q_{\mu-\nu}\psi^S\,,$$

and become geometrical numbers on taking the angular averages, which give

$$\langle T_\mu T_\nu\rangle = \tfrac{1}{5}\delta_{\mu,-\nu}(-1)^\lambda T_\lambda T_{-\lambda}\,.$$

Similarly, the L_1 contributions to the density matrix can be evaluated under the approximation of angular averages. The results are summarized in Table III along with the experimental figures[25] as well as the corresponding results of the Stodolsky-Sakurai model.[21] It appears that, except for $\rho_{3,-1}$, the predictions of this model are not very different from those of Stodlosky-Sakurai. Agreement with experiment can at most be called fair. Even for $\rho_{3,-1}$ the present data, which have large uncertainties, are not inconsistent with zero.

5. PROCESS $PB \to VB$

As these processes probably involve the largest number of amplitudes, it is necessary to evolve a consistent set of notations. The general matrix elements of the meson-quark operator for the production of a vector meson with polarization m by a pseudoscalar meson P

TABLE III. Predictions for density-matrix parameters of $Y_{0\,3/2^-}{}^*$ and comparison with experiment.

Theory	Interaction	$\rho_{3,3}$	Re$\rho_{3,-1}$	Re$\rho_{3,1}$
$QQ\bar{Q}$ model	M_2	$\frac{1}{8}$	0	0
	E_1	$\frac{5}{32}$	0	0
	L_1	$\frac{1}{4}$	0	0
Stodolsky-	M_2	$\frac{1}{8}$	$\sqrt{\frac{3}{8}}$	0
Sakurai	E_1	$\frac{3}{8}$	$-\sqrt{\frac{3}{8}}$	0
model	L_1	0	0	0
Experimental values[a] at $P_{\rm lab}=1.8$ to 2.2 BeV/c		0.073±0.052	0.039±0.05	0.057±0.043

[a] Reference 25. Data at all production angles were included in the fit.

[25] Orin I. Dahl *et al.*, Phys. Rev. 163, 1377 (1967).

can be written as

$$\langle\mu',m|\mu\rangle' \equiv \langle\psi^S\chi_\mu'; V_m|u_A'\tilde{A}+u_B'\tilde{B}|\psi^S\chi_\mu'; P\rangle, \quad (5.1)$$

$$\langle\mu',m|\mu\rangle'' \equiv \langle\psi^S\chi_\mu''; V_m|u_A''\tilde{A}+u_B''\tilde{B}|\psi^S\chi_\mu''; P\rangle. \quad (5.2)$$

Here ψ^S is the spatial wave function (same for initial and final states); (χ',χ'') are the spin-$\frac{1}{2}$ functions of $[2,1]_a$ and $[2,1]_s$ symmetries, respectively. The numbers u_A', u_A'', u_B', u_B'' are the $SU(3)$ matrix elements of the operators $u_{\beta\alpha}^{(\pm)}$ of Eq. (2.2) and can be read from Table I of QMB. For convenience we shall use the same symbols for the meson-quark amplitudes defined in Sec. 2 and the corresponding meson-baryon amplitudes obtained by evaluating the orbital matrix elements of Eqs. (2.3) and (2.4). Then in the notation of Sec. 2, the combinations of the various amplitudes that appear in this process are

$$X_1^{(\pm)} = \tfrac{1}{3}[(u_A''-3u_A')A_1^{(\pm)}+(u_B''-3u_B')B_1^{(\pm)}], \quad (5.3)$$

$$X_2^{(\pm)} = \tfrac{1}{3}[(u_A''-3u_A')a^{(\pm)}+(u_B''-3u_B')b^{(\pm)}], \quad (5.4)$$

$$X_3^{(-)} = \tfrac{1}{3}[(u_A''+u_A')\{A_1^{(-)}-a^{(-)}\} + (u_B''+u_B')\{B_1^{(-)}-b^{(-)}\}]. \quad (5.5)$$

These parameters give rise to a set of *six* linearly independent amplitudes, which are listed in Table IV.

We first note the following $SU(3)$-type sum rules in the density matrices, analogous to the amplitude sum rules:

$$\rho^V(\pi^+p \to K^{*+}\Sigma^+) = \rho^V(K^-p \to \omega\Sigma^0), \quad (5.6)$$

$$\rho^V(\pi^-p \to \omega n) = \rho^V(K^-p \to \bar{K}^{*0}n), \quad (5.7)$$

$$\rho^V(K^+p \to K^{*+}p) = \rho^V(K^-p \to \rho^0\Lambda), \quad (5.8)$$

$$\rho^V(\pi^-p \to K^{*0}\Lambda) = \rho^V(K^-p \to \omega\Lambda). \quad (5.9)$$

The ω state considered in these equations is based on the "ideal mixing angle".[20] These relations, which should be interpreted as valid for every separate density-matrix element, hold for a general mixture of (+) and (−) terms in the process $PB \to VB$. Additional $SU(3)$-type relations in the forward direction are

$$\rho^V(K^-p \to \bar{K}^{*0}n) = \rho^V(K^-p \to \omega\Lambda), \quad (5.10)$$

$$\rho^V(K^+p \to K^{*+}p) = \rho^V(K^-p \to \rho^-\Sigma^+). \quad (5.11)$$

The experimental status of Eqs. (5.9)-(5.11) is shown in Table V in respect of certain individual values $\rho_{m,m'}{}^V$. The pattern of agreement may at best be described as fair, though it must be remembered that the experimental figures are not strictly at zero angle, but are averaged over certain small value of t.

TABLE IV. The six independent amplitudes $\langle\mu,m|\mu'\rangle$ for the process $P+B \to V+B$, in terms of combinations defined in Eqs. (5.3)-(5.5). The other six can be obtained from the relation $\langle\mu,m|\mu'\rangle = (-1)^{\mu+m+\mu'}\langle-\mu,-m|-\mu'\rangle$. For other notations see text.

$\langle\mu,m\|\mu'\rangle$	Amplitude
$\langle\frac{1}{2},+1\|\frac{1}{2}\rangle$	$\frac{1}{4}\sqrt{2}k^2\sin\theta[4\cos\theta X_2^{(+)}+(X_1^{(-)}+2X_2^{(-)}+3X_3^{(-)})]$
$\langle\frac{1}{2},0\|\frac{1}{2}\rangle$	$-\frac{1}{2}[X_1^{(+)}+\frac{1}{3}k^2(3\cos^2\theta-1)X_2^{(+)}$ $+k^2\cos\theta(X_1^{(-)}+2X_2^{(-)})]$
$\langle\frac{1}{2},-1\|\frac{1}{2}\rangle$	$-\frac{1}{4}\sqrt{2}k^2\sin\theta$ $\times[4\cos\theta X_2^{(+)}+(X_1^{(-)}+2X_2^{(-)}-3X_3^{(-)})]$
$\langle\frac{1}{2},1\|-\frac{1}{2}\rangle$	$\sqrt{2}k^2\sin^2\theta X_2^{(+)}$
$\langle\frac{1}{2},0\|-\frac{1}{2}\rangle$	$-\frac{1}{2}k^2\sin\theta[4\cos\theta X_2^{(+)}+(X_1^{(-)}-X_2^{(-)})]$
$\langle\frac{1}{2},-1\|-\frac{1}{2}\rangle$	$-\frac{1}{2}\sqrt{2}[X_1^{(+)}-\frac{2}{3}k^2(3\cos^2\theta-1)X_2^{(+)}$ $-k^2\cos\theta(X_1^{(-)}-X_2^{(-)})]$

This model does not predict relations between density-matrix elements $\rho_{m,m'}{}^V$ for a specific process. However, if one uses only the (+) terms or the (−) terms, it is possible to deduce the relation

$$\rho_{0,0}{}^V(\pi^-p \to \rho^0 n) = \tfrac{2}{3}. \quad (5.12)$$

It may be of some interest to compare this prediction with experiment. In this connection Miller *et al.*[26] made an analysis of the density-matrix elements for this process, after taking account of the interference between the $T=0$ and $T=1$ contributions to the final $\pi^+\pi^-$ state. If we ignore the contribution of the $T=0$ part of $\rho_{0,0}{}^V$, the trace relation is simply

$$\rho_{0,0}{}^V + 2\rho_{1,1}{}^V = 1. \quad (5.13)$$

This gives the estimate

$$\rho_{0,0}{}^V - \rho_{1,1}{}^V = \tfrac{1}{2} \quad (5.14)$$

for the process $\pi^-p \to \rho^0 n$ at $\theta=0$, which seems to compare very favorably with the value ($\approx 0.64 \pm 0.13$) that can be read from Fig. 30 of Ref. 26. It may be noted that the validity of this result in our $QQ\bar{Q}$ model is a result of the dynamical assumptions (i) of spin and $SU(3)$ independence of the $Q\bar{Q}$ force, and (ii) preponderance of the $Q\bar{Q}$ force over QQ. It does not depend on the (more controversial) assumption (iii) discussed in

TABLE V. Experimental verification of density-matrix sum rules (5.9)-(5.11) for the processes $PB \to VB$. The data are given for all the elements, $\rho_{0,0}$, $\rho_{1,-1}$, and $\mathrm{Re}\rho_{1,0}$.

Process	P_{lab} (BeV/c)	$\rho_{0,0}$	$\rho_{1,-1}$	$\mathrm{Re}\rho_{1,0}$	Process	P_{lab} (BeV/c)	$\rho_{0,0}$	$\rho_{1,-1}$	$\mathrm{Re}\rho_{1,0}$
$\pi^-p \to K^{*0}\Lambda$ [a]	3.8−4.2	0.266±0.11	0.164±0.08	···	$K^-p \to \omega\Lambda$ [b]	4.1	$0.1_{-0.10}^{+0.16}$	0.18±0.16	···
$K^-p \to \bar{K}^{*0}n$ [c]	4.1	0.5 ±0.1	−0.03 ±0.08	−0.5 ±0.05	$K^-p \to \omega\Lambda$ [d]	3.5	0.28±0.15	0.15±0.1	0.08±0.08
$K^+p \to K^{*+}p$ [c]	3.5	0.27 ±0.08	0.26 ±0.07	0.12±0.05	$K^-p \to \rho^-\Sigma^+$ [d]	3.5	0.17±0.08	0.17±0.08	0.01±0.04

[a] Reference 25. [b] J. Mott *et al.*, Phys. Rev. Letters **18**, 355 (1967). [c] Reference 27. [d] Reference a (iii) of Table I.

[26] D. H. Miller *et al.*, Phys. Rev. **153**, 1423 (1967).

Sec. 2, about the s-wave nature of the $Q\bar{Q}$ force. As for the possible effects of interference between the (+) and (−) terms, one would expect these to be small if one type can be shown to dominate over the other. Indeed, as will be seen later in this section, it is possible to fit the density-matrix results mainly with (−) terms, with a little mixture of (+) terms.

A result of the form (5.12) within the additivity model could not be discerned in the work of Ref. 7. A calculation performed by the present authors, however, answers this question in the negative. A result of this type may therefore be thought to have a bearing on the dynamical aspects of the $QQ\bar{Q}$ model, such as the validity of the assumptions (i) and (ii) listed in Sec. 2.

We note in passing a sum rule, obtained by using only (+) terms, viz.,

$$\rho_{1,0}{}^V=\sqrt{2}\cot\theta\rho_{1,-1}{}^V. \qquad (5.15)$$

However, it is not in good agreement with some recent data.[26,27] This may be regarded as an argument in favor of the dominance of the (−) terms over (+) terms.

As further tests of this model, we have made a detailed comparison of the element $\rho_{0,0}{}^V$ at $\theta=0$ for the processes $\pi N\to\rho N$ and $KN\to K^*N$ with available data. For this purpose we have made use of the equality (2.13) but not of the inequalities (2.14) and (2.15). The results with pure (+) or pure (−) amplitudes are, respectively, represented by

$$\rho_{0,0}{}^{(+)}=\frac{1}{3}\frac{[3a_0+8(b_0\bar{d}^{(+)}+b_1\bar{f}^{(+)})]^2}{9a_0^2+32(b_0\bar{d}^{(+)}+b_1\bar{f}^{(+)})^2}, \qquad (5.16)$$

$$\rho_{0,0}{}^{(-)}=\frac{1}{3}\frac{[a_0+2(b_0\bar{d}^{(-)}+b_1\bar{f}^{(-)})]^2}{a_0^2+2(b_0\bar{d}^{(-)}+b_1\bar{f}^{(-)})^2}, \qquad (5.17)$$

where

$$\bar{d}^{(+)}=\frac{k^2d^{(+)}}{D^{(+)}-F^{(+)}},\quad \bar{d}^{(-)}=\frac{d^{(-)}}{D^{(-)}-F^{(-)}}, \qquad (5.18)$$

$$\bar{f}^{(+)}=\frac{k^2f^{(+)}}{D^{(+)}-F^{(+)}},\quad \bar{f}^{(-)}=\frac{f^{(-)}}{D^{(-)}-F^{(-)}}, \qquad (5.19)$$

and (a_0,b_0,b_1) are certain linear combinations of the $SU(3)$ coefficients u_A', u_A'', u_B', u_B''. These expressions are seen to be identical through the simple correspondence

$$\tfrac{4}{3}\bar{d}^{(+)}\rightleftarrows\bar{d}^{(-)},\quad \tfrac{4}{3}\bar{f}^{(+)}\rightleftarrows\bar{f}^{(-)}, \qquad (5.20)$$

so that the predictions of the theory in respect of the parameter $\rho_{0,0}(\theta=0)$ are exactly the same with (+) or (−) amplitudes. A typical fit to the data with only (−) amplitudes is obtained with

$$\bar{d}^{(-)}=-\tfrac{7}{2},\quad \bar{f}^{(-)}=+\tfrac{1}{2}. \qquad (5.21)$$

[27] Manuel G. Doncel (unpublished).

TABLE VI The density-matrix elements $\rho_{0,0}{}^V$ at $\theta=0$ for the processes $\pi p\to\rho p$ and $Kp\to K^*p$, for only the (−) amplitudes, as well as for the combination of (+) and (−) amplitudes described in Sec. V. The experimental values (which are averaged over small values of momentum transfer) and the corresponding laboratory momenta are also listed.

		$\rho_{0,0}{}^V$		
			$QQ\bar{Q}$ model	
Process	Lab momentum (BeV/c)	Experimental	with (+) and (−) amplitudes	with (−) amplitudes alone
$\pi^+p\to\rho^+p$	4[a] 8[a]	0.7 ±0.08 0.54±0.07	0.57	0.67
$\pi^-p\to\rho^-p$	4[b]	0.53±0.12	0.51	0.60
$\pi^-p\to\rho^0n$	2.75[c]	0.77±0.09	0.58	0.67
$K^+p\to K^{*+}p$	3.5[d] 5.0[d]	0.25±0.1 0.2 ±0.05	0.25	0.33
$K^-p\to K^{*-}p$	4.1[d] 5.4[d]	0.4 ±0.1 0.43±0.13	0.48	0.58
$K^-p\to K^{*0}n$	4.1[d] 5.5[d]	0.5 ±0.1 0.7 ±0.1	0.46	0.56

[a] Aachen-Berlin-CERN Collaboration, Phys. Letters 22, 533 (1967).
[b] Aachen-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Nuovo Cimento 31, 729 (1964).
[c] Reference 26.
[d] Reference 27.

The results which are shown in Table VI are seen to be only in fair agreement with experiment. A counterpart of Eq. (5.21) with (+) amplitudes is

$$\bar{d}^{(+)}=-21/8,\quad \bar{f}^{(+)}=+\tfrac{3}{8}. \qquad (5.22)$$

Clearly the agreement can be greatly improved by a suitable admixture of (+) and (−) amplitudes. In this respect, results of Sec. 4 for $PB\to PB^*$ processes suggest that (−) amplitudes are definitely more useful than (+) amplitudes (which give zero for such processes). Therefore, we look for a better fit with the data using a dominance of (−) amplitudes. Now, the density matrix involves the further parameter

$$R=(D^{(+)}-F^{(+)})/[k^2(D^{(-)}-F^{(-)})], \qquad (5.23)$$

which is an index of the mixture of (+) and (−) amplitudes. A typical fit with $R=-0.1$, which is also shown in Table VI, indicates the greatly improved nature of the agreement. A comparison of Eqs. (5.21) and (5.22) with the inequalities (2.14) and (2.15) shows that except for the (probably spurious) agreement with $d^{(+)}\gg f^{(+)}$, the other inequalities are either not satisfied, or can not be tested. This fact probably speaks against the simple s-wave assumption [assumption (iii)], but does not invalidate assumptions (i) and (ii).

As for the angular distributions of the density matrices, these are given by rather unwieldy expressions involving a large number of parameters, even if only one type of amplitudes, say (−), were considered. As this would prevent any simple conclusions from being drawn on the basis of comparison with experiment, we have refrained from such an analysis for the case $PB\to VB$.

Finally, we note that the analysis has been carried out by considering all the parameters $A^{(\pm)}$, etc., as *relatively real,* in order to avoid having a much larger number of independent parameters as against totally inadequate experimental data. This, however, may not be a serious handicap since the data on these matrix elements are only with respect to their real parts.[28]

6. PROCESSES $PB \to VB^*$

As in Sec. 5, the analysis of these processes can be made in terms of the matrix element

$$\langle \mu', m | \mu \rangle^s = \langle \psi^S \chi_\mu{}^s ; V_m | u_A{}^s \tilde{A} + u_B{}^s \tilde{B} | \psi^S \chi_\mu{}'' ; P \rangle , \quad (6.1)$$

where the spatial function ψ^S is the same as before, but the spin-$\frac{3}{2}$ function χ^s now appears in the final state. The $SU(3)$-matrix elements $u_A{}^s$ and $u_B{}^s$ of the operators $u_{\beta\alpha}{}^{(\pm)}$ between octet and decuplet states can be read off from Table II of QMB. Again, as in Sec. 5, we use the same notation for the orbital matrix elements of the meson-quark amplitudes as for the latter themselves.[29] In this case, only the following combinations of the amplitudes are relevant for the various processes:

$$Y_1{}^{(\pm)} = (u_A{}^s + 2u_B{}^s) A_1{}^{(\pm)} , \quad (6.2)$$

$$Y_2{}^{(\pm)} = (2u_A{}^s + u_B{}^s) a^{(\pm)} + (u_A{}^s - u_B{}^s) b^{(\pm)} . \quad (6.3)$$

These parameters give rise to a set of amplitudes listed in Table VII for a given $PB \to VB^*$ process [without explicit reference to the actual $SU(3)$ indices]. These amplitudes obey the symmetry relation

$$\langle -\mu', -m | -\mu \rangle^s = (-1)^{\mu + m - \mu'} \langle \mu', m | \mu \rangle^s . \quad (6.4)$$

For comparison with experiment, we first look for possible sum rules between the V-meson density-matrix elements $\rho_{m,m'}{}^V$ and the corresponding decuplet elements $\rho_{m,m'}{}^D$. A general relation for a given process which takes into account both (+) and (−) quark-meson amplitudes, and is valid for all angles, is

$$\tfrac{4}{3}[\rho_{3,3}{}^D + \sqrt{3}\rho_{3,-1}{}^D] = \rho_{1,1}{}^V + \rho_{1,-1}{}^V . \quad (6.5)$$

This relation, which has been checked in sufficient detail for all the processes $K^- p \to \rho Y_1^*$, $\pi p \to \rho N^*$, $\pi p \to \omega N^*$, and $K p \to K^* N^*$ by Bialas and Zalewski,[11] is fairly well satisfied within the experimental inaccuracies. However, the satisfaction of this relation in our model means something more than what was implied by the authors of Ref. 11, viz., that it is a test of the additivity or independent quark model. Since in the $QQ\bar{Q}$ model, we are taking account of all the multiple scattering effects within the $QQ\bar{Q}$ system, the validity of this result clearly goes beyond the simple additivity assumption.

More specialized relations between the density-matrix elements are obtained if one considers the (+) or (−) meson-quark amplitudes separately. Thus for (+) amplitudes only, the following relations are derived

$$\rho_{1,1}{}^V = \tfrac{4}{3}\rho_{3,3}{}^D , \quad (6.6)$$

$$\rho_{1,-1}{}^V = (4/\sqrt{3})\rho_{3,-1}{}^D . \quad (6.7)$$

It may be of interest to note that the $SU(6)_W$ model also predicts the stronger relations (6.6) and (6.7) for the process $K p \to K^* N^*$. However, the experimental data in Table VIII do not seem to provide any convincing tests in this regard. A second relation, obtainable only with (+) amplitudes is

$$\rho_{1,0}{}^V = 2(\sqrt{\tfrac{2}{3}})\rho_{3,1}{}^D , \quad (6.8)$$

whose experimental status is again rather uncertain.[30] Using (−) amplitudes alone, one does not obtain the specialized relations (6.6) and (6.7) except in the forward direction $\theta = 0$, where (6.6) is true and (6.7) reduces to the trivial result $0 = 0$. Another result, which is true *only* for (−) amplitudes at all angles, is

$$\rho_{3,1}{}^D = 0 . \quad (6.9)$$

This appears to be roughly consistent with the corresponding experimental density-matrix elements for the processes $\pi^+ p \to \rho^0 N^{*++}$ and $K^+ p \to K^{*0} N^{*++}$.[27,31] The angular distributions predicted by pure (−) amplitudes for the density matrices $\rho_{m,m}{}^{V,D}$ are as follows:

$$\rho_{3,3}{}^D = \frac{3}{8} \frac{1 + \cos^2\theta}{1 + \alpha + \cos^2\theta} , \quad (6.10)$$

$$\rho_{3,1}{}^D = 0 , \quad (6.11)$$

$$\rho_{3,-1}{}^D = -\tfrac{1}{8}\sqrt{3} \frac{\sin^2\theta}{1 + \alpha + \cos^2\theta} , \quad (6.12)$$

TABLE VII. The five independent amplitudes $\langle \mu', m | \mu \rangle$ for the process $PB \to VB^*$ in terms of the combinations defined in Eqs. (6.2) and (6.3). The others can be obtained from the relations $\langle \frac{3}{2}, m | \frac{1}{2} \rangle = \sqrt{3} \langle \frac{1}{2}, m | -\frac{1}{2} \rangle$, $\langle \frac{1}{2}, 1 | \frac{1}{2} \rangle = -\langle \frac{1}{2}, -1 | \frac{1}{2} \rangle$, and the symmetry relation (6.4).

$\langle \mu', m \mid \mu \rangle$	Amplitude
$\langle \frac{3}{2}, 1 \mid \frac{1}{2} \rangle$	$2 Y_2{}^{(+)} \sin^2\theta$
$\langle \frac{3}{2}, 0 \mid \frac{1}{2} \rangle$	$-\sqrt{2}[\frac{1}{2} Y_1{}^{(-)} \sin\theta - \frac{1}{2} Y_2{}^{(-)} \sin\theta + 2 Y_2{}^{(+)} \sin\theta \cos\theta]$
$\langle \frac{3}{2}, -1 \mid \frac{1}{2} \rangle$	$-[Y_1{}^{(+)} - Y_1{}^{(-)} \cos\theta + Y_2{}^{(-)} \cos\theta - \frac{2}{3} Y_2{}^{(+)} (3\cos^2\theta - 1)]$
$\langle \frac{1}{2}, 1 \mid \frac{1}{2} \rangle$	$(-\sqrt{\frac{1}{3}})[Y_1{}^{(-)} \sin\theta + 2 Y_2{}^{(-)} \sin\theta + 4 Y_2{}^{(+)} \sin\theta \cos\theta]$
$\langle \frac{1}{2}, 0 \mid \frac{1}{2} \rangle$	$(\sqrt{\frac{2}{3}})[Y_1{}^{(+)} + Y_1{}^{(-)} \cos\theta + 2 Y_2{}^{(-)} \cos\theta + \frac{4}{3} Y_2{}^{(+)} (3\cos^2\theta - 1)]$

[28] There would, however, be some contributions to the elements that would be missed by considering the amplitudes as relatively real. More specifically, in our simplified analysis we have ignored the contributions of terms of the form (Ima) (Imb), in the evaluation of quantities like Re(a^*b).

[29] This is clearly justified, since the spatial wave function of the baryon decuplet is the same as for the baryon octet.

[30] At 8 BeV/c this relation is fairly well satisfied for the process $\pi^+ p \to \rho^0 N^{*++}$, where the experimental numbers for the left-hand side and the right-hand side of Eq. (6.8) are, respectively, -0.119 ± 0.025 and -0.121 ± 0.05. On the other hand, at 4 BeV/c the respective sides are -0.06 ± 0.03 and -0.016 ± 0.04. In these data, the $|t|$ value is averaged up to 0.3 (BeV/c)^2.

[31] D. Brown *et al.*, Phys. Rev. Letters **19**, 664 (1967).

TABLE VIII. Comparison of sum rules (6.6) and (6.7) for density-matrix parameters [with (+) amplitude only] with available data for the processes $K^+p \to K^{*0}N^{*++}$, $\pi^+p \to \rho^0 N^{*++}$, and $\pi^+p \to \omega^0 N^{*++}$.

Process	P_{lab} (BeV/c)	$\lvert t\rvert$ interval (BeV/c)^2	Relation $\rho_{1,1}{}^V$	$=\frac{4}{3}\rho_{3,3}{}^D$	Relation $\rho_{1,-1}{}^V$	$=(4/\sqrt{3})\rho_{3,-1}{}^D$
$K^+p \to K^{*0}N^{*++}$	3[a]	0.1	0.06 ±0.04	0.0 ±0.07	0.08±0.08	0.04±0.15
		0.2	0.1 ±0.5	0.12±0.1	−0.02±0.05	−0.23±0.18
		0.72	0.28 ±0.08	0.27±0.11	0.22±0.08	0.14±0.2
	3.5[a]	0.1	0.15 ±0.05	0.15±0.12	−0.13±0.08	−0.25±0.18
		0.25	0.13 ±0.05	0.0 ±0.13	0.03±0.09	−0.14±0.16
		0.64	0.19 ±0.05	−0.03±0.1	0.08±0.08	−0.16±0.16
	5[a]	0.1	0.09 ±0.04	0.23±0.09	−0.07±0.07	−0.09±0.18
		0.27	0.17 ±0.04	0.28±0.11	0.09±0.08	0.09±0.21
$\pi^+p \to \rho^0 N^{*++}$	4[b]	0.07	0.08 ±0.03	0.13±0.05	0.02±0.03	0.05±0.08
		0.8	0.42 ±0.02	0.27±0.05	0.04±0.04	0.2 ±0.09
	8[c]	0.3	0.11 ±0.02	0.06±0.04	−0.03±0.02	0.03±0.06
$\pi^+p \to \omega^0 N^{*++}$	4[c]	<0.6	0.26 ±0.05	0.2 ±0.01	0.13±0.05	0.039±0.18
	8[c]	<0.6	0.365±0.05	0.32±0.11	0.17±0.08	0.09±0.09

[a] The data are averaged up to $\lvert t\rvert < 0.1$ (BeV/c)^2 for all cases. See Ref. 27.
[b] See Ref. 32. The data are compared at $\lvert t\rvert < 0.07$ (BeV/c)^2.
[c] Reference a of Table VI.

$$\rho_{0,0}{}^V = \frac{\sin^2\theta + \alpha\cos^2\theta}{1+\alpha+\cos^2\theta}, \tag{6.13}$$

$$\rho_{1,0}{}^V = \frac{1}{\sqrt{2}}\frac{(1-\alpha)\sin\theta\cos\theta}{1+\alpha+\cos^2\theta}, \tag{6.14}$$

$$\rho_{1,-1}{}^V = -\tfrac{1}{2}\alpha\frac{\sin^2\theta}{1+\alpha+\cos^2\theta}, \tag{6.15}$$

$$\sqrt{\alpha} = \frac{Y_1{}^{(-)}+2Y_2{}^{(-)}}{Y_1{}^{(-)}-Y_2{}^{(-)}}. \tag{6.16}$$

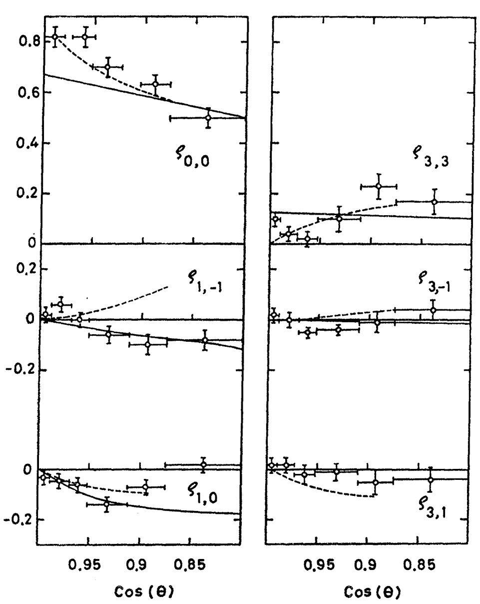

FIG. 1. The density-matrix elements of ρ^V and ρ^D, as functions of $\cos\theta$, for the process $\pi^+p \to \rho^0 N^{*++}$. The results of the present model are shown by solid lines and those of the OPE model by dashed lines. The experimental points are taken from Ref. 32.

The rest of the discussion in this section concerns the comparison of individual matrix elements with experiment using a dominance of (−) amplitudes as in Sec. 5. The value of the parameter α for $\pi p \to \rho N^*$ is geometrical, viz.,

$$\alpha = 4,$$

since $Y_1{}^{(-)} = 0$ for this process. For $\pi p \to \omega N^*$, we first take the physical ω in terms of the "ideal mixing angle,"[20] as

$$\omega = -(\sqrt{\tfrac{1}{3}})\omega_0 + (\sqrt{\tfrac{2}{3}})\omega_8,$$

in which case we have, from Eqs. (6.2), (6.3), and (6.16),

$$\sqrt{\alpha} = \frac{A_1{}^{(-)}+4a^{(-)}+2b^{(-)}}{A_1{}^{(-)}-2a^{(-)}-b^{(-)}}. \tag{6.17}$$

Using the same ratios (5.21) for the parameters as used for $PB \to VB$, we have $\alpha = 2.56$. In Figs. 1 and 2 we compare the angular distributions for the processes $\pi^+p \to \rho^0 N^{*++}$ and $\pi^+p \to \omega^0 N^{*++}$, respectively, with the available data. The solid lines are the predictions of our model and are found to be in fairly good agreement with recent data[31] for $0 \leqslant \lvert t\rvert \leqslant 0.8$ (BeV/c)^2. Indeed, the agreement looks better than the one for the one-pion-exchange (OPE) model with absorption,[32] the curves for which are also included for comparison.

A closer examination is of interest for the density-matrix elements at $\theta = 0$. For $\pi p \to \omega N^*$, we have the geometrical numbers

$$\rho_{0,0}{}^V = \tfrac{2}{3}, \quad \rho_{3,3}{}^D = \tfrac{1}{8},$$
$$\rho_{3,1}{}^D = \rho_{3,-1}{}^D = \rho_{1,0}{}^V = \rho_{1,-1}{}^V = 0.$$

For the other $SU(3)$ processes, these quantities also depend on $\bar{d}^{(-)}$ and $\bar{f}^{(-)}$ via the parameter α of Eq. (6.16). In Table IX are listed the values of $\rho_{0,0}{}^V$ and $\rho_{3,3}{}^D$ at $\theta = 0$ for the processes $\pi^+p \to \rho^0 N^{*++}$, $\pi^+p \to$

[32] Aachen-Berlin-Birmingham-Bonn-Hamburg-London (I.C.)-München Collaboration, Phys. Rev. 138, B897 (1965).

$\omega^0 N^{*++}$, $K^+p \to K^{*0}N^{*++}$, and $K^-p \to K^{*-}N^{*+}$ against their experimental values. The agreement looks quite satisfactory for all the measured cases except for the process $K^+p \to K^{*0}N^{*++}$, where the calculated value $\rho_{0,0}{}^V = 0.33$ differs significantly from the experimental value of 0.9. It may be noted that the $SU(6)_W$ prediction for this number is also 0.33,[33] in agreement with our result. This discrepancy has been shown to be remedied in the peripheral model by using appropriate physical masses in the propagators for the exchanged particles.[27] However, the $QQ\bar{Q}$ model by itself cannot throw further light on this question without reference to a more detailed mechanism for the quark interactions.

7. SUMMARY AND CONCLUSIONS

We have tried to present a fairly detailed comparison of the $QQ\bar{Q}$ model with experimental results for several meson-baryon processes. The specific processes considered are $PB \to PB$, $PB \to PB^*$, $PB \to PB^{(-)}$, $PB \to VB$, $PB \to VB^*$, where B, B^* are the **8** and **10** members of the **56** baryons and $B^{(-)}$ a negative-parity baryon belonging to the $(\mathbf{70}, 1^-)$ representation of $SU(6) \otimes O(3)$. The predictions are of the following types:

(1) sum rules for total and differential cross sections,
(2) density-matrix sum rules for a given process,
(3) angular distribution of density-matrix elements, with special reference to their zero-angle behavior. These predictions can not only be tested with experiment, but can also be compared with those of contemporary models, especially the additivity assumption for quark amplitudes [7–11] and $SU(6)_W$ symmetry considered by previous authors.[27,33]

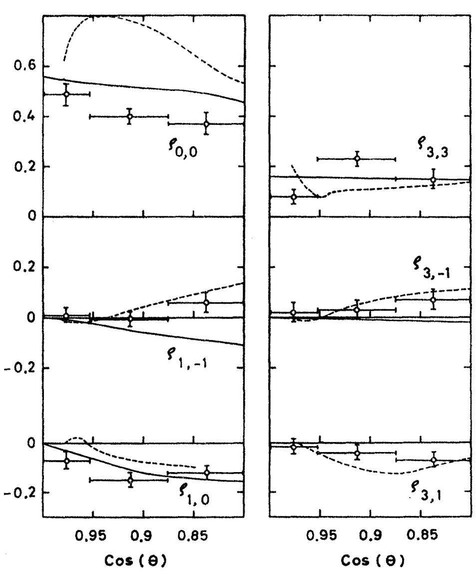

FIG. 2. The density-matrix elements of ρ^V and ρ^D, as functions of $\cos\theta$, for the process $\pi^+p \to \omega^0 N^{*++}$. The results of the present model are shown by solid lines and those of the OPE model by dashed lines. The experimental points are taken from Ref. 32.

[33] M. G. Doncel and E. de Rafael, Nuovo Cimento 42, 426 (1966).

TABLE IX. Predictions of the $QQ\bar{Q}$ model for the density-matrix elements $\rho_{0,0}{}^V$ and $\rho_{3,3}{}^D = \frac{3}{8}(1-\rho_{0,0}{}^V)$ at $\theta = 0$, using only $(-)$ amplitudes (as described in Sec. 6) for the processes $\pi^+p \to \rho^0 N^{*++}$, $K^+p \to K^{*0}N^{*++}$, and $\pi^+p \to \omega^0 N^{*++}$ as well as the corresponding experimental values averaged over small momentum transfers.

Process	Lab momentum (BeV/c)	$\rho_{0,0}{}^V$ Experiment	$\rho_{0,0}{}^V$ Theory	$\rho_{3,3}{}^D$ Experiment	$\rho_{3,3}{}^D$ Theory
$\pi^+p \to \rho^0 N^{*++}$	4[a]	0.77 ±0.04	0.66	0.08 ±0.03	0.12
	8[a]	0.77 ±0.04		0.05 ±0.03	
$\pi^+p \to \omega^0 N^{*++}$	2.77[b]	0.56 ±0.01	0.56	0.15 ±0.01	0.165
	4[a]	0.47 ±0.05		0.15 ±0.04	
	8[a]	0.26 ±0.10		0.24 ±0.08	
$K^+p \to K^{*0}N^{*++}$	3[c]	0.88 ±0.04	0.33	0.0 ±0.07	0.25
	3.5[c]	0.7 ±0.05		0.11 ±0.12	
	5[c]	0.82 ±0.03		0.18 ±0.07	

[a] Reference a of Table VI. Data for reactions (1) and (2) are averaged over the $|t|$ values up to 0.3 (BeV/c)^2 and 0.6 (BeV/c)^2, respectively.
[b] S. S. Yamamoto *et al.*, Phys. Rev. 140, B730 (1965).
[c] Reference 27.

An essential feature of the $QQ\bar{Q}$ model is the appearance of two types of meson-quark amplitudes, the $(+)$ and $(-)$ types. Physically, the $(-)$ amplitudes come about from p-wave structures in the initial and final meson-quark ($QQ\bar{Q}$) wave functions, while the $(+)$ amplitudes are mainly the result of overlaps between corresponding s-wave structures. To keep the number of parameters at a reasonably low level, we have examined the possibility of fitting the data with only one type of amplitudes. In this respect we have been guided by the consideration that for certain processes, especially $PB \to PB^*$, which are observed to be strong experimentally, only the $(-)$ amplitudes contribute while the $(+)$ amplitudes vanish exactly. This suggests that we look for fits with predominantly $(-)$ amplitudes, and add small mixtures of the $(+)$ amplitudes only if necessary. In several cases we find that the predictions with only $(+)$ or only $(-)$ amplitudes are *identical*; for example, the processes $PB \to VB$ and $PB \to VB^*$ have the same density-matrix structures in terms of either $(+)$ or $(-)$ amplitudes. With regard to the processes $PB \to VB^*$, the $(+)$ amplitudes give rise to certain extra density-matrix sum rules whose experimental status is however rather uncertain. From these considerations, we have taken the $(-)$ amplitudes as the main basis of parametrization. This gives quite satisfactory experimental fits for most processes, except for $PB \to VB$, where a small ($\sim 10\%$) admixture of $(+)$ amplitudes is indicated.

We note further that the cross-section sum rules derived in Sec. 3 are completely general, in that these include the effects of both $(+)$ and $(-)$ amplitudes. We would also like to stress that as far as meson-baryon processes are concerned, these sum rules are more general than corresponding results deducible from the pure additivity assumption inasmuch as the present model allows for multiple-scattering effects within the $QQ\bar{Q}$ system.

Another result (found in Sec. 6), which does not depend on the existence of only (+) or only (−) amplitudes, is the density-matrix relation (6.5) for the process $PB \to VB^*$. In this connection, we repeat the remarks made in Sec. 6 that since the relation (6.5) survives the multiple scattering effects within the $QQ\bar{Q}$ system, its validity is again more general than its deduction from a pure additivity model would seem to suggest.

Other results found in the last section depend on additional assumptions, one of which is the dominance of (−) amplitudes. Since even this assumption leaves as many as six free parameters, we have here an opportunity to test the more specific dynamical assumptions of the $QQ\bar{Q}$ model which reduce the number of independent (−) amplitudes still further. In this respect, we have tried to examine separately the effects of the three dynamical assumptions noted in Sec. 2. Now, the two assumptions, viz., (i) dominance of the $Q\bar{Q}$ force over QQ force, and (ii) spin and $SU(3)$-spin independence of the $Q\bar{Q}$ force, which reduce the effective number of (−) amplitudes to *four*, seem to be very consistent with the experimental results. However, the third assumption, viz., a dominance of the s-wave $Q\bar{Q}$ force, does not find support from experiment, since the large value of the ratio $d^{(-)}/f^{(-)}$ which is needed to fit the density-matrix data goes against the spirit of this assumption.

Certain "geometrical" results obtained only with (−) amplitudes are the angular distributions for the density matrices in $PB \to VB^*$ and the sum rule

$$\rho_{3,3}{}^D = \tfrac{3}{8}(1-\rho_{0,0}{}^V),$$

valid for all $SU(3)$ processes of this type in the forward direction $\theta=0$. Indeed, the angular distributions of the density matrices predicted for the processes $\pi p \to \rho N^*$ and $\pi p \to \omega N^*$ not only agree well with experiment, but the agreement looks even better than for the OPE model with absorption.[32] It may be noted that the angular distribution involves only one free parameter α [Eq. (6.16)] for the process $\pi p \to \omega N^*$, while for the process $\pi p \to \rho N^*$ this parameter has a fixed value $\alpha=4$. The zero-angle values of the density matrices are equally satisfactory except for $Kp \to K^*N^*$, a discrepancy also shared by the $SU(6)_W$ results.

The analysis as a whole suggests that certain general features of this model seem to stand experimental testing fairly well. The most important one is the very classification of multiple-scattering effects into two parts, the one within the $QQ\bar{Q}$ system taking precedence over the multiple-scattering effects on the meson by the quark constituents of baryons, by virtue of the assumption that the $Q\bar{Q}$ force is much stronger than QQ force. Another hypothesis, which also survives experimental testing fairly well, is the approximate spin and $SU(3)$ independence of the $Q\bar{Q}$ force. A third feature of the $QQ\bar{Q}$ model, which concerns the form of parametrization of the various amplitudes, suggests that the (−) amplitudes are experimentally preferred over the (+) amplitudes with respect to most processes that can distinguish between their individual effects.

The predictions of this model have many points of similarity with those of the additivity assumption as well as $SU(6)_W$, though the dynamical features are different. Thus both the $QQ\bar{Q}$ model as well as the additivity assumption give rise to the same density-matrix sum rule (6.5). On the other hand, the additivity model of Ref. 7 predicts zero values for certain density-matrix elements for vector-meson production, such as $\rho_{1,0}$, while the $QQ\bar{Q}$ model does not. This feature of the $QQ\bar{Q}$ model facilitates a meaningful comparison with experiment even for nonzero values of θ with quite encouraging results, while the additivity model makes sense for these elements only in the forward direction.

Comparison of the $QQ\bar{Q}$ model with the predictions of $SU(6)_W$ puts the former in a more favorable position in relation to experiment for many $PB \to VB^*$ processes, as can be seen from the details of Sec. 6. However, for "bad" cases like $Kp \to K^*N^*$, while the $SU(6)_W$ model contains a built in mechanism for suitable corrections,[27] the present model cannot answer such questions without further dynamical assumptions on the mechanism of the QQ and $Q\bar{Q}$ forces (e.g., vector-meson exchange).

The $QQ\bar{Q}$ model is, in principle, capable of making predictions on double-density matrices, just as certain authors have done for the additivity model. However, in the latter analysis, the interesting sum rules are those in which the amplitudes of all the related processes $MB \to MB$, $BB \to BB$, and $B\bar{B} \to B\bar{B}$ are involved. The present analysis in terms of only meson-baryon (MB) processes, does not yield any interesting sum rule involving MB processes alone, except for the (uninteresting) $SU(3)$-type relations. Moreover, double density matrices also involve the imaginary parts of the amplitudes which are known to be quite appreciable.[25] This would effectively double the number of parameters compared with the analysis of single density matrices, so that hardly any physical insight would be expected from a study of such matrices, unless a more economical model were considered.

Finally, we should like to mention a different type of $QQ\bar{Q}$ model, which has been suggested by Watson[34] recently, for an understanding of backward scattering in certain reactions. While backward scattering is usually attributed to baryon exchange between a meson and a baryon, this author considers the process to occur as a result of a simultaneous exchange of a diquark and an antiquark between the respective hadrons. This effectively involves a diquark-antiquark scattering process (presumably through a baryon exchange)

[34] P. J. S. Watson, Phys. Letters **25B**, 287 (1967).

occurring between the exchanged constituents of the hadrons. The present $QQ\bar{Q}$ model is of a rather complementary nature, however. Here a quark of the baryon scatters against the full $Q\bar{Q}$ structure of the meson with or without rearrangement, instead of (as in Watson) a diquark of the baryon scattering against the antiquark of the meson. Moreover, our $QQ\bar{Q}$ amplitude is not just approximated by a baryon-exchange term, but analyzed in a very general manner in terms of certain scalar functions which have been called (+) and (−) amplitudes in the text.

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Systematic Renormalization Procedure for the N-Quantum Approximation*

ARTHUR HALPRIN
Department of Physics, University of Delaware, Newark, Delaware

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We reformulate the central idea of Greenberg's N-quantum approximation (NQA) as a power-counting procedure. This allows for a more direct application of the NQA idea. Applied to the Lee model, this reformulation yields results identical to those obtained by Pagnamenta. The procedure is also applied in the lowest two orders to the relativistic $A(x)^3$ interaction. The lowest-order results differ in a nontrivial way from those obtained by Greenberg. The power-counting procedure cannot be applied to the $A(x)^3$ model quite as straightforwardly as to the Lee model. However, with the Lee-model results at hand, the modifications necessary to produce equations that have a finite term-by-term iterative solution suggest themselves quite naturally.

I. INTRODUCTION

THE N-quantum approximation (NQA) is a nonperturbative method for calculating the Heisenberg field operators of a specified quantum field theory.[1,2] It employs an expansion of the Heisenberg field in terms of normal-ordered products of a complete set of asymptotic field operators, wherein the expansion coefficients are essentially vertex functions or scattering amplitudes for the various processes allowed by the specific Lagrangian.[3] In general, there are an infinite number of distinct scattering processes, and the approximation that is made is to retain only a finite number of them in this expansion. Unfortunately, a straightforward truncation leads to difficulties with the conventional renormalization program, and therefore some effects of the omitted terms in the expansion must be recovered. We shall refer to such a recovery operation as NQA renormalization.

Greenberg[1] has investigated the first nontrivial truncation of the $A(x)^3$ interaction and has devised a NQA renormalization scheme for that case. However, only a partial rationale for using that particular procedure is provided; we refer specifically to the "third go-around." Pagnamenta[4] has used a similar NQA renormalization procedure in a treatment of the static Lee model,[5] but a third go-around is not required there because of the simplicity of the model.

In Sec. II, we reformulate the NQA for the Lee model as a quasiperturbative approximation in which the rules for quasipower counting lead automatically to renormalized equations. Such a formulation allows for a greater variety of approximations than those entertained in the straightforward NQA. For example, the two- and three-quantum approximations of Pagnamenta are identical to the third- and fifth-order approximations of the quasiperturbative formulation, respectively. Our main point, however, is the automatic renormalization.

The quasiperturbative approach is applied in Sec. III to the $A(x)^3$ model.[6] In third order, which corresponds to a two-quantum approximation, we obtain a nonlinear integral equation for the vertex function that is renormalized except for a self-energy term. This remaining difficulty is dealt with by imposing a behavior on the mass renormalization counter term

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