Diffractive 2π Production and π-π Interactions

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It is shown that diffractive production reactions of the type $\pi X \to 3\pi X$ may serve as a useful source of information on π-π interactions. An analysis of the available experimental information is made using two simple models for the diffractive production. The results indicate the absence of a strong "π" $\to 3\pi$ vertex with a completely symmetric isospin coupling. This suggests that σ terms do not contribute much more than $(0.3\text{–}0.4)m_\pi^{-1}$ to the a_0 scattering length.

I. INTRODUCTION

An application of the "soft pion method" to low-energy $\pi\pi$ scattering yielded[1] scattering lengths

$$a_0 = 0.2 m_\pi^{-1}, \quad a_2 = -0.06 m_\pi^{-1}, \tag{1}$$

which are significantly smaller than the values suggested by peripheral pion-production experiments. The interpretation of these experiments is quite ambiguous.[2] It is also recognized, however, that the soft-pion calculation might fail if sizable "intrinsic" $I=0$ S-wave interactions were present.[3] If dominated by a "σ resonance," such interactions contribute the term

$$g_\sigma\left(\frac{\delta_{ab}\delta_{cd}}{s-m_\sigma^2}+\frac{\delta_{ac}\delta_{bd}}{t-m_\sigma^2}+\frac{\delta_{ad}\delta_{bd}}{u-m_\sigma^2}\right) \tag{2}$$

to the scattering amplitude $T(\pi_a(q_1)+\pi_b(q_2) \to \pi_c(q_3) + \pi_d(q_4))$, which for large σ masses[4] has the effective form

$$\lambda(\delta_{ab}\delta_{cd}+\delta_{ac}\delta_{bd}+\delta_{ad}\delta_{bc}) \tag{3}$$

in the region of low s, t, u. The term given in Eq. (3) would contribute[1]

$$\delta a_0 = 5\lambda/32\pi m_\pi, \quad \delta a_2 = 2\lambda/32\pi m_\pi \tag{4}$$

to a_0 and to a_2, respectively.

Obviously $g_\sigma \pi\cdot\pi\sigma$ or $\lambda(\pi\cdot\pi)^2$ interactions affect also the unphysical "π" $\to 3\pi$ vertex. In the following, we will try to obtain information on this vertex by studying diffractive 2π production in high-energy reactions[5] of the type

$$\pi^-(q)+X(p) \to \pi^-(q_1)+\pi^-(q_2)+\pi^+(q_3)+X(p') \tag{5}$$

in the "threshold region" $v \equiv (\sum q_i)^2 \approx 9m_\pi^2$.

The concept of diffractive production is rather qualitative. Roughly speaking, it corresponds to the assumption that only momentum and possibly orbital angular momentum, but no internal quantum numbers, are exchanged with the target X, and that in the case of composite targets such as nuclei the amplitudes for production on the individual nucleons add up coherently. There is, as we will see below, considerable freedom in embodying these ideas in specific models which would quantitatively correlate reaction (5) and the "π" $\to 3\pi$ vertex. Nevertheless, for the specific purpose of investigating the possible existence of interactions of the type (2) or (3), reaction (5) has one important advantage. $\pi\pi$ scattering at threshold could proceed even in the complete absence of such "intrinsic" S-wave interaction by "induced" interactions, e.g., ρ exchange in the t and u channel.[6] For reaction (5), on the other hand, a ρ intermediate state would manifest itself as an emission of a real P-wave pion pair, and would be strongly suppressed in the above-mentioned "threshold" region, where all pions are relatively almost at rest.

The rate of reaction (5) depends in that region mainly on the symmetric interactions (2) and (3). We find that sizable interactions of this type [corresponding to $\delta a_0 \approx (\frac{1}{2}-1)m_\pi^{-1}$ in Eq. (4)] are in qualitative contradiction with the experimental data, no matter which particular model for the diffraction production is used, and could therefore be excluded.

II. THEORETICAL ESTIMATES

The amplitude for reaction (5) can be decomposed into noninterfering subamplitudes for production of 3π states with given mass $v^{1/2}$, angular momentum J, parity, isospin I, and symmetry type of isospin coupling. [For diffractive production, parity $=(-1)^{J+1}$, $I=1$.] We distinguish, in particular, $f_{\square\square\square} \equiv f_S$, the part of the production amplitude with the completely symmetric isospin structure such as in Eq. (3).

$$f = f_S + \cdots \text{ (other types of isospin coupling)}$$
$$= \sum_{J=0}^{\infty} (f_S)_J + \cdots. \tag{6}$$

* Work supported in part by the U.S. Air Force Office of Research, Air Research and Development Command under Contract No. AF 49(638)-1545.

[1] S. Weinberg, Phys. Rev. Letters 17, 616 (1967).

[2] V. Hagopian, W. Selove, J. Alitti, J. P. Baton, and M. Neveau-René, Phys. Rev. 145, 1128 (1966).

[3] Such interactions may be necessary in order to explain the η-3π decay and are not excluded by the various successful applications of the soft-pion method. See, e.g., W. A. Bardeen, Lowell S. Brown, B. W. Lee, and H. T. Nieh, Phys. Rev. Letters 18, 1170 (1967).

[4] Experiments indicate that if a σ resonance exists its mass must be large. See, e.g., Ref. 2.

[5] M. L. Good and W. D. Walker, Phys. Rev. 120, 1857 (1960).

[6] The values of a_0 and a_2 obtained from ρ exchange are quite close to those predicted by the soft-pion method. These connections were emphasized, e.g., by J. J. Sakurai, Phys. Rev. Letters 17, 552 (1966). See also R. Rockmore and T. Yao, *ibid.* 18, 501 (1967).

The production cross section separates in a similar fashion:

$$d\sigma_{\pi\to 3\pi}=d\sigma_{\pi\to 3\pi}{}^{S}+\cdots=\sum_J (d\sigma_{\pi\to 3\pi}{}^{S})_J+\cdots. \quad (7)$$

In particular, the contribution of f_S alone yields a *lower bound* for $d\sigma$. We observe that:

(a) $(f_S)_{J=0}$ contains a constant part where all π-π pairs are in a relative S state in the 3π c.m. system. All other parts involve D-wave π-π pairs and are strongly damped in the region of interest, where $v\approx 9m_\pi{}^2$. The constant part, which corresponds to an effective interaction like that in Eq. (3) above, may therefore be dominant.

(b) Since the pion pole (in the "v" channel) contributes to $(f_S)_{J=0}$, Fig. 1(a) may be very important in the threshold region.

We discuss now two models for f_S in the threshold region which incorporate both the basic features of a diffractive production and remarks (a) and (b) above.

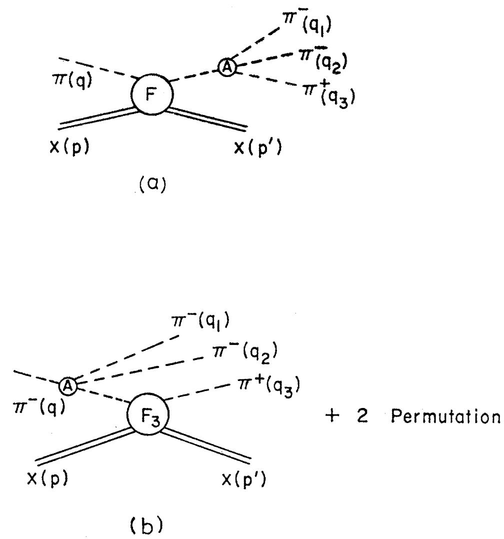

FIG. 1. Pion-pole diagrams.

Model A

Using lab momenta and energies, we have[7] in the diffractive region $[t\equiv(p-p')^2\ll s,\ v\ll s\approx 2M_X q$, where $q=|\mathbf{q}|]$,

$$\begin{aligned} v-m_\pi{}^2&\approx 2q(p_0-p_0'),\\ v_i-m_\pi{}^2&\equiv(q-q_k-q_j)^2-m_\pi{}^2\approx -2q_i(p_0-p_0'). \end{aligned} \quad (8)$$

The direct (v channel) and three-crossed π poles are therefore approached simultaneously. Model A—which is quite similar to the models used to explain the $\rho\pi$ enhancement[7] in reaction (5) for higher values of v—consists simply of the sum of the four pion-pole diagrams illustrated in Fig. 1. The F (F_i) blobs indicate diffractive π-X scattering:

$$F=iqg_\pi(t) \quad [F_i=iq_i g_\pi(t)] \quad (9)$$

and A_S is the part of the "π" $\to 3\pi$ vertex with completely symmetric isospin coupling. Using (8) and (9), we find that the four diagrams combine to

$$f_S=iqg_\pi(t)(A_S-\sum A_{iS})/(v-m_\pi{}^2), \quad (10)$$

where A_S and (A_{iS}) represent the on-shell but unphysical π-3π vertex. The various subenergies $s_{ij}\equiv(q_i+q_i)^2$ satisfy, therefore,

$$\sum s_{ij}=s(\pi^+\pi^-)+s(\pi^+\pi^-)+s(\pi^-\pi^-)=4m_\pi{}^2. \quad (11)$$

At least one s_{ij} has to be continued away from the physical region $s_{ij}\geqslant 4m_\pi{}^2$.[8] If A_S can be represented over the v threshold region by the "effective" constant interaction of Eq. (3), $A_S(\pi^-\to\pi^-\pi^-\pi^+)\approx 2\lambda$, and we have

$$f_S{}^{(A)}=iqg_\pi(t)4\lambda/(v-m_\pi{}^2). \quad (12)$$

Calculating now $d\sigma^S(\pi\to 3\pi)$, we find the following lower bound for $N(v_0,t_0)$, the number of all $\pi X\to 3\pi X$ reactions which are expected to occur in the range $t<t_0$, $9m_\pi{}^2<v<v_0$:

$$N(t_0,v_0)\geqslant\frac{(4\lambda)^2}{2!}N(t_0)(2\pi)^{-6}I(v_0)\times\frac{\int_0^{t_0}dt(d\sigma/dt)(\pi X\to\pi X)}{\int_0^{t_0}dt(d\sigma/dt)(\pi X\to 3\pi X)}, \quad (13)$$

where $N(t_0)=$No. of $\pi X\to 3\pi X$ events with $t<t_0$, v_0 unrestricted,

$$I(v_0)=\int_{9m_\pi{}^2}^{v_0}dv\ \phi_3(v)/(v-m_\pi{}^2)^2 \quad (14)$$

and $\phi_3(v)$ is the covariant 3-meson phase space.

Model B

Model A does not include diagrams in which more than one of the three pions present after the dissociation scatter on the target X [see, e.g., Fig. 2(a)]. These diagrams represent the mutual screening or "shadow effect"[9] of the three pions in the final state. If this

[7] See, e.g,. L. Stodolsky, Phys. Rev. Letters **18**, 973 (1967).

[8] A reasonable procedure motivated by the assumption of weak $I=2$ π-π interactions is to continue only in $s(\pi^-\pi^-)$, leaving $s(\pi^+\pi^-)$ and $s'(\pi^+\pi^-)$ at their actual physical values $[s(\pi^+\pi^-)=(q_1+q_3)^2$, $s'(\pi^+\pi^-)=(q_2+q_3)^2$ in A_S; $s(\pi^+\pi^-)=(q_1+q_3)^2$, $s'(\pi^+\pi^-)=(q-q_1)^2\equiv t_1$ in A_{1S}, etc.]. In principle, Eq. (10) would then yield a lower bound for the rate of reaction (5) for given s, t, v, s_{ij}, and t_i in terms of $|A_S-\sum A_{iS}|^2$, where A and A_i are evaluated at the above-mentioned arguments.

[9] R. J. Glauber, Phys. Rev. **100**, 242 (1955); and in *Lectures in Theoretical Physics*, edited by W. E. Brittin *et al.* (Interscience Publishers, Inc., New York, 1959), Vol. I, p. 315.

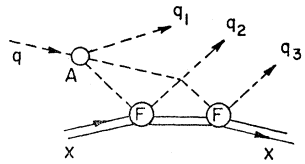

FIG. 2. A shadow-correction diagram.

shadow correction is large, then the three diagrams of Fig. 1(b) do not represent adequately the scattering in the final state and should therefore be replaced by the general "3π"$X \to$ "3π"X diffraction scattering of Fig. 3. In the region of interest, the intermediate and final 3π states will be approximated [remembering remark (a) above] by the unique 0^- state Y which is completely symmetrical and constant in relative $\pi\pi$ momenta, and Eq. (12) will be replaced by

$$f_S = 2iq[g_{\pi X}(t) - g_{YX}(t)]/(v - m_\pi^2). \qquad (15)$$

Using an optical impact parameter representation,[9] we have

$$g_{\pi X}(t) = \pi \int bdb\ J_0(t^{1/2}b)\rho_{\pi X}(b) \qquad (16)$$

and a similar representation with $\rho_{\pi X} \to \rho_{YX}$ is assumed for $g_{YX}(t)$. Model B consists essentially of the estimate

$$\rho_{YX}(b) = 1 - [1 - \rho_{\pi X}(b)]^3, \qquad (17)$$

which is motivated by the following heuristic consideration: The state "Y" represents in configuration space a localized[10] noninteracting 3-pion "clump." The transmission factor $t_{YX}(b) \equiv 1 - \rho_{YX}(b)$ is estimated by $t_{YX}(b) \approx t_{\pi X}(b)^3$, which is appropriate to the independent transmission of all the pions. The model-B expression for f_S is therefore[11]

$$f_S^{(B)}(t) = \left\{ 2\lambda i q_\pi \int bdb\ J_0(t^{1/2}b) \right.$$
$$\left. \times [2\rho_\pi(b) - 3\rho_\pi^2(b) + \rho_\pi^3(b)] \right\} \Big/ (v - m_\pi^2). \qquad (18)$$

We note that in the limit of complete absorption $\rho_\pi(b) = \theta(b_0 - b)$, and $f_S^{(B)}$ vanishes, whereas for almost π-transparent targets [$\rho_\pi(b) \ll 1$] the shadow corrections are negligible, and we recover Eq. (12). In general, $f_S^{(B)}(t) < f_S^{(A)}(t)$, and if model B is used, the lower bound of Eq. (13) is suppressed by the factor

$$r(t_0) = \frac{\int^{t_0} dt [f^{(B)}(t)]^2}{\int^{t_0} dt [f^{(A)}(t)]^2}$$
$$= \frac{|\int bdb [2\rho_\pi(b) - 3\rho_\pi^2(b) + \rho_\pi^3(b)] U(t_0^{1/2}b)|^2}{|\int bdb\ 2\rho_\pi(b) U(t_0^{1/2}b)|^2}, \qquad (19)$$
$$U(t_0^{1/2}b) = \int_0^{t_0} dt\ J_0(t^{1/2}b).$$

Unlike the model-A predictions, Eq. (19) depends on the details of the target shape $\rho_\pi(b)$ which, in principle, can be reconstructed from the data. For $t_0 = 0$ we find the simple relation

$$r(t_0) > [1 - 3(\sigma_{\mathrm{el}}/\sigma_{\mathrm{tot}}) + 2(\sigma_{\mathrm{el}}/\sigma_{\mathrm{tot}})^2]^2. \qquad (20)$$

Equation (20) readily follows from the expression for the elastic and total cross section in an absorptive model,

$$\sigma_{\mathrm{el}} = \tfrac{1}{2}\pi \int bdb\ \rho_\pi^2(b), \quad \sigma_{\mathrm{tot}} = \pi \int bdb\ \rho_\pi(b), \qquad (21)$$

and from the inequality

$$\int bdb\ \rho^3(b) \int bdb\ \rho(b) > \left[\int bdb\ \rho^2(b) \right]^2, \qquad (22)$$

which holds for any positive absorption $\rho(b)$.

Model B represents essentially the infinite sum of all multiple scatterings of the various pions on the target X. However, it still contains $\pi\pi$ interactions (or the effective constant λ) to first order only. We have no simple way of including terms of higher order in λ. (This would essentially amount to modifying our assumption of independent transmission of the three pions.) Note, however, that the actual values of λ considered will be very small. Also, to first order in the diffraction scattering, the effect of the $\pi\pi$ scattering (Fig. 4) will be to change the phase of the amplitude f_S rather than its magnitude. We therefore feel justified in neglecting higher-order $\pi\pi$ interactions, at least in the present crude treatment.

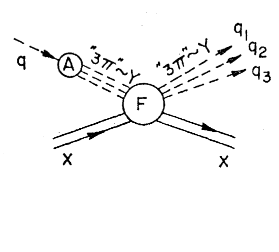

FIG. 3. The general "3π"$X \to$ "3π"X diffractive diagram.

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[10] In an effective-σ model [Eq. (2)], the localization is exact only within $\Delta b \approx m_\sigma^{-1}$, and the resulting corrections of order $(\partial\rho/\partial b) m_\sigma^{-1}/\rho$ are small for nucleon targets and negligible for nuclear targets if $m_\sigma \gtrsim 700$ MeV.

[11] Significant changes resulted when the mutual shadow corrections were taken into account. Although rather heuristic considerations were used, it is suggested that shadow corrections should also not be neglected when constructing models to explain the ρ-π Enhancement as a kinematic effect (see Ref. 7 for a description of such models).

III. COMPARISON WITH EXPERIMENT

We proceed now to compare the bounds on $N(t_0,u_0)$ predicted by models A and B with the experimental information available at present. Reaction (5) was studied at $q_{\rm lab}=16$ GeV/c for C_2F_5Cl targets[12] and $q_{\rm lab}=7$, 8 GeV/c for proton targets.[13,14] There are strong indications for diffractive production in the first case. The differential cross section $d\sigma_{\pi\to3\pi}/dt$ was very sharply peaked at small t, having the form [Ref. 12(a)]

$$\frac{d\sigma_{\pi\to3\pi}}{dt}=\left(\frac{d\sigma_{\pi\to3\pi}}{dt}\right)_{t=0}e^{-80t} \tag{23}$$

typical of diffraction from the nucleus as a whole, and almost half of the events occurred for $|t|<m_\pi^2$.

Also,

$$l\equiv\int_0^{m_\pi^2}\frac{d\sigma_{\pi\to3\pi}}{dt}\bigg/\int\frac{d\sigma_{\pi\to2\pi}}{dt}$$

or

$$\equiv\int_0^{m_\pi^2}\frac{d\sigma_{\pi\to3\pi}}{dt}\bigg/\int\frac{d\sigma_{\pi\to4\pi}}{dt}\approx20.$$

Since $\pi\to2\pi$ and $\pi\to4\pi$ processes cannot proceed via diffraction production, this is strongly suggestive of predominance of this mechanism in the range $t<t_0=0.0225$ GeV^2 used by the experimentalist to define the "coherent sample." For the case of the reaction on nucleons, only $\sim5\%$ of the events had $t<m_\pi^2$, and the ratio l defined above was close to 1.

Thus, there is no clear-cut evidence for the predominance of diffractive production in this case. By identifying all $\pi p\to3\pi p$ events which did not show unmistakable evidence to the contrary (such as the production of the N_{33} resonance) as diffractive events, we are effectively overestimating $N(t_0,v_0)$, making lower bounds like Eq. (13) less effective. Nevertheless, the πN data are quite useful in providing a rough check of our results, and also the detailed information required for estimating the model-B suppression factor of Eq. (19) is available.

Model A: π-Nucleus Data

The number of events in the coherent sample $N(t_0=0.0225$ GeV$^2)$ was ~740 [Ref. 12(c)]. The integrated $\pi\to3\pi$ cross section over this region is ~30 mb per C_2F_5Cl molecule. {The value 40 mb was quoted [Refs. 12(a) and (b)] for $t_0\approx0.08$, and Eq. (23) reduces this value to 30.} The corresponding integrated π-nucleus elastic cross section is estimated[15] to be 600 mb per C_2F_5Cl molecule. Using the above estimates in Eq. (13) and also expressing λ in terms of the more convenient and familiar equivalent scattering length δa_0 of Eq. (4), we find $(m_\pi\equiv1)$

$$\begin{aligned} &N(v_0=16)>70(\delta a_0)^2, && N_{\rm exp}=0;\\ &N(v_0=25)>300(\delta a_0)^2, && N_{\rm exp}=5;\\ &N(v_0=36)>650(\delta a_0)^2, && N_{\rm exp}=17.\end{aligned} \tag{24}$$

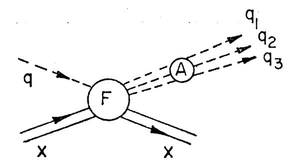

FIG. 4. The diagrammatic representation of the final-state interaction with the diffraction scattering treated to first order.

Thus model A, if taken seriously, would seem to exclude any effective constant interaction that is not bound by

$$\delta a_0<0.12. \tag{25}$$

It would be preferable from the theoretical point of view to restrict the comparison to values of v_0 which are indeed close to threshold $(v_0<16)$. Experimentally, there are no events at all in this region, and such a comparison by itself would be of small statistical significance. Improved statistics would allow us to restrict comparison to $v_0\approx16$, with resulting improved bounds on δa_0. We note also that the variation of $N_{\rm exp}(v_0)$ with v_0 deviates somewhat from that theoretically expected if the inequality (13) is saturated (for example, $N_{\rm exp}(v_0=36)/N_{\rm exp}(v_0=25)\approx3.4$, versus 2.2 expected). This may indicate that the tail of the A_1 or $\rho\pi$ enhancement does contribute[16] at the higher values of v_0. If we could reliably subtract off this contribution, our bounds for δa_0 could improve. This also may be feasible once better data are available.

Model A: π-Nucleon Data

In Ref. 13 only one $\pi N\to3\pi N$ nonisobar event (which corresponds to 8 μb cross section) occurs with

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[12] Orsay-Ecole Polytechnique–Milan–Saclay–Berkeley collaboration, Allard *et al.*, (a) Phys. Letters **12**, 143 (1964); (b) *ibid.* **19**, 431 (1965); (c) Nuovo Cimento **46A**, 737 (1966).

[13] Neal M. Cason, Phys. Rev. **148**, 1282 (1966), $(q_L=7$ GeV/$c)$.

[14] Aachen-Berlin-Cern collaboration, Phys. Letters **20**, 82 (1966), $(q_L=8$ GeV/$c)$.

[15] There is no experimental information on elastic π-nucleus (with $A\approx20$) and at $q_{\rm lab}\approx16$ GeV/c. p-nucleus data available at present [see, e.g., *High Energy Physics and Nuclear Structure*, edited by G. Alexander (North-Holland Publishing Co., Amsterdam, 1967), in particular the talk by B. P. Gregory] imply $\sigma_{\rm tot}(p-C_2F_5Cl)\approx3600$ mb. The corresponding π total cross section is presumably somewhat smaller. $\sigma_{\rm tot}(\pi p)/\sigma_{\rm tot}(pp)\approx0.6$, and because of the shielding effect of the various nucleons we believe it is reasonable to estimate $\sigma_{\rm tot}(\pi-C_2F_5Cl)/\sigma_{\rm tot}(p-C_2F_5Cl)\approx0.8$. Also, $\sigma_{\rm el}(\pi p)/\sigma_{\rm tot}(\pi p)\approx0.2$, assuming again a somewhat more "opaque" nucleus $\sigma_{\rm el}(\pi-C_2F_5Cl)/\sigma_{\rm tot}(\pi-C_2F_5Cl)\approx0.25$ and $\sigma_{\rm el}(\pi-C_2F_5Cl)\approx720$. Using the form $d\sigma/dt=e^{-80t}(d\sigma/dt)_{t=0}$ [t in (GeV/c)^2], we finally arrive at the above estimate for $\int_0^{0.0225}dt[d\sigma_{\rm el}(\pi-C_2F_5Cl)/dt]$.

[16] There is also a possibility that the experimental resolution causes the broadening of the $\rho\pi$ enhancement peak with some resulting "overflow" of events into the region of interest. The apparent $N_{\rm exp}(t_0,u_0)$ may therefore be an overestimate of the number of "genuine" threshold events, so that our bounds on δa_0 could perhaps be made more stringent.

a value of $v_0<42$. Choosing t_0 so that[17]

$$\int^{t_0} \frac{d\sigma_{\mathrm{el}\,\pi p}}{dt} \approx 4 \text{ mb},$$

we find the bound

$$\delta a_0 < 0.3. \tag{26}$$

In Ref. 14, 10 events corresponding to 12.5 μb occurred for $v_0<36$, and instead of (26) we find now

$$\delta a_0 < 0.4. \tag{26$'$}$$

Model B: π-Nucleus Data

The bounds that can be put on δa_0 deteriorate now by the factor $1/\sqrt{r}(t_0)$, where $r(t_0)$ is given in Eq. (19). Unfortunately, there is not enough experimental information to allow a direct calculation of $r(t_0)$. Quite extensive information on proton-nuclei interactions is available[15] and enables us to make a reasonable estimate of $r(t_0)$. If we assume that for the light nuclei considered ($A\approx 20$), a Gaussian form,

$$\rho(b) \approx \rho_0 e^{-b^2/b_0^2}, \tag{27}$$

with $\rho_0 = 1-0.8$ and $b_0 \approx 80$ (GeV)$^{-2}$,[15] is an adequate description of the C_2F_5Cl nuclei for 16-GeV/c pions, then we find $1/\sqrt{r}(t_0)\approx 2.5$–3, and the bound (25) becomes

$$\delta a_0 < 0.3\text{–}0.35. \tag{28}$$

As an independent check we observe that the reasonable ratio[15] $\sigma_{\mathrm{el}}/\sigma_{\mathrm{tot}} = \frac{1}{4}$ yields, in Eq. (20),

$$1/\sqrt{r}(t=0) = 2.5.$$

Model B: π-Nucleon Data

In the π-nucleon case, $r(t_0)$ can in principle be calculated quite accurately. Our estimate yields $1/\sqrt{r}(t=0.0225)=1.5$,[18] and Eqs. (26) and (26′) become

$$\delta a_0 < 0.45, \quad \delta a_0 < 0.6. \tag{29}$$

As emphasized before, the πN data are somewhat less useful for our present analysis, and we believe (28) and (25) to be more reliable estimates than (29) and (26) or (26′). The fact that the results for π-N and N-nucleus do not differ too widely is encouraging.

We have no preference as to which of the two models used above is more reliable, and the differences between (28) and (25) reflect, in our opinion, the theoretical ambiguity involved.[11] However, even if the weaker bound (28) is adopted, significant intrinsic isospin symmetric interactions in the $\pi \to 3\pi$ are excluded.

IV. SUMMARY AND DISCUSSION

We have seen above that information pertaining to the unphysical $\pi \to 3\pi$ vertex may be extracted from diffraction production experiments, and that, in particular, the symmetric part of the amplitude is on the average very weak in the threshold region. Without appealing to specific models, no correlation can be made between this result and the question of the size of π-π scattering lengths. Any model in which the symmetric interaction is slowly varying in the region of low s, t, u would, however, yield such a correlation, and our results would then support small π-π scattering lengths. The σ model with large m_σ, mentioned in the Introduction, and also dispersive models with a large S-wave subtraction constant λ,[19] fall in the above category.

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[17] This choice corresponds to the assumption that ~70% of the elastic πp scattering at 7–8 GeV/c is "diffractive."

[18] This suppression factor is close to the bound $1/\sqrt{r}(t=0) < 1.45$, which follows with $\sigma_{\mathrm{el}}(\pi N)/\sigma_{\mathrm{tot}}(\pi N)\approx 0.2$ from Eq. (20).

[19] G. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).