Tests for $|\Delta I| \leq 1$ in Electromagnetic Interactions†

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We present a brief survey of possible experiments with up to three hadrons to test the hypothesis that $|\Delta I| \leqslant 1$ in electromagnetic interactions. The prime emphasis is upon experiments utilizing electron-positron storage rings. In a note added in proof, the survey is extended to colliding-beam experiments of the type $e^+ + e^- \rightarrow (2n+1)\pi$.

I. INTRODUCTION

As has been noted by Dombey and Kabir[1] and others,[2-8] there is not much experimental evidence as to the isospin transformation properties of the electromagnetic-hadron vertices. One often assumes that

$$|\Delta I| \leqslant 1 \tag{1}$$

based upon meager experimental evidence. It is the purpose of this article to outline the most unambiguous tests of (1).

In order to determine the isospin properties of the electromagnetic interaction as unambiguously as possible one needs experimental data on interactions with a physical photon or with a virtual photon originating from a lepton-antilepton vertex:

$$\gamma \text{ (virtual or real)} \rightarrow \text{assorted hadrons.} \tag{2}$$

Reaction (2) is to be considered as representing the whole set of possibilities formed by crossing the various channels, e.g., reaction (3),

$$\gamma \rightarrow \pi + \rho, \tag{3}$$

is representative of the following types of reactions that might be produced in the laboratory:

$$l + \bar{l} \rightarrow \pi + \rho, \tag{3$'$}$$

$$\rho \rightarrow \pi + l + \bar{l}, \tag{3$''$}$$

$$\rho \rightarrow \pi + \gamma. \tag{3$'''$}$$

Kinematically, one cannot have solely a physical photon in the entrance (exit) channel. Study of reaction (3‴) has previously been suggested by Grishin *et al.*[8]

Throughout this article C conservation will be assumed valid for electromagnetic interactions. If C violations do occur in the reactions discussed below, these violations should be apparent (if there are final-state interactions) in experiments where the reaction and its charge conjugate are separately investigated. If C is conserved, the hadrons must be in an eigenstate of C with

$$C = -1. \tag{4}$$

II. SURVEY OF POSSIBLE HADRON CHANNELS

As hypercharge is conserved in strong and electromagnetic interactions, we know that ΔI must be integer. Therefore, to test for $\Delta I > I$ currents we need to examine hadron systems which are capable of forming isospin states of at least 2. It will be assumed throughout this paper that there exist no useful $I \geqslant 2$ single hadron states.

Case A. All but two of the hadrons individually have zero isospin. We shall consider two subcases: (i) two nucleon isobars and (ii) two $I=1$ particles.

(i) If one of the two particles in question is an $N_{3/2}^*$, the other isobar may have $I=\frac{1}{2}$ or $\frac{3}{2}$:

$$\gamma \rightarrow N_{3/2}^* + \bar{N}_{1/2}^* \text{ or charge conjugate,} \tag{5}$$

$$\gamma \rightarrow N_{3/2}^* + \bar{N}_{3/2}^*. \tag{6}$$

Benayoun[9] has studied reactions (5) and (6) as test of ΔI rules. He finds that for (5) only $I=1$ and 2 states can be formed. If (1) is true, then only $I=1$ is permitted and he finds

$$-F^i(\bar{p}N_+^*) = F^i(\bar{n}N_0^*), \tag{7}$$

where the superscript i indicates the particular coupling or form factor considered, and the parentheses indicate the particles in the final state. Because different members of an isospin multiplet may have slightly different masses, experimental reaction rates are not completely equal for equal magnitudes of the matrix elements. It is useful to define "reduced" reaction rates Γ_r, which are the experimentally observed rates divided by the appropriate phase-space factors.[10] As a notational convenience, only the relevent hadrons and their charge states will be listed within the parentheses immediately following Γ_r, rather than the whole reaction. We then

* National Science Foundation Graduate Fellow.

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[2] P. P. Divakaran, V. Gupta, and G. Rajasekaren, Phys. Rev. **166**, 1792 (1968).

[3] M. Veltman, Phys. Letters **24B**, 587 (1967).

[4] S. Alder, Phys. Rev. Letters **18**, 519 (1967); **18**, 1036(E) (1967).

[5] R. J. Cence *et al.*, Phys. Rev. Letters **19**, 1393 (1967).

[6] C. Baltay *et al.*, Phys. Rev. Letters **19**, 1498 (1967).

[7] G. Shaw, Nucl. Phys. **B3**, 338 (1967).

[8] V. G. Grishin *et al.*, Yadern. Fiz. **4**, 126 (1966) [English transl.: Soviet J. Nucl. Phys. **4**, 90 (1967)].

[9] M. Benayoun, Nuovo Cimento **51A**, 832 (1967).

[10] For a discussion of phase space see, for example, G. Källén, *Elementary Particle Physics* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1964).

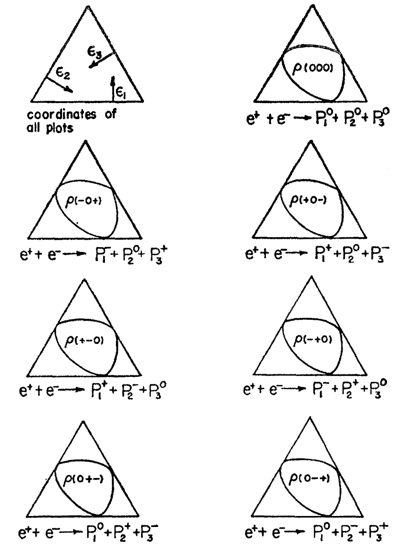

FIG. 1. Dalitz plots of the reaction, $e^+ + e^- \rightarrow P_1 + P_2 + P_3$, where the P_i are isovector particles and their masses satisfy $m_1 \leq m_2 \leq m_3$.

have

$$\Gamma_r(\bar{p}N_+^*) = \Gamma_r(\bar{n}N_0^*) . \tag{8}$$

One variation of reaction (5) is

$$\gamma + d \rightarrow N + N_{3/2}^* . \tag{9}$$

It was suggested as one of the few tests of (1) that would employ conventional techniques (as opposed to colliding-beam experiments) by Dombey and Kabir[1] and by Shaw.[7] One expects to find

$$\Gamma_r(pN_{3/2,0}^*) = \Gamma_r(nN_{3/2+}^*) \tag{9'}$$

if (1) is true and one can experimentally distinguish the $N_{3/2}^*$ resonance from possible πN background events in an $I = \frac{1}{2}$ state. Because the $N_{3/2}^*$ resonance is broad, the background may be large.

For reaction (6) Benayoun has found that the elimination of $I = 2$ and 3 contributions to the final hadron state results in the respective relations[11]:

$$F_{++}^i + F_+^i - F_0^i - F_-^i = 0 , \tag{10}$$

$$F_{++}^i + 3F_+^i + 3F_0^i + F_-^i = 0 . \tag{11}$$

Here we have simplified our notation by subscripting only the charge of the $N_{3/2}^*$. If hypothesis (1) is valid, then (10) and (11) are true simultaneously and we can set up four relations, in each of which one of the charge amplitudes has been eliminated:

$$F_{++}^i + 2F_+^i + F_0^i = 0, \tag{12a}$$

$$F_+^i + 2F_0^i + F_-^i = 0 , \tag{12b}$$

$$F_{++}^i - 3F_0^i - 2F_-^i = 0 , \tag{12c}$$

$$2F_{++}^i + 3F_+^i - F_-^i = 0 . \tag{12d}$$

Constructing triangle inequalities, we obtain a total of four relations, e.g., Eq. (12d) yields

$$|2\sqrt{[\Gamma_r(N_{++}^*\bar{N}_{--}^*)]} - 3\sqrt{[\Gamma_r(N_+^*\bar{N}_-^*)]}| \leqslant \sqrt{[\Gamma_r(N_-^*\bar{N}_+^*)]} \leqslant 2\sqrt{[\Gamma_r(N_{++}^*\bar{N}_{--}^*)]} + 3\sqrt{[\Gamma_r(N_+^*\bar{N}_-^*)]} . \tag{13}$$

The reader can readily verify that we are justified in stating inequalities of the above form without regard to whether the experimental transition rates have been integrated or summed over undetected variables, e.g., helicities and/or additional particles with zero isospin.

(ii) The other groups of interactions of interest in case A are when both of the hadrons in question have $I = 1$. If Eq. (4) is valid, the $I = 2$ state would have negative G parity. Thus we are lead to consider interactions where the total G parity of the hadrons is negative or indeterminate.

If the G parity is negative, e.g., reaction (3), then assumptions (1) and (4) predict that only the $I = 0$ channel can contribute. Consulting a tabulation of Clebsch-Gordan coefficients of this state leads one to predict the following definite ratio of transition:

$$\Gamma_r(\rho^+\pi^-) = \Gamma_r(\rho^0\pi^0) = \Gamma_r(\rho^-\pi^+) . \tag{14}$$

Experiments utilizing reactions (3″) and/or (3‴) apparently would be difficult because of the background superimposed upon the large ρ width.

If the G parity is indeterminate, as, for example, in the reactions

$$\gamma \rightarrow \Sigma + \bar{\Sigma} , \tag{15}$$

$$\gamma \rightarrow \pi + \pi + \Lambda + \bar{\Lambda} , \tag{16}$$

then (1) and (4) imply nothing more than there should be no $I = 2$ component in the final state. Because we have more than one accessible I state, we can only expect to form predictions in the form of inequalities.

The $I = 2$ state formed by two $I = 1$ particles is

$$|I = 2\rangle = (\sqrt{\tfrac{1}{3}})|m_1 = 1, m_2 = -1\rangle - (\sqrt{\tfrac{1}{3}})|0,0\rangle + (\sqrt{\tfrac{1}{3}})|-1,1\rangle . \tag{17}$$

Because the various I states are orthogonal and we require that there be no contribution from the above state, we obtain, for example,

$$F^i(\Sigma^0\bar{\Sigma}^0) = F^i(\Sigma^+\bar{\Sigma}^-) + F^i(\Sigma^-\bar{\Sigma}^+) , \tag{18}$$

from which we can construct the following inequality relation:

$$|\sqrt{[\Gamma_r(\Sigma^+\bar{\Sigma}^-)]} - \sqrt{[\Gamma_r(\Sigma^-\bar{\Sigma}^+)]}| \leqslant \sqrt{[\Gamma_r(\Sigma^0\bar{\Sigma}^0)]} \leqslant \sqrt{[\Gamma_r(\Sigma^+\bar{\Sigma}^-)]} + \sqrt{[\Gamma_r(\Sigma^-\bar{\Sigma}^+)]} . \tag{19}$$

Case B. Three hadrons with $I \neq 0$. The G parity, if it can be defined, can simplify the analysis of an interaction by reducing the number of accessible isospin

[11] M. Benayoun, thesis, University of Paris, 1967 (unpublished). Although analogous expressions appear in Ref. 9, they contain a misprint. We have changed some of the signs to be consistent with the Condon and Shortley sign conventions.

channels. We shall divide the following discussion into three subcases: (i) $G=-1$, (ii) $G=+1$, and (iii) the hadrons are not in an eigenstate of G. By far the most restricted subcase is (i).

(i) Three isovector particles can only be in the unique $I=0$ state if (1) and (4) are true and $G=-1$. This unique state is an isoscalar which is formed by the triple product of the three isovectors. Experimentally, there should be no observed transitions to all neutral hadrons [implied by Eq. (4) alone]; interactions producing charged isovectors will be observed to be independent of the particular charge that each isovector is assigned.

One experimental consequence of these results is that the Dalitz plots of three distinguishable isovector particles produced by electron-positron annihilations will bear the following relationships. When all plots are constructed with their axes parallel for each corresponding isovector particle (see Fig. 1) and experimental events are classified by the charge state of the first two isovectors and placed in the corresponding plot (thus there are seven plots for distinguishable particles), then all plots where two particles are charged will have the same density:

$$\rho(-0+)=\rho(+0-)=\rho(+-0)=\rho(-+0)=\rho(0+-)=\rho(0-+). \quad (20)$$

The plot of neutrals which will have zero density everywhere

$$\rho(000)=0. \quad (21)$$

If we only assume (4), then we still have relation (21) and equal density distributions between plots of charge conjugate reactions:

$$\rho(-0+)=\rho(+0-),\quad \rho(+-0)=\rho(-+0),\quad \rho(0+-)=\rho(0-+). \quad (22)$$

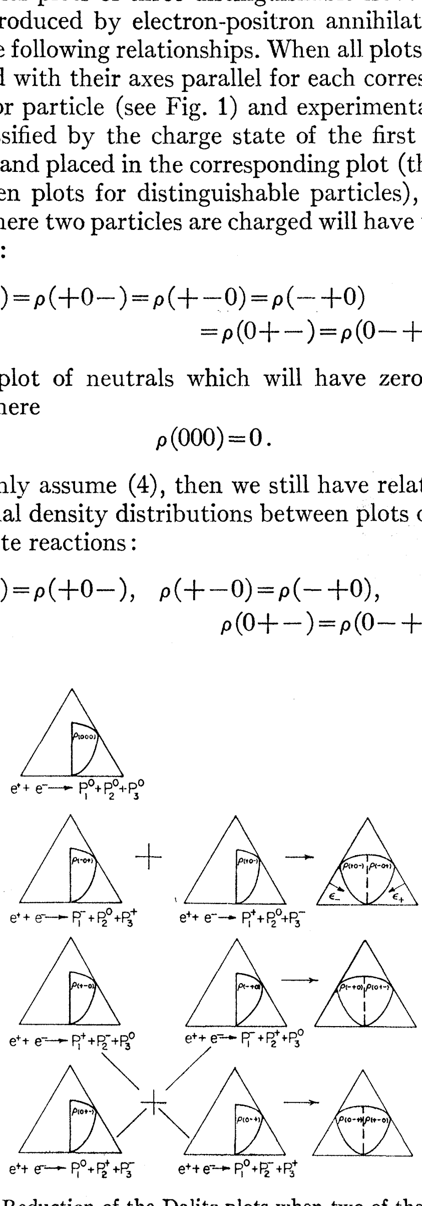

FIG. 2. Reduction of the Dalitz plots when two of the isovector particles (P_1 and P_3) belong to the same isomultiplet. We require $\epsilon_1 \geqslant \epsilon_3$.

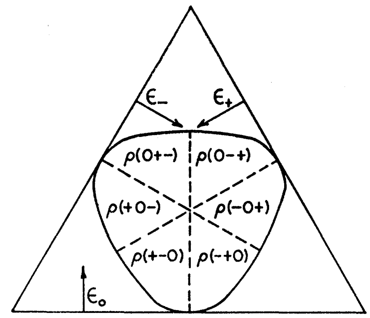

FIG. 3. Reduction of the six charged plots to one when all three isovector particles are from the same isomultiplet. We require $\epsilon_1 \geqslant \epsilon_2 \geqslant \epsilon_3$.

If two of the isovector particles belong to the same isomultiplet, we have seven "half"-plots (see Fig. 2). (We can make the two "indistinguishable" particles "distinguishable" by requiring that the kinetic energy of the first of the two be greater than that of the second.) Naturally, relations (20) and (21) still follow from assuming Eqs. (1) and (4).

Similarly, when all three isovector particles belong to the same isomultiplet, e.g., the 3π case, we have seven "sixth"-plots (Fig. 3); we require that the kinetic energies are ordered.[12] So, for the reaction

$$e^+ + e^- \to 3\pi \quad (23)$$

we do not expect to see any $\pi^+ = \pi^-$ asymmetry if C is conserved [i.e., relation (22)], and there should be no $3\pi^0$ events. If, furthermore, (1) is good, then the density distribution is threefold symmetric about the center of the Dalitz plot [i.e., relation (20)].

It appears that reaction (23) might provide one of the cleanest tests of (1) [and of (4)] in electromagnetic interactions. At moderate energies the Dalitz plot of 3π production will look very much like the decay of the ω meson if (1) and (4) are valid. At higher energies in the center-of-mass system, ρ-meson bands will appear in the plot but the criterion of the threefold symmetry is not affected. Several events of reaction (23) have been seen at 775 MeV.[13] To observe interference effects from an admixture of $|\Delta I|=2$ at the ω resonance, one needs final-state interactions in the $I=2$ channels to shift the phase. The reason is that the relative phases of the form factors governing transitions to $I=0$ and $I=2$ must vanish in the absence of final-state interactions because of the *PCT* theorem. At the ω-resonance energy the phase of the form factor related to the $I=0$ transition will be shifted in phase by 90°. If one can neglect final-state interactions in the $I\neq 0$, 3π channels, then there will be no interference effects appearing in the Dalitz plot due to $|\Delta I|=2$ terms in the electromagnetic interaction, whereas C-violating effects, if they occur, would show large interference, because the form factors of the

[12] T. D. Lee, Phys. Rev. **139**, B1415 (1965).
[13] J. E. Augustin *et al.*, Phys. Rev. Letters **20**, 126 (1968).

C-violating terms, the $I=1$ and 3 ones, would be in phase with the form factor of the resonant $I=1$ channel.

For a detailed analysis of the constraints placed upon the form factors governing various I-state production mechanisms of 3π, the reader is referred to Zemach.[14]

(ii) We now consider the subcase where the three isovectors are in a $+1$ eigenstate of the total G-parity operator. Assuming (4), we find that only three $I=1$ and the unique $I=3$ states are present. If (1) is valid also, then the $I=3$ contribution should vanish. The $I=3$ state is

$$|I=3\rangle=(\sqrt{\tfrac{1}{10}})[|+-0\rangle+|-+0\rangle+|+0-\rangle+|-0+\rangle+|0+-\rangle+|0-+\rangle]+(\sqrt{\tfrac{2}{5}})|000\rangle. \quad (24)$$

Again, using the orthogonality of the various I channels, Eq. (24) implies

$$F_{+-0}{}^i+F_{-+0}{}^i+F_{+0-}{}^i+F_{-0+}{}^i+F_{0+-}{}^i+F_{0-+}{}^i+2F_{000}{}^i=0. \quad (25)$$

Because we have assumed that the final state has $G=+1$, we may reformulate Eq. (25) to obtain

$$F_{000}{}^i+F_{-0+}{}^i+F_{+-0}{}^i+F_{0+-}{}^i=0. \quad (26)$$

Equation (26) in turn yields the following two inequalities based upon the Dalitz plots [relation (22) is implied by assumption (4)]:

$$|\sqrt{[\rho(000)]}-\sqrt{[\rho(-0+)]}|\leqslant\sqrt{[\rho(+-0)]}+\sqrt{[\rho(0+-)]}, \quad (27)$$

$$|\sqrt{[\rho(+-0)]}-\sqrt{[\rho(0+-)]}|\leqslant\sqrt{[\rho(000)]}+\sqrt{[\rho(-0+)]}. \quad (28)$$

(iii) When the final state is not an eigenfunction of the total G-parity operator, we must include in our examination particles which have nonzero strangeness and/or baryon number. As the number of different I channels increases, the number of different reaction rates that need to be experimentally determined and compared increases in order to test (1). There are likely to be severe experimental complications due to background if any of the hadrons are not stable with respect to the strong interactions. Consequently, we shall limit our discussion to the following possible experimental reactions:

$$e^++e^-\rightarrow\Sigma+\bar{\Sigma}+\pi, \quad (29)$$

$$e^++e^-\rightarrow\Sigma+\bar{N}+K, \text{ or charge conjugate}, \quad (30)$$

$$e^++e^-\rightarrow\Xi+\bar{\Xi}+\pi, \quad (31)$$

$$e^++e^-\rightarrow N+\bar{N}+\pi, \quad (32)$$

$$e^++e^-\rightarrow K+\bar{K}+\pi. \quad (33)$$

Reactions (30), (31), (32), and (33) are very similar in their isospin analyses; they consist of two isospinors and one isovector particle. We shall consider all these reactions simultaneously. For definiteness, we shall write out the analysis for reaction (30).

For reaction (30) we find that only the unique $I=2$ state is inconsistent with (1) and (4). Thus we can form only one constraint upon the reaction amplitudes:

$$F^i(\Sigma^+\bar{p}K^0)+\sqrt{2}F^i(\Sigma^0\bar{n}K^0)+\sqrt{2}F^i(\Sigma^0\bar{p}K^+)+F^i(\Sigma^-\bar{n}K^+), \quad (34)$$

which immediately leads to the following inequalities:

$$|\sqrt{[2\Gamma_r(\Sigma^0\bar{n}K^0)]}-\sqrt{[2\Gamma_r(\Sigma^0\bar{p}K^+)]}|\leqslant\sqrt{[\Gamma_r(\Sigma^+\bar{p}K^0)]}+\sqrt{[\Gamma_r(\Sigma^-\bar{n}K^+)]}, \quad (35)$$

$$|\sqrt{[\Gamma_r(\Sigma^+\bar{p}K^0)]}-\sqrt{[\Gamma_r(\Sigma^-\bar{n}K^+)]}|\leqslant\sqrt{[2\Gamma_r(\Sigma^0\bar{n}K^0)]}+\sqrt{[2\Gamma_r(\Sigma^0\bar{p}K^+)]}. \quad (36)$$

In the remaining reactions (31), (32), and (33), we note that the two isospinors are each other's antiparticle. Detailed kinematic analyses of these reactions under assumption (4) may lead to further experimental consequences. In reaction (33) the final state consists of three pseudoscalar particles which are in a $J^P=1^-$ state. The phenomenological analysis is particularly straightforward and we can append to the above inequalities (35) and (36) the additional relation valid for the symmetry axes of the Dalitz plots (i.e., when $p_\pi\cdot p_K=p_\pi\cdot p_{\bar{K}}$):

$$|\sqrt{[\Gamma_r(K^+K^-\pi^0)]}-\sqrt{[\Gamma_r(K_LK_S\pi^0)]}|\leqslant\sqrt{[2\Gamma_r(K^+\bar{K}^0\pi^-)]}=\sqrt{[2\Gamma_r(K^0K^-\pi^+)]}\leqslant\sqrt{[\Gamma_r(K^+K^-\pi^0)]}+\sqrt{[\Gamma_r(K_LK_S\pi^0)]}, \quad (37)$$

because on the symmetry axis the $I_{K\bar{K}}=I=1$ contribution vanishes; the absence of this I channel requires $F(K^+\bar{K}^0\pi^-)=F(K^0K^-\pi^+)$.

Reaction (29) has four accessible channels under assumption (1), viz., the unique $I=0$ and the three $I=1$ states. The absence of the unique $I=3$ state and the two $I=2$ states places three constraints upon the reaction amplitudes:

$$F_{+0-}{}^i+F_{0+-}{}^i+F_{-0+}{}^i+F_{0-+}{}^i+F_{+-0}{}^i+F_{-+0}{}^i+2F_{000}{}^i=0, \quad (38)$$

$$F_{+0-}{}^i+F_{0+-}{}^i-F_{-0+}{}^i-F_{0-+}{}^i=0, \quad (39)$$

$$F_{+0-}{}^i-F_{0+-}{}^i-F_{-0+}{}^i+F_{0-+}{}^i+2F_{+-0}{}^i+2F_{-+0}{}^i=0. \quad (40)$$

Because the reaction has both Σ and $\bar{\Sigma}$, we expect from C conservation to observe that

$$\rho(-0+)=\rho^r(0+-),\quad \rho(+-0)=\rho^r(+-0),\quad \text{and}\quad \rho(+0-)=\rho^r(0-+), \quad (41)$$

where the superscript r indicates that the variables of the first two particles have been interchanged. One may solve the three simultaneous relations (38)–(40) to obtain a constraint where any pair of charge states are absent. In turn one may then write down a set of inequalities associated with this derived constraint. There appears little to recommend reaction (29) as a test of

[14] C. Zemach, Phys. Rev. 133, B1201 (1964).

assumption (1), however, because one needs excellent statistics so that one may compare partial reaction rates within each charge configuration. C conservation guarantees that almost every inequality relation that we form will be trivially satisfied if we use *total* transition rates to each of the seven possible charge configurations; the only relations that would not be trivially satisfied are summarized by

$$|A-C| \leqslant B \ [\text{for } I \neq 2] \tag{42}$$

and

$$A+B+C \geqslant N \ [\text{for } I \neq 3], \tag{43}$$

where

$$A=\surd[\Gamma_{r\text{tot}}(\Sigma^-\bar{\Sigma}^0\pi^+)]=\surd[\Gamma_{r\text{tot}}(\Sigma^0\bar{\Sigma}^+\pi^-)], \tag{44}$$

$$B=\surd[\Gamma_{r\text{tot}}(\Sigma^+\bar{\Sigma}^-\pi^0)]=\surd[\Gamma_{r\text{tot}}(\Sigma^-\bar{\Sigma}^+\pi^0)], \tag{45}$$

$$C=\surd[\Gamma_{r\text{tot}}(\Sigma^+\bar{\Sigma}^0\pi^-)]=\surd[\Gamma_{r\text{tot}}(\Sigma^0\bar{\Sigma}^-\pi^+)], \tag{46}$$

$$N=\surd[\Gamma_{r\text{tot}}(\Sigma^0\bar{\Sigma}^0\pi^0)]. \tag{47}$$

The simultaneous absence of both the $I=2$ and the $I=3$ channels yields no additional inequality relations for the total transition rates.

III. CONCLUSIONS

A survey of various possible reactions that provide direct tests of the $|\Delta I| \leqslant 1$ hypothesis in electromagnetic interactions indicates the great usefulness of definite G-parity states if C is conserved. The one feasible reaction with not more than three hadrons and where there are no difficulties from background (due to the finite width of any of the hadrons) is reaction (23). In an e^+-e^- colliding-beam experiment, the 3π events may be recorded on a single complete Dalitz plot, where the density should have a high degree of symmetry.

Note added in proof. In general, the more hadrons produced, the more complex the analysis. However, in reactions of the form

$$e^+ + e^- \rightarrow (2n+1)\pi \tag{48}$$

one is able to predict certain linear relationships between the reaction rates of various charge configurations by means of Shmuskevich's principle[15] and C conservation [Eq. (4)].

A state composed of an odd number of pions will possess negative G parity and so if Eqs. (1) and (4) are true the state will be an eigenstate of the isospin operator with eigenvalue $I=0$. Thus such a state is "unpolarized" in isospin space.

Shmuskevich's original principle[15] may be briefly stated: In a reaction where the incident beam and the target are isotopically unpolarized each emergent beam is also unpolarized in isospin space. Specifically, if one of the emergent beams is composed of pions, there will be equal numbers of π^+, π^-, and π^0 emerging. More accurately, there will be equal numbers of each type of pion in the limit of equal masses of particles within each isomultiplet. Because the masses are not quite equal in fact, one may "reduce" the experimental rates by assuming that the transition amplitudes are at most slowly varying functions of the 4-momenta and that the major effect of the inequalities of the masses is the unequal phase space available.

To clarify how the above principles lead to definite ratio predictions, we shall provide two examples.

For our first example, let us rephrase reaction (48) to read

$$e^- + e^+ \rightarrow \pi_1 + \pi_2 + \cdots + \pi_{2n+1}, \tag{49}$$

where the subscripted numbers are chosen by the constraint

$$\mathcal{E}_1 \leq \mathcal{E}_2 \leq \cdots \leq \mathcal{E}_{2n+1}, \tag{50}$$

where $\mathcal{E}_i$ is the energy of the ith particle in the center-of-mass system (which would also be the laboratory frame of reference for a colliding-beam experiment). We can write out immediately $2n+1$ relations using Shmuskevich's principle:

$$\Gamma_r(\pi_i=\pi^-)=\Gamma_r(\pi_i=\pi^0)=\Gamma_r(\pi_i=\pi^+), \tag{51}$$

where we mean by $\Gamma_r(\pi_i=\pi^c)$ the reduced reaction rate summed over all charge configurations consistent with the ith pion having a charge c. Naturally, $\Gamma_r(\pi_i=\pi^-)=\Gamma_r(\pi_i=\pi^+)$ is guaranteed by requirement (4) alone.

As our second example, we shall write out a relation among the total (reduced) cross sections σ_j for the production of exactly j π^- mesons in reaction (48). By Shmuskevich's principle the total number of π^- and π^0 produced should be equal. If, in a particular event, j π^- mesons are produced, then there must be exactly j π^+ mesons produced. Hence we expect to find that if $|\Delta I| \leq 1$ and $C=-1$ are true, then

$$\sum_{j=1}^{n}(2n+1-3j)\sigma_j=0.$$

The summation need not include $j=0$ because $\sigma_0=0$ by relation (4).

ACKNOWLEDGMENTS

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[15] I. Shmuskevich, Dokl. Akad. Nauk SSSR **103**, 235 (1955). am indebted to Frank Zerilli for his translation of this reference.