Quark Model and Masses of Baryons and Mesons

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The procedure here, in addition to taking the usual additivity of amplitudes in the quark model, consists of the following: (i) The expectation values of the matrix elements of the potential for the appropriate quark wave function give us the corrections to the mass matrix; (ii) the amplitudes of the matrix elements of the potential can be analytically continued from the quark-quark to the quark-antiquark channel to describe mesons; (iii) the bound-state quark wave functions are assumed to exist and within a multiplet the quark wave functions do not differ widely; (iv) there is a $T_3{}^3$ type of mass breaking in quark-quark space. The results are in good agreement with experimental data.

I. INTRODUCTION

THE quark model[1] has been applied to many problems with a fair degree of success.[2] In the same model, we want to consider mass relations starting from the interaction Hamiltonian

$$V=A(\alpha,\beta\to\alpha,\beta)+B(3,\alpha\to 3,\alpha)+B(\alpha,3\to\alpha,3)+C(\alpha,\beta\to\beta,\alpha)-\tfrac{1}{2}C(\alpha,3\to 3,\alpha)-\tfrac{1}{2}C(3,\alpha\to\alpha,3)+D(\alpha,\beta\to\beta,\alpha)\,. \quad (1)$$

In the above, the repeated Greek letters stand for 1,2,3 (respectively, $\mathcal{P}$, $\mathfrak{N}$, and λ quarks) and are to be summed. The independent amplitudes A, B, C, and D consist really of eight invariant functions with the dynamical factors suppressed. The values of these functions for the appropriate quark wave functions are denoted by a, b, c, and d, respectively.

We next diagonalize V for the three-quark systems. This gives us the $SU(6)$ mass results[3] for the octet and decuplet baryons with the usual $SU(6)$ vectors as eigenstates of V. We then take $\mathfrak{N}$ and λ quarks as equivalent for electromagnetic interactions. This gives us the relations of Glashow and Socolow[4] for the electromagnetic mass differences, excepting two which disagree with experimental results.

The model is extended to include baryons of the 70-plet. We obtain mixing of isotopic-spin multiplets of different $SU(3)$ representations automatically, together with many earlier results.[5–7] The quark-quark amplitudes are then analytically continued to the quark-antiquark channel for consideration of meson masses. The constants a, b, c, and d are now evaluated for the meson wave functions. The mixing of pseudoscalar and vector mesons occurs in a natural way, and with ω-ϕ mixing as input, we get the desired η-X mixing.[8]

It seems that although the present model is lacking in the dynamical details, it is able to give the relations deduced from other models only when they agree with experimental results, and otherwise not.

II. BARYONS

We shall consider here the system of three quarks giving the 56-plet and 70-plet in the nonrelativistic limit obtained as eigenstates of V. The subscripts $\pm$ indicate the spin state of the quark. v is a finite matrix with matrix elements given in terms of a_d, a_e, b_d, b_e, c_d, c_e, d_d, and d_e, where the subscripts d and e correspond to spin-nonflip and spin-flip amplitudes. We note, e.g.,

$$v|\mathcal{P}_+\mathfrak{N}_-\rangle=a_d|\mathcal{P}_+\mathfrak{N}_-\rangle+a_e|\mathcal{P}_-\mathfrak{N}_+\rangle+(c_d+d_d)|\mathfrak{N}_+\mathcal{P}_-\rangle+(c_e+d_e)|\mathfrak{N}_-\mathcal{P}_+\rangle\,. \quad (2)$$

For three-quark states, we have to consider terms like the above for all possible pairs of quarks. Statistics are not taken into consideration, since the unknown wave function suppresses a part of the symmetry properties.

A. 56-Plet

We now obtain the eigenstates of v for the 56-plet. For this purpose, let $s|\alpha_\pm\beta_\pm\gamma_\pm\rangle$ denote symmetrization of the $|\alpha_\pm\beta_\pm\gamma_\pm\rangle$ state with the minimum number of terms.[9] The eigenstates of v are degenerate with respect to rotations in spin or isotopic-spin space. Hence, from the equation

$$v|\mathcal{P}_+\mathcal{P}_+\mathcal{P}_+\rangle=[3(a_d+d_e+c_e)+3(a_e+d_d+c_d)]|\mathcal{P}_+\mathcal{P}_+\mathcal{P}_+\rangle \quad (3)$$

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[1] M. Gell-Mann, Phys. Letters 8, 214 (1964); G. Zweig, CERN report (unpublished).

[2] E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu 2, 105 (1965) [English transl.: JETP Letters 2, 65 (1965)]; H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966); H. J. Lipkin, *ibid.* 16, 1015 (1966); H. J. Lipkin, F. Scheck, and H. Stern, Phys. Rev. 152, 1375 (1966); J. J. Kokkedee and L. Van Hove, Nuovo Cimento 43, 711 (1966); J. L. Friar and J. S. Trefil, *ibid.* 49, 642 (1967).

[3] M. A. B. Bég and V. Singh, Phys. Rev. Letters 13, 418 (1964).

[4] S. L. Glashow and R. Socolow, in *Proceedings of the Seminar in High-Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965), p. 423; H. R. Rubinstein, Phys. Rev. Letters 17, 41 (1966).

[5] I. P. Gyuk and S. F. Tuan, Phys. Rev. Letters 14, 121 (1965).

[6] R. H. Dalitz, in *Proceedings of the International Conference on High-Energy Physics, Berkeley, 1966* (University of California Press, Berkeley, 1967).

[7] D. L. Katyal, V. S. Bhasin, and A. N. Mitra, Phys. Rev. 161, 1546 (1967).

[8] It seems to be required in the quark model that η be very nearly a strange quark-antiquark pair—a result which we obtain here.

[9] J. L. Friar and J. S. Trefil, Ref. 2.

we obtain, with m as the mass of the nonstrange quarks,

$$N^*=3(m+a_d+d_e+c_e)+3(a_e+d_d+c_d). \quad (4)$$

Similarly we obtain, with δm as the excess of mass for the strange quark,

$$Y_1^*=3(m+a_d+d_e+c_e)+3(a_e+d_d+c_d) + (\delta m+2b_d-c_e)+(2b_e-c_d), \quad (5)$$

$$\Xi^*=3(m+a_d+d_e+c_e)+3(a_e+d_d+c_d) +2(\delta m+2b_d-c_e)+2(2b_e-c_d), \quad (6)$$

$$\Omega^-=3(m+\delta m+a_d+d_e+a_e+d_d+2b_d+2b_e). \quad (7)$$

Here the usual decuplet assignment of particles is obtained in finding the eigenstates of v.

We next consider the baryon octet. We find that the states p, Σ^+, Λ, and Ξ^0 of spin component $\frac{1}{2}$, taken as respectively proportional to $s|\mathcal{P}_+\mathcal{P}_-\mathcal{N}_+\rangle-2s|\mathcal{P}_+\mathcal{P}_+\mathcal{N}_-\rangle$, $s|\mathcal{P}_+\mathcal{P}_-\lambda_+\rangle-2s|\mathcal{P}_+\mathcal{P}_+\lambda_-\rangle$, $s|\mathcal{P}_+\mathcal{N}_-\lambda_+\rangle-s|\mathcal{P}_-\mathcal{N}_+\lambda_+\rangle$, and $s|\mathcal{P}_+\lambda_+\lambda_-\rangle-2s|\mathcal{P}_-\lambda_+\lambda_+\rangle$, are eigenstates of v. This gives the respective masses as

$$N=3(m+a_d+d_e+c_e), \quad (8)$$

$$\Sigma=3(m+a_d+d_e+c_e)+(\delta m+2b_d-c_e)-\tfrac{1}{2}(2b_e-c_d), \quad (9)$$

$$\Lambda=3(m+a_d+d_e+c_e) +(\delta m+2b_d-c_e)+\tfrac{1}{2}(2b_e-c_d), \quad (10)$$

$$\Xi=3(m+a_d+d_e+c_e) +2(\delta m+2b_d-c_e)+\tfrac{1}{2}(2b_e-c_d). \quad (11)$$

The above results are equivalent to the $SU(6)$ formula[3]

$$M=M_0+M_1Y+M_2[I(I+1)-\tfrac{1}{4}Y^2]+M_3J(J+1), \quad (12)$$

where

$$\begin{aligned} M_0&=3(m+a_d+d_e+c_e)-\tfrac{3}{4}(a_e+d_d+c_d) +(\delta m+2b_d-c_e)+\tfrac{1}{8}(2b_e-c_d),\\ M_1&=-(\delta m+2b_d-c_e)-\tfrac{1}{4}(2b_e-c_d),\\ M_2&=-\tfrac{1}{2}(2b_e-c_d),\\ M_3&=(a_e+d_d+c_d)+\tfrac{1}{2}(2b_e-c_d). \end{aligned} \quad (13)$$

The above include the results for octet and decuplet baryons deduced by Federman *et al.*[10] under different assumptions. Further, the relationship they require for equal spacing of decuplets is true here, since we have taken the $T_3{}^3$ type of mass-breaking term. The model presented here seems to generate broken $SU(6)$ symmetry.

The electromagnetic corrections of the 56-plet are next calculated. For the $(\mathcal{N},\lambda)$ doublet we introduce, similarly to what has been done before, the matrix elements w_{dd}, w_{ed}, w_{de}, and w_{ee}, where the first and second subscripts stand for spin and unitary spin, respectively. We also take the matrix elements $w_{ij}{}^{(1)}$ and $w_i{}^{(2)}$, where the superscript indicates the number of $\mathcal{P}$ quarks in the interaction. The electromagnetic corrections to the masses of the particles in Eqs. (4)–(11) are given in the Appendix. These give the following relations,[4] many of which are $SU(6)$ results, and are compared with the available experimental data.[11]

$SU(3)$ results:

$$6.5\simeq\Xi^--\Xi^0=\Sigma^--\Sigma^+-n+p\simeq 6.88\text{ MeV}, \quad (14)$$

$$M_{\text{tr}}(\Lambda,\Sigma^0)=\tfrac{1}{6}\sqrt{3}[w(\Sigma^0)+3w(\Lambda) -2w(n)-2w(\Xi^0)]. \quad (15)$$

$SU(6)$ results:

$$N^{*0}-N^{*+}=Y_1{}^{*0}-Y_1{}^{*+}=n-p\simeq 1.3\text{ MeV}, \quad (16)$$

$$N^{*++}+N^{*0}-2N^{*+}=Y_1{}^{*+}+Y_1{}^{*-}-2Y_1{}^{*0} =\Sigma^++\Sigma^--2\Sigma^0\simeq 1.7\text{ MeV}, \quad (17)$$

$$7.9\pm6.8\simeq N^{*-}-N^{*++}=3(n-p)\simeq 3.9\text{ MeV}, \quad (18)$$

$$5.8\pm3.9\simeq Y_1{}^{*-}-Y_1{}^{*+} =\Sigma^++\Sigma^--2\Sigma^0+2(n-p)\simeq 4.3\text{ MeV}, \quad (19)$$

$$0.45\pm0.85\simeq N^{*0}-N^{*++} =2(n-p)-(\Sigma^++\Sigma^--2\Sigma^0)\simeq 0.9\text{ MeV}, \quad (19')$$

$$4.9\pm3\simeq\Xi^{*-}-\Xi^{*0}=N^{*-}-N^{*0} \quad (20)$$

$$=n-p+\Sigma^++\Sigma^--2\Sigma^0\simeq 3.0\text{ MeV}, \quad (20')$$

$$8.5\pm3.7\simeq(5/3)(\Xi^{*-}-\Xi^{*0})+\tfrac{2}{3}(N^{*0}-N^{*++}) =\Sigma^--\Sigma^+-2(\Sigma^0-\Sigma^+)+3(n-p)\simeq 5.7\text{ MeV}. \quad (21)$$

[See Ref. 12 for Eq. (21).]

The two $SU(6)$ relations[4] $\Sigma^0-\Sigma^+=n-p$ and $\Xi^--\Xi^0=\Sigma^--\Sigma^0$ are not obtained, and they are also not well satisfied. Equation (17) may be compared with the results of Biswas *et al.*,[13] who obtain the values 4.79 and 5.7 MeV as compared to the value 1.7 MeV. The experimental value is very uncertain and is compatible with both estimates.

Furthermore, we get the relation

$$3\Lambda+\Sigma^++\Sigma^--\Sigma^0=n+p+\Xi^-+\Xi^0 \quad (22)$$

as true, including the electromagnetic corrections. But we note that (22) is no improvement over merely averaging the masses of electromagnetic multiplets for comparison with the strong-interaction results. The inaccuracy here is much larger than in Eqs. (14)–(21), where the errors in the strong interactions cancel.

B. 70-Plet

Now we shall consider the **70** representation to accommodate more resonances. We avoid a possible mixing between the two **70** representations in the product space **6×6×6** by taking the projection operators P and P' which describe orthogonal spaces. We

[10] P. Federman, H. R. Rubinstein, and I. Talmi, Phys. Rev. Letters 22, 208 (1965).

[11] A. H. Rosenfeld, A. Barbaro-Galtieri, W. J. Podolsky, L. R. Price, M. Roos, and W. J. Willis, Rev. Mod. Phys. 39, 1 (1967).

[12] P. D. De Souza and D. B. Lichtenberg, Phys. Rev. 161, 1513 (1967).

[13] S. N. Biswas, S. K. Bose, K. Datta, J. Dhar, Yu. V. Novozhilov, and R. P. Saxena, Trieste Report, 1967 (unpublished).

take[14]

$$P=\tfrac{1}{2}[e+(12)]-\tfrac{1}{6}\sum R,$$
$$P'=\tfrac{1}{2}[e-(12)]-\tfrac{1}{6}\sum \delta_R R, \tag{23}$$

where e is the identity, (12) is a permutation between indices 1 and 2, and R is any permutation of the three indices. Here,

$$\mathbf{70}=(4,8)+(2,10)+(2,8)+(2,1). \tag{24}$$

There will be no mixing of the first multiplet on the right-hand side of (4) with any other, but the corresponding isotopic multiplets with spin $\frac{1}{2}$ will get mixed. The mass eigenvalues of V for the different multiplets are calculated. This gives us the corresponding masses, with the notation of Ref. 5, as

$$N_\gamma=3(m+a_d+a_e), \tag{25}$$

$$\Sigma_\gamma=3(m+a_d+a_e)+\delta m+2(b_d+b_e)+\tfrac{1}{2}(c_d+c_e), \tag{26}$$

$$\Lambda_\gamma=3(m+a_d+a_e)+\delta m+2(b_d+b_e)-\tfrac{1}{2}(c_d+c_e), \tag{27}$$

$$\Xi_\gamma=3(m+a_d+a_e)+2\delta m+4(b_d+b_e)-\tfrac{1}{2}(c_d+c_e), \tag{28}$$

$$\bar{N}=3(m+a_d), \tag{29}$$

$$\bar{N}^*=3(m+a_d+c_d+d_d), \tag{30}$$

$$\bar{\Omega}=3(m+\delta m+a_d+2b_d+d_d), \tag{31}$$

$$\bar{\Sigma},\ \bar{Y}_1^*=3(m+a_d)+\delta m+2b_d+(5/4)c_d+\tfrac{3}{2}d_d \mp[(b_e-\tfrac{1}{2}c_e)^2+(9/4)(d_d+\tfrac{1}{2}c_d)^2]^{1/2}, \tag{32}$$

$$\bar{\Xi},\bar{\Xi}^*=3(m+a_d)+2\delta m+4b_d+\tfrac{1}{4}c_d+\tfrac{3}{2}d_d \mp[(b_e-\tfrac{1}{2}c_e)^2+(9/4)(d_d+\tfrac{1}{2}c_d)^2]^{1/2}, \tag{33}$$

$$\bar{\Lambda},\Lambda'=3(m+a_d)+\delta m+2b_d-(5/4)c_d-\tfrac{3}{2}d_d \mp[(b_e+\tfrac{1}{2}c_e)^2+(9/4)(d_d+\tfrac{1}{2}c_d)^2]^{1/2}. \tag{34}$$

We now consider a comparison of the above results with experimental data.[11] We find that if we only make use of resonances belonging probably to $\mathbf{70}^-$, the constants are more or less the same as calculated from $\mathbf{56}^+$. Therefore we have considered both $\mathbf{56}^+$ and $\mathbf{70}^-$ with the same constants, and find that there is a reasonable fit with the known J^P, including a few predictions of $J^P=\frac{1}{2}^-$ where these are not positively determined. This agreement is surprising, since the wave functions for $\mathbf{56}^+$ and $\mathbf{70}^-$ have opposite parities and hence must be different.

We collect together in Table I the predicted and experimental masses of $\mathbf{56}^+$ and $\mathbf{70}^-$ with an average value of the constants taken in many places where they can be determined in more than one way. The constants are taken as $m+a_d=483$ MeV, $a_e=25$ MeV, $\frac{1}{2}\delta m+b_d=103$ MeV, $b_e=-46$ MeV, $c_d=-18$ MeV, $c_e=-22$ MeV, $d_d=93$ MeV, and $d_e=-148$ MeV.

TABLE I. Baryon resonances.

Particle, resonance	I	Predicted spin	Assigned spin, parity	Predicted mass (MeV)	Expt. mass (MeV)
N	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}^+$	939	939
Σ	1	$\frac{1}{2}$	$\frac{1}{2}^+$	1204	1193
Λ	0	$\frac{1}{2}$	$\frac{1}{2}^+$	1130	1115
Ξ	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}^+$	1358	1317
N^*	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{3}{2}^+$	1239	1236
Y_1^*	1	$\frac{3}{2}$	$\frac{3}{2}^+$	1393	1385
Ξ^*	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}^+$	1547	1530
Ω^-	0	$\frac{3}{2}$	$\frac{3}{2}^+$	1701	1680
N_γ	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}^-$	1524	1525
Σ_γ	1	$\frac{3}{2}$	$\frac{3}{2}^-$	1618	1660
Λ_γ	0	$\frac{3}{2}$	$\frac{3}{2}^-$	1658	1700
Ξ_γ	$\frac{1}{2}$	$\frac{3}{2}$	$\cdots$	1772	1810
$\bar{N}^*$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}^-$	1674	1670
$\bar{\Lambda}$, Λ'	0	$\frac{1}{2}$	$\frac{1}{2}^-$	1400, 1676	1405, 1670
$\bar{N}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}^-$	1449	1570($\Gamma=130$)
$\bar{Y}_1^*$, $\bar{\Sigma}$	1	$\frac{1}{2}$	$\cdots$	1903, 1641	1915, 1680
$\bar{\Xi}^*$, $\bar{\Xi}$	$\frac{1}{2}$	$\frac{1}{2}$	$\cdots$	2127, 1865	2270, 1935
$\bar{\Omega}$	0	$\frac{1}{2}$	$\cdots$	2346	(?)

If $\Xi_\gamma(1810)$ has the required J^P, then there are 15 particles of $\mathbf{56}^+$ and $\mathbf{70}^-$ which have the desired spin-parity assignments, and thus seven of them may be regarded as verifications. The scheme as extended also requires the remaining resonances to have the predicted J^P assignments. We have included a (1680) resonance[15] as different from 1660 and 1780.

With the mass formulas (25)–(34), the mass relations due to Gyuk and Tuan[5] can be derived, except for their Eq. (4), which seems to be wrong. The model thus generates results of a broken $SU(6)$ symmetry with predicted mixing of different isotopic-spin multiplets. We may also note that we obtain Eqs. (3.9)–(3.11) of Ref. 7 between $\mathbf{56}^+$ and $\mathbf{70}^-$ in our model. The advantage of this model over the Dalitz[6] model and a subsequent development[7] is that there are no low-lying resonances. On the other hand, our model does not explain the parity of $\mathbf{70}^-$. This, together with the equality of constants for $\mathbf{56}^+$ and $\mathbf{70}^-$, seems surprising. However, it can be understood if the forces have extremely short range or the quarks have extremely small momenta. This model (including the results of the next section) is at present in concord with all experimental facts, and in this sense seems to be better than all the earlier models.[4–7]

III. MESONS

In Eq. (1) we have described the quark-quark channel in terms of the eight invariant functions A, B, C, and D. We now describe the quark-antiquark channel by analytic continuation of these functions for specific spin and unitary-spin states.

[14] M. Hamermesh, *Group Theory* (Addison-Wesley Publishing Co., Inc., Reading, Mass., 1962).

[15] M. Derrik, T. Fields, J. Loken, R. Ammar, R. E. P. Davis, W. Kropac, J. Mott, and F. Schweingruber, Phys. Rev. Letters **18**, 266 (1967).

In order to write down the matrix V for the crossed channel, we start from the part of V that describes only strange quarks, given as

$$V_2=(A_d+2B_d+D_d)(|\lambda_+\lambda_+\rangle\langle\lambda_+\lambda_+| + |\lambda_+\lambda_-\rangle\langle\lambda_+\lambda_-| + (+\rightleftarrows -)) + (A_e+2B_e+D_e)(|\lambda_+\lambda_+\rangle\langle\lambda_+\lambda_+| + |\lambda_+\lambda_-\rangle\langle\lambda_-\lambda_+| + (+\rightleftarrows -)).$$

Explicitly, the crossed channel considered is $1,2\rightarrow 3,4$ to $1,\bar{4}\rightarrow 3,\bar{2}$. This equation in the crossed channel becomes[13]

$$V_2=(A_d+2B_d+D_d)(|\lambda_+\bar{\lambda}_-\rangle\langle\lambda_+\bar{\lambda}_-| + |\lambda_+\bar{\lambda}_+\rangle\langle\lambda_+\bar{\lambda}_+| + (+\rightleftarrows -)) + (A_e+2B_e+D_e)(|\lambda_+\bar{\lambda}_-\rangle\langle\lambda_+\bar{\lambda}_-| - |\lambda_+\bar{\lambda}_-\rangle\langle\lambda_-\bar{\lambda}_+| + (+\rightleftarrows -)).$$

We take the expectation values in the above equation with meson quark-antiquark wave functions, and thus obtain the finite matrix v_2 as

$$v_2=(a_d+2b_d+d_d)P_t(\lambda\bar{\lambda}) + [2(a_e+2b_e+d_e)+(a_d+d_d+2b_d)]P_s(\lambda\bar{\lambda}), \quad (35)$$

where $P_t(\lambda\bar{\lambda})$ and $P_s(\lambda\bar{\lambda})$ are, respectively, the spin triplet and singlet projection operators for the $(\lambda\bar{\lambda})$ system.

We next consider the matrix V_1 consisting of a strange quark and a nonstrange quark. As before, this gives in the crossed channel

$$v_1=(a_d+b_d)(P_{K^*}+P_{\bar{K}^*})+[2(a_e+b_e)+(a_d+b_d)] \times(P_K+P_{\bar{K}})+\tfrac{1}{2}\sqrt{2}(c_d+2d_d) \times[\tfrac{1}{2}\sqrt{2}(|\mathcal{P}_+\bar{\mathcal{P}}_+\rangle - |\mathcal{N}_+\bar{\mathcal{N}}_+\rangle)\langle\lambda_+\bar{\lambda}_+| + \cdots] +\sqrt{2}(c_d+2d_d+2d_e+c_e) \times[\tfrac{1}{2}(|\mathcal{P}_+\bar{\mathcal{P}}_-\rangle - |\mathcal{N}_+\bar{\mathcal{N}}_-\rangle - |\mathcal{P}_-\bar{\mathcal{P}}_+\rangle + |\mathcal{N}_-\bar{\mathcal{N}}_+\rangle)\tfrac{1}{2}\sqrt{2}(\langle\lambda_+\bar{\lambda}_-| - \langle\lambda_-\bar{\lambda}_+|)+\text{H.c.}]. \quad (36)$$

In Eq. (36) P_{K^*} is the projection operator for the K^* subspace, and similarly for the operators $P_{\bar{K}^*}$, P_K, and $P_{\bar{K}}$. We note that the last two terms in Eq. (36) describe the coupling between nonstrange quark-antiquark states and strange quark-antiquark states.

Similarly, for the nonstrange quarks we obtain in the crossed channel

$$v_0=a_dP_{tt}+(a_d+2c_d+2d_d)P_{ts}+(a_d+2a_e)P_{st} + (a_d+2a_e+2c_d+4c_e+2d_d+4d_e)P_{ss}, \quad (37)$$

where P_{tt}, P_{ts}, P_{st}, and P_{ss} stand, respectively, for the spin-triplet–isospin-triplet, spin-triplet–isospin-singlet spin-singlet–isospin-triplet, and spin-singlet–isospin-singlet projection operators of the nonstrange quark-antiquark system.

We now obtain from Eqs. (36) and (37),

$$\rho=(2m+a_d), \quad (38)$$

$$\pi=(2m+a_d)+2a_e, \quad (39)$$

$$K^*=(2m+a_d)+(\delta m+b_d), \quad (40)$$

and

$$K=(2m+a_d)+(\delta m+b_d)+2(a_e+b_e). \quad (41)$$

Further, Eq. (36) shows that spin-triplet states of a strange quark-antiquark pair do not mix with the nonstrange isosinglet provided

$$c_d+2d_d=0. \quad (42)$$

When this is so, Eqs. (35) and (37) give us

$$\phi=(2m+a_d)+2(\delta m+b_d)+d_d \quad (43)$$

and

$$\omega=(2m+a_d)+c_d. \quad (44)$$

In $SU(3)$ space this gives us the desired ω-ϕ mixing. Also by (40), (43), and (44) we now get

$$\tfrac{1}{2}(\omega+\rho)+\phi=2K^*, \quad (45)$$

which is well satisfied.

We next consider the mixing of $(\lambda\bar{\lambda})(s)$, the strange quark-antiquark pair spin singlet, and $(q\bar{q})(ss)$, the nonstrange quark-antiquark–spin-isospin singlet. The mass matrix in this subspace is now given as

$$[2K-\pi-\tfrac{1}{2}(\omega-\rho)+2d_e]|(\lambda\bar{\lambda})(s)\rangle\langle(\lambda\bar{\lambda})(s)| +\sqrt{2}(c_e+2d_e)[|(\lambda\bar{\lambda})(s)\rangle\langle(q\bar{q})(ss)|+\text{H.c.}] +[\pi+\tfrac{1}{2}(\omega-\rho)+4c_e]|(q\bar{q})(ss)\rangle\langle(q\bar{q})(ss)|. \quad (46)$$

Equation (46) gives the two mass states $|\eta\rangle$ and $|X\rangle$. Taking these masses as eigenvalues, we obtain in this channel

$$d_e=2 \quad \text{or} \quad -135\ \text{MeV}. \quad (47)$$

When we put

$$|X\rangle=\cos\theta|(q\bar{q})(ss)\rangle+\sin\theta|(\lambda\bar{\lambda})(s)\rangle$$

and

$$|\eta\rangle=\cos\theta|(\lambda\bar{\lambda})(s)\rangle-\sin\theta|(q\bar{q})(ss)\rangle, \quad (48)$$

Eq. (47) gives us

$$\tan\theta=1.6 \quad \text{or} \quad 0.3. \quad (49)$$

The first value in (49) gives the mixing angle of 22° between the $SU(3)$ octet and singlet mesons, as could have been anticipated for a linear mass formula.[16] However, the second mixing angle gives η as almost completely $(\lambda\bar{\lambda})$. This conclusion has already been found necessary to explain the production[9] of η and π^0, pair annihilation,[17] and the electromagnetic decay.[18]

In Eq. (47), if $d_e=2$ MeV, then since $d_d=-\tfrac{1}{2}(\omega-\rho)$ is also small, it is more likely that D is zero in this channel. If so, we get equality of the ω and ρ masses and the interesting mass relation

$$\eta X=(\eta+X-2K+\pi)(2K-\pi)-\tfrac{1}{8}(\eta+X-2K)^2, \quad (50)$$

[16] G. Alexander, H. J. Lipkin, and F. Scheck, Phys. Rev. Letters 17, 412 (1966).

[17] M. Elitzur, H. R. Rubinstein, and H. Stern, Phys. Rev. Letters 17, 420 (1966).

[18] D. G. Sutherland, CERN Report No. Th. 761, 1967 (unpublished); F. A. Berends, A. Donnachie, and G. C. Oades, CERN Report No. Th. 792, 1967 (unpublished).

which is well satisfied. For reasons already mentioned, however, $\tan\theta=0.3$ seems more reasonable to us.

APPENDIX

The following are the electromagnetic corrections to the masses of the baryons:

$$w(p)=2(w_{dd}^{(1)}+w_{ee}^{(1)})-(w_{de}^{(1)}+w_{ed}^{(1)})+(w_d^{(2)}+w_e^{(2)}),$$

$$w(n)=2(w_{dd}^{(1)}+w_{ee}^{(1)})-(w_{de}^{(1)}+w_{ed}^{(1)})+(w_{dd}+w_{ee})+(w_{de}+w_{ed}),$$

$$w(\Sigma^+)=w(p),$$

$$w(\Sigma^0)=2(w_{dd}^{(1)}+w_{ee}^{(1)})+(w_{dd}+w_{ee})-\tfrac{1}{2}(w_{de}+w_{ed})+\tfrac{1}{2}(w_{ed}^{(1)}+w_{ed}^{(1)}),$$

$$w(\Sigma^-)=3(w_{dd}+w_{ee}),$$

$$w(\Lambda)=2(w_{dd}^{(1)}+w_{ee}^{(1)})+(w_{dd}+w_{ee})+\tfrac{1}{2}(w_{de}+w_{ed})-\tfrac{1}{2}(w_{de}^{(1)}+w_{ed}^{(1)}),$$

$$w(\Lambda,\Sigma^0)=\tfrac{1}{2}\sqrt{3}[(w_{de}^{(1)}+w_{ed}^{(1)})-(w_{de}+w_{ed})],$$

$$w(\Xi^-)=3(w_{dd}+w_{ee})=w(\Sigma^-),$$

$$w(\Xi^0)=w(n),$$

$$w(N^{*++})=3(w_d^{(2)}+w_e^{(2)}),$$

$$w(N^{*+})=2(w_{dd}^{(1)}+w_{ee}^{(1)})+(w_d^{(2)}+w_e^{(2)})+2(w_{de}^{(1)}+w_{ed}^{(1)}),$$

$$w(N^{*0})=2(w_{dd}^{(1)}+w_{ee}^{(1)})+2(w_{de}^{(1)}+w_{ed}^{(1)})+(w_{dd}+w_{ee}+w_{de}+w_{ed}),$$

$$w(N^{*-})=3(w_{dd}+w_{ee}+w_{de}+w_{ed}),$$

$$w(Y_1^{*+})=w(N^{*+}),$$

$$w(Y_1^{*0})=w(N^{*0})=w(\Xi^{*0}),$$

$$w(Y_1^{*-})=w(N^{*-})=w(\Xi^{*-})=w(\Omega^-).$$

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Local Field Theory and Isospin Invariance. III. Interactions of Self-Conjugate Isofermion Fields*

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Interactions involving self-conjugate fields of arbitrary spin and half-integral isospin are studied. (The abbreviation SCIF is used for "self-conjugate isofermion.") In the absence of interactions, SCIF theories are nonlocal but relativistically covariant. In the presence of interactions, the covariance is lost because the interaction Hamiltonian density fails to commute with itself at spacelike separations. Typical interactions involving SCIF's either contain no pair (creation and destruction) terms or only pair terms. Thus, in general, crossing symmetry is lost. A model is exhibited in which the "invariant" scattering amplitude is not a Lorentz-invariant function of Lorentz scalars. In a crossed channel the scattering amplitude vanishes identically. Strong, electromagnetic, and weak interactions of SCIF's are studied with emphasis on the experimental properties of such objects. SCIF's can only interact in pairs with photons and normal hadrons because of selection rules, independently of the proportionality factor between charge Q and isospin component I_3. A SCIF is stable under strong and electromagnetic interactions. SCIF's can be produced in pairs through strong and electromagnetic interactions, the latter only if aided by strong interactions. If the SCIF charges are half-integral, the current-current weak interaction can lead to SCIF pair production with the aid of the strong interactions, when the SCIF field contributes to the weak current. The possibility of the W meson being a SCIF is investigated. Identifying $W^{\pm}$ with the $I_3=\pm\frac{1}{2}$, $Q=\pm1$ members of a SCIF isospin doublet, we have two distinct types of coupling. The charged current J^+ can be coupled to (W^+, W^{+*}) or to (W^{-*}, W^-) ($W^{+*}\not\equiv W^-$ for SCIF theories). Neither $J\cdot W$ coupling is CPT invariant but the effective $J\cdot J$ coupling is CP, T, and CPT invariant if only one of the possibilities is employed. (We specialize to the case of a T-invariant theory.) If instead we couple one current to (W^+, W^{+*}) and another to (W^{-*}, W^-), the effective current-current interaction involving cross terms is CP-noninvariant. This CP-violating interaction is noncovariant and involves an energy dependence roughly of the form $\Delta E/m_W$, where ΔE is the mean energy transferred by the currents.

I. INTRODUCTION

THE principle of microscopic causality (sometimes called local commutativity) is one of the key postulates of modern field theory.[1] Since the precise mathematical statement of this postulate involves regions of space-time not yet accessible experimentally it is of interest to examine possible consequences of the violation of the principle. An interesting set of theories violating the principle, yet not artificially constructed by the introduction of nonlocal form factors, is made

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[1] R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics and All That* (W. A. Benjamin, Inc., New York, 1964), Chap. 3.