Electromagnetic Mass Differences and Regge Phenomenology

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(Received 22 March 1968)

We examine the electromagnetic mass differences of hadrons by a phenomenological analysis of the forward virtual Compton amplitude. In addition to the usual elastic contribution, we consider in detail the contribution from A_2^0 Regge exchange for $I=1$ mass differences and possible contributions from non-Regge behavior for this weak amplitude in the asymptotic region. In the approximation of neglecting the inelastic resonant spectrum in Compton scattering, the contribution of the pure Regge exchange to the subtraction term for any $I=1$ mass difference can be related to a ratio of the A_2^0 residue functions (hence independent of the target) and the relevant elastic form factors. This analysis suggests that a large Regge contribution requires comparable longitudinal and transverse electroproduction cross sections at high energy for small spacelike virtual photon mass. The tadpole model is also examined from the observation that certain linear combinations of $I=1$ mass differences with just the A_2^0 Regge pole removed can be formulated in terms of unsubtracted dispersion relations and are presumably satisfied by retaining just the elastic contribution. The F/D ratio that we calculate in this way for the tensor-meson couplings to baryons agrees with that obtained from high-energy experiments. From a superconvergence relation it is argued that there is a fixed pole in the helicity-flip crossed-channel Compton amplitude at $J=0$, $I=1$, the experimental consequences of which are discussed.

I. INTRODUCTION

In this paper, we analyze the electromagnetic mass differences in hadron multiplets according to the Cottingham formulation[1] which related these quantities to integrals over the forward virtual Compton amplitudes for scattering with transverse and longitudinal photons. In accordance with the suggestion of Harari,[2] we consider separately those electromagnetic splittings which transform like isospin $I=1$ or 2, since we assume that the virtual Compton amplitude for $I=1$ at high energies contains a contribution from the A_2^0 Regge pole, while for $I=2$ there is no large Regge tail. We also include in our treatment the contribution from the non-Regge behavior of fixed poles with $J=0, I=1,2$, $C=1$, $P=(-1)^J$, which are admissable in right-signature, nonsense amplitudes for weak processes, and nonanalytic pieces in the partial-wave sense amplitudes of the foım δ_{J0}, which can be present[3] and contribute to the asymptotic behavior.

First, we examine the contributions to a typical $I=1$ mass splitting and note that the Regge region $\cos\theta_t \to \infty$ for virtual Compton scattering never distinctly contributes in the Cottingham expression, since the integral is for the region $|\cos\theta_t|\leqslant 1$. However, Regge behavior does dictate the issue of subtractions in the amplitudes $t_{1,2}^{(1)}(q^2,\nu)$, invariant amplitudes related to the transitions induced by longitudinal and transverse photons, and here suggests that $t_1^{(1)}(q^2,\nu)$ requires a subtraction, while $t_2^{(1)}(q^2,\nu)$ does not.

We introduce a new function

$$H^{(1)}(q^2,\nu)=t_1^{(1)}(q^2,\nu)-C^{(1)}(q^2)\nu^2t_2^{(1)}(q^2,\nu)\,,$$

which obeys an unsubtracted dispersion relation since the A_2^0 Regge pole is removed in this combination and $C^{(1)}(q^2)$, a ratio of the A_2^0 Regge residues, is a universal function for $I=1$ mass differences independent of the target. $C^{(1)}(q^2)$ can be obtained from high-energy electroproduction total cross sections for scattering from protons and neutrons and depends only on the $\nu\to\infty$ limit of $\sigma_T^{(1)\,\mathrm{long}}(q^2,\nu)/\sigma_T^{(1)\,\mathrm{trans}}(q^2,\nu)$. Then we obtain the subtraction term $t_1^{(1)}(q^2,0)=H^{(1)}(q^2,0)$ from the dispersion relation for $H^{(1)}(q^2,\nu)$ and find that the limit $q^2\to 0$ of $q^2t_1^{(1)}(q^2,0)$ has no bearing on the possibility of sign reversal in electromagnetic $I=1$ mass differences. Moreover, the A_2^0 Regge pole, as is suggested by this analysis, which neglects large inelastic resonant contributions, will only contribute a large amount to $I=1$ mass differences if

$$\lim_{\nu\to\infty}[\sigma_T^{(1)\,\mathrm{long}}(q^2,\nu)/\sigma_T^{(1)\,\mathrm{trans}}(q^2,\nu)]\approx 1$$

for small spacelike $q^2>0$, a proposition which can be experimentally tested. There is also the possibility of fixed poles or Kronecker δ's which contribute unknown pieces.

We next examine the tadpole model[4] in this formulation, identifying the A_2^0 as the "tadpole" piece, by taking certain combinations of mass splittings in an $SU(3)$ multiplet such as $2F\Delta M^N-(F+D)\Delta M^\Xi$, where F and D are couplings of A_2^0 to the baryons. The A_2^0 contribution is just removed in this combination of mass splittings, which can therefore be computed

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[1] W. N. Cottingham. Ann. Phys. (N. Y.) 25, 424 (1963); M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters 2, 7 (1959).
[2] H. Harari, Phys. Rev. Letters 17, 1303 (1966).
[3] D. Gross and H. Pagels, Phys. Rev. Letters 20, 961 (1968).
[4] S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964).

using only *unsubtracted* dispersion relations. Assuming no fixed pole in $t_1^B(q^2,\nu)$ (or that its F/D ratio is the same as that for the A_2^0), we can obtain the F/D ratio retaining just the elastic contribution, and we find good agreement with the $F/D\approx -2$ obtained by an analysis of tensor-meson coupling in high-energy scattering.[5] Finally, we observe that we can construct a superconvergence relation for the residue of the fixed pole in $t_2^{(1)}(q^2,\nu)$. Simple saturation of this sum rule indicates that it is badly violated unless the fixed pole is indeed present. We discuss the experimental consequences of the existence of such a fixed pole in electroproduction experiments.

II. CONTRIBUTIONS TO ELECTROMAGNETIC MASS DIFFERENCES

A. Cottingham Formula

Central to our analysis of electromagnetic mass differences will be the Cottingham[1] formula, which we now discuss. We shall assume the validity of second-order perturbation theory,[6] in which case the self-energy of a hadron is given by

$$\Delta M=-\frac{1}{8\pi^2}\int_{-\infty}^{+\infty}\frac{d^4q}{q^2-i\epsilon}T_{\mu\nu}(q^2,\nu)g^{\mu\nu}. \tag{2.1}$$

Here $e_\mu e_\nu{}^*T_{\mu\nu}(q^2,\nu)$ is the forward virtual Compton amplitude of a photon of mass q^2, polarization e_μ, and lab energy $q_0=\nu$ scattering from a hadron of mass M and momentum p, with $p^2=-M^2$, $p\cdot q=-M\nu$. Lorentz invariance, current conservation, which here has the expression $q^\mu T_{\mu\nu}=q^\nu T_{\mu\nu}=0$, and the assumed discrete symmetries of the electromagnetic interactions allow us to write

$$T_{\mu\nu}(q^2,\nu)=t_1(q^2,\nu)(q^2g_{\mu\nu}-q_\mu q_\nu)+t_2(q^2,\nu)\times\left(\nu^2g_{\mu\nu}+\frac{q^2}{M^2}p_\mu p_\nu+\frac{\nu}{M}(p_\mu q_\nu+p_\nu q_\mu)\right). \tag{2.2}$$

That only two invariant amplitudes enter this general expression follows from the fact that we consider the Compton amplitude as summed on hadron spins, so that $t_{1,2}(q^2,\nu)$ are just related to the two transitions induced by longitudinal and transverse photons.

By rotating the contour of integration over q_0 from the real to imaginary axis in Eq. (2.1), Cottingham has shown that one can express ΔM in terms of experimentally accessible scattering amplitudes for electroproduction. The rotation of the contour will induce no additional pieces coming from the arc at ∞, provided that ΔM is finite, i.e., that $q^2\delta T_{\mu\mu}(q^2,\nu)\to 0$, $q_0\to\infty$. Transforming $\nu\to i\nu$, doing the trivial angular integration in Eq. (2.1), and using Eq. (2.2), one has the Cottingham formula

$$\begin{aligned}\Delta M&=-\frac{1}{4\pi}\int_0^\infty\frac{dq^2}{q^2}\int_{-q}^{+q}d\nu(q^2-\nu^2)^{1/2}\\&\quad\times[3q^2t_1(q^2,i\nu)-(q^2+2\nu^2)t_2(q^2,i\nu)]\\&=-\frac{1}{4\pi}\int_0^\infty q^2dq^2\int_{-1}^{+1}d(iz_t)(1+z_t^2)^{1/2}\\&\quad\times[3t_1(q^2,z_t)-(1-2z_t^2)t_2(q^2,z_t)],\end{aligned} \tag{2.3}$$

where $z_t=\cos\theta_t=\nu/\sqrt{(-q^2)}$ is the scattering angle in the barycentric frame for the t-channel process $\gamma+\gamma\to A+\bar{A}$. Writing the Cottingham formula in terms of the variable z_t serves to emphasize that, quite irrespective of the issue of required subtractions in the dispersion relations for $t_{1,2}(q^2,\nu)$ in the variable ν (which is presumed to be established by the Regge behavior of these amplitudes as $z_t\to\infty$), the Regge region $z_t\to\infty$ never explicitly enters the expression for the mass shift, since $|z_t|\leqslant 1$ and the integration in Eq. (2.3) never singles out the Regge region.[7] Rather, what is important in the calculation of ΔM is the behavior of the amplitudes as a function of the photon mass as $q^2\to+\infty$, which is determined by form factors, the behavior of Regge residue functions, or the residues of fixed poles in this limit, about which Regge phenomenology says nothing.

B. Crossing Relations

In establishing the Regge behavior for the amplitudes $t_{1,2}(q^2,\nu)$ for virtual Compton scattering, it is necessary to examine the crossing relations from direct channel (s) to the crossed channel (t). Here we utilize the helicity amplitudes.[8] In our application, we require only the combination of helicity amplitudes summed on the hadron spins, so we shall surpress these helicity indices and denote by $+$, $-$, and 0 the helicity states of the virtual photons for the scattering process. It is instructive to consider also the crossing relations for nonforward scattering, $t\neq 0$, where t is the momentum

[5] V. Barger and M. Olsson, Phys. Rev. Letters **18**, 294 (1967); K. V. L. Sarma and D. D. Reeder (to be published); K. V. L. Sarma and G. H. Renninger, Phys. Rev. Letters **20**, 399 (1968).

[6] Using the first-order perturbation theory is not merely a convenience, but an assumption. It is possible that, by inserting data from electroproduction experiments into the Cottingham formula, the resulting integrals diverge, suggesting a breakdown of the perturbative approach.

[7] In a recent attempt to calculate the n-p mass difference, Y. Srivastava [Phys. Rev. Letters **20**, 232 (1968)] writes $t_1^{(1)}(q^2.\nu)$ as the pole term, plus a term $\beta(q^2)\nu^{\alpha_{A_2}}$, which he uses in the Cottingham formula to calculate the inelastic contribution to the mass difference. In our opinion, this is not justified, since in the Cottingham formula one is never in the asymptotic (large-z_t) region. Furthermore, his attempt to calculate $\beta(q^2)$ from a finite-energy sum rule for $t_1^{(1)}(q^2)$ neglects fixed poles at $J=0$, additional terms in the Legendre expansion at $J=-\frac{3}{2}$, daughters at $J=-\frac{1}{2}$, $-\frac{3}{2}$, and a pole near $J=-\frac{3}{2}$ required by Mandelstam symmetry [since $\alpha_{A2}(0)\approx\frac{1}{2}$]. Furthermore, he chooses the cutoff at threshold—which leads to the unphysical result that the residue function has a singularity as $q^2\to 0$. This gives an artificial enhancement of "inelastic contribution."

[8] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

transfer squared for the direct-channel process. Later we take $t \to 0$.

The crossing relations on the four helicity amplitudes $F_{++}{}^s$, $F_{+-}{}^s$, $F_{0+}{}^s$, and $F_{00}{}^s$ are then

$$\begin{aligned} F_{++}{}^s(s,t)+F_{+-}{}^s(s,t)&=F_{++}{}^t(s,t)+F_{+-}{}^t(s,t)\,,\\ F_{++}{}^s(s,t)-F_{+-}{}^s(s,t)&=-\cos^2\theta\,[F_{++}{}^t(s,t)-F_{+-}{}^t(s,t)]\\ &\quad+(4/\sqrt{2})\cos\theta\sin\theta\,F_{0+}{}^t(s,t)+\sin^2\theta\,F_{00}{}^t(s,t)\,,\\ F_{0+}{}^s(s,t)&=[(\cos\theta\sin\theta)/\sqrt{2}][F_{++}{}^t(s,t)-F_{+-}{}^t(s,t)\\ &\quad+F_{00}{}^t(s,t)]-(1-2\cos^2\theta)F_{0+}{}^t(s,t)\,,\\ F_{00}{}^s(s,t)&=\sin^2\theta\,[F_{++}{}^t(s,t)-F_{+-}{}^t(s,t)]-\cos^2\theta\,F_{00}{}^t(s,t)\\ &\quad+[(4\sin\theta\cos\theta)/\sqrt{2}]F_{0+}{}^t(s,t)\,, \end{aligned}\tag{2.4}$$

where $\cos\theta=t(M\nu-q^2)/M[(\nu^2+q^2)t(t+4q^2)]^{1/2}$ is the crossing angle and $2M\nu=s-M^2+\frac{1}{2}(t+2q^2)$. The crossing relations as specified above will be quite different depending on whether we take the limit according to $t\to 0$, $q^2\to 0$ or $q^2\to 0$, $t\to 0$ (corresponding to physical forward Compton scattering), since the crossing angle $\cos\theta\to 0$ for the first case and $\cos\theta\to 1$ for the second. Such noncommutivity of limits can have no dynamical consequences, since the invariant amplitudes are indifferent to the order of the limits. It is worth remarking in this context that the purely longitudinal t-channel amplitude $F_{00}{}^t(s,t)$ has a kinematical factor $4q^2/(4q^2+t)$ [similarly, $F_{0+}{}^t(s,t)$ has a factor $(4q^2/(4q^2+t))^{1/2}$], so that, as the photon mass $q^2\to 0$ for $t\neq 0$, $F_{00}{}^t\to 0$ as a longitudinal amplitude should; however, in the limit $t\to 0$ and then $q^2\to 0$, the amplitude $F_{00}{}^t$ need not vanish.[9]

In our application to the mass-shift calculations, we take the limit $t\to 0$ with $q^2\neq 0$, in which case the crossing relations are easily obtained from Eq. (2.4) by setting $\cos\theta=0$:

$$\begin{aligned} 2F_{++}{}^t(s,0)&=F_{++}{}^s(s,0)+F_{00}{}^s(s,0)\,,\\ 2F_{+-}{}^t(s,0)&=F_{++}{}^s(s,0)-F_{00}{}^s(s,0)\,,\quad (t=0,\ q^2\neq 0) \end{aligned}\tag{2.5a}$$

and we also have the conspiracy relations on t-channel amplitudes

$$\begin{aligned} F_{0+}{}^t(s,0)&=0\,,\\ F_{00}{}^t(s,0)-F_{++}{}^t(s,0)&=F_{+-}{}^t(s,0)\,,\quad (t=0,\ q^2\neq 0) \end{aligned}\tag{2.5b}$$

as follows from the vanishing of the helicity-flip amplitudes $F_{0+}{}^s(s,t)$, $F_{+-}{}^s(s,t)$ for forward scattering.

To establish the invariant amplitudes free of kinematical singularities from the helicity amplitudes $F_{++}{}^s(s,0)$ and $F_{00}{}^s(s,0)$, corresponding to forward scattering with purely transverse and longitudinal photons, we note that rotational invariance informs us of the presence of the kinematical factor $\sin^2\theta_t=1+\nu^2/q^2$ in the helicity-flip amplitude $F_{+-}{}^t(s,0)$, so that $\bar{F}_{+-}{}^t(s,0)=F_{+-}{}^t(s,0)/(\nu^2+q^2)$ has no kinematical singularities as $\nu^2\to -q^2$. Hence we have from Eq. (2.5) that the amplitudes

$$t_1(q^2,\nu)=\frac{F_{++}{}^s(s,0)+(\nu^2/q^2)F_{00}{}^s(s,0)}{\nu^2+q^2}\,,\tag{2.6a}$$

$$t_2(q^2,\nu)=\frac{F_{++}{}^s(s,0)-F_{00}{}^s(s,0)}{\nu^2+q^2}\tag{2.6b}$$

are free of kinematical singularities and correspond to the choice of Cottingham given in Eq. (2.2). We note that there are no kinematical singularities in $t_{1,2}(q^2,\nu)$ as $q^2\to 0$, since $F_{00}{}^s\sim q^2$ in this limit.[10] In terms of the t-channel amplitudes, we have

$$t_2(q^2,\nu)=\frac{2F_{+-}{}^t(s,0)}{\nu^2+q^2}\,,\tag{2.7a}$$

$$-q^2t_1(q^2,\nu)=\nu^2t_2(q^2,\nu)+F_{00}{}^t(s,0)\,.\tag{2.7b}$$

C. Regge Behavior and Subtractions

To study the contributions to the mass shift arising from intermediate states appearing in the virtual Compton amplitudes, we shall consider dispersion relations for $t_{1,2}(q^2,\nu)=t_{1,2}{}^*(q^2,-\nu)$ for fixed q^2. The question of subtractions in the dispersion relations arises quite naturally in the Regge phenomenology and, as was first emphasized by Harari,[2] can be expected to differ depending on whether one considers mass differences with pure $I=1$ or $I=2$, I being the isospin in the crossed channel. For a particular isospin, pure Regge behavior would lead one to suspect an asymptotic limit for the t-channel amplitudes as $z_t=\nu/\sqrt{(-q^2)}\to\infty$ of

$$\begin{aligned} F_{+-}{}^t&\to z_t{}^{\alpha^{(I)}(0)}\,,\\ F_{00}{}^t&\to z_t{}^{\alpha^{(I)}(0)}\,, \end{aligned}\tag{2.8}$$

where $\alpha^{(I)}(0)$ is the $t=0$ intercept of the leading Regge trajectory with $C=1$, $P=(-1)^J$, $G=-1$. For $I=1$ this would correspond to the $A_2{}^0(1308)$ with $\alpha_{A_2{}^0}{}^{(1)}(0)\approx 0.4$, while for $I=2$ we assume, in accord with the conjecture of Alfaro, Fubini, Rosetti, and Furlan,[11] that $\alpha^{(2)}(0)\leqslant 0$.

From the relations between the invariant amplitudes and t-channel helicity amplitudes [Eq. (2.7)] and using Eq. (2.8), we have that pure Regge behavior implies for the invariant amplitudes as $\nu\to\infty$

$$\begin{aligned} t_1{}^{(I)}(q^2,\nu)&\to\beta_1{}^{(I)}(q^2)\nu^{\alpha^{(I)}(0)}\,,\\ t_2{}^{(I)}(q^2,\nu)&\to\beta_2{}^{(I)}(q^2)\nu^{\alpha^{(I)}(0)-2}\,, \end{aligned}\tag{2.9}$$

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[9] This follows just from current conservation, which implies for a longitudinal photon $\epsilon_L{}^\mu\langle A|J_\mu(0)|B\rangle=(-q^2/|\mathbf{q}|)^{1/2}\langle A|J_0(0)|B\rangle$, where, for our application to a t-channel amplitude, $|\mathbf{q}|=-q^2+\frac{1}{4}t$. This kinematical factor is 0 or 1, depending on whether one takes $q^2\to 0$ or $t\to 0$ first.

[10] Taking the limit $q^2\to 0$ in the crossing relations at $t=0$, Eqs. (2.5) and (2.7) imply, since $F_{00}{}^s=0$, that $F_{++}{}^s=2F_{++}{}^t=2F_{+-}{}^t=F_{00}{}^t$. Had we taken the limit $q^2\to 0$, $t\to 0$, then $F_{++}{}^s=F_{+-}{}^t$, $F_{++}{}^t=F_{00}{}^t=0$. We also note that the combination of helicity amplitudes given by Eq. (2.6) is only free of kinematical singularities at $t=0$. For $t\neq 0$ a separate treatment is required.

[11] V. de Alfaro, S. Fubini, G. Rossetti, and G. Furlan, Phys. Letters 21, 576 (1966).

where $\beta_1^{(I)}(q^2)$ are the Regge residue functions at $t=0$. From the above assumptions on Regge behavior we have that $t_1^{(1)}(q^2,\nu)$ obeys a once-subtracted dispersion relation, while $t_2^{(1)}(q^2,\nu)$, $t_1^{(2)}(q^2,\nu)$ obey unsubtracted dispersion relations and $t_2^{(2)}(q^2,\nu)$ obeys a superconvergent dispersion relation.

If we now relax the condition of pure Regge behavior and admit the possibility of fixed poles contributing to the asymptotic behavior, our conclusions are modified. Such fixed poles in the partial-wave amplitudes in the t channel can, in general, occur in weak processes at nonsense points for right-signature amplitudes.[12] Here a fixed pole could appear in the partial-wave decomposition of $F_{+-}{}^t(s,t)$ at $J=0$ for $I=1,2$. Further, the presence of such fixed poles in $F_{+-}{}^t(s,t)$ will in general require a nonanalytic piece of the form δ_{J0} in the partial-wave amplitudes for $F_{00}{}^t(s,t)$, $F_{++}{}^t(s,t)$, here at a sense, right-signature point. That fixed poles in weak processes impose nonanalytic behavior in the J plane for other amplitudes is a general consequence of the conspiracy relation Eq. (2.5b) and will be discussed in detail elsewhere.[3] Suffice it to remark here that if there is a fixed pole at $J=0$ in the partial-wave amplitude for $F_{+-}{}^t(s,0)$, then the conspiracy relation (2.6) considered as $\nu\to\infty$ will have a term behaving like a constant on the right-hand side, and hence the same constant must appear on the left-hand side. Since there can not be a fixed pole in the sense amplitudes, the only alternative is a nonanalytic piece in the partial wave δ_{J0} which contributed the constant as $\nu\to\infty$.

In the presence of fixed poles our conclusion, Eq. (2.9), now becomes modified to

$$\begin{aligned} t_1^{(I)}(q^2,\nu) &\to R_1^{(I)}(q^2)+\beta_1^{(I)}(q^2)\nu^{\alpha^{(I)}(0)}, \\ t_2^{(I)}(q^2,\nu) &\to R_2^{(I)}(q^2)\nu^{-2}+\beta_2^{(I)}(q^2)\nu^{\alpha^{(I)}(0)-2}, \end{aligned} \tag{2.10}$$

where $R_{1,2}^{(I)}(q^2)$ are related to the residues of the fixed pole and coefficient of the δ_{J0}. In what follows, let us specialize to $I=1$ mass differences; the $I=2$ mass differences we shall comment on later. For this case we have that $t_{1,2}^{(1)}(q^2,\nu)$ satisfy

$$t_1^{(1)}(q^2,\nu)=t_1^{(1)}(q^2,0)+\frac{16M^3\nu^2f_1^{(1)}(q^2)}{q^2(q^4-4M^2\nu^2)} +\frac{\nu^2}{\pi}\int_{\nu_t^2}^{\infty}\frac{\mathrm{Im}t_1^{(1)}(q^2,\nu')d\nu'^2}{\nu'^2(\nu'^2-\nu^2)}, \tag{2.11a}$$

$$t_2^{(1)}(q^2,\nu)=\frac{4Mq^2f_2^{(1)}(q^2)}{(q^4-4M^2\nu^2)} +\frac{1}{\pi}\int_{\nu_t^2}^{\infty}\frac{\mathrm{Im}t_2^{(1)}(q^2,\nu')d\nu'^2}{\nu'^2-\nu^2}, \tag{2.11b}$$

[12] J. B. Bronzan, I. S. Gerstein, B. W. Lee, and F. E. Low, Phys. Rev. Letters **18**, 32 (1967); Phys. Rev. **157**, 1448 (1967); V. Singh, Phys. Rev. Letters **18**, 36 (1967); H. D. I. Abarbanel, F. E. Low, I. J. Muzinich, S. Nussinov, and J. H. Schwarz, Phys. Rev. **160**, 1329 (1967); A. H. Mueller and T. L. Trueman, *ibid.* **160**, 1296 (1967); **160**, 1306 (1967).

where $t_1^{(1)}(q^2,0)$ is the subtraction term introduced by Harari[2] and $f_{1,2}^{(1)}(q^2)$ are the residues of the elastic-pole-term contributions and are related to form factors. The presence of fixed poles or Kronecker δ's at $J=0$ does not alter the representations (2.11).

In order to study the contributions to the mass shift in detail, we introduce the new function

$$H^{(1)}(q^2,\nu)=t_1^{(1)}(q^2,\nu) -[\beta_1^{(1)}(q^2)/\beta_2^{(1)}(q^2)]\nu^2t_2^{(1)}(q^2,\nu), \tag{2.12}$$

which has the physical significance of just having the Regge behavior from the $A_2{}^0$ trajectory removed. The price that one pays for this improved high-energy behavior is the introduction of the ratio $\beta_1^{(1)}(q^2)/\beta_2^{(1)}(q^2)$ of the Regge residues. Because we are summing on the spins of the target, we may simply apply the factorization theorem on Regge residues, which informs us that

$$\begin{aligned} \beta_1^{(1)}(q^2)&=\beta_H^{(1)}(1/q^2)\gamma_L^{(1)}(q^2), \\ \beta_2^{(1)}(q^2)&=\beta_H^{(1)}[\gamma_T^{(1)}(q^2)-\gamma_L^{(1)}(q^2)], \end{aligned} \tag{2.13}$$

where $\beta_H^{(1)}$, independent of q^2, refers to the coupling of the $A_2{}^0$ trajectory to the target hadron, and $\gamma_{T,L}^{(1)}(q^2)$ refer only to the coupling of the $A_2{}^0$ trajectory to transverse and longitudinal photons defined according to Eq. (2.6). Hence the ratio

$$C^{(1)}(q^2)=\frac{\beta_1^{(1)}(q^2)}{\beta_2^{(1)}(q^2)}=\frac{\gamma_L^{(1)}(q^2)}{q^2[\gamma_T^{(1)}(q^2)-\gamma_L^{(1)}(q^2)]} \tag{2.14}$$

is a universal function for all $I=1$ mass differences independent of the target hadron.

Using the assumption (2.10), we have for asymptotic behavior of $H^{(1)}(q^2,\nu)$ as $\nu\to\infty$

$$H^{(1)}(q^2,\nu)\to R_1^{(1)}(q^2)-C^{(1)}(q^2)R_2^{(1)}(q^2),$$

so that the dispersion relation for $H^{(1)}(q^2,\nu)$ is

$$H^{(1)}(q^2,\nu)=R_1^{(1)}(q^2)-C^{(1)}(q^2)R_2^{(1)}(q^2) +\frac{4Mq^2f_1^{(1)}(q^2)}{q^4-4M^2\nu^2}-\frac{C^{(1)}(q^2)q^44Mf_2^{(1)}(q^2)}{4M^2(q^4-4M^2\nu^2)} +\frac{1}{\pi}\int_{\nu_t^2}^{\infty}\frac{\mathrm{Im}H^{(1)}(q^2,\nu')d\nu'^2}{\nu'^2-\nu^2}. \tag{2.15}$$

It follows that the subtraction term appearing in Eq. (2.11a) can now be related to our other functions, since from Eq. (2.12) we have $t_1^{(1)}(q^2,0)=H^{(1)}(q^2,0)$, so that

$$q^2t_1^{(1)}(q^2,0)=4Mf_1^{(1)}(q^2)+q^2R_1^{(1)}(q^2) -\frac{\gamma_L^{(1)}(q^2)}{\gamma_T^{(1)}(q^2)-\gamma_L^{(1)}(q^2)}\left[R_2^{(1)}(q^2)+\frac{q^2}{M}f_2^{(1)}(q^2)\right] +q^2H_{\mathrm{cont}}^{(1)}(q^2,0), \tag{2.16}$$

where

$$H_{\text{cont}}^{(1)}(q^2,0)=\frac{1}{\pi}\int_{\nu_t^2}^{\infty}\frac{\operatorname{Im}H^{(1)}(q^2,\nu')d\nu'^2}{\nu'^2}$$

represents a contribution from inelastic continuum states with just the Regge behavior removed, and hence is convergent. From this expression for the subtraction term one has immediately the result of Harari:

$$q^2t_1^{(1)}(q^2,0)=4Mf_1^{(1)}(0)\,,\quad q^2\to 0 \tag{2.17}$$

since the other contributions vanish in this limit on purely kinematical grounds, e.g., $\gamma_L^{(1)}(0)=0$, $\gamma_T^{(1)}(0)\neq 0$. However, contrary to the suggestion of Ref. 2, the sign of $4Mf_1^{(1)}(0)$ has no direct bearing on sign reversal for the $I=1$ mass differences if we now consider the contributions to the mass shift in the Cottingham formula.

In classifying the contributions to ΔM in terms of intermediate states, we shall follow Harari[2] and write

$$\Delta M=\Delta\bar{M}^{\text{elastic}}+\Delta\bar{M}^{\text{inelastic}}+\Delta M^{\text{sub}}, \tag{2.17$'$}$$

where $\Delta\bar{M}^{\text{elastic, inelastic}}$ are defined in Ref. 2, Eq. (11), assuming that $t_1^{(1)}(q^2,\nu)$ needs a subtraction. ΔM^{sub} is the contribution of the subtraction term and is given by an integral over $q^2t_1^{(1)}(q^2,0)$ specified by Eq. (2.16). Hence we shall have, in general, from the various terms in Eq. (2.16)

$$\Delta M^{\text{sub}}=\Delta M'^{\text{elastic}}+\Delta M^{\text{fixed pole}}+\Delta M^{\text{Regge}}+\Delta M'^{\text{inelastic}}, \tag{2.17$''$}$$

with $\Delta M'^{\text{elastic, inelastic}}$ integrals over $4Mf_1^{(1)}(q^2)$ and $q^2H_{\text{cont}}^{(1)}(q^2,0)$, respectively, and the fixed-pole and Regge contributions are defined by Eqs. (2.18) and (2.19). Now, writing

$$\Delta M^{\text{elastic, inelastic}}=(\Delta\bar{M}+\Delta M')^{\text{elastic, inelastic}},$$

we combine Eqs. (2.17′) and (2.17″):

$$\Delta M=\Delta M^{\text{elastic}}+\Delta M^{\text{inelastic}}+\Delta M^{\text{fixed}}+\Delta M^{\text{Regge}}. \tag{2.17$'''$}$$

The quantity $\Delta M^{\text{elastic}}$ which appears in this expression is the same as that obtained by Cottingham,[1] who assumed unsubtracted dispersion relations. What we observe here is that the issue of subtracted or unsubtracted dispersion relations has no bearing on the proper elastic-pole-term contribution to the mass shift ΔM, for what has happened in writing Eq. (2.17′) is that part of the elastic-pole-term contribution got shuffled into the subtraction term. For this reason, the sign of $q^2t_1^{(1)}(q^2,0)$ as $q^2\to 0$, which is specified by the Born term, can have no bearing on the usual conclusions obtained by considering just the elastic piece in the unsubtracted formalism.

So far, we have been perfectly general. Now we can make the heuristic approximation $\Delta M^{\text{inelastic}}=0$, consistent with assuming that there are no large direct-channel resonances—or that they cancel in their contribution to $\Delta M^{\text{inelastic}}$, but will retain the other pieces in Eq. (2.17‴). Later, we shall comment on this approximation. There is now, in addition to the elastic contribution, the contribution from the Regge pole and fixed poles according to[13]

$$\Delta M^{\text{Regge}}+\Delta M^{\text{fixed}}=-\tfrac{3}{8}\int_0^{\infty}dq^2\times[q^2t_1^{(1)}(q^2,0)-4Mf_1^{(1)}(q^2)],$$

so that we shall identify

$$\Delta M^{\text{Regge}}=-\frac{3}{8M}\int_0^{+\infty}dq^2\times\frac{\gamma_L^{(1)}(q^2)}{\gamma_L^{(1)}(q^2)-\gamma_T^{(1)}(q^2)}q^2f_2^{(1)}(q^2)\,, \tag{2.18}$$

$$\Delta M^{\text{fixed pole}}=-\tfrac{3}{8}\int_0^{+\infty}dq^2\,q^2R_1^{(1)}(q^2)+\frac{\gamma_L^{(1)}(q^2)}{\gamma_L^{(1)}(q^2)-\gamma_T^{(1)}(q^2)}R_2^{(1)}(q^2)\,, \tag{2.19}$$

which, with the neglect of the continuum, are the only other contributions. In the absence of any simple model or experimental information on the functions $R_{1,2}^{(1)}(q^2)$ and $\gamma_T^{(1)}(q^2)/\gamma_L^{(1)}(q^2)$ which appear in these expressions, we can not hope to estimate the sign or magnitude of these contributions. What is encouraging is that ΔM^{Regge} is related to $q^2f_2^{(1)}(q^2)$, which is a charge form factor, and hence is present for all charged particles and does not depend on their possessing a magnetic moment.

The ratio $\gamma_T^{(1)}(q^2)/\gamma_L^{(1)}(q^2)$ can be determined from experiments on the scattering of high-energy electrons by protons and neutrons. Since the fixed poles do not contribute to absorptive parts or total cross sections, we shall have

$$\frac{\gamma_L^{(1)}(q^2)}{\gamma_T^{(1)}(q^2)}=\lim_{\nu\to\infty}\frac{\sigma_T^{(1)\,\text{long}}(q^2,\nu)}{\sigma_T^{(1)\,\text{trans}}(q^2,\nu)}, \tag{2.20}$$

where the cross sections are for longitudinal and transverse photons and the difference $\sigma_T^{(1)}=\sigma_p-\sigma_n$. According to factorization, the ratio (2.20) is the same for all targets, transforming like $I=1$; it is a universal function.

We can now discuss under what conditions the Regge contribution given by ΔM^{Regge} above will be large. For the case of the nucleon mass difference $\Delta M=M_p-M_n=-1.3$ MeV, the elastic contribution can be computed using the observed dipole fit to the form factors and gives the classic wrong sign $\Delta M^{\text{elastic}}\approx+0.8$ MeV. Just as an example, let us assume a dominant longitudinal

[13] This is just Harari's ΔM_1^{sub}, with the elastic contribution taken out.

coupling of the A_2^0 (for which there is no experimental or theoretical justification), so that $\gamma_L^{(1)}(q^2) \gg \gamma_T^{(1)}(q^2)$, $q^2 \neq 0$, and hence

$$\Delta M^{\text{Regge}} = -\tfrac{3}{8} M \int_0^\infty dq^2\, q^2 f_2^{(1)}(q^2) = -0.3 \text{ MeV}.$$

This has the right sign to reverse the elastic piece but not the magnitude. What is evidently required to enhance the Regge contribution is comparable longitudinal and transverse couplings $\gamma_L^{(1)}(q^2) - \gamma_T^{(1)}(q^2) \approx 0$ for some small $q^2 > 0$.[14] Only the region for small q^2 is relevant, since the entire contribution to the integral equation (2.18) comes from this region because of the sharp falloff of the form factors in $f_2^{(1)}(q^2)$. The proposition that $\gamma_L^{(1)}(q^2)/\gamma_T^{(1)}(q^2) \approx 1$ for small $q^2 > 0$ can be experimentally tested through the relation (2.20).

Besides the pure Regge-pole contribution to mass shifts, there can also be the contributions from the residue functions $R_{1,2}^{(1)}(q^2)$. The residue of the fixed pole $R_2^{(1)}(q^2)$ in the helicity-flip amplitude F_{+-}^t can be calculated from a superconvergence relation as we discuss in Sec. III. A phenomenological analysis of this superconvergence relation strongly indicates that this residue does not vanish and the fixed pole is indeed present; however, the presence of this fixed pole does not spoil the results of the tadpole model irrespective of the F/D ratio of the fixed-pole coupling to the baryon octet, as we shall show in Sec. III. A similar phenomenological analysis of superconvergence relations for $I=2$ mass differences indicates no fixed pole, or at least a small residue, $R_2^{(2)}(q^2) \approx 0$, supporting our conviction that just the elastic pieces account for these mass differences.

About the other residue $R_1^{(1)}(q^2)$ appearing in the amplitude $t_1^{(I)}(q^2,\nu)$ no such simple conclusions can be made, primarily because its Regge behavior is worse than $t_2^{(I)}(q^2,\nu)$ and it is difficult to cancel off the pure Regge piece reliably by taking linear combinations of amplitudes. If $R_1^{(1)}(q^2) \neq 0$ and contributes significantly to $\Delta M^{\text{fixed pole}}$ through Eq. (2.19), then the results of the tadpole model will in general be spoiled, there being no reason to expect the F/D ratios for $R_1^{(1)}(q^2)$ to be the same as that for the A_2^0. On the other hand, the success of the tadpole model can be taken as evidence that $R_1^{(1)}(q^2) \approx 0$. We briefly discuss the experimental consequences of fixed poles in Sec. IV.

Finally, a comment is necessary regarding the inelastic contributions which we have neglected. These states are just those open channels for the photon incident on the hadron, and in lieu of experimental data for these reactions, one has recourse only to a phenomenological analysis. This may be unreliable, particularly since an infinite set of states can in general contribute (making up the Regge tail). However, restricting one's attention to just the low-lying states, in the case of Compton scattering from the baryon octet, one expects the dominant contribution from the large magnetic dipole transition to the decuplet. In the narrow-resonance approximation, and keeping only the magnetic dipole transition, the decuplet contributes a pole piece according to

$$^{10}t_1(q^2,\nu) = {}^{10}t_2(q^2,\nu) = \frac{\alpha\mu^{*2}(q^2)(M^{*2}-M^2+q^2)}{\pi M(M^{*2}-M^2+q^2)^2 - 4M^2\nu^2},$$

where M^* is the resonance mass and M the target-baryon mass. Here $\mu^*(q^2)$ is the same transition moment defined by Dalitz and Sutherland[15] (with a possible q^2 dependence included) and for the proton has the experimental value $\mu_p^*(0) = 3.38 \pm 0.06$. Assuming the same dipole fit to $\mu^*(q^2)$ as for $G_p^M(q^2)$, suitably normalized at $q^2=0$, and $SU(3)$ for the transition moments, one finds for a typical decuplet contribution to an $I=1$ baryon mass splitting $\Delta M^{\text{inelastic}} \approx 0.05$ MeV, completely negligible. For this reason, our present hopes of understanding $I=1$ mass differences rests with the high-energy Regge piece, Eq. (2.18), which presumably represents a suitable sum over the entire inelastic spectrum in accord with the principles supporting finite-energy sum rules.

The complete neglect of the resonant inelastic spectrum is a more dangerous assumption in this approach, because the contributions to the subtraction term $t_1^{(1)}(q^2,0)$ from the inelastic states in $H_{\text{cont}}^{(1)}(q^2,0)$ given in Eq. (2.16) contains an integral over $\nu^2 C^{(1)}(q^2) \times \text{Im} t_2^{(1)}(q^2,\nu)$. If $C^{(1)}(q^2)$ is large, which is desirable if the elastic piece is to give a sizeable contribution, this inelastic term may also contribute a comparable amount to ΔM^{Regge}.

Furthermore, the complete neglect of the continuum from the standpoint of finite-energy sum rules is inconsistent with the presence of Regge behavior, for this is to be generated out of the continuum. What we have done in classifying the contributions to ΔM—which are simply convenient definitions—is to call $\Delta M^{\text{inelastic}}$ the contribution of the spectrum with the high-energy Regge tail removed and ΔM^{Regge} the contribution of the Regge tail to the subtraction term. It may turn out—as we are suggesting—that the contribution of what we call $\Delta M^{\text{inelastic}}$ is small, perhaps because of resonances cancelling each other, while the dominant contribution from the spectrum contributes to ΔM^{Regge}. It should be remarked that if we assumed the *complete* absence of the continuum spectrum (minus Regge background) and kept only the nucleon pole in the *n*-*p* mass difference, then we can establish a sum rule relating nucleon form factors to $\gamma_L^{(1)}(q^2)/\gamma_T^{(1)}(q^2)$. This, however, is extremely unreliable,[7] and in fact is kinematically inconsistent, since this calculation gives $\gamma_L^{(1)}(q^2)/\gamma_T^{(1)}(q^2)$ nonvanishing as $q^2 \to 0$.

[14] If $\gamma_T(q_0^2) = \gamma_L(q_0^2)$ for some $q_0^2 > 0$, then $C^{(1)}(q^2)$ is singular at q_0^2 and this requires special treatment. We assume that this is not the case.

[15] R. H. Dalitz and D. G. Sutherland, Phys. Rev. **146**, 1180 (1966).

III. RELATIONS AMONG BARYON MASS SPLITTINGS

In Sec. II, we have seen that without knowledge of Regge residues that characterize the asmyptotic behavior of the virtual photon scattering amplitude, it was impossible to calculate the nonelastic contribution to the electromagnetic mass splitting. In this section, we show that certain combinations of the mass splittings for different baryons in the octet are given in terms of amplitudes that need no subtractions.

The asymptotic behavior of $t_1^{(1)B}(q^2,\nu)$ and $t_2^{(1)B}(q^2,\nu)$ (B denotes a particular baryon N, Σ, or Ξ) is given, in the absence of fixed poles or Kronecker δ's, by

$$t_1^{(1)B}(q^2,\nu) \to (1/q^2)\gamma_L(q^2)\gamma_B\nu^{\alpha_{A_2}(0)}+O(\nu^{-\epsilon}),$$
$$t_2^{(1)B}(q^2,\nu) \to -[\gamma_L(q^2)-\gamma_T(q^2)]\gamma_B\nu^{\alpha_{A_2}(0)-2}+O(\nu^{-2-\epsilon}),\quad \epsilon>0, \quad (3.1)$$

where γ_B is the factorized Regge residue of the A_2 and the appropriate baryon, and is independent of q^2. Since the only dependence of the asymptotic behavior on the particular baryon occurs through γ_B, we can cancel out the leading asymptotic behavior by taking the combinations

$$A_i^{BB'}(q^2,\nu)=\gamma_{B'}t_i^{(1)B}(q^2,\nu)-\gamma_B t_i^{(1)B'}(q^2,\nu). \quad (3.2)$$

In the absence of fixed poles at $J=0$, $A_1^{BB'}(q^2,\nu)$ satisfies an unsubtracted dispersion relation and $A_2^{BB'}(q^2,\nu)$ is superconvergent. Therefore the same combination of baryon mass shifts, i.e., $\gamma_{B'}\Delta M^B-\gamma_B\Delta M^{B'}$, can be calculated by saturation of the unsubtracted dispersion relations for $A_i^{BB'}(q^2,\nu)$ with low-lying states. In particular, if we keep only the "elastic" part, i.e., the baryon pole, we derive

$$\gamma_{B'}\Delta M^B-\gamma_B\Delta M^{B'}=\gamma_{B'}(\Delta M^B)^{\text{elastic}}-\gamma_B(\Delta M^{B'})^{\text{elastic}}, \quad (3.3)$$

or

$$\frac{\delta M^B}{\delta M^{B'}}=\frac{\Delta M^B-(\Delta M^B)^{\text{elastic}}}{\Delta M^{B'}-(\Delta M^{B'})^{\text{elastic}}}=\frac{\gamma_B}{\gamma_{B'}}. \quad (3.4)$$

In the presence of fixed poles at $J=0$ in t_1, $A_1^{BB'}(q^2,\nu)$ has the asymptotic behavior

$$A_1^{BB'}(q^2,\nu) \underset{\nu\to\infty}{\longrightarrow} \gamma_{B'}R_1^{(1)B}(q^2)-\gamma_B R_1^{(1)B'}(q^2),$$

and this will add an unknown term to the right-hand side of Eq. (3.3). A fixed pole in t_2 does not affect the above. The quantity δM^B is essentially what has been called the "tadpole" contribution to the mass splittings,[4] and we have therefore shown that the ratio of the "tadpoles" is given by the ratio of the coupling of A_2 to the various baryons. Note that our definition of tadpole requires that one calculate the elastic contribution to the mass splitting by using an *unsubtracted* dispersion relation.[16]

The couplings of the A_2 trajectory to the baryons can be determined experimentally. Barger and Olsson,[5] in an analysis of total cross section, have shown that it is consistent to describe the Regge couplings of the tensor mesons (A_2, f_0, K^{**}) by an F/D ratio—i.e., that $SU(3)$ is a good symmetry for the residue functions. They find for the tensors a common $(F/D)_T=-2.0\pm0.5$.

More recently, Reeder and Sarma[5] have analyzed hypercharge exchange reactions for meson-baryon scattering and determined the F/D ratios of the $K^{**}(1420, J^P=2^+)$ residue function, which is assumed to be the same as for A_2, namely, $(F/D)^{**}=-1.8$, consistent with the above. A third calculation of the F/D ratio for the K^{**}, by Sarma and Renninger,[5] gives $(F/D)_{K^{**}}=-1.6\pm0.2$. We therefore assume that the A_2 couplings are given by $SU(3)$ and the F/D ratio

$$(F/D)_{A_2}=-1.8\pm0.2. \quad (3.5)$$

Therefore, for the combination of baryon couplings that enter into Eq. (3.4) we have

$$\gamma_N=(F+D)\gamma=\gamma,$$
$$\gamma_\Sigma=2F\gamma=(4.5\pm0.6)\gamma, \quad (3.6)$$
$$\gamma_\Xi=(F-D)\gamma=(3.5\pm0.5)\gamma.$$

To compare this with Eq. (3.3), we calculate $(\Delta M^B)^{\text{elastic}}$ using $SU(3)$ for the magnetic moments of the Σ and Ξ and a universal dipole form factor

$$\frac{G_E^B(q^2)}{G_E^B(0)}=\frac{G_M^B(q^2)}{G_M^B(0)}=\frac{1}{(1+q^2/0.71\ \text{BeV}^2)^2}. \quad (3.7)$$

The contribution of the decuplet was also calculated using $SU(3)$, the known $\Delta N\gamma$ coupling, and the same dipole form factors. The decuplet does not contribute to ΔM^N and its contribution to ΔM^Σ and ΔM^Ξ is extremely small (≈ 0.05 MeV). The results are summarized in Table I. The agreement with Eq. (3.3) is quite satisfactory; the main uncertainty is in the experimental determination of ΔM^Ξ and the F/D ratio of the A_2 couplings. For comparison, we have also calculated the elastic part of the $\Delta I=2$ mass difference of the Σ's. As shown in Table I, this is in good agreement with experiment. This indicates that the pole approximation is good if, as in this case, one can write unsubtracted dispersion relations for t_i.

We have thus calculated the ratio of the baryon "tadpole" under the assumption of no fixed poles in t_1 at $J=0$. Since we shall argue, in Sec. IV, for the existence of a fixed pole in t_2 at $J=0$, this assumption is somewhat shaky. However, the success of the calculation indicates that if there is a fixed pole in t_1, it gives a small contribution to the above combination of mass splittings. (This could be the case even for a strongly coupled fixed pole in t_1 if its couplings to the baryons had the same F/D ratio as the A_2.)

[16] S. Okubo, Phys. Rev. Letters **18**, 256 (1967); Y. Liu and S. Okubo [Nuovo Cimento **52**, 1186 (1967)] also consider tadpole contributions from A_2 exchange. However, in this analysis, it is assumed that the subtraction term $t_1^{(1)}(q^2,0)$ transforms like a tadpole, for which we find no justification, since it properly contains part of the elastic piece.

TABLE I. Tadpole contributions to mass differences.

	$\Delta M_{\rm exp}$ (MeV)	$(\Delta M)_{\rm elastic}$ (MeV)	δM^B (MeV)	$\delta M^B/\delta M^N$	γ_B/γ_N
$N=p-n$	-1.29	$+0.79$	-2.08	1	1
$\Sigma=\Sigma^+-\Sigma^-$	-7.97 ± 0.11	-0.22	-7.75 ± 0.1	3.72 ± 0.1	4.5 ± 0.5
$\Xi=\Xi^0-\Xi^-$	-6.5 ± 1.0	-1.1	-5.4 ± 1.0	2.6 ± 0.5	3.5 ± 0.5
$\Sigma^++\Sigma^--2\Sigma^0$	1.79 ± 0.3	1.54			

IV. SUPERCONVERGENCE RELATION AND FIXED POLE

In the absence of a fixed pole at $J=0$ in $t_2^{(1)}$, the function $\nu A_2^{BB'}(q^2,\nu)$ would satisfy a nontrivial superconvergence relation. However, in general, this superconvergence relation evaluates the residue of the fixed pole. Thus, if

$$\nu A_2^{BB'}(q^2,\nu) \underset{\nu\to\infty}{\longrightarrow} [\gamma_{B'}R_2^{(1)B}(q^2)-\gamma_B R_2^{(1)B'}(q^2)]\nu^{-1},$$

we have

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\nu\, \nu \operatorname{Im}A_2^{BB'}(q^2,\nu) =\gamma_{B'}R_2^{(1)B}(q^2)-\gamma_B R_2^{(1)B'}(q^2), \quad (4.1)$$

where $R_2^{(1)B}$ is proportional to the residue of the fixed pole at $J=0$ of $t_2^{(1)B}$. This relation allows one to calculate the ratio $\gamma_{B'}/\gamma_B$:

$$\frac{\gamma_B}{\gamma_{B'}}=\left[\frac{1}{\pi}\int_{-\infty}^{+\infty} d\nu\, \nu\operatorname{Im}t_2^{(1)B}(q^2,\nu)-R_2^{(1)B}(q^2)\right]\Big/ \left[\frac{1}{\pi}\int_{-\infty}^{+\infty} d\nu\, \nu\operatorname{Im}t_2^{(1)B'}(q^2,\nu)-R_2^{(1)B'}(q^2)\right]. \quad (4.2)$$

Written in this form, one sees that what we have done is to take a ratio of finite-energy sum rules[17] for νt_2^B and $\nu t_2^{B'}$, which are not superconvergent, and use factorization. The finite-energy sum rule for $\nu t_2^{(1)B}(q^2,\nu)$ yields

$$\frac{1}{\pi}\int_0^N d\nu^2 \operatorname{Im}t_2^{(1)B}(q^2\nu)-R_2(q^2)=\beta(q^2)\gamma_B N^{\alpha_{A_2}(0)}, \quad (4.3)$$

and Eq. (4.2) is derived by taking ratios of two such equations for different particles and letting the cutoff approach infinity. This procedure can be carried out not only in the present case of virtual Compton scattering, but also for all amplitudes where one trajectory prevents superconvergence. The advantage is, of course, that one can then (in theory if not in practice) let the cutoff got to infinity and derive sum rules relating the ratio of Regge residues to various particles to a convergent ratio of infinite quantities.

Returning to (4.2), we evaluate this relation at $q^2=0$ and investigate whether the fixed pole is indeed present. The contribution of the baryon pole to the integral is simply

$$\frac{1}{\pi}\int_0^\infty d\nu^2 \operatorname{Im}t_2^{(1)B}(q^2,\nu)=\frac{q^2 f_2^{(1)B}(q^2)}{M_B} =\frac{\alpha}{\pi M_B}\frac{[G_E^B(q^2)]^2+q^2[G_M^B(q^2)]^2/4M_B^2}{1+q^2/4M_B^2}. \quad (4.4)$$

Thus, if we were to neglect the continuum and set $R_2^{(1)B}=0$, we would derive, at $q^2=0$,

$$\frac{\gamma_B}{\gamma_{B'}}=\frac{M_{B'}}{M_B}\frac{[G_E^B(0)]^2}{[G_E^{B'}(0)]^2}, \quad (4.5)$$

or

$$\gamma_\Sigma=0,\ \gamma_\Xi=-(M_N/M_\Xi)\gamma_N. \quad (4.6)$$

In the limit of $SU(3)$ symmetry for the baryon masses, this corresponds to pure D coupling for the A_2—in violent disagreement with experiment, or with the ratio of "tadpoles" as calculated previously. The addition of the decuplet contribution has little effect at $q^2=0$, and is in the wrong direction. Thus, without a fixed pole, Eq. (4.2) predicts a small and positive F/D ratio for the A_2 couplings. Unless we assume that the rest of the continuum enters strongly to alter this result, we are forced to the conclusion that $t_2^{(1)}$ has a fixed pole at $J=0$, that this fixed pole couples singificantly at $q^2=0$, and that its F/D ratio is not the same as the F/D ratio of the A_2 trajectory. This is not unreasonable, since t_2 is proportional to the t-channel helicity-flip amplitude and $J=0$ is therefore a nonsense value for t_2 (right signature). Fixed poles at right signature at nonsense values of the angular momentum are allowed for weak amplitudes, since unitarity is linear and provides no mechanism for removing them.[12] In fact, in the case of noncommuting vector currents, one is forced to have a fixed pole at $J=1$ by current algebra. In our case, current algebra does not seem to provide any information about the existence or the residue of a fixed pole at $J=0$.[18]

The dependence of $R_2^{(1)B}(q^2)$ on the mass of the virtual photon should be trivial. In particular, the residue cannot have poles at the masses of vector mesons, $q^2=-m_V^2$, since this would imply a fixed pole in strong amplitudes at a right-signature value.

It should be possible to detect the presence of this fixed pole in Compton scattering. In particular, at large enough momentum transfers, where the A_2 tra-

[17] K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 625 (1967); R. Gatto, *ibid.* 18, 803 (1967); A. Logunov, L. Soloviev, and A. Tavkhelidze, Phys. Letters **24B**, 181 (1967); M. Virasoro, Nuovo Cimento **51A**, 227 (1967); R. Dolen, D. Horn, and C. Schmid, Phys. Rev. Letters **19**, 402 (1967).

[18] The presence or absence of fixed poles may be related to the existence of the commutator $[J_\mu(x),J_\mu(0)]\delta(x_0)$, discussed by J. D. Bjorken [Phys. Rev. **148**, 1467 (1966)], although we have not been able to establish this in detail.

jectory falls below $J=0$ [$t\approx -0.6$ (BeV/c)^2], the fixed pole should dominate the asymptotic behavior of the difference of the differential cross section for protons and neutrons. Hence we expect

$$\lim_{s\to\infty} s \frac{d\sigma^{(1)}(s,t)}{d\Omega}\bigg|_{t\lesssim -0.6\ (\mathrm{BeV}/c)^2} \to F(t)\,,$$

independent of s, for electroproduction from the nucleons, where $F(t)$ is proportional to the residue of the fixed pole or the coefficient of the δ_{J0} in sense amplitudes.

ACKNOWLEDGMENT

We would like to thank Professor Ivan Muzinich for several enlightening discussions.

PHYSICAL REVIEW VOLUME 172, NUMBER 5 25 AUGUST 1968

Electromagnetic Perturbations on πNN and πNN^* Couplings in the Chew-Low Model: General Features*

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(Received 18 March 1968)

Electromagnetic perturbations on πNN and πNN^* couplings are studied in the N-N^* reciprocal bootstrap model. In the present paper we confine ourselves to rather general features, making the linear-D approximation for simplicity. There are several self-consistent coupling shifts, much as in the analogous $SU(3)$ reciprocal bootstrap. It is shown that, except for even-J exchanges in the t channel, the "driving terms" are orthogonal to these self-consistent coupling shifts. Thus, as in the $SU(3)$ case, no simple predictions can be made for coupling shifts in the linear-D approximation.

I. INTRODUCTION

SYMMETRY-BREAKING perturbations on the Chew-Low model have been much studied,[1–7] in the hope that (i) a unique set of perturbations would be approximately self-consistent; (ii) the set would be "driven" by the electromagnetic, weak, or semistrong interactions, thus allowing the prediction that observed mass and coupling shifts should be in the same ratio as the approximately self-consistent perturbations of the model. A unique set of approximately self-consistent perturbations, resembling the experimental results, was indeed found for electromagnetic and strong mass splittings of the $J=\frac{1}{2}^+$ octet B and the $J=\frac{3}{2}^+$ decouplet Δ,[1–4] and for the parity-violating part of weak decays $B\to B+\Pi$.[5] On the other hand, several different self-consistent perturbations were discovered for the parity-conserving part of the weak decays $B\to B+\Pi$, so no predictions could be made for this amplitude.[6,7] A similar situation was found for strong perturbations on the $BB\Pi$ and $\Delta B\Pi$ couplings,[6,7] except that in this case it was possible to achieve predictions by noting that the B and Δ mass shifts would preferentially "drive" a particular set of the self-consistent coupling shifts.[6]

We wish to report here on general features of an analogous study of electromagnetic coupling shifts in the $SU(2)$ version of the Chew-Low model. The same disease occurs as in $SU(3)$ coupling shifts: There are several different sets of self-consistent coupling shifts. In addition, we find that the one-photon exchange contribution to πN scattering, and contributions such as γN and $\gamma\pi N$ intermediate states in the s and u channels, only "drive" those sets of coupling shifts which are *not* self-consistent. This supports the conclusion of the related $SU(3)$ studies: The perturbed Chew-Low model does not predict any simple pattern of parity-conserving coupling shifts unless the mass shifts impose one.

In the present paper we derive the above-mentioned results in the linear-D approximation, where the mathematics is simple, and discuss how the results are related to general properties of the crossing matrix. In the following paper,[8] an attempt is made to obtain a rough estimate of the coupling shifts in spite of these

* Work supported in part by the U. S. Atomic Energy Commission. Prepared under Contract No. AT(11-1)-68 for the San Francisco Operations Office, U. S. Atomic Energy Commission.

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‖ Supported in part by National Science Foundation Fellowship and by American Association of University Women Fellowship (1966–67). This paper is based on a thesis submitted by N. S. Thornber to the California Institute of Technology in partial fulfillment of the requirements for the Ph.D. degree.

[1] R. E. Cutkosky and P. Tarjanne, Phys. Rev. 133, B1292 (1964).

[2] R. H. Capps, Phys. Rev. 134, B1396 (1964).

[3] R. Dashen and S. Frautschi, Phys. Rev. Letters 13, 497 (1964).

[4] R. Dashen and S. Frautschi, Phys. Rev. 137, B1331 (1965).

[5] R. Dashen, S. Frautschi, and D. Sharp, Phys. Rev. Letters 13, 777 (1964).

[6] R. Dashen, Y. Dothan, S. Frautschi, and D. Sharp, Phys. Rev. 143, 1185 (1966).

[7] B. Diu, H. Rubinstein, and R. Van Royen, Nuovo Cimento 43A, 961 (1966).

[8] N. S. Thornber, following paper, Phys. Rev. 172, 1395 (1968).