Diffraction-Disassociation Model of Very-High-Energy Nucleon-Nucleon Interactions and the Diffusion of Cosmic Rays through the Atmosphere

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A simple model of very-high-energy nucleon-nucleon interactions is constructed on the assumption that at sufficiently high energies the dominant processes can be described in terms of the production of two centers—fireballs—which carry the same quantum numbers (except angular momentum) as the corresponding nucleons, and that the cross section for producing fireballs of a definite invariant mass will decrease with energy at the same rate as the elastic scattering cross section. The rate of decrease is assumed to be derived from the dominance of the exchange of a Pomeranchuk Regge trajectory. Fireball mass spectra are defined by the model without the introduction of arbitrary parameters. Meson production multiplicities and nucleon inelasticity factors, which are derived from the fireball mass distributions with minimal additional assumptions, are in accord with experimental observations of nucleon-nucleon interactions. This description of nucleon-nucleon interactions is then used to construct a model of the propagation of cosmic rays through the atmosphere. The nucleon attenuation lengths, the muon energy spectra, and the muon charge ratios deduced from the model are in qualitative agreement with observations.

INTRODUCTION

THE description of the dynamics of the elementary particle interactions in terms of the properties of the Regge poles in the complex angular-momentum plane suggests that at very high energies reactions are dominated by the exchange of Regge poles with the largest real component of angular momentum. Although all cross sections for the production of well-defined states will decrease with energy, the slowest variation with energy would seem to be associated with the exchange of the Pomeranchuk pole which is responsible for diffraction scattering in this description of dynamics. This suggests that at very high energies only those partial cross sections contribute substantially to the total cross section which decrease with energy as slowly as the elastic scattering cross section, and then only reactions mediated by the exchange of the Pomeranchuk pole are important in the high-energy limit. In other terminology, the dominant process is diffraction disassociation[1] of the interacting nucleons described by the exchange of the Pomeranchuk pole between the nucleons. There is definite evidence for the existence of such transitions from the study of nucleon-nucleon interactions at laboratory energies near 30 GeV,[2–4] though this kind of reaction is presumably not dominant until higher energies are reached.

The diagram of Fig. 1 represents a typical high-energy reaction according to the description of this model. State a is defined by its invariant mass M_a, angular momentum J_a, and by other quantum numbers which are the same as those of the nucleon N_a. If the nucleon N_a is a proton, the charge of state a will be $+1$, the total isotopic spin I will be equal to $\frac{1}{2}$, I_3 will be equal to $+\frac{1}{2}$, the hypercharge Y will be equal to $+1$, and the state will belong to an SU_3 octet as does the proton. State b will have the same quantum numbers as the nucleon N_b except for the spin J_b. States a and b will be states of the continuum, not discrete states, though the production of quasistationary states might be important. We will call these states isobars or fireballs, as they are presumably to be identified with isobars when their masses are small and with the phenomena called fireballs when their masses are larger.

We shall attempt to show that if the simplest form of the Regge-pole description of elementary particle reactions is valid for these reactions, the description itself almost completely defines much of the character of the reactions: In particular, the mass spectrum of the fireballs is essentially determined. We specifically assume that the dominant processes which describe the interaction of two nucleons at very high energies are such that the particles which result from the interaction can be sensibly separated into two groups, where each group can be identified with one of the two original nucleons. The resultant system will have approximately the same energy and momentum in the center-of-mass system as the nucleon, and will have the same quantum numbers (except for angular momentum). Each fireball will have a definite center of mass. We further assume

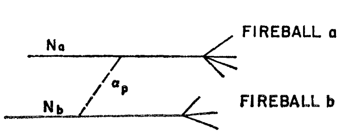

FIG. 1. Diagram of typical reaction showing the production of fireballs by the exchange of the Pomeranchuk trajectory between two nucleons.

[1] M. L. Good and W. D. Walker, Phys. Rev. **120**, 1857 (1960).

[2] A. P. Contegouris, S. C. Frautschi, and How-Sen Wong, Phys. Rev. **129**, 974 (1963).

[3] G. Cocconi, A. N. Diddens, E. Lillethun, G. Manning, A. E. Taylor, T. G. Walker, and A. M. Weatherall, Phys. Rev. Letters **7**, 450 (1961).

[4] E. W. Anderson, E. J. Bleser, G. B. Collins, T. Jufii, J. Menes, F. Turkot, R. A. Carrigan, Jr., R. M. Edelstein, N. C. Hien, T. J. McMahon, and I. Nadelhaft, Phys. Rev. Letters **16**, 855 (1966).

that the partial cross sections for the production of such sets of fireballs will decrease with energy at the same rate as the cross section for elastic scattering and that rate is defined by the simple Regge-pole model. This implies that in the asymptotic region no cross section decreases more slowly with energy than the elastic scattering cross section and that all of the cross sections which decrease more quickly are now so small that they can be neglected.

POMERANCHUK TRAJECTORY MODEL

According to the elementary Regge-pole description of interactions, the cross section for any two-body process j will have the form

$$d\sigma_j/dt = g_j^2(t)(s/s_0)^{2[\alpha(t)-1]}, \qquad (1)$$

where s is the square of the center-of-mass energy, s_0 is a scale factor to be evaluated later, t is the square of the four-momentum transfer, and $\alpha(t)$ is the value of the dominant Regge trajectory evaluated at t. At very high energies, reactions mediated by the Pomeranchuk pole should dominate. If we express the Pomeranchuk trajectory as

$$\alpha(t) = \alpha(0) + \alpha'(t)t = 1 + \alpha'(t)t \approx 1 + \alpha'(0)t,$$

where the presumption that the total cross section is constant requires that $\alpha(0)=1$ and the other approximations should be valid over the small range of t where the cross sections are substantial, we can write the differential cross section in the form

$$d\sigma_j/dt = g_j^2(t)\exp[2\alpha'(t)\ln(s/s_0)]t. \qquad (2)$$

The total cross section for the reaction j will then be equal to

$$\sigma_j = \int_{-\infty}^{t''} \frac{d\sigma_j}{dt} dt = \frac{g_j^2(t')}{2\alpha' \ln(s/s_0)}, \qquad (3)$$

where t' is an appropriate mean value of t and t'' is the maximum kinematically allowed value of t for the reaction j at the energy s. If the function $g_j^2(t)$ does not vary strongly with t, or if the energy squared s is sufficiently great so that the value of t'' is near zero, $g_j^2(t')$ will be nearly independent of s. The partial cross section σ_j then decreases with energy inversely with the logarithm of the energy, a result particularly well known when applied to elastic scattering. This is true of all partial cross sections according to this description of the reactions.

We have then the seemingly paradoxical situation that the total cross section is constant but all of the partial cross sections decrease with energy. This contradiction is removed when it is noted that the number of states available energetically increases with energy and then the number of partial cross sections increases with energy. The actual variation of a partial cross section with respect to the energy in the center-of-mass system is suggested by the graph of Fig. 2.

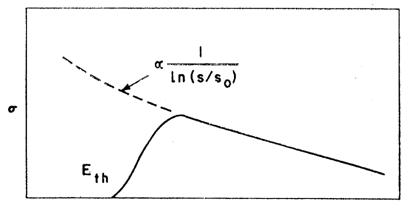

FIG. 2. Typical variation of a partial cross section with energy for a transition involving the exchange of the Pomeranchuk trajectory. The threshold energy is E_{th}.

The constant value of the total cross section, which implies a balance between the well-defined decrease of the partial cross sections and the well-defined increase in the magnitudes of the masses energetically available, suggests that the model places important constraints on the mass spectrum of the fireball or isobar states at energies in the asymptotic region. Let us assume that the total cross section at an energy $s_a^{1/2}$ is equal to σ_a and represents a certain spectrum of invariant masses M_f and M_b. At a higher energy $s_a'^{1/2}$, the cross sections for the production of the invariant masses kinematically accessible at the energy $s_a^{1/2}$ will have decreased uniformly by a specific amount to a new lower value of σ_a'. The decrease in that part of the total cross section $\sigma_a - \sigma_a'$ will be compensated precisely by the partial cross sections for the production of heavier isobars than were kinematically available at the energy $s_a^{1/2}$. The cross section for the production of isobars which are heavier than those kinematically available at the energy $s_a^{1/2}$ and lighter than the maximum masses kinematically accessible at the energy $s_a'^{1/2}$ is then essentially determined by the model.

In order to exploit these relations fully it is necessary to consider the relation between the masses of the isobars produced in the forward direction and the masses of the backwards isobars. The description suggested by the diagram of Fig. 1 would seem to require that the masses are independent beyond the constraint of kinematic factors; the Regge trajectory should carry no information beyond the restrictions on the change in quantum numbers. It should then be a good approximation to write a differential cross section as

$$d\sigma(M_f, M_b) = P(M_f)P'(M_b)dM_f dM_b, \qquad (4)$$

where M_f and M_b are the masses of the backwards and forwards isobars in the laboratory system: From the symmetry of the problem $P(M) = P'(M)$.

The total cross section can be written as

$$\sigma = \int_{M_n} \int_{M_n} d\sigma(M_f, M_b) \qquad (5)$$

or

$$\sigma = \int_{M_n} \int_{M_n} P(M_f)P(M_b)dM_f dM_b, \qquad (6)$$

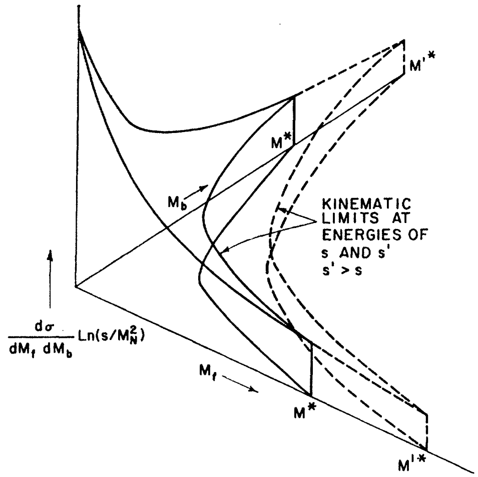

FIG. 3. A schematic drawing showing the variation of $d\sigma/(dM_f dM_b)$ as suggested by this model. Here M_f is the invariant mass of the forward fireball and M_b is the mass of the backward fireball produced by the nucleon-nucleon interaction.

where M_n is the nucleon mass and the upper limits of the integrals are the largest masses acceptable according to the kinematic constraints of the reaction at the energy s, but are not otherwise defined.

We can understand the most important variations of the integration limits by considering the least magnitude of four-momentum transfer required to produce masses M_f and M_b in nucleon-nucleon collisions. If one of the final states is a nucleon, say, M_b, then for sufficiently large values of the center-of-mass energy $s^{1/2}$, the least magnitude of four-momentum transfer squared t' required to produce an isobar of mass M_f is

$$t'\approx (M_n/s)^2(M_f^2-M_n^2)^2.$$

For $M_f \gg M_n$,

$$t'\approx (M_n/s)^2 M_f^4. \quad (7)$$

If both fireballs M_b and M_f are rather larger than M_n and s is sufficiently large, the minimum four-momentum transfer will be such that

$$M_b M_f \approx (st')^{1/2}. \quad (8)$$

For values of t' such that $t'\approx M_n^2$, the relations (7) and (8) are in agreement.

The magnitude of the four-momentum transfer at the energetic threshold of two-particle states is very high, so we expect that the cross section at threshold will be very small. The cross section will presumably not be appreciable until an energy $s^{1/2}$ is reached such that the momentum transfer required is small. It is then necessary to choose some effective threshold which will be dependent upon the choice of minimum magnitude of momentum transfer. If the partial cross sections are described accurately by a relation such as Eq. (2), the cross section will reach a value of about $1/e$ of the final value for a value of t such that

$$t=-1/[2\alpha' \ln(s/s_0)].$$

Then, setting $t=t'$, from Eq. (8) we obtain

$$M_b M_f \approx \{s/[2\alpha' \ln(s/s_0)]\}^{1/2}.$$

Using the notation M^* for the maximum value of either M_f or M_b, we have

$$M_b = M^* M_n/M_f,$$

where

$$M^* M_n = \{s/[2\alpha' \ln(s/s_0)]\}^{1/2}. \quad (9)$$

The total cross section can be written as

$$\sigma=\int_{M_n}^{M^*} dM_f \int_{M_n}^{M^*M_n/M_f} dM_b P(M_f)P(M_b).$$

Since the total cross section is independent of energy, this integral must be independent of s, although from Eq. (9) limits of integration depend upon s, and from Eq. (3) it follows that

$$P(M_f)P(M_b)dM_f dM_b \propto 1/[2\alpha' \ln(s/s_0)]$$

and the integrand also depends upon s. Writing

$$P(M)=B(M)/[2\alpha' \ln(s/s_0)]^{1/2},$$

we have the equation

$$\int_{M_n}^{M^*} dM_f \int_{M_n}^{M^*M_n/M_f} dM_b B(M_f)B(M_b) = \sigma\times 2\alpha' \ln(s/s_0). \quad (10)$$

This equation can be solved for $B(M)$ in principle, where $B(M)$ represents the relative probability of finding the fireball with a mass M.

Though we were not able to solve Eq. (10) to find an analytic form for $B(M)$ over the whole range of s, the form

$$B(M)=(\alpha'\sigma)^{1/2}/M[\ln(M/M_n)]^{1/2} \quad (11)$$

can be demonstrated to be a solution in the asymptotic limit: $s\rightarrow\infty$. Using this form for $B(M)$, relation (10) goes to

$$2\sigma\alpha' \int_{M_n}^{M^*} \frac{1}{M}\left[\frac{\ln(M^*/M_n)}{\ln(M/M_n)}-1\right]^{1/2} dM,$$

which for large s and then large M^* goes to

$$2\sigma\alpha' \int_{M_n}^{M^*} (1/M)[\ln(M^*/M_n)/\ln(M/M_n)]^{1/2} dM = 4\sigma\alpha' \ln(M^*/M_n).$$

Using Eq. (9) for M^*,

$$4\sigma\alpha' \ln(M^*/M_n) = 2\sigma\alpha'\{\ln(s/M_n^2) - \ln[2M_n^2\alpha' \ln(s/s_0)]\} = 2\sigma\alpha' \ln(s/s_0).$$

This equality will hold asymptotically if $s_0=M_n^2$, and Eq. (10) is satisfied asymptotically if $B(M)$ has the form of relation (11): Q.E.D.

In summary, according to this model the cross section for the production of a pair of fireballs of mass M_f and

M_b by a high-energy nucleon-nucleon interaction where the center-of-mass energy is $s^{1/2}$ will be equal to

$$d\sigma/(dM_f dM_b) = P(M_f)P(M_b), \qquad (12)$$

where

$$P(M) = \sigma^{1/2}M^{-1}[2\ln(M/M_n)\ln(s/M_n^2)]^{-1/2}.$$

The cross section for the production of a fireball with a mass between M and $M+dM$ in the forward (or backward) direction will be

$$\sigma(M) = \tfrac{1}{2}\sigma(dM/M)[\ln(M/M_n)\ln(M^*/M_n)]^{-1/2}. \qquad (13)$$

The character of the two-dimensional fireball mass spectrum derived from this model is suggested by the schematic diagram of Fig. 3. Both the smoothness of the upper surface and the sharpness of the kinematic limits are artificial, resulting from the physical approximations of the model. The kinematic limits must actually be much more diffuse and the smooth surface represents an attempt to average over the structure which must obtain as a result of the existence of discrete isobar states.

The mathematical approximations which were made are inadequate at energies as low as 30 GeV in the laboratory and quite good at energies as high as 1000 GeV. This alone indicates only that relation (11) is not a good representation of $B(M)$ for small values of M, which is hardly surprising on physical grounds anyway, as the discrete character of the states available must then be important.

Any attempt to evaluate the energy where the physical assumptions are likely to be valid is subject to the usual difficulties of defining or bounding the asymptotic region. It would appear that the exchange of the Pomeranchuk pole is important but not dominant at energies as low as 30 GeV in the laboratory system, however. At energies below 1000 GeV the decay products from the forward and backward fireballs are not well separated kinematically, and final-state interactions between these particles are probably not negligible. For very heavy fireballs the separation is not sharp until extremely high energies are reached. These final-state interactions—for example, the scattering of a meson emitted in some sense from the forward fireball by a meson emitted from the backwards fireball—may not affect most conclusions derived from the model, however.

COMPARISON WITH EXPERIMENTAL EVIDENCE—NUCLEON-NUCLEON INTERACTIONS

All of the information which presently exists concerning the interactions of very-high-energy particles is derived from observations of the interactions of the primary cosmic rays or the interactions of the high-energy secondary products of the primary interactions. Cosmic-ray experiments are difficult and the measurements are seldom as precise or as well defined as desirable. Nevertheless, many of the broad qualitative features of very-high-energy interactions are now well understood through the analysis of such cosmic-ray experiments and some important quantitative results which bear on the details of the interactions have been established. It appears to us that all of these results are consistent with characteristics of high-energy reactions which are defined by this diffraction-disassociation model.

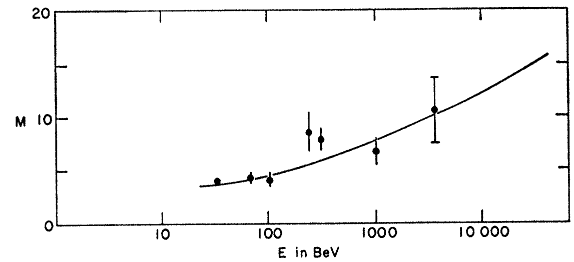

FIG. 4. Multiplicity of charged particles produced in nucleon-nucleon interactions as a function of the laboratory energy of the incident nucleon. The points show experimental results (see Ref. 16); the solid line shows the results of this model.

There has been considerable theoretical and experimental interest in the variation with nucleon energy of the multiplicity of particles produced in nucleon-nucleon interactions. Since most values of this quantity represent conclusions derived from the measurements of the total number of charged particles produced in the collision nucleons with nuclei, and the total energy of the incident nucleon is seldom known precisely, the reliability of the extension of the results of these measurements to the characteristics of nucleon-nucleon collisions at definite energies is not certain. The points[5] on Fig. 4 show the results of various workers concerning their conclusions as to the average number of charged particles produced in nucleon-nucleon interactions at different energies of the incident nucleon. Except at the lowest energies the measurements plotted on the graph represent measurements of multiplicities resulting from the interactions of fast nucleons with light nuclei, and resulting multiplicities may well be a little larger than for pure samples of nucleon-nucleon collisions.

Although the diffraction-disassociation model proposed here does not explicitly predict the multiplicity as a function of nucleon energy, this multiplicity can be derived from the model prediction of the fireball mass spectra and further information extracted from the experimental measurements of very-high-energy interactions. It has now been established that the mean transverse momentum of particles produced in the interaction of any strongly interacting particles with nucleons at high energies is about 400 MeV and is independent of the interaction energy. If the particles emitted during the decay of the fireball are emitted nearly isotropically in the center-of-mass system of the

[5] Y. Pal and B. Peters, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 33, 1 (1964), see especially p. 40.

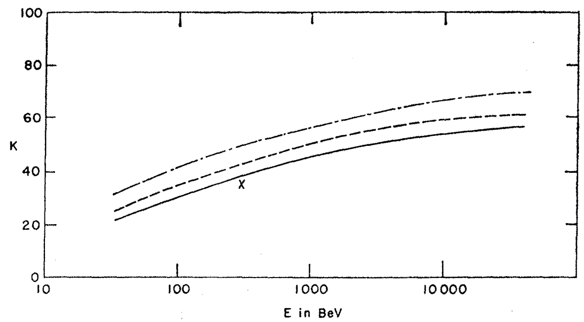

FIG. 5. Nucleon inelasticities derived from the model of nucleon-nucleon interactions. The dashed line represents the results when the mesons are emitted isotropically; the solid line gives the results for a distribution of the form 1–0.5 cosθ. The upper curve represents an estimate of the inelasticity for nucleon-nucleus interactions with nitrogen or oxygen. The cross shows the experimental result of Dobrotin *et al.* (see Ref. 7).

fireball, the mean total energy of these particles will be about 500 MeV for pions, about 700 MeV for K mesons, and the nucleons ejected during the fireball decay will have a total energy only a little greater than 1 GeV.

If some of the relative angular momentum of the incident particles is retained in the spin of the fireball, the decay angular distributions will not be isotropic but may approach the classical limit where the number of particles emitted per unit solid angle will be proportional to $1/\sin\theta$, where θ is the angle with respect to the direction of the incident nucleon. The mean energy of the particles in the center-of-mass system of the fireball will then be slightly higher.

Typically, then, a fireball would be expected to decay such that the decay products consist of one nucleon carrying off a total energy of about 1000 MeV in the rest frame of the fireball and a number of mesons which carry off an average total energy E_p near 500 MeV. To the extent that we can neglect the large fluctuations which result from the distribution of energies taken off by the resultant particles, we could estimate the mean multiplicity of particles emitted from the fireball according to a relation of the form

$$N=(M_f-E_n)/E_p+1, \qquad (14)$$

where N is the total number of particles emitted from the fireball with mass M_f.

In fact, the distribution of particle energies does affect the multiplicities and an expression such as Eq. (14) somewhat overestimates the multiplicities. We have chosen to attempt a more realistic calculation by assigning a distribution of decay energies which may simulate nature more closely. The specific distribution which was used was chosen for convenience in calculation using Monte Carlo techniques as well as for a reflection of prejudices we hold concerning the description of the decays of massive states. Explicitly, we assume in our calculation that the fireball decays by emitting mesons sequentially where the meson energy is chosen by a random number generator to have a probability expressed by

$$P(E)=\exp(-E/E_0);$$

here E is the total energy of the meson in the fireball system and E_0 was chosen to be 400 MeV. If the energy E chosen randomly was so large as to leave a residual lighter than the mass of a nucleon plus a meson, the decay was assumed to be a two-body decay to a nucleon and meson, and recalculated on that basis.

The decay was assumed to be isotropic where the angle was chosen using a random-number generator. The recoil of the fireball was considered, and the fireball was allowed to decay again and again until a last two-body decay to a nucleon and a meson concluded the decays. The distribution of transverse momenta from the decays of fireballs calculated in this way is consistent with observed distributions and a mean transverse momentum of 400 MeV/c.

Although such a decay scheme represents an implicit approximation to an isobar model, where the emission of each meson represents the transition of the fireball from one state to another, the actual distributions which result are found to be remarkably close to those derived from a statistical model; that is, the energy distributions of the mesons from all cases such that a fireball of mass M decays into N mesons are very close to the distributions calculated analytically on the assumption that probability of a configuration is proportional to the volume of relativistic phase space.[6] We have no particular insight into this result.

Two-thirds of the mesons emitted from the fireballs are assumed to be charged. This ratio will hold accurately for the production of pions if the target nucleons are half neutrons and half protons and charge independence of the strong interactions is assumed. For the production of nucleons and K mesons, particles with an isotopic spin equal to one-half, the ratio of charged to neutral particles is not determined by these conditions. The ratio is probably less than two-thirds for the most part, however; at high multiplicities where we might consider that the isotopic spin vector is nearly randomly oriented we might expect the ratio of charged to neutral particles to approach 1.

The solid line in Fig. 4 then represents the values of the average multiplicities of charged particles produced in the nucleon-nucleon interactions of high-energy protons with an equal mixture of target neutrons and protons calculated by Monte Carlo methods according to the model presented here. The masses of the fireballs are selected randomly according to the distributions of Eq. (13) and the fireball decay spectra were calculated as above. The agreement of the model calculations with observations can be considered rather good, especially when it is considered that the experimental measurements are actually concerned with collisions with light nuclei where there will be some contamination of the

[6] R. Adair, H. Kasha, and C. J. B. Hawkins (unpublished).

nucleon-nucleon results with multiple nucleon interactions increasing the observed multiplicity.

At very high energies where the multiplicities are large and nucleon-nucleon production and K-meson emission from massive fireballs are particularly important, the curve of Fig. 4 may overestimate the multiplicity by factors of the order of 15%. The heavy particles will carry away more energy than a pion and the ratio of charged particles to neutral particles will be smaller than for the pion.

It has long been known that fast nucleons typically retain most of their energy in the course of nucleon-nucleon interactions. A quantity called the inelasticity, which is the proportional energy lost by the incident nucleon—largely through the production of high-energy mesons and baryon pairs—is usually used as a parameter to describe the interactions. In the fireball model proposed here, the mean inelasticity for an event such that the incident nucleon forms a fireball with mass M_f moving in the forward direction will be expressed approximately as

$$K = 1 - E_n'/M_f, \tag{15}$$

where E_n' is the average energy of emission of the nucleon in the rest system of the fireball. If we take the mean momentum of the nucleon in the fireball system as 500 MeV, the value of E_n' will be about equal to 1.10 GeV. Since the energy lost by the nucleon in the formation of the backwards fireball is neglected in the approximation of Eq. (14), the actual values of K should be somewhat higher than given by Eq. (15). This is an appreciable effect at low energies—for interactions of nucleons with energies less than 200 GeV—but is negligible for high-energy interactions according to this model.

Since this diffraction-disassociation model results in definite predictions for the mass spectra of the fireballs for the nucleon-nucleon interactions of different energies, the model also predicts definite inelasticities. Again it is desirable to calculate the inelasticities on the basis of the model of isotropic fireball decay which was discussed. The dashed curve of Fig. 5 shows the mean inelasticities calculated in this way from this model as a function of the laboratory energy of the incident nucleon. Here the mean inelasticity is defined as the energy lost by a large sample of incident nucleons divided by the total incident energy. Although the calculated values are not inconsistent with the experimental observations, the accuracy of those observations is probably not sufficient to establish the increase in inelasticity with energy which is characteristic of the model.

The model presented here also predicts definite fireball mass distributions for the interactions of nucleons of any given energy. Then, following the arguments of the last paragraphs, the model predicts distributions of inelasticity factors. For each interaction there are two inelasticity factors even as there are two

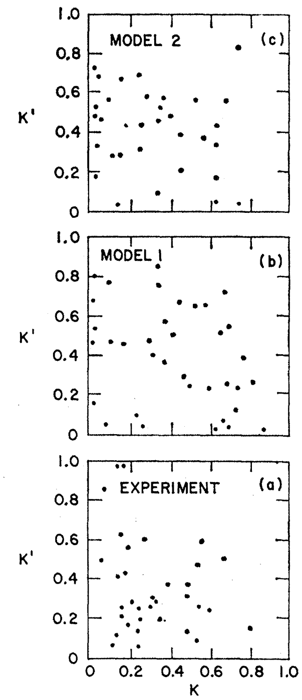

FIG. 6. Forward and backward inelasticity distributions for nucleon-nucleon interactions. The distribution *a* represents the measurements of Dobrotin and Slavatinski (see Ref. 7). The distributions *b* and *c* represent the results of Monte Carlo calculations using the model of nucleon-nucleon interactions. Model 1 represents the pions as being emitted isotropically in the center-of-mass system of the fireball while the calculations of model 2 were made assuming the pions are emitted preferentially backwards with a distribution of the form of 1–0.5 cosθ.

fireballs; the two factors are related kinematically through the Lorentz transformations which connect the system where the target nucleus is at rest with the frame where the incident nucleon is at rest. Observed distributions of inelasticity factors can then be related statistically to fireball mass distributions.

Dobrotin and Slavatinsky,[7] at the Lebedev Institute, have made extensive measurements of these inelasticity coefficients using a calorimeter in connection with a cloud chamber in a magnetic field. The set of events observed by this group were initiated by incident nucleons with a median energy near 300 GeV. Since the distribution of fireball masses suggested from the model presented here is not very sensitive to the energy of the incident nucleon, good energy resolution is not important.

The results of the experiment of Dobrotin *et al.* for the 34 events which they observed are shown in the plot of Fig. 6(a), where for each event the inelasticity of the forwards nucleon is plotted against the inelasticity of the backwards nucleon. For comparison we have

[7] N. A. Dobrotin and S. A. Savatinski, in *Proceedings of the Tenth Annual International Conference on High-Energy Physics, Rochester, 1960*, edited by E. C. G. Sudarshan, J. H. Tinlot, and A. C. C. Melissinos (Interscience Publishers, Inc., New York, 1961), p. 819.

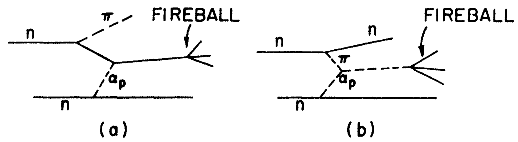

FIG. 7. Reactions which contribute to an anisotropy in the decays of the fireballs.

plotted as Fig. 6(b) the equivalent quantities for 34 randomly selected events generated by the Monte Carlo programs following the recipes of the model presented here: 17 events were chosen such that the incident energy was 150 GeV and 17 events were selected with the incident energy equal to 450 GeV. While there are very important qualitative similarities between the experimental distributions and the calculated distribution—a large portion of the events of both samples show a strong asymmetry in the inelasticity factors for the forwards and backwards nucleons—it is clear that the mean inelasticity factor of about 0.35 observed experimentally, and shown as a cross on Fig. 5, is smaller than the value of 0.42 derived from the model.

If the validity of the model is to be retained, better agreement with the experimental results can be obtained only by postulating that the nucleon is emitted preferentially forwards in the system of the fireball. Since the fireball is not a stationary state, no pseudoscalar is defined by such an asymmetry, and parity conservation is not manifestly violated by such a presumption.

An asymmetry of this nature, where the nucleon is emitted preferably forward in the center-of-mass system of the fireball, seems generally plausible. Contributions to the production amplitude from Pomeranchuk exchanges such as those suggested by the diagrams of Fig. 7 are likely to be important and coherent with the contribution from the archetype amplitude described by the diagram of Fig. 1.

The nucleons will be emitted anisotropically with respect to the center of mass of the forward particles (the residue of the incident nucleon) for either of the transitions described by the diagrams of Fig. 7(a) or 7(b). The transition in Fig. 7(a) describes a reaction such that one pion is emitted forwards in that system while a fireball is emitted backwards. The nucleon, emitted isotropically from the fireball, will be emitted backwards in the center of mass of the whole forwards system. The effect will not be extremely strong, however. According to the description of Fig. 7(b), the nucleon is emitted in the forward direction while a fireball is emitted backwards. This will likely be a relatively strong effect resulting in an over-all asymmetry in the decays of the fireballs such that the nucleon is preferably emitted in the forward direction.

This asymmetry was introduced into the Monte Carlo calculations of the decay of the fireballs by imposing an angular distribution of the form

$$dN/d\Omega = 1 - A\cos\theta$$

on the pions emitted from the fireball, where θ is the angle with respect to the forward direction. With A taken arbitrarily as 0.5, the inelasticities are decreased to the values shown by the solid line in Fig. 5. The plot of Fig. 6(c) shows the improved agreement with Dobrotin and Slavatinsky's data from the modified distributions.

There is other evidence concerning mean inelasticities which suggests values near 0.40 for the nucleon-nucleus inelasticity in light nuclei but the uncertainties are large.

COMPARISON WITH EXPERIMENTAL OBSERVATIONS—PROPAGATION OF COSMIC RAYS THROUGH THE ATMOSPHERE

The general character of the propagation of cosmic rays through the atmosphere provides further tests of any model of high-energy processes. Since the atmosphere is made up of light nuclei—nitrogen and oxygen—rather than free nucleons, any interpretation of the measurements on cosmic rays must rely upon some model of the multiple interactions in a light nucleus which cannot be really negligible. As a result of this kind of uncertainty it is not important that the calculations based on the model agree with observations with great accuracy, but the general character of the observations should be reflected in results derived from the model.

The calculation proceeds by selecting a nucleon of definite energy and definite charge using a random-number generator. The nucleon energy spectrum chosen initially for this investigation is

$$dN/dE = 2.25E^{-2.67}\ \text{nucleons/cm}^2\ \text{sr sec GeV},$$

where the exponent is taken from the measured proton spectra at sea level by the Durham group using the approximation, suggested by Pal and Peters,[8] that the spectrum is not changed by the diffusion through the atmosphere. The constant was chosen to fit the measured flux of primaries at the top of the atmosphere[9] where the geomagnetic cutoff is about 17 GeV. This flux of protons has been measured by many groups to be about 100 protons/m^2 sr sec, when corrections are applied for reentrant splash albedo.[10] We assume that 74% of the nucleons enter the atmosphere as free protons and 26% as light nuclei with a neutron-proton ratio near 1.[11] Although the neutrons are bound to protons in light nuclei, we treat the contributions of the nucleons as if they were incoherent.

Each nucleon is then followed through a history of

[8] Y. Pal and B. Peters, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 33, 1 (1964).
[9] P. C. Agrawal, S. V. Damle, G. S. Gokhak, G. Joseph, P. K. Kunte, M. G. K. Menon, and R. Sunderrajan, in Proceedings of the Ninth Symposium on Cosmic Rays, Elementary Particle Physics, and Astrophysics, Bombay, 1965, p. 58 (unpublished) This paper also reviews earlier work.
[10] F. R. McDonald, Phys. Rev. 109, 1367 (1958).
[11] C. J. Waddington, Progr. Nucl. Phys. 8, 3 (1960).

collisions. If the nonelastic cross section for nucleon-nucleon interactions is taken as 30 mb, the mean free path of the incident nucleons would correspond to about 56 g/cm^2 in the absence of structure in the nucleon distribution—that is, in the absence of nuclei. We take into account the existence of the light nuclei by imposing a hierarchy of structure, where the mean free path for collision with a structure (nucleus) is 56 g, but the existence of the condensations of nucleons, which we know as nuclei, results in shielding of nucleons to an extent which leads to a mean free path between actual collisions in the atmosphere of 88 g, in agreement with the absorption cross section of light nuclei as measured for nucleons with energies near 30 GeV.

For each collision, fireball masses are calculated according to the spectra suggested by the model proposed here, and the fireball is considered to decay into mesons again according to the model. Charges are assigned to the mesons following prescriptions designed to insure charge independence. These particular recipes will be discussed in detail later. The mesons are followed through matter in the nucleus, and between nuclei. The mean free path for meson-nucleon interaction is taken as $\frac{3}{2}$ the nucleon-nucleon mean free path in accordance with the well-known relations between high-energy nucleon-nucleon cross sections and meson-nucleon cross sections. The meson will have a certain probability of decaying before it makes a collision. If the meson decays, the muon energy is selected, knowing the meson-muon decay energy and angular distribution, and the muon charge and energy are recorded; the muon energy is corrected for ionization losses in passage to sea level.

Of course, for each distribution of path length, decay length, emission angle, etc., a random-number generator is used with appropriate algorithms to simulate the distributions.

In choosing to restrict our analyses to some aspect of the spectra of particles with high energy—over 20 GeV for muons, over 50 GeV for nucleons—we are able to exploit the steepness of the primary energy spectrum, and all other spectra, and simplify our calculations considerably by neglecting certain classes of secondary events. In particular, we are able to consider all interactions of mesons with nucleons as absorptive in character; we can eliminate from our calculation any meson which interacts with a nucleon, as the neglect of mesons produced by meson-nucleon interactions leads to no considerable error. This follows from a combination of the steepness of the nucleon energy spectrum and the high degree of inelasticity of meson-nucleon collisions. Meson-nucleon collisions are highly inelastic in the sense that only rarely will a meson emerge from an interaction with an appreciable fraction of the laboratory energy of the incident nucleon. Since the nucleon and meson spectra vary sharply with energy, the number of mesons in any energy interval which are produced through the secondary interaction of a high-energy meson will be negligible compared with the number of mesons in that energy interval which are produced directly through a nucleon-nucleon interaction.

Such a treatment is not justified for nucleons: Nucleon-nucleon interactions are often nearly elastic and the nucleon which emerges from the decay of the fireball is followed to the next interaction if the nucleon energy is above the threshold of interest. As a result of the comparatively small inelasticities of the nucleons, the population of nucleons in a definite energy interval at a definite depth in the atmosphere is derived from primary nucleons which have not made any interaction with the atmosphere and nucleons which result from the decays of fireballs produced by higher-energy primary nucleons. This secondary population is not at all negligible compared to the primary population; therefore, the nucleon flux is attenuated more slowly by the atmosphere than might be anticipated from the measure of the mean free path. This effect is then reflected in the measured attenuation length of the nucleon spectra in the atmosphere of about 120–145 g/cm^2 compared to the mean free path of about 88 g/cm^2.

Since the uncertainty in the treatment of the interactions of nucleons and mesons in the nuclei which make up the real atmosphere constitutes a fundamental limitation to the comparison of this model of nucleon-nucleon interactions to observations of the propagation of cosmic rays through the atmosphere, further discussion of the treatment of nucleon-nucleus interactions which is used in these calculations is useful. We treat the nuclei formally as cylindrical clusters of nucleons with the axis of the cylinder in the direction of the flux, and adjust the thickness of the clusters so that the shielding is such as to reduce the nucleon-nucleon mean free path of 56 g/cm^2 to the measured nucleon-nucleus mean free path of 88 g/cm^2 for nitrogen and oxygen. This sets the thickness of the cluster nucleus as 1.0 mean free path. A nucleon which interacts with the nucleus will interact with but one nucleon 58% of the time. If we consider the nucleon emitted by the fireball as a continuation of the identity of the original nucleon, we can consider that the nucleon will make more than one nucleon-nucleon interaction in the nucleus about 42% of the time.

Taking the meson-nucleon cross section as about $\frac{2}{3}$ the nucleon-nucleon cross section, the cylindrical nuclei will be about 0.67 mean free paths thick to mesons and the mean free path for mesons in the atmosphere will be about 115 g.

To the extent that multiple interactions with the nucleons in the nucleus can be considered as incoherent separate accidents, we believe that no serious errors are injected by the use of such a simple model of the nucleus. If coherent interactions of the incident nucleus with several target nuclei—that is, many-body interactions—are important, the model may represent nature inadequately.

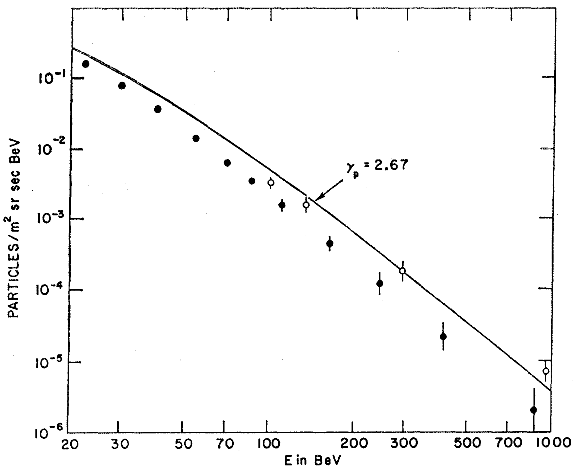

FIG. 8. The muon flux from the zenith as measured at sea level. The solid points represent the results of the Durham group (see Ref. 12) and the open points are from the Manchester results (see Ref. 13). The solid curve shows the flux as calculated on the basis of this model.

The differential energy spectrum of the charged muons which reach sea level from the vertical was then calculated from the description of nucleon-nucleon interactions introduced here and the model of diffusion through the atmosphere outlined in this section. For clarity we restate the parameters used in the calculation: The exponent of the energy dependence which describes the primary nucleon spectrum was taken as -2.67; the mean free path for protons in the atmosphere was taken as 88 and 110 g/cm² for pions; and the angular distribution of the pions emitted from the fireball was taken as $1+A\cos\theta$, where A was taken as -0.50 to fit the results of Dobrotin *et al.*—this is the only arbitrary parameter in the model, the other parameters being essentially dictated by observations.

The solid line of Fig. 8 shows the calculation distribution of muon flux versus muon energy, while the points show the flux measured by Wolfendale and his collaborators at Durham[12] and by the Manchester group.[13] The discrepancies in the experimental measurements at the higher energies—beyond 200 GeV—suggest the difficulties and uncertainties of these measurements and we will be primarily concerned with comparisons between theory and observation at energies under 200 GeV. The discrepancies between the calculated results and the observed values of the Durham group do appear to be significant, however, the calculated values being higher.

The same calculations led to an attenuation length of 145 g/cm² for the integral proton flux over 50 GeV. This is to be compared with the results of the Durham group[14] of about 120 g/cm² for fluxes under 50 GeV and the measurements of Baradzei[15] *et al.* which suggest a value of about 148 g/cm² at an energy of about 1000 GeV. Again a discrepancy is suggested, since the value of the Durham group is probably more nearly relevant to our calculations than the results at 1000 GeV.

TABLE I. The average number of pions with various charges and neutrons and protons emitted in the decay of a fireball of charge 1 and isotopic spin $\frac{1}{2}$ for various decay modes: (a) The fireball decays to a nucleon and a meson; (b) the fireball decays to an isospin-$\frac{1}{2}$ isobar and a meson and the isobar then decays to a nucleon and meson; and (c) the fireball decays to an isospin-$\frac{3}{2}$ isobar and a meson and the isobar decays to a nucleon and meson.

	π^+	π^0	π^-	p	n
(a) $I^{1/2}\rightarrow n$	2/3	1/3	0	1/3	2/3
(b) $I^{1/2}\rightarrow I^{1/2}\rightarrow n$	8/9	6/9	4/9	5/9	4/9
(c) $I^{1/2}\rightarrow I^{3/2}\rightarrow n$	7/9	6/9	5/9	7/9	2/9

These discrepancies—if the discrepancies are indeed real—between the results of the calculations and the experimental observations, are not easily removed by changing the parameters in the models of the nucleon-nucleon interaction. The muon flux would be reduced if the multiplicity deduced from the model is already somewhat smaller than the estimates derived from experiment as shown in Fig. 4. If the pions are produced with lower energies in the laboratory system, the differential muon flux would also be reduced but the nucleon would then have to carry off more of the total energy, the nucleon inelasticity would be reduced, and the attenuation length would be increased—but the calculated attenuation length of 145 g already seems rather too great. In general, we believe that it is very difficult, perhaps impossible, to reconcile the observed multiplicities, the observed muon flux, and the observed attenuation length with any plausible model of nucleon-nucleon interactions and retain the basic parameters used to describe the primary flux and the mean free path through the atmosphere.

Since the description of the nucleon-nucleon interaction in terms of the exchange of the Pomeranchuk trajectory demands that the fireball have the same charge and isotopic spin as the incident nucleon, and the charge distribution of the primary nucleons is well known, it would appear that the charge distribution of the mesons produced by the interaction would largely be determined.

The charges of the mesons emitted by the fireball were calculated using a random-number generator to produce the probabilities required by the fireball quantum numbers and charge independence. The numbers of Table I show the charge distributions, determined by charge independence, of the nucleons and mesons which are emitted in the decay of a fireball into a meson and a nucleon, or a nucleon and two

[12] P. J. Hayman and A. W. Wolfendale, Proc. Phys. Soc. (London) **80**, 710 (1962).
[13] J. E. R. Holmes, B. G. Owen, and A. L. Rodgers, Proc. Phys. Soc. (London) **78**, 496 (1962).
[14] G. Brooke, P. J. Hatman, P. J. Taylor, and A. W. Wolfendale, J. Phys. Soc. Japan Suppl. **A3**, 311 (1962).
[15] L. T. Baradzei, V. I. Rubtsor, Y. A. Smordin, M. V. Solovgov, and B. V. Tolkachev, J. Phys. Soc. Japan Suppl. **17**, 433 (1962).

mesons, assuming that the incident nucleon is a proton and then that the charge quantum numbers of the fireball are the same as the quantum numbers of a proton. The probabilities are reversed according to charge symmetry if the incident nucleon is a neutron. For decays of the fireball into two mesons we assume that the fireball, with isospin equal to $\frac{1}{2}$, decays into an isobar and a meson, where the isospin of the isobar is either $\frac{1}{2}$ or $\frac{3}{2}$, and then the isobar decays into a meson and nucleon. For definiteness, in our calculation we assumed that the isospin of the intermediate isobar was $\frac{1}{2}$ half the time and $\frac{3}{2}$ half the time. For fireballs which emitted more than two mesons, we used simple formulas which are in accord with charge independence and should be good approximations to reality. For the production of N pions in the decay of a fireball with a charge of $+1$ and isospin of $\frac{1}{2}$, the probable numbers of protons, neutrons, and pions of various charges are given by the relations

$$N(p)=\tfrac{1}{2}+1/(2N+1)\,,\qquad N(n)=\tfrac{1}{2}-1/(2N+1)\,,$$
$$N(\pi^+)=\tfrac{1}{3}N+N^2/(2N+1)\,,\quad N(\pi^0)=\tfrac{1}{3}N\,,$$
$$N(\pi^-)=\tfrac{1}{3}N-N^2/(2N+1)\,.$$

If the fireball is initiated by a neutron, and has then charge zero, the relations are reversed according to charge symmetry.

If the interactions in the atmosphere of all nucleons with energies greater than some threshold energy $E_{\rm th}$ are considered, about 50% of these interactions involve nucleons which have made one previous interaction. The charge of these nucleons is rather well moderated by the interactions, and it is a reasonably good approximation to consider that these are half protons and half neutrons, while the original flux was composed of 87% protons and 13% neutrons. As a result of this effect, the effective interacting flux is then about $\frac{2}{3}$ protons and $\frac{1}{3}$ neutrons. Then from one meson emission, the positive-to-negative charge ratio of the pions emitted from the fireball will be about 2:1. For the emission of two mesons, the positive-to-negative ratio will be about 1.20:1. The charge ratio of mesons emitted with larger multiplicities from the fireballs will be even closer to 1. Since the interaction cross sections for negative and positive mesons with the nuclei in the atmosphere will be about the same (in the absence of Coulomb effects, which are surely unimportant at very high energies, charge independence requires them to be the same), the charge ratios of the pions at production will be reflected in the charge ratios of the muons which result from the decays of the pions.

The curve plotted in Fig. 9 shows the muon charge ratio[5,11,12] at sea level, for muons incident from the vertical, calculated from the models of the nucleon-nucleon interaction and of the diffusion of particles through the atmosphere, which were described here. The statistical error in these Monte Carlo calculations ranges from a few percent at the lower energies to about 10% at the highest energies plotted here. The rather constant value of the ratio with energy stems partially from the cancellation of two effects. For the very-high-energy events, the average multiplicity of the fireballs is larger and the relative importance of fireballs which emit only one meson is reduced, an effect which tends to reduce the charge ratio. However, the relative importance of secondary interactions of nucleons is less important since the secondary interactions, which are representative of an enriched neutron component, take place where the atmosphere is more dense and the mesons so produced are more likely to interact before decaying to muons. This meson flux, with a charge ratio near 1, is then suppressed relative to the flux derived from the primary nucleon flux, which is mainly composed of protons and which produces a higher ratio of positive to negative muons.

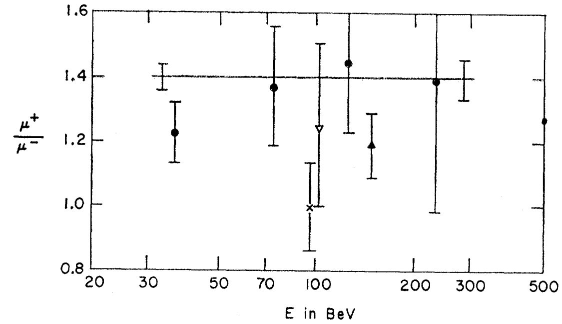

FIG. 9. The muon charge ratio for muons which reach sea level from the zenith plotted against muon energy. The points are taken from the review of Pal and Peters (see Ref. 16) and represent the results of experimental observations. The solid curve is derived from this model.

The points on Fig. 9 represent the experimental values of the muon charge ratio. Though the errors are large, the experimental results on the ratio of positive to negative muons seem to suggest a value for the charge ratio near 1.25, a value which is rather lower than the Monte Carlo results from this model of about 1.40. This result, together with the large muon fluxes predicted by the model, may mean that the model, as used here, overemphasizes the importance of light isobars.

Nevertheless, the generally large values of the muon charge ratios which are observed can be considered to substantiate some of the general features of the models proposed here. In particular, as emphasized by Pal and Peters,[5,16] it appears likely that light isobars are produced in the forward direction in nucleon-nucleon interactions and that these isobars carry off most of the energy of the incident nucleon as well as retain the charge quantum numbers of that nucleon. The observed muons are derived from pions which result from the

[16] B. Peters, in *Proceedings of the 1962 Annual International Conference on High-Energy Nuclear Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 623.

decay of these isobars into nucleons and one or two mesons. This result does not constitute any strong evidence for the main thesis of the model introduced here, however: that almost all of the interactions can be described in terms of the production of fireballs with the same energy and quantum numbers as the incident nucleon.

SUMMARY AND COMMENTS

The consequences of the simple Pomaranchuk model used to discuss nucleon-nucleon interactions at very high energies were shown to be in reasonable accord with the limited direct information available concerning these interactions. The model of nucleon-nucleon interactions was then used as the basis of a model of the diffusion of cosmic rays through the atmosphere. Discrepancies between the theoretical predictions and the observations of cosmic-ray phenomena, which seem to be somewhat outside of the uncertainties of the observations and the uncertainties in the calculations, are seen.

In particular, the calculated muon flux appears to be greater than the observed flux by a factor of about 1.8; the observed nucleon attenuation length through the atmosphere appears to be smaller than the calculated attenuation length; and the positive-to-negative muon charge ratio which is observed seems to be smaller than the calculated ratio. All of these discrepancies are removed if the probability of producing the lighter isobars is reduced by a factor of about 2 with respect to the probability calculated according to Eq. (13). Since the high-energy muon flux is derived primarily from the lighter isobars, this flux would be reduced in the calculations. Since the charge ratio is highest for muons produced by these light isobars, the charge ratio would be reduced. And since the nucleons emitted by these light isobars are most highly elastic, the inelasticity would be increased and the calculated attenuation length would be reduced.

A relaxation of the assumption that K mesons do not contribute very much to the flux of mesons does not improve the agreement between the calculations and the observations. It is known that the lighter isobars, with $Y=1$ and $I=\frac{1}{2}$, do not decay into K mesons with a high probability and it is these isobars which contribute mainly to the muon flux kinematically, whether the decays are to K mesons or to pions. In detail, the nucleon inelasticity factors and the nucleon attenuation length are not changed: The muon flux is reduced only slightly; and since the production of K^- mesons can only occur from these isobars if they are very heavy, the positive-to-negative muon charge ratio is increased, worsening the agreement with the measured values.

We note that such a reduction in the cross section for the production of the very lightest isobars is not in violation of the conceptual basis of the nucleon-nucleon interaction model. The model does predict the spectrum of heavy fireballs, where the approximation can be made that the fluctuations which must result from discrete states can be ignored, but the essentially continuous concepts used in the calculation are not relevant to the light, discrete isobars with masses less than 2.0 GeV/c^2.

While a reduction in the probability of production of the lighter isobars or fireballs improves the fit of the theoretical model to the observed characteristics of the cosmic-ray flux—with, of course, the addition of one more parameter—the agreement between the predictions of the nucleon-nucleon model and the results of Dobrotin and Slavatinsky[7] is then considerably weakened. It is our feeling, at this time, that the uncertainties in the whole body of data are sufficiently great so that these seeming contradictions should not yet be considered of fundamental importance.

ACKNOWLEDGMENTS

This work borrows heavily from early work, which was not published, by H. Kasha and the author on the diffusion of cosmic rays through the atmosphere. I also wish to thank Dr. Kasha for the benefit of continuous discussion and criticism of this work. Many of the basic views presented in this paper are derived, explicitly and implicitly, from the works of Professor B. Peters and Professor Yash Pal. Further correspondence with Professor Pal and Professor Peters during the preparation of this work has been invaluable. I also wish to thank Professor Marc Ross and Professor L. Jones for the benefit of discussions on these problems. Professor Jones's work (private communication) on nucleon-nucleon interactions, which is more general and more nearly relevant to lower-energy processes inasmuch as all single-particle exchanges are considered, appears to complement this work, which is primarily concerned with reactions at very high energy.