Neutron Hole States in $Z=40$ Nuclei Studied with the (p,d) Reaction on ^{90}Zr, ^{91}Zr, and ^{92}Zr†

J. B. BALL AND C. B. FULMER
Oak Ridge National Laboratory, Oak Ridge, Tennessee
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The (p,d) reaction on ^{90}Zr, ^{91}Zr, and ^{92}Zr was studied at a proton energy of 31 MeV. Deuteron spectra were measured with a broad-range magnetic spectrograph. The reaction on ^{90}Zr excites strongly the ground state of ^{89}Zr ($l=4$) and excited states at 0.588 MeV ($l=1$), 1.094 MeV ($l=1$), and 1.450 MeV ($l=3$). These states are shown to be consistent with pickup of neutrons from the $1g_{9/2}$, $2p_{1/2}$, $2p_{3/2}$, and $1f_{5/2}$ shell-model levels, respectively. On this basis definite spin-parity assignments are made to these levels. Small portions of the strength for pickup of these neutrons are also observed for higher excited states. The reaction on ^{91}Zr excites the states corresponding to the creation of the $1g_{9/2}$ hole and the consequent coupling of this hole to a $2d_{5/2}$ neutron. All six of the expected one-particle one-hole states are observed. The ^{92}Zr target, which was included to study the effect of this isotopic impurity in the ^{91}Zr target, yielded data on a number of excited states in ^{91}Zr.

I. INTRODUCTION

THE nuclei in the region of the zirconium isotopes have been subject to rather extensive treatment on the basis of simple shell-model descriptions.[1–5] The results of such studies indicate that, for this particular region of nuclei, the low-lying states can be well described by assuming the participation of relatively few shell-model levels.

The study reported in this paper is part of a series of experiments designed to examine the detailed shell-model structure of nuclei in this mass region and, more explicitly, the nature of the effective two-body residual interactions that give rise to the observed properties of these nuclei.

The basic idea of this experiment can be seen by reference to Fig. 1. In the simple description of the ground state of ^{90}Zr, the neutrons fill completely the levels up to and including the $1g_{9/2}$ to form a major closed shell at $N=50$. The two protons beyond the filled $2p_{3/2}$ level are distributed between the $2p_{1/2}$ and $1g_{9/2}$ levels. This is indicated schematically in Fig. 1(a). The ground state of ^{91}Zr is very similar except for the presence of one additional neutron. The lowest available level for this neutron is the $2d_{5/2}$ as indicated in Fig. 1(b).

The (p,d) reaction at 17 MeV and above has been shown to proceed as a one-step direct interaction process.[6,7] In the case of a ^{90}Zr target we would thus expect to see states corresponding to creation of single neutron holes in the filled shells. We should expect the lowest state to correspond to pickup of one of the $1g_{9/2}$ neutrons, the next state to be characteristic of pickup of a $2p_{1/2}$ neutron, etc.

In the case of the ^{91}Zr target, the lowest state excited in the (p,d) reaction will, of course, correspond to pickup of the single neutron in the $2d_{5/2}$ level. The next major strength we expect is when sufficient reaction energy is expended to remove one of the $1g_{9/2}$ neutrons. This should occur in the same region of reaction Q as that observed for the ^{90}Zr ground-state transition. In the case of the ^{91}Zr target, however, the total strength is not expected to be observed in a single state. The $1g_{9/2}$ neutron hole will couple to the single $2d_{5/2}$ neutron to form a sextuplet of states with spin-parity 2+, 3+, 4+, 5+, 6+, and 7+. This case, shown in Fig. 1(c), is particularly interesting because of its analogy with the simple interpretation of the low-lying ^{92}Nb states as indicated in Fig. 1(d). The observed levels[8] in ^{92}Nb manifest the proton-neutron interaction between the $2d_{5/2}$ and $1g_{9/2}$ shell-model levels. It was the primary purpose of this experiment to examine the nature of this interaction between these same two shell-model orbitals when the two particles were of the same type, i.e., the neutron-neutron interaction. It was hoped that differences in the deduced interactions might give evidence for the importance of exchange terms in the identical-particle case.

The ^{90}Zr(p,d) reaction was studied as a "calibration" to determine the position of the single-particle hole states and to judge the ability of the distorted-wave calculations to reproduce the shapes and magnitudes of

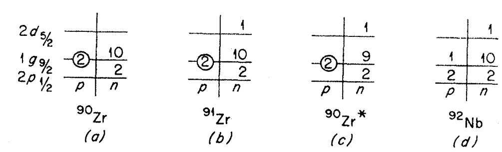

FIG. 1. Schematic shell-model description of (a) ground state of ^{90}Zr, (b) ground state of ^{91}Zr, (c) one-particle, one-hole excited states in ^{90}Zr, and (d) low-lying states in ^{92}Nb.

† Research sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation.
[1] B. F. Bayman, A. S. Reiner, and R. K. Sheline, Phys. Rev. **115**, 1627 (1959).
[2] I. Talmi and I. Unna, Nucl. Phys. **19**, 225 (1960).
[3] K. H. Bhatt and J. B. Ball, Nucl. Phys. **63**, 286 (1965).
[4] N. Auerbach and I. Talmi, Nucl. Phys. **64**, 458 (1965).
[5] J. Vervier, Nucl. Phys. **75**, 17 (1965).
[6] C. D. Goodman and J. B. Ball, Phys. Rev. **118**, 1062 (1960).
[7] J. C. Legg and E. Rost, Phys. Rev. **134**, B752 (1964).
[8] R. K. Sheline, C. Watson, and E. W. Hamburger, Phys. Letters **8**, 121 (1964); R. F. Sweet, K. H. Bhatt, and J. B. Ball, *ibid.* **8**, 131 (1964); J. B. Ball and M. R. Cates, *ibid.* **25B**, 126 (1967).

the angular distributions. It was anticipated that an experimental sum rule could be established by requiring the six $l=4$ states in the ^{91}Zr reaction to sum to the single $l=4$ state in the ^{90} Zr reaction. The ^{92}Zr(p,d) reaction was included to identify any spurious peaks in the ^{91}Zr spectra from the ^{92}Zr impurity in the target.

II. EXPERIMENTAL DETAILS

The reactions were studied using a 31-MeV proton beam from the Oak Ridge Isochronous Cyclotron. The deuterons were detected with a broad-range magnetic spectrograph. The spectrograph and beam preparation system have been described previously.[9]

The energy of the incident protons was chosen as a compromise between keeping the energy low for the best absolute resolution and using a high enough energy to avoid overlapping the elastic and inelastic protons on the region of interest in the deuteron spectra. The analyzed beam had an energy spread ($\Delta E/E$) of 1/1000. Typical currents on target were 30–50 nA with a spot size of 1×10 mm and an angular divergence of ± 8 mrad.

The deuteron spectra were recorded on 50-μ-thick Kodak NTB emulsions. Aluminum absorbers were used to eliminate triton tracks. A spectrograph entrance angle of $\pm 1.5°$ and a scanning zone of 1 in. were used: This corresponds to a solid angle of $\sim 5\times 10^{-4}$ sr. Overall resolution was typically 18 keV for the deuteron groups.

The targets were prepared by the ORNL Isotopes Division by rolling isotopically enriched samples of the metal. The isotopic abundances for the targets are given in Table I. The nominal thickness was 0.6 mg/cm^2 for each of the targets.

III. DISTORTED-WAVE ANALYSIS

The original intent of using the ^{90}Zr reaction to calibrate the distorted-wave Born-approximation (DWBA) calculations was complicated by the presence of the two $l=1$ transitions to states in ^{89}Zr only 0.5 MeV apart. These states are shown in Fig. 2, labeled 2 and 3.

At the time these data were taken, the lower state was reasonably well established[10] to have spin and parity $\frac{1}{2}-$ but there was little evidence for the correct assignment of the higher state. It was felt that the classification of this state was important to this study,

TABLE I. Isotopic abundances of targets.

	Composition (%)				
Target	^{90}Zr	^{91}Zr	^{92}Zr	^{94}Zr	^{96}Zr
^{90}Zr	98.6	0.8	0.4	0.2	...
^{91}Zr	7.0	87.0	5.2	0.8	...
^{92}Zr	2.3	0.9	95.7	1.0	0.1

[9] J. B. Ball, IEEE Trans. Nucl. Sci. **NS-13**, 340 (1966).
[10] *Nuclear Data Sheets*, compiled by K. Way *et al.* (U. S. Government Printing Office, National Academy of Sciences—National Research Council, Washington, D. C.), NRC 60-3-85; H. J. Kim and R. L. Robinson, Phys. Rev. **151**, 920 (1966).

since if both states were $\frac{1}{2}-$ then this fractionation of the $2p_{1/2}$ strength would suggest similar fractionation of the $g_{9/2}$ strength. Such strong splitting of the $p_{1/2}$ strength would also be contrary to the simple shell-model description of these nuclei.

Possibly the easiest test to make of the data presented here, is to determine if the first $l=1$ state exhausts the expected $p_{1/2}$ strength. If it does, then the higher state must be due to $p_{3/2}$ pickup.

To obtain reasonable confidence, it would also be desirable to show that the ground-state transition exhausts the expected $g_{9/2}$ strength. This means that we must perform the distorted-wave calculations in such a way that comparison between different l transfers are meaningful. In the past, it has been customary to employ a cutoff in the radial integral in order to reproduce the observed structure in the (p,d) angular distribu-

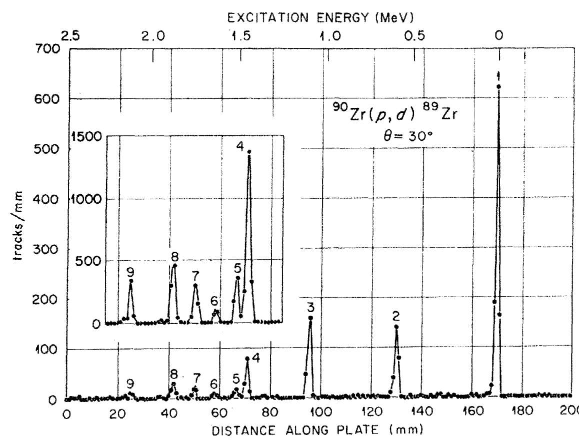

FIG. 2. Spectrum for ^{90}Zr$(p,d)^{89}$Zr at 30°. The ground state corresponds to a Q value of -9.77 ± 0.04 MeV. The insert shows the region of the weaker states taken with higher counting statistics.

tions. The problems with extracting consistent spectroscopic factors from a calculation with radial cutoff is illustrated in Fig. 3. In this figure we show relative spectroscopic factors extracted as a function of cutoff radius for $1g_{9/2}$ pickup and $2p_{3/2}$ pickup. The DWBA parameters used and the angular range included in the comparison are the same as used in treating the data below. Although the precise ratios are subject to the exact criteria of the fitting procedure, the indicated trends persist. For example, if only the magnitude of the first maximum is used to determine the spectroscopic factor, the oscillations in the $2p_{3/2}$ curve are even more pronounced.

The DWBA calculations used in this work to extract the spectroscopic factors do not employ a radial cutoff. Both the finite range and nonlocality have been included in the local-energy approximation. These have much the same effect on increasing the magnitude of the angular structure as does the arbitrary cutoff procedure, but are applicable to all l transfers in a consistent fashion. The bound-state wave functions modified by these corrections were calculated with the FANLFR[11] program. The angular distributions were calculated with the code JULIE.[12] Angular distributions for each l transfer were calculated for several Q values so that an energy correction could be determined.

The parameters used in the DWBA calculations are shown in Table II. The proton parameters are taken from the work of Fricke *et al.*[13] The geometrical parameters are the averaged best-fit parameters that were obtained from analysis of 40-MeV proton elastic scattering data. These were used by Fricke *et al.* to obtain good fits to 30-MeV proton elastic scattering data and by Fulmer *et al.*[14] to obtain good fits to 61-MeV proton

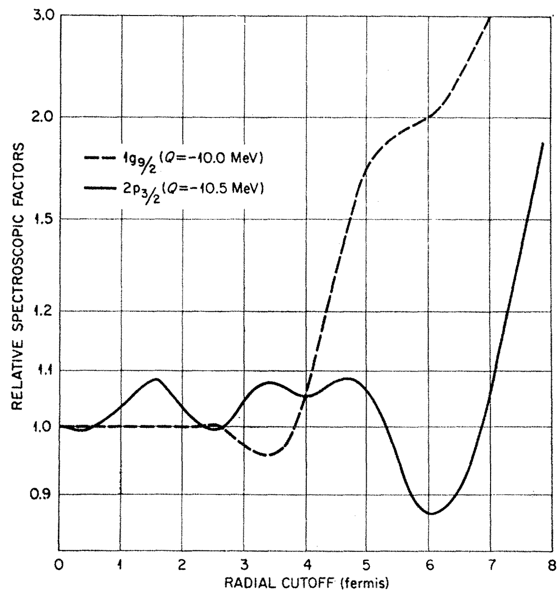

FIG. 3. Relative spectroscopic factors for $1g_{9/2}$ and $2p_{3/2}$ neutron pickup as a function of cutoff of the radial integral. Each curve has been normalized to unity at zero cutoff.

elastic scattering data from targets over a wide range of nuclei. The proton well depths are interpolated from the 30-MeV results of Ref. 13. The value of V_S is the value obtained from the 40-MeV fit to ^{90}Zr polarization and elastic scattering data by Fricke *et al.*

The deuteron parameters listed in Table II were obtained in the preliminary analysis of 34.4-MeV deuteron elastic scattering data by Newman *et al.*[15] These parameters differ from those obtained from the averaged family of optical-model parameters reported in Ref. 15 principally in the value of the Coulomb radius parameters r_c. A few DWBA calculations were made with the

TABLE II. Optical-model parameters used in the DWBA calculations. The notation for the parameters is the same as that of Satchler.[a]

	Proton	Deuteron
V (MeV)	49.9	104.3
r_0 (F)	1.16	1.07
a (F)	0.75	0.78
W (MeV)	4.0	0.0
W_D (MeV)	5.0	13.9
r_0' (F)	1.37	1.26
a' (F)	0.63	0.80
r_c (F)	1.25	1.0
V_S (MeV)	6.9	7.0
r_S (F)	1.06	1.07
a_S (F)	0.74	0.78

[a] G. R. Satchler, Nucl. Phys. **A92**, 273 (1967).

averaged family parameters; these made no appreciable difference in the results.

The bound-state wave functions were calculated (with the code FANLFR) by using a Woods-Saxon potential with a radius parameter $r_0=1.16$ F and a diffusivity $a=0.75$ F. A Thomas-type term was used for the spin-orbit potential with a spin-orbit strength, $\lambda=25$.

IV. RESULTS AND DISCUSSION

A. ^{90}Zr$(p,d)^{89}$Zr Reaction

A typical spectrum for this reaction has already been shown in Fig. 2. The calculated angular distribution for an $l=4$ transition is compared with the data in Fig. 4.

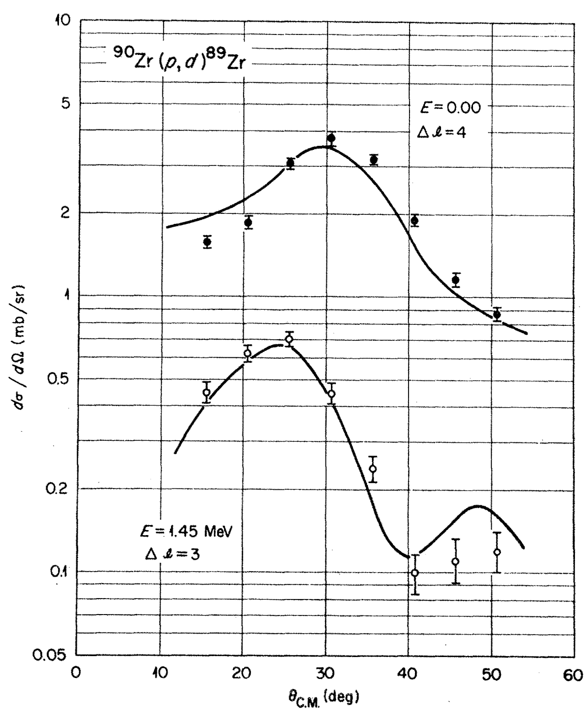

FIG. 4. Angular distribution of the deuteron groups leading to the ground state and third excited state in the reaction ^{90}Zr$(p,d)^{89}$Zr. The solid curves are the DWBA calculation.

[11] J. K. Dickens and F. G. Perey (unpublished).
[12] R. M. Drisko (unpublished).
[13] M. P. Fricke, E. E. Gross, B. J. Morton, and A. Zucker, Phys. Rev. **156**, 1207 (1967).
[14] C. B. Fulmer, J. B. Ball, A. Scott, and M. L. Whiten, Phys. Letters **24B**, 505 (1967).
[15] E. Newman, L. C. Becker, B. M. Preedom, and J. C. Hiebert, Nucl. Phys. **A100**, 225 (1967).

The magnitude of the calculated curve was chosen to give a best fit to the experimental points on the basis of minimizing the expression

$$\chi^2=\sum\frac{(S\sigma_{\rm th}-\sigma_{\rm ex})^2}{\Delta\sigma_{\rm ex}{}^2}. \quad (1)$$

The quantity S (or more correctly C^2S) is then the spectroscopic factor for the pickup reaction exciting this state. It should be noted that in applying the DWBA calculations with no cutoffs and with finite range and nonlocality corrections (in the local-energy approximation) we have used no arbitrary normalizations. We have assumed the relationship[16]

$$\sigma(\theta)_{\rm ex}=\tfrac{3}{2}D_0{}^2(C^2S)\sigma(\theta)_{\rm JULIE} \quad (2)$$

to hold rigorously. The value of the overlap integral $D_0{}^2$, has been taken to be 1.6 from calculations including the effect of the D-state admixture in the deuteron wave function.[17]

The spectroscopic factor for the ground-state transition is found to be 9.5 ± 1.3. The (p,d) reaction can excite both $T_<$ and $T_>$ states in the final nucleus. If we assume that the $1g_{9/2}$ level is completely filled for neutrons, then the total spectroscopic strength of 10 will be divided among these states. The sum of the spectroscopic factors for the $T_<$ states is given by[18]

$$S_<=\nu-\pi/(2T+1), \quad (3)$$

where ν is the total number of neutrons occupying the shell-model level under consideration, π is the number of protons in this same level, and T is the isotopic spin of the target nucleus. If we take the probability of finding a proton pair in the $1g_{9/2}$ level to be 36% for the ^{90}Zr ground state then the sum of the $T_<$ spectroscopic strengths is expected to be 9.93. Within the errors on the experimental spectroscopic factor the transition to the ground state exhausts the major portion of the total $T_<$ strength.

The quoted errors on the spectroscopic factors in this paper are somewhat arbitrary. The cross-section measurements are no better than 10%, with nominal 5% errors assigned to counting statistics, plate reading, target thickness, and charge collection. An additional 10% error is assigned to the extraction of the spectroscopic strength from the DWBA calculations. This is largely a matter of intuition and cannot be defended in any rigorous fashion. We do not mean to imply that the DWBA calculations are accurate within an absolute error of 10%, but that the relative magnitudes between different l values are good to about 10%, and it is the relative strengths in which we are primarily interested here. If one chooses to believe the DWBA predictions to any accuracy of only 30%, e.g., then hopefully this larger inaccuracy is systematic and will affect all l values in the same manner.

The angular distribution for deuterons leading to the first excited state is shown in Fig. 5. The theoretical curve is for an $l=1$ transition and the spectroscopic factor obtained is 1.7 ± 0.2. Since this level has been established previously as a $\frac{1}{2}-$ level and since the $2p_{1/2}$ level is the next available in the shell-model scheme, we assume that this level arises from the pickup of one of the $2p_{1/2}$ neutrons. The sum of the expected $T_<$ spectroscopic strengths given by Eq. (3) is 1.87.

The angular distribution for the second excited state, at 1.094 MeV, is also shown in Fig. 5. It is seen to have an $l=1$ character very similar to that of the first excited state. Since the first excited state exhausts a major portion of the expected $2p_{1/2}$ strength, this second state cannot be ascribed to pickup of a $2p_{1/2}$ neutron. Even rather large errors (30%) would lead to a total strength for the two states that exceeds the sum-rule limit. Hence, we conclude that this second $l=1$ state must be due to pickup of a $2p_{3/2}$ neutron and therefore must be a $\frac{3}{2}-$ state. This conclusion is in good agreement

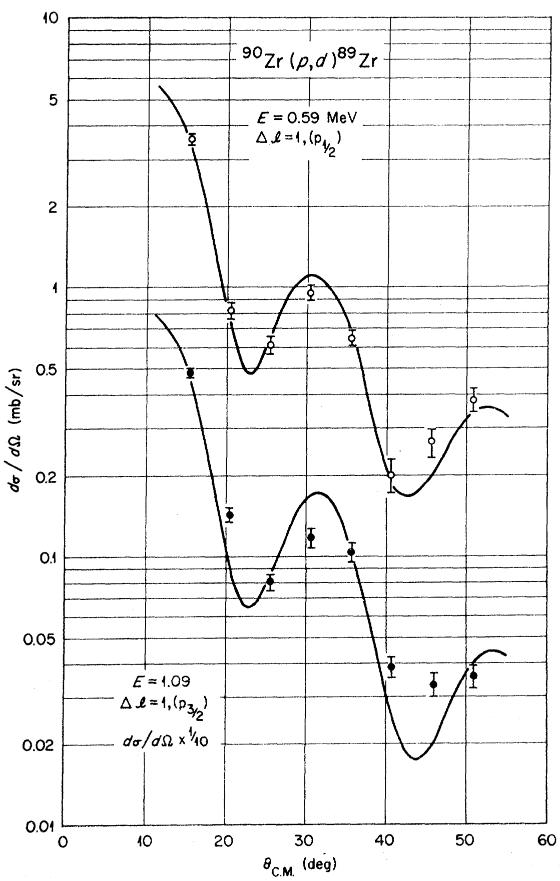

FIG. 5. Angular distribution of the deuteron groups leading to the first and second excited states in the reaction ^{90}Zr$(p,d)^{89}$Zr. The solid curves are the DWBA calculation.

[16] R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240, 1962 (unpublished).
[17] R. M. Drisko (private communication).
[18] J. B. French and M. H. Macfarlane, Nucl. Phys. 26, 168 (1961).

with the recent (^{3}He,α) work of Bassani *et al.*,[19] Rundquist,[20] Fou *et al.*,[21] and the (p,n) resonance studies of Kim and Robinson.[22]

The theoretical curve shown in Fig. 5 with the data from the 1.094-MeV state is the DWBA prediction for the pickup of a $2p_{3/2}$ neutron. The extracted spectroscopic factor is 2.4±0.3. The calculated sum-rule limit from Eq. (3) is 3.60. Thus, while the strength to the 1.094-MeV state is more than half of that expected, it does not exhaust the sum rule and we should expect to observe one or more additional states corresponding to $2p_{3/2}$ pickup.

The calculated angular distribution for an $l=3$ transition is compared with the data for the third excited state, at 1.450 MeV, in Fig. 4. It is apparent from the figure, that an $l=3$ and $l=4$ pickup are readily distinguishable. The theoretical curve is the calculation for the pickup of a $1f_{5/2}$ neutron with a spectroscopic factor of 3.0±0.4. The sum-rule limit given by Eq. (3) is 5.4 so that we again expect appreciable strength in higher states. Although a spin assignment of $\frac{3}{2}$ was suggested for this state in Ref. 22, our assignment of $\frac{5}{2}-$ is not incompatible with their results.

It is interesting to note that the first four states excited by the (p,d) reaction exhaust a major portion of the single-particle strength expected for creating neutron holes in the filled levels between $N=28$ and $N=50$. Thus, up to about 1.5 MeV, the simple idea of creating single holes in filled neutron levels agrees rather well with the experimental results. Above this energy we begin to observe the small additional pieces of the transition strengths that serve as a warning of a more complicated situation. The data for all of the observed levels are given in Table III.

The 1.626-MeV state, although weakly excited, has a definite $l=2$ angular distribution (see following section). This state can arise from pickup of one of a pair of $2d_{5/2}$ neutrons present in the ground state of ^{90}Zr in the form of a two-particle, two-hole excitation from the shell-model levels we have previously assumed to be filled. This transition is, therefore, a test of the deviation from absolute closure implied by the simple model. The small spectroscopic factor of 0.05 indicates that a neutron pair is present in the $2d_{5/2}$ level with a probability of only 2.5%. We interpret this as evidence that $N=50$ is an extremely good closed shell.

The state at 1.513 MeV is assigned as $\frac{9}{2}+$ since this strength from $1g_{7/2}$ pickup would not be expected below the $2d_{5/2}$ pickup seen at 1.626 MeV. The observed strength does not exceed, within the errors, the sum-rule limit for $1g_{9/2}$ pickup. This state is unreported previously.

TABLE III. Levels observed in ^{89}Zr with the ^{90}Zr(p,d) reaction.

Level No.	E (MeV)	l	$j\pi$	C^2S
1	0.000	4	$\frac{9}{2}+$	9.6±1.3
2	0.588±0.005	1	$\frac{1}{2}-$	1.7±0.2
3	1.094	1	$\frac{3}{2}-$	2.4±0.3
4	1.450	3	$\frac{5}{2}-$	3.0±0.4
5	1.513	4	$\frac{9}{2}+$	0.33
6	1.626	2	$\frac{5}{2}+$	0.05
7	1.740	1	$(\frac{3}{2},\frac{1}{2})-$	0.22, 0.24
8	1.866	1	$(\frac{3}{2},\frac{1}{2})-$	0.40, 0.42
9	2.098	3	$\frac{5}{2}-$	0.66

It is tempting to try to approach the sum-rule limit by assigning both the higher $l=1$ transfers to pickup of a $2p_{3/2}$ neutron. The data, however, do not justify this assignment and one (but not both) could be due to $2p_{1/2}$ pickup. Spectroscopic factors are given for both possibilities.

It must be noted that the $2p_{3/2}$ and $1f_{5/2}$ strengths are not exhausted by the observed levels. No additional strongly excited levels were observed in this work (below 3.3 MeV) so that we must assume that either the remaining strength lies above this energy, the remaining strength is spread over many weakly excited final states, or the calculation is deficient in accounting for the strength of these higher excited states. The first of these possibilities could not be explored in this work because of experimental limitations imposed by the bombarding energy.

B. ^{91}Zr(p,d)^{90}Zr Reaction

A spectrum for this reaction is shown in Fig. 6. The lower-energy part of the spectrum is dominated by the ground-state transition from pickup of the $2d_{5/2}$ neutron. The calculated angular distribution for this level is compared with the data in Fig. 7. An interesting feature of the angular distribution is the lack of a well-defined first maximum. This agrees well with the calculation. A summary of the observed level energies and extracted spectroscopic factors is given in Table IV.

The spectroscopic factor for the ground-state transition, shown in Fig. 7, was found to be 0.98. This is in good agreement with the value of 1.0 from the simple model implied in Fig. 1.

An arrow in Fig. 6 marks the position of the known first excited 0+ state in ^{90}Zr. The probability of exciting this state with the (p,d) reaction is seen to be extremely low. In fact, no evidence is seen for this state and from the highest-intensity spectrum that was taken (15°) we can set an upper limit of $S\leq 0.001$ for the spectroscopic factor for $l=2$ pickup exciting this level. This extremely small spectroscopic factor for the higher 0+ state indicates that the ratio of $(p_{1/2})^2$ to $(g_{9/2})^2$ protons in the ground state of ^{91}Zr is not drastically different from the ratio present in the ^{90}Zr ground state. The upper limit

[19] G. Bassani, J. Picard, N. Saunier, and G. Souchere, Contribution to the International Conference on Nuclear Structure, Tokyo, 1967 (unpublished).

[20] D. E. Rundquist, thesis, University of Illinois, 1966 (unpublished).

[21] C. M. Fou, R. W. Zurmuhle, and J. M. Joyce, Phys. Rev. **155**, 1248 (1967).

[22] H. J. Kim and R. L. Robinson, Phys. Rev. **162**, 1036 (1967).

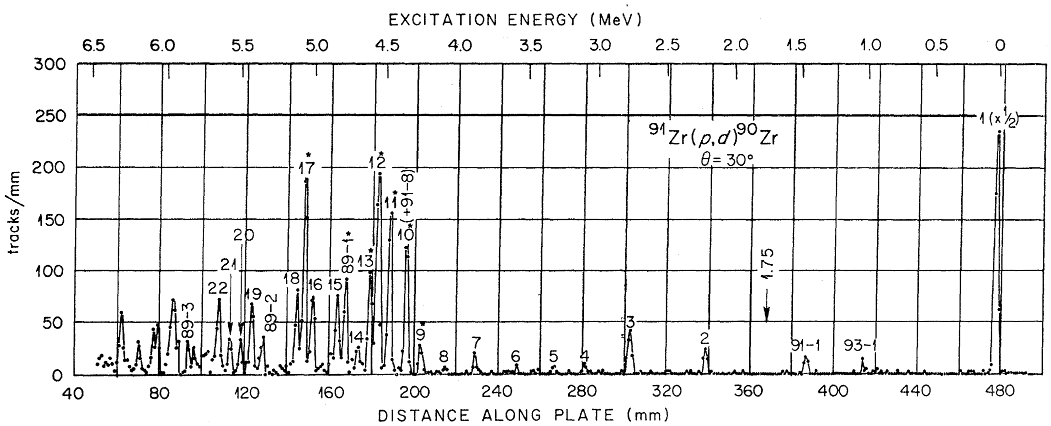

FIG. 6. Spectrum for ^{91}Zr$(p,d)^{90}$Zr at 30°. The ground state corresponds to a Q value of -5.00 ± 0.04 MeV. The arrow at 1.75 MeV marks the location of the known excited 0+ state in ^{90}Zr. Peaks marked with an asterisk are those showing $l=4$ character. Peaks due to isotopic zirconium impurities are labeled by the state number in the final nucleus [e.g., 91-1 is peak No. 1, ground state, from the ^{92}Zr$(p,d)^{91}$Zr reaction].

set here for the spectroscopic factor is consistent with the change in proton distribution deduced from the work of Preedom.[23] Model calculations[24] reproducing his observed change between ^{90}Zr and ^{92}Zr predict that the ground state of ^{91}Zr contains 62.4% $(p_{1/2})_0{}^2$ and 36.2% $(g_{9/2})_0{}^2$ components compared to 64.4 and 35.6%, respectively, in ^{90}Zr. These numbers give a predicted spectroscopic factor of 0.0001 for exciting the higher 0+ level.

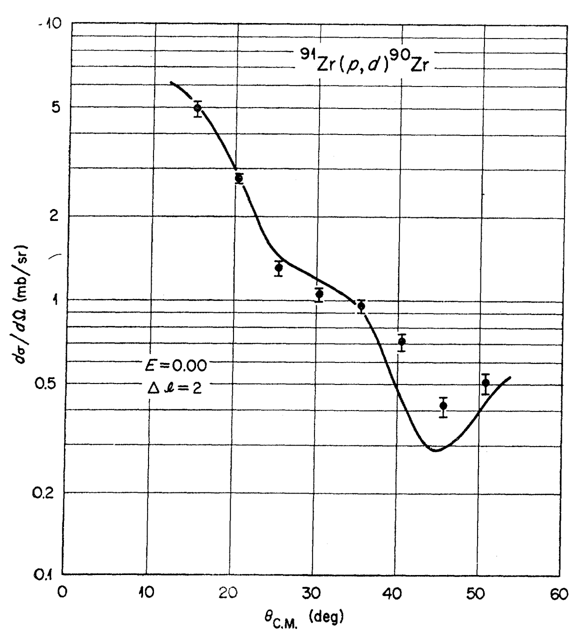

FIG. 7. Angular distribution of the deuteron group leading to the ground state in the reaction ^{91}Zr$(p,d)^{90}$Zr. The solid curve is the DWBA calculation.

[23] B. M. Preedom, thesis, University of Tennessee, 1967 (unpublished).

[24] J. B. Ball, B. M. Preedom, M. R. Cates, and E. Newman, Contribution to the International Conference on Nuclear Structure, Tokyo, 1967 (unpublished).

The state weakly excited at 2.184 MeV is the known 2+. This state is usually interpreted as a recoupling of the $g_{9/2}$ proton pair. This state can be reached in this reaction through the presence of a small amount of $(\pi g_{9/2})_2{}^2$ coupled to the $d_{5/2}$ neutron in the ^{91}Zr ground state. The ^{91}Zr ground-state wave function from the calculations mentioned in the preceding paragraph contains 1.3% of this configuration. This gives a predicted spectroscopic factor of 0.013 in fair agreement with the value of 0.04 obtained in this experiment.

The state at 2.744 MeV is probably the collective 3− known to be at this energy. The angular distribution is

TABLE IV. Levels observed in ^{90}Zr with the ^{91}Zr(p,d) reaction.

Level No.	E (MeV)	l	Δj	C^2S
1	0.000	2	$\frac{5}{2}$	0.98
2	2.184±0.005	2	$\frac{5}{2}$	0.04
3	2.744	...	...	
4	3.069	...	...	
5	3.296	...	...	
6	3.557	...	...	
7	3.840	...	...	
8	4.030	...	...	
9	4.220	4	$\frac{9}{2}$	0.26
10	4.320	4	$\frac{9}{2}$	1.27
11	4.443	4	$\frac{9}{2}$	1.85
12	4.528	4	$\frac{9}{2}$	2.20
13	4.578	4	$\frac{9}{2}$	0.96
14	4.65 (2?)	...	...	
15	4.81±0.01	1	$(\frac{1}{2}, \frac{3}{2})$	0.37, 0.34
16	4.98	1	$(\frac{1}{2}, \frac{3}{2})$	0.42, 0.38
17	5.05	4	$\frac{9}{2}$	2.05
18	5.10	1	$(\frac{1}{2}, \frac{3}{2})$	0.41, 0.37
19	5.42	1	$(\frac{3}{2}, \frac{1}{2})$	0.47, 0.51
20	5.50	1	$(\frac{3}{2}, \frac{1}{2})$	0.18, 0.19
21	5.56	1	$(\frac{3}{2}, \frac{1}{2})$	0.20, 0.22
22	5.64	1	$(\frac{3}{2}, \frac{1}{2})$	0.41, 0.44
23	5.95	1	$(\frac{3}{2}, \frac{1}{2})$	0.58, 0.62

rather ambiguous and may be a combination of $l=1$ and $l=3$ from existing small components of $(p_{1/2})^{-1}(d_{5/2})^1$, $(p_{3/2})^{-1}(d_{5/2})^1$, and $(f_{5/2})^{-1}(d_{5/2})^1$ couplings. The state at 3.069 MeV is probably the known 4+ attributed to the $(g_{9/2})^2$ proton configuration and is consistent with the same interpretation as given the 2.184-MeV state. Except for the ground state, the levels observed below 4 MeV are weakly excited and it is difficult to draw any detailed conclusions from their angular distributions.

The states of particular interest for this work are those marked with an asterisk in Fig. 6. These are the states showing $l=4$ angular distributions corresponding to pickup of a $g_{9/2}$ neutron. The $l=4$ ground-state transition from the ^{90}Zr impurity in the ^{91}Zr target is seen between peaks 14 and 15. Peak number 10 is slightly enhanced by a degeneracy in energy with the strong $l=4$ transition from the ^{92}Zr impurity.

As expected, we observe that the $l=4$ strength is now divided among six final states in ^{90}Zr. The spectroscopic factors for these states, listed in Table IV, seem to show a monotonic behavior. The strength, however, is not particularly well distributed in a $2J+1$ fashion. A comparison of the relative strengths with $2J+1$ is given in Table V. Although spin assignments are made in the table for the basis of comparison, the deviations from $2J+1$ suggest that the intensities are affected by configuration mixing. If such configuration mixing is present then the assigned values must be regarded with considerable suspicion until additional experimental information is available about these levels.

It is perhaps surprising that these hole states stay as pure as they apparently do and that we see only six states. Within our resolution, we see no evidence of any of these states being doublets nor any additional $l=4$ states in the next 1.5 MeV of excitation. We thus conclude that these states are relatively pure 1p-1h excitations.

TABLE V. Comparison of relative spectroscopic factors for the $l=4$ particle-hole states in ^{90}Zr with a $2J+1$ distribution of the strength.

Level No.	E (MeV)	$S_i/\sum S_i$	Assumed J	$2J+1/60$
9	4.220	0.03	2	0.08
10	4.320	0.15	4	0.15
11	4.443	0.21	5	0.18
12	4.528	0.26	7	0.25
13	4.578	0.11	3	0.12
17	5.050	0.24	6	0.22
	$\sum S(l=4)=8.6$			

The fact that the $l=4$ spectroscopic factors sum only to 8.6 may indicate that some of the strength was missed. This strength is, however, well within the experimental inaccuracies, the same as the ^{90}Zr strength. The center of gravity of the $l=4$ strength in ^{91}Zr is $Q=-9.60$ MeV, which agrees well with the ^{90}Zr value of $Q=-9.79$ MeV.

It is difficult to draw any conclusions from the $l=1$ transfers since more transitions are seen than are expected from coupling a $p_{1/2}$ and a $p_{3/2}$ hole to the $d_{5/2}$ neutron. The fact that these two couplings produce some states of the same spin and parity will lead to additional mixing. We have listed the spectroscopic factors for both $2p_{1/2}$ and $2p_{3/2}$ transfer in Table IV.

C. ^{92}Zr$(p,d)^{91}$Zr Reaction

A spectrum for this reaction is shown in Fig. 8. The number of final states is noticeably less than observed when the odd $d_{5/2}$ neutron was present but the spectrum is not as simple as that seen for the ^{90}Zr reaction. Thus the presence of the $d_{5/2}$ neutron pair gives rise to some splitting of the transition strengths.

The main purpose of including this reaction was to study the effect of the ^{92}Zr impurity in the ^{91}Zr target.

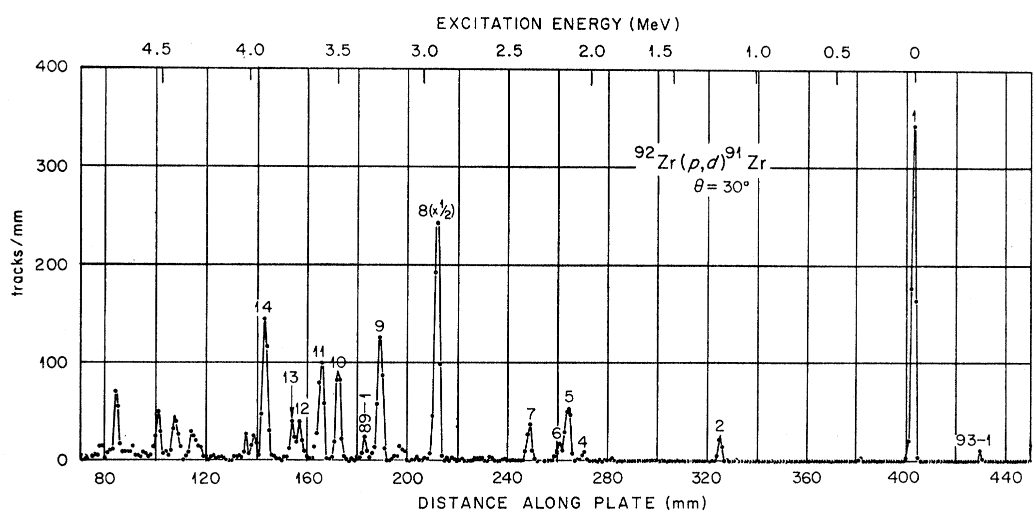

FIG. 8. Spectrum for ^{92}Zr$(p,d)^{91}$Zr at 30°. The ground state corresponds to a Q value of -6.44 ± 0.04 MeV.

TABLE VI. Levels observed in ^{91}Zr with the ^{92}Zr(p,d) reaction.

State No.	E (MeV)	l	$j\pi$	C^2S
1	0.000	2	$\frac{5}{2}+$	1.86±0.33
2	1.204±0.005	0	$\frac{1}{2}+$	0.06
3	1.47±0.01	···	···	very weak
4	2.04±0.01	2	$\frac{3}{2}+$	0.07
5	2.135±0.005	4	$\frac{9}{2}+$	0.78
6	2.180	4	$\frac{7}{2}+$	0.20
7	2.350	···	···	
8	2.895	4	$\frac{9}{2}+$	7.9
9	3.225	1	$(\frac{1}{2},\frac{3}{2})-$	1.00, 0.91
10	3.468	1	$(\frac{1}{2},\frac{3}{2})-$	0.82, 0.76
11	3.567	1	$(\frac{1}{2},\frac{3}{2})-$	1.14, 1.04
12	3.695	4	$\frac{9}{2}+$	0.45
13	3.738	2	$(\frac{5}{2},\frac{3}{2})+$	0.14, 1.06
14	3.890	(3, 4)	···	
15	4.20±0.01	(1)	$(\frac{1}{2}-,\frac{3}{2}-)$	0.05
16	4.31	1	$(\frac{3}{2},\frac{1}{2})-$	0.26, 0.28
17	4.40	1	$(\frac{3}{2},\frac{1}{2})-$	0.42, 0.45
18	4.50	1	$(\frac{3}{2},\frac{1}{2})-$	0.44, 0.47
19	4.73	1	$(\frac{3}{2},\frac{1}{2})-$	0.63, 0.67
20	4.82	1	$(\frac{3}{2},\frac{1}{2})-$	0.14, 0.15

The energies of the states and their spectroscopic factors are given in Table VI. Many of these levels have been observed in the ^{90}Zr$(d,p)^{91}$Zr reaction by Cohen and Chubinsky.[25] The l assignments and spectroscopic factors for the stripping reaction provide additional information in interpreting the results of the pickup reaction. Cohen and Chubinsky also reported the observation of the levels up to number 6 in the ^{92}Zr$(d,t)^{91}$Zr reaction, but data were taken at only one angle.

Within the quoted errors on the deduced spectroscopic factors for this work, the ground-state strength agrees with the value of 2.0 expected if the neutron pair occupies only the $2d_{5/2}$ level. The amount of deviation from this simple picture can be judged by examining some of the strengths to other low-lying states.

The first excited state at 1.204 MeV is assigned spin and parity $\frac{1}{2}+$ from the strong $l=0$ transition observed in the (d,p) reaction.[25] This state presumably corresponds to exciting the odd $2d_{5/2}$ neutron in the ground state of ^{91}Zr into the $3s_{1/2}$ level. Such a state can be excited by the pickup reaction if the ground state of ^{92}Zr contains an admixture of two-particle excitation of the neutron pair from the $2d_{5/2}$ to $3s_{1/2}$ level. The small spectroscopic factor of 0.06 for exciting the 1.204-MeV state indicates a probability of only 3% for finding the neutron pair in the $3s_{1/2}$ level.

The level at 2.040 MeV is strongly excited in the (d,p) reaction with an $l=2$ transfer and assigned spin and parity $\frac{3}{2}+$. The small spectroscopic factor seen in the present work agrees with this interpretation and sets the probability of finding the neutron pair in the $2d_{3/2}$ level at about 4% for the ^{92}Zr ground state.

The level observed at 2.18 MeV with a weak $l=4$ transition probably corresponds to the strong $l=4$ level seen in the stripping reaction. This level is thus the $g_{7/2}$ pickup and the spectroscopic factor indicates about a 10% admixture of this pair in the ^{92}Zr ground state.

These additional ground-state components, within the experimental errors, still sum to ~2. When this sum is normalized to 2.0, the indicated distribution of the neutron pair from this work is 85% $2d_{5/2}$, 9% $1g_{7/2}$, 3% $3s_{1/2}$, and 3% $2d_{3/2}$. These admixtures are about the same as those suggested by Ref. 25 and indicate a significant deviation from the simple model which considers only the $2d_{5/2}$ level.

The remaining $l=4$ transitions do not correlate with strong $l=4$ stripping transitions and are assumed to be due to pickup of $1g_{9/2}$ neutrons from the $N=50$ core. In particular, a major portion of the $l=4$ pickup strength appears in a state at 2.895 MeV which is not seen in the stripping reaction. This state is quite close in Q value to the ground-state transition in the ^{90}Zr(p,d) reaction; this suggests that it is primarily the 2p-1h state with one neutron hole in the $1g_{9/2}$ level and a pair of zero-coupled neutrons in the $2d_{5/2}$ level. To the extent that this $\frac{9}{2}+$ state will mix with the $\frac{9}{2}+$ state made by coupling the $g_{9/2}$ hole with the 2+ state of the neutron pair, it will also be possible to excite the higher state. To first order, the 2p-1h state arising primarily from the 2+ neutron coupling will be about 0.90 MeV higher than the 0+ coupling (the 0+ to 2+ splitting observed in ^{92}Zr). The $l=4$ state observed at 3.695 MeV may be a likely candidate for such a level. The spectroscopic factor of 5% of the 2.895 level indicates a reasonably small amount of mixing between these two states. The center of gravity of the $l=4$ strength is at $Q=-9.57$ MeV.

Again, the number of $l=1$ states and the inability to distinguish between $p_{1/2}$ and $p_{3/2}$ transfer makes it difficult to make any interpretation for these states. Spectroscopic factors for both transitions are given in Table VI.

V. PARTICLE-HOLE STATES IN ^{90}Zr

Six states were observed in the ^{91}Zr$(p,d)^{90}$Zr reaction with $l=4$ angular distributions. These levels, presumably the 1p-1h states corresponding to the coupling of the $1g_{9/2}$ neutron hole to the $2d_{5/2}$ neutron, were listed in Table V with tentative spin assignments made on the basis of relative intensities.

The assignment of spins to these levels by comparison of their relative intensities with a $2J+1$ distribution is based on the assumption that the states have a pure particle-hole character. Although it is possible for the strengths to be redistributed through mixing of other configurations with these levels, it is very unlikely that such mixing causes significant deviations in this particular case.

There are two types of possible configuration mixings;

[25] B. L. Cohen and O. V. Chubinsky, Phys. Rev. 131, 2184 (1963).

those that result in components that can be populated in the pickup reaction and those that cannot. The first arises from components in the ^{91}Zr ground-state wave function of the type $[(\pi g_{9/2})_4{}^2(\nu d_{5/2})]_{5/2}$. Pickup of the $d_{5/2}$ neutron would excite the state that is due principally to the 4+ coupling of the proton pair. This character has been ascribed to the known 4+ state at 3.08 MeV and the weak state, labeled number 4 in Fig. 6, may be this state. If so, the spectroscopic factor is $\leq 1\%$ of that for the ground-state transition. This configuration cannot then contribute significant mixing to the higher particle-hole 4+. Pickup of a $g_{9/2}$ neutron would lead to configurations of the type $[(\pi g_{9/2})_4{}^2(\nu d_{5/2})\times(\nu g_{9/2})^{-1}]_J$ and these components would similarly be expected to produce, at most, 1% effects on the observed strengths. There is no evidence in these studies for any significant mixture of neutron-excited configurations in the ^{91}Zr ground state. For example, the next level available for the odd neutron is the $3s_{1/2}$ level which will give rise to a $[(\pi g_{9/2})_2{}^2(\nu s_{1/2})]_{5/2}$ component. No contribution from $l=0$ transfer is observed in the transition to the 2+ state at 2.18 MeV.

There are also possible components of the type $[(\pi g_{9/2})_6{}^2(\nu d_{5/2})(\nu g_{9/2})^{-1}]_J$, which cannot be excited by the pickup reaction. Such components can mix into all six of the states of interest here. However, since the excitation of the protons from their 0+ coupling to the 6+ coupling requires over 2 MeV of energy, the unperturbed levels with this character will lie this much higher than the unperturbed particle-hole states. We thus expect the mixing to be rather small. In addition, the effect of such mixing will be to reduce all of the strengths rather than shift strength from one level to another.

We thus conclude that the assigned spins are probably correct and these levels are a good representation of the $(\nu d_{5/2})(\nu g_{9/2})^{-1}$ particle-hole interaction. The energy spectrum of these states is shown in Fig. 9 and compared to the proton-neutron interactions derived from the spectrum of ^{92}Nb. Since the particle-hole and particle-particle interactions are simply related, standard techniques of vector algebra[26] have been used to convert the $(\nu d_{5/2})(\pi g_{9/2})$ interaction deduced from ^{92}Nb to the corresponding $(\nu d_{5/2})(\pi g_{9/2})^{-1}$ particle-hole interaction. The gross features of the $(\nu d_{5/2})(\nu g_{9/2})^{-1}$ spectrum resemble the $(\nu d_{5/2})(\pi g_{9/2})^{-1}$ spectrum. The overall energy splitting is almost identical and both spectra have one level separated from and higher than the other five. The specific spin assignments, however, show no apparent correlation. The most obvious disagreement is the difference in the positions of the 2+ and 6+ levels in the two spectra. This lack of correspondence between the two spectra would seem to indicate that the $T=1$ component of the residual two-body force, which gives rise to the identical-particle spectrum, does not play a dominant role in determining the non–identical-particle spectrum where both $T=0$ and $T=1$ components contribute.

[26] D. M. Brink and G. R. Satchler, Nuovo Cimento 4, 549 (1956).

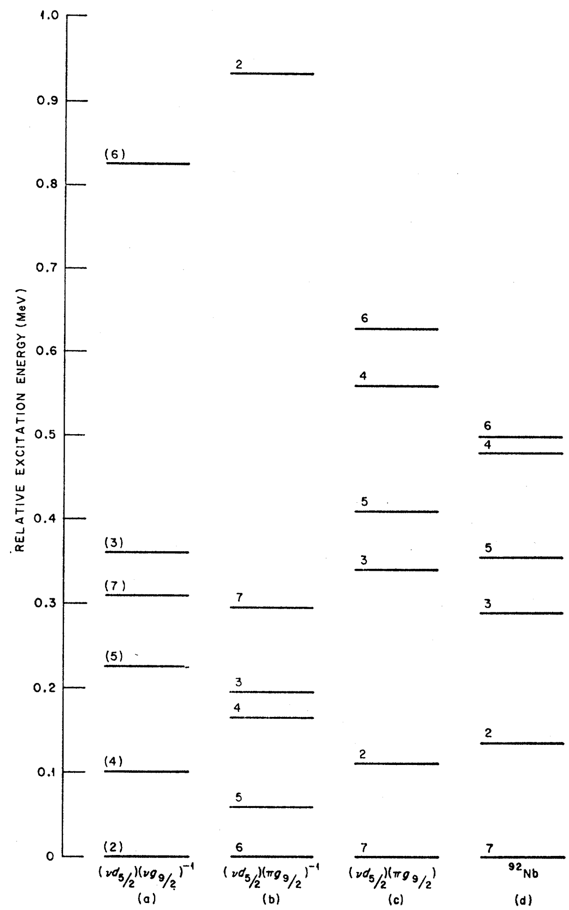

FIG. 9. (a) Particle-hole spectrum deduced from this work for the $(\nu d_{5/2})(\nu g_{9/2})^{-1}$ interaction. These assignments must be considered very tentative. (b) Particle-hole spectrum for the $(\nu d_{5/2})\times(\pi g_{9/2})^{-1}$ interaction as calculated from the particle-particle interaction shown in (c). (c) $(\nu d_{5/2})(\pi g_{9/2})$ particle-particle interaction deduced from the observed spectrum of ^{92}Nb shown in (d).

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