results for the energy of the T=1 analog to the  ${}^{40}Sc$ ground state is shown in Fig. 3. The present result for the energy of this analog level differs from that of Berg and Kashy<sup>5</sup> by 1.9±1.3 keV, of which about 0.5 keV may be due to the differential pileup effect mentioned above. However, the difference of  $10.2\pm3.4$  keV from the result of Armini et al.<sup>6</sup> appears to be too large to be accounted for by such an effect.

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# Nuclear-Reaction Studies in the Strontium Isotopes: Neutron Particle-Hole States in $Sr^{88}$ and the $Sr^{87}(d,p)Sr^{88}$ Reaction\*

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The levels of Sr<sup>88</sup> have been studied with the Sr<sup>87</sup> (d,p)Sr<sup>88</sup> reaction at a bombarding energy of 7.5 MeV using the MIT-ONR Van de Graaff generator and multiple-gap spectrograph. Seventy-three levels below 8.516-MeV excitation were observed, 32 of which displayed direct-stripping angular distributions. A distorted-wave Born-approximation analysis for the latter transitions was performed, and the values of the transferred orbital angular momentum  $l_n$  and spectroscopic strengths  $(2J+1)S_{l_n,j}/(2J_0+1)$  were extracted. Sum-rule strengths were subsequently calculated and compared with shell-model limits. Locations of neutron particle-hole state multiplets were determined and compared with a simple model involving singleparticle neutron states, determined from the  $Sr^{88}(d,p)Sr^{89}$  reaction, weakly coupled to a single  $1_{g_{9/2}}$  hole in the major N = 50 shell. A comparison of the data is made with the previously known level structure of Sr<sup>88</sup>.

## I. INTRODUCTION

HIS paper is part of a series of nuclear-reaction studies in the strontium isotopes. It presents the results of the  $Sr^{87}(d,p)Sr^{88}$  reaction at a bombarding energy of 7.5 MeV. Previous publications in the series<sup>1-3</sup> have dealt with the level structure of Sr<sup>89</sup>, and subsequent papers will report on reactions from the other stable isotopes of strontium.

The lowest-lying level structure of Sr<sup>88</sup> has been studied recently both experimentally<sup>4-9</sup> and theoretically.<sup>10,11</sup> From a simple shell-model point of view,

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the ground state of Sr<sup>88</sup> is described as having a closed neutron shell at N = 50, filling up to the  $2d_{5/2}$  orbit, and a semiclosed proton shell at Z=38, filling up to the  $2p_{1/2}$  orbit. Since it would require more energy to promote a neutron in this wave function over the major shell gap than it would to excite a proton, it is expected that the lowest-lying states in Sr<sup>88</sup> will be comprised largely of proton core-excited configurations. Some partial corroboration of this expectation has been recently reported by Kavaloski et al.4 using the  $Y^{89}(d, He^3)Sr^{88}$  reaction. The neutron core-excited configurations should begin appearing only above 4 or 5 MeV in Sr<sup>88</sup> with the possible exception of small fragments at lower excitations arising from admixture in collective states such as the first 2<sup>+</sup> at 1.84 MeV and 3<sup>-</sup> at 2.75 MeV. To date, no information on such neutron-excited configurations is available.

The objective of the present work is to determine the location of the neutron particle-hole states in Sr<sup>88</sup> built on a single  $g_{9/2}$  hole in the N = 50 core. This was done by means of the  $Sr^{87}(d,p)Sr^{88}$  reaction, which transfers a single neutron to the Sr<sup>87</sup> target having  $J_0^{\pi} = \frac{9}{2}^+$ . The use of high resolution and an isotopically pure target has enabled 73 levels to be observed up to 8.516-MeV excitation in Sr<sup>88</sup>. The level structure and spectroscopic information extracted have been compared to data on Sr<sup>88</sup> reported previously by other investigators<sup>4–9</sup> using different reactions.

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<sup>&</sup>lt;sup>1</sup> E. R. Cosman, H. A. Enge, and A. Sperduto, Phys. Rev. 165, 1175 (1968).

<sup>&</sup>lt;sup>2</sup> E. R. Cosman, H. A. Enge, and A. Sperduto, Phys. Letters 22,  $195_{\rm F}$  (1966).



FIG. 1. The proton spectrum from the  $Sr^{87}(d,p)Sr^{88}$  reaction measured in the multiple-gap spectrograph at  $\theta_{lab} = 45^{\circ}$  and incident energy of 7.5 MeV. The proton groups are labeled with numbers used to identify the corresponding states in  $Sr^{88}$  listed in Table I. The spectrum has a distinctive feature that the transitions below  $E_x = 4.0$  MeV in  $Sr^{88}$  are few and extremely weak, whereas above 4.0 MeV many strong proton groups appear. The latter are ascribed to neutron particle-hole states discussed in the text

### **II. EXPERIMENTAL METHODS AND RESULTS**

The details of the experimental procedure used here have been described in a previous publication and, therefore, only a brief summary will be given now. The 7.5-MeV deuteron beam from the MIT-ONR Van de Graaff accelerator was used to bombard a Sr<sup>87</sup> target consisting of a  $25-\mu g/cm^2$  layer of enriched Sr<sup>87</sup> evaporated onto a  $5-\mu g/cm^2$  Formvar backing. The isotopic analysis of the strontium in the target was Sr<sup>84</sup>, <0.05%; Sr<sup>86</sup>, 1.33%; Sr<sup>87</sup>, 85.95%; and Sr<sup>88</sup>, 12.72%. The reaction protons were measured simultaneously at 24 different angles in the MIT multiple-gap spectrograph<sup>12</sup> on Kodak NTB 50-µ nuclear emulsions. Because of the  $J_0^{\pi} = \frac{9}{2}$  spin of the target and resultant statistical decrease in yield by a factor of 10, a necessarily long 4000- $\mu$ C (d,p) exposure was made to ensure favorable counting statistics. A separate  $Sr^{87}(d,d)$  elastic scatter-

12 H. A. Enge and W. W. Buechner, Rev. Sci. Instr. 34, 155 (1963),

ing run was made with 3.0-MeV deuterons to determine the target thickness.

Figure 1 shows a typical  $Sr^{87}(d,p)Sr^{88}$  proton spectrum recorded at  $\theta_{lab} = 45^{\circ}$ , and Figs. 2 and 3 display angular distributions for some of the most prominent transitions seen. In most cases the data were extracted only for the forward angles; however, where a more complete shape was helpful for the distorted-wave Born-approximation (DWBA) analysis, the back-angle yields were also examined.

A listing of all the levels identified to be from  $Sr^{88}$  is given in Table I and a complete explanation is given in the table caption. The ground-state Q value for the  $Sr^{87}(d,p)Sr^{88}$  reaction was determined to be  $Q_0 = 8.865$  $\pm 0.005$  MeV.

## III. DWBA ANALYSIS

The theoretical (d, p) stripping curves used to compare with the data were calculated using the computer code

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FIG. 2. Angular distributions of some  $Sr^{87}(d,p)Sr^{88}$ transitions. At the top right of each drawing is the number used to identify the corresponding state in Table I. The circles represent the experimental data, and the solid curves are derived from DWBA calculations assuming the indicated  $l_n$  and Q values

JULIE.<sup>13</sup> The program employed surface absorption, a neutron well of the Woods-Saxon type, zero-range interactions, no spin-orbit force, and no lower cutoffs in the radial integrals. The deuteron and proton distorted waves were calculated in optical wells identical to those used to analyze the  $Sr^{88}(d,p)Sr^{89}$  reaction in Ref. 1.<sup>14</sup>

optical wells of the form

$$U(r) = -\frac{V}{1+e^{x}} + iW\frac{d}{dx'}\frac{1}{1+e^{x'}} + V_{c}(r,r_{c}),$$

where  $x = (r - r_0 A^{1/3})/a$ ,  $x' = (r - r_0' A^{1/3})/a'$ , and  $r_c = r_{0c} A^{1/3}$ . The Coulomb potential  $V_c$  is derived from a uniformly charged sphere of radius  $r_c$ . The parameters for both particles were extrapolated from those determined by Perey [F. G. Perey, Phys. Rev. 131, 745 (1963)] and were, for the deuteron, V=96.0 MeV,  $r_0=1.15$  F,

Because of the nonzero target spin,  $J_0^{\pi} = \frac{9}{2}$ , the only restrictions on the values of  $l_n$  and j, the orbital and total angular momenta of the transferred neutron, are  $j=l_n\pm\frac{1}{2}$ ,  $\mathbf{j}=\mathbf{J}_f-\mathbf{J}_0$ , and  $\pi_f=(-)^{l_n}$ , where  $J_f$  and  $\pi_f$ are the spin and parity of the residual state in Sr<sup>88</sup>. Although more than one  $l_n$  value could then characterize a transition, the strong stripping states in the present reaction appeared to have angular-distribution shapes which could be described well with only one  $l_n$ . The fitting procedure used here involved matching the calculated DWBA reaction function  $\sigma(E_d,Q,l_n,\theta)$  to the average shape of the experimental angular distribution around the position of the first forward-angle maximum

 $\overline{a=0.81 \text{ F}, W=80 \text{ MeV}, a'=0.68 \text{ F}, r_0'=1.34 \text{ F}, \text{ and } r_{0c}=1.30 \text{ F};}$ and for the proton, V=55 MeV,  $r_0=1.25$  F, a=0.65 F, W=54MeV, a'=0.47 F,  $r_0'=1.25$  F, and  $r_{0c}=1.25$  F. Use of other deuteron parameters was tried in Ref. 1 and further discussion on the final choice was given in that paper.

<sup>&</sup>lt;sup>13</sup> R. H. Bassel, K. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240 (unpublished) available from Office of Technical Services, Department of Commerce, Washington 25, D. C. <sup>14</sup> The deuteron and proton wave functions were calculated from



(see Figs. 2 and 3). The spectroscopic strength  $(2J_f+1)S_{l_n,j}/(2J_0+1)$  could then be extracted from the relationship

$$\frac{d\sigma}{d\Omega} = 1.48 \left(\frac{2J_f + 1}{2J_0 + 1}\right) S_{l_n, j\sigma}(E_d, Q, l_n, \theta).$$

Their values are given in Table I.

#### IV. DISCUSSION

### A. Level Scheme, Sum Rules, and Neutron Particle-Hole States

As seen from the  $Sr^{s7}(d,p)Sr^{s8}$  spectrum in Fig. 1, the proton yield as a function of excitation energy in  $Sr^{s8}$ has a striking feature. Namely, below 4.0 MeV, the transitions seen are low in intensity and few in number; whereas, between 4.0 and 7.5 MeV, many strong proton groups appear with an average spacing of about 100 keV. The upper three graphs in Fig. 4, which show the spectroscopic strengths from Table I plotted against excitation energy in  $Sr^{s8}$ , illustrate that these strong proton peaks correspond primarily to  $l_n=0$  and 2 neutron-stripping transfers. Furthermore, it is evident from Fig. 4 that there are two distinct clusters of  $l_n=2$ levels and one cluster of  $l_n=0$  levels, each cluster of states being spread over about 1 MeV.

These features would indicate that, except for the  $0^+$ ground state and  $2^+$  state at  $E_x = 1.839$  MeV, there are no levels containing configuration strength of the type  $\{(l, j)_n (1g_{9/2})_n^{-1}\}_J$  below 4.0 MeV in Sr<sup>88</sup>. The groundstate transition is the expected filling of the  $g_{9/2}$  shell by an  $l_n=4$  neutron transfer. Within the limits of error incurred from the experiment and DWBA analysis, its experimental value of  $S_{l,j} = 1.20$  is consistent with the expected value of 1.0, assuming a pure  $(1g_{9/2})_n^{-1}$  ground state for Sr<sup>88</sup>. From the weak  $l_n = 2$  stripping pattern seen for the 2<sup>+</sup> state at 1.839 MeV, we conclude that it necessarily has a small fragment of the type  $\{(2d_{5/2})_n\}$  $\times (1g_{9/2})_n^{-1}_{2^+}$ , with  $S_{l,j}=0.25$ . The only other states seen in this region, Nos. 2, 3, and 4 at  $E_x = 2.738$ , 3.208, and 3.583 MeV, respectively, have extremely weak yields and nonstripping angular distributions. Their spins have been assigned elsewhere as  $3^-$ ,  $(2^+)$ ,  $(3^+, 2^+)$ ,  $l_n = 2$  and 0, respectively (see Table IV).

The strong transitions above 4.0 MeV undoubtedly correspond to the configurations  $\{(2d_{5/2})_n(1g_{9/2})_n^{-1}\}_J$ ,  $\{(2d_{3/2})_n(1g_{9/2})_n^{-1}\}_J$ , and  $\{(3s_{1/2})_n(1g_{9/2})_n^{-1}\}_J$ . The simplest model for the level structure in this case is a single  $g_{9/2}$  neutron hole weakly coupled to the singleparticle neutron levels of Sr<sup>89</sup>. Assuming no interactions of the resulting states among themselves or with other degrees of freedom in Sr<sup>88</sup>, this would produce deTABLE I. Sr<sup>88</sup> levels up to 8.5 MeV from the Sr<sup>87</sup>(d, p)Sr<sup>88</sup> reaction. The excitation energies and Q values are arithmetic averages of energies determined at a minimum of four reaction angles. The uncertainties in these values were estimated as  $\pm 5$  keV standard error for the lowest states and  $\pm 10$  keV for the highest states. The values of  $(d\sigma/d\Omega)_{max}$  and  $\theta_{max}$  are given for a level whenever it is possible, and  $l_n$  and  $(2J+1)S_{ln,j}/(2J_0+1)$  are listed for those displaying stripping angular distributions. An ns in the column under  $l_n$  means that the corresponding angular distribution showed a nonstripping pattern. Uncertainty in an  $l_n$  assignment is indicated by an enclosure in parentheses.

Level No.	$\overset{E_x}{({ m MeV})}$	Q (MeV)	$ heta_{ m max}$ (deg)	${(d\sigma/d\Omega)_{ m max} \over ({ m mb}/{ m sr})}$	$l_n$	$\left(\frac{2J+1}{2J_0+1}\right)S_{ln,j}$
0	0	8.865	32	0.049	4	1.20
1	1.839	7.026	0	(0.080	2	0.126
2	2.738	6.127	•••	< 0.015	ns	
3	3.208	5.657	•••	< 0.015	ns	
4	(3.590)	(5.275)	•••	(<0.005)	(ns)	
5	4.035	4.830	50	0.20	2	0.279
6	4.294	4.571	50	0.28	2	0.376
7	4.408	4.455	51	0.67	2	0.875
8	4.450	4.415	51	0.064	2	0.083
9	4.464	4.396	•••	< 0.030	(ns)	
10	4.514	4.351	51	0.84	2	1.08
11	4.551	4.313	•••	< 0.050	?	
12	4.636	4.229	51	0.45	2	0.564
(13) <sup>a</sup>	4.748	4.121	51	0.66	2	0.805
14	4.789	4.083	•••	< 0.050	?	
15	4.876	3.992	0	(0.68)	0	0.230
16	4.920	3.946	•••	< 0.010	ns	
17	5.101	3.767	52	0.91	2	1.04
18	5.133	3.726	•••	< 0.030	3	
19	5.157	3.703		< 0.030	(ns)	
20	5.199	3.663	•••	< 0.030	(ns)	
21	5.418	3.450	0	(0.350)	0	0.105
22	5.463	3.402	0	1.892	0	0.563
23	5.506	3.355	0	0.094	0	0.027
24	5.665	3.200	26	0.041	(1)	0.029
25	5.719	3.146	0	2.803	0	0.789
(26) <sup>a</sup>	5.775	3.093	0	1.452	0	0.405
27	5.830	3.025	•••	< 0.040	(ns)	
28	5.876	2.983	•••	< 0.050	(ns)	
29	6.008	2.862	51	0.045	2	0.042
30	6.043	2.842	37	0.198	ns	
31	6.149	2.722	51	0.094	2	0.088
32	6.214	2.654	0	0.121	0	0.031
33	6.237	2.619	25	0.208	(1)	0.127
34	6.306	2.556	•••	0.050	2	
35	6.340	2.531	51	0.100	2	0.091

generate multiplets of levels with about the same relative spacings as the corresponding states in Sr<sup>89</sup>. A degenerate multiplet would be displaced from the Sr<sup>88</sup> ground state by  $\Delta$ , the energy required to create the particle-hole configuration, which includes the higherorder residual effects arising from the interaction of the hole and particle with the remaining nucleons. Introducing interactions among the members of a multiplet would remove their degeneracy and allowing admixture with other possible configurations would produce fractionization of the strength of individual members. The  $Sr^{87}(d,p)Sr^{88}$  strength function in Fig. 4 appears to reveal such multiplets of levels, and a suggestive matching of these clusters with parent states in Sr<sup>89</sup> is indicated by the accompanying plot from the  $Sr^{88}(d,p)Sr^{89}$ reaction,<sup>1</sup> together with the dashed vertical guide lines.

In this simple picture, the lowest group of  $l_n=2$  states centered near  $E_x=4.57$  MeV and the small fragment seen in the 2<sup>+</sup> vibration at  $E_x=1.839$  MeV would have large components of the type  $\{(2d_{5/2})_n \times (g_{9/2})_n^{-1}\}_J$  and correspond to the  $2d_{5/2}$  ground state

of Sr<sup>89</sup>. The number of states in this group is eight compared to six expected just from counting the different possible values of J and points to the type of admixture described above. This increase in complexity precludes the possibility of applying a (2J+1) rule to the observed strengths with the objective of determining the spins of the individual states in the multiplet. A further prediction of this simple model would be that the summed spectroscopic strength  $\sum_{\alpha} (2J+1)S_{l_n,j^{\alpha}}/(2J_0+1)$ , taken over states  $\alpha$  in the Sr<sup>88</sup> multiplet and the value  $S_{l_{n},j}$ , measured in the  $Sr^{88}(d,p)Sr^{89}$  reaction<sup>1</sup> for the  $d_{5/2}$  ground-state transition, should be equal. The deduced values are 5.23 and 4.76, respectively, and therefore support the prediction to within errors expected from either experiment. Finally, the energy centroid  $\Delta = \sum_{\alpha} E_{\alpha} S_{l_n,j^{\alpha}} / \sum_{\alpha} S_{l_n,j^{\alpha}}$  associated with this cluster was determined to be 4.57 MeV.

Similar remarks can be made about the other groups seen in Fig. 4. The  $l_n=0$  transitions at about  $E_x=5.6$ MeV are expected to have large  $\{(3s_{1/2})_n(g_{9/2})_n^{-1}\}_{J=4,5}$ components which are associated with the strong  $3s_{1/2}$ 

Level No.	(MeV)	Q (MeV)	$ heta_{\max}$ (deg)	$(d\sigma/d\Omega)_{ m max}$ (mb/sr)	$l_n$	$\left(\frac{2J+1}{2J_0+1}\right)S_{ln,j}$
36 37 38 39	6.376 6.420 6.465 6.515	2.491 2.447 2.402 2.351	52 (0)	0.09) 0.215 0.80 (0.205)	(2) (2) 2 (0)	(0.076) (0.175) 0.675 (0.049)
40 41 42 43 (44)	6.564 6.612 6.629 6.687 (6.710)	2.303 2.253 2.238 2.181 2.149	52  	0.36 <0.10 <0.060 <0.050 (<0.050)	2 (ns) ns ns	0.311
45 46 47	6.740 6.752 6.826	2.128 2.128 2.113 2.042	52 52	0.91	2 2	0.748 0.159
48 49 50 51	6.869 6.931 6.958 7.026	1.999 1.937 1.899 1.832	53 53	<0.100 0.39 0.58 0.090	? 2 2 ns	0.311 0.456
(52) 53 54 (55)	7.088 7.109 7.136	1.772 1.754 1.728	(35)	<0.020 <0.200 <0.300	2	0.200
(55)* 56 57 58	7.195 7.251 7.337 7.426	1.673 1.617 1.534 1.430	53 81	0.48	2 4	1.137
59 60 61 62	7.561 7.594 7.640 7.674	1.297 1.280 1.219 1.185				
63 64 65	7.742 7.839 7.889	1.116 1.026 0.976				
66 67 68 69	7.967 8.003 8.103 8.142	0.901 0.876 0.765 0.726				
70 71 72	8.450 8.493 8.516	0.414 0.372 0.349				

TABLE I. (continued).

• The three levels, Nos. 13, 26, and 55, are put in parentheses because they may arise from the presence of the Sr<sup>88</sup> contaminant in the target. Their Q values (4.121, 3.093, and 1.673, respectively) and  $l_n$  assignments are in close agreement to three levels seen in the Sr<sup>88</sup>( $d_r$ )Sr<sup>89</sup> reaction (Nos. 0, 1, and 8 in Ref. 1, with Q = 4.133, 3.102, and 1.678 MeV, respectively). Their yields, however, are not in complete agreement with the expected contaminant groups and, therefore, because an unambiguous discrimination could not be made, the states were included in the table on a tentative basis.

higher  $l_n=2$  group, located around  $E_x=6.7$  MeV in  $Sr^{88}$ , is probably associated with three strong d states in Sr<sup>89</sup>, level Nos. 3, 4, and 8 of Ref. 1 at  $E_x = 1.931$ , 2.000, and 2.455 MeV. Level No. 3 in Sr<sup>89</sup> is thought to have spin  $\frac{5}{2}$ , and Nos. 4 and 8,  $\frac{3}{2}$ . Therefore, it is

single-particle state at  $E_x = 1.031$  MeV in Sr<sup>89</sup>. The likely that the states of this second  $l_n = 2$  group consist of both types  $\{(2d_{5/2})_n(1g_{9/2})_n^{-1}\}_J$  and  $\{(2d_{3/2})_n$  $\times (1g_{9/2})n^{-1}$ . Their sum strength is 3.49, in close agreement to the value of 3.76 which is the analogous sum for the three Sr<sup>89</sup> states. Since a separation of the two types just mentioned cannot be made unambigu-

 TABLE II. A summary of information on neutron particle-hole states extracted from the present experiment. For comparison, the locations of the corresponding single-particle states in Sr<sup>89</sup> are also given. A detailed explanation is given in the text.

$\mathrm{Sr}^{87}(d,p)$	$Sr^{88}$ Present work (21+1)		$\mathrm{Sr}^{88}(d,p)\mathrm{Sr}^{89}$ Cosman <i>et al.</i> <sup>a</sup>							
Particle-hole config.	$\sum_{\alpha} \frac{(2J+1)}{(2J_0+1)} S_{l,j}^{\alpha}$	(MeV)	Level No.	$E_x^{89}$ (MeV)	Config.	$\sum_{\alpha} (2J+1) S_{l,j}^{\alpha}$	$\Delta - E_x^{89}$ (MeV)			
$\frac{\{(2d_{5/2})_n(1g_{9/2})_n^{-1}\}_J}{\{(3s_{1/2})_n(1g_{9/2})_n^{-1}\}_J}$	5.23 <sup>b</sup> 2.20°	4.57	0	0.0	2d 5/2	4.76	4.57			
$\{(2d_{3/2})_n(1g_{9/2})_n^{-1}\}_J$	3.49 <sup>d</sup>	(6.72)°	$\left\{\begin{array}{c} 3\\ 4\\ 0\end{array}\right\}$	1.931 2.000	$\left. \begin{array}{c} 2d_{5/2} \\ 2d_{3/2} \\ 2d_{3/2} \end{array} \right\}$	3.76	4.53)e			
$\{(1g_{7/2})_n(1g_{9/2})_n^{-1}\}_J$		(7.4) <sup>f</sup>	10	2.455 2.671	$2d_{3/2}$ ) $1g_{7/2}$	5.89	(4.73) <sup>f</sup>			

\* Reference 1. <sup>b</sup> States included in the sum are Nos. 1, 5, 6, 7, 8, 10, 12, 13, and 17. <sup>c</sup> States included in the sum are Nos. 15, 21, 22, 23, 25, 26, 32, and 39. <sup>d</sup> States included in the sum are Nos. 29, 31, 35, 36, 37, 38, 40, 46, 47, 49, 50, and 55. <sup>e</sup> The states in this group may not all be of the type  $\{(2d_{3/2})_n(1g_{3/2})_n^{-1}\}$  (see text) and, therefore, this value is only an approximation. <sup>f</sup> This is only an estimate since not all the  $l_n = 4$  strength was seen (see text).



FIG. 4. The upper three graphs in this figure display the spectroscopic strengths as a function of excitation energy in Sr<sup>88</sup> for the indicated  $l_n$  values determined from the DWBA analysis of the Sr<sup>87</sup>(d,p)Sr<sup>88</sup> reaction. The bottom graph shows a similar plot determined in Ref. 1 from the Sr<sup>88</sup>(d,p)Sr<sup>89</sup> reaction. The clusters of states in Sr<sup>88</sup> seen with  $l_n = 0$  and 2 are interpreted as multiplets of neutron particle-hole states having the configuration designated by their adjacent labels. These states might be constructed by coupling a (1g<sub>9/2</sub>) neutron hole to the states of Sr<sup>89</sup>, and a suggested correspondence of the Sr<sup>88</sup> and Sr<sup>89</sup> level structure is indicated by the dashed vertical guidelines.

ously, the corresponding values of  $\Delta$  cannot be computed. However, as a simplification, we will assume that the  $\{(2d_{5/2})_n(1g_{9/2})_n^{-1}\}$  contributions are small compared to those for the  $\{(2d_{3/2})_n(1g_{9/2})_n^{-1}\}$  configuration. This is supported by the fact that the  $d_{5/2}$ state No. 3 in Sr<sup>89</sup> represents only 14% of the summed strength of the triplet Nos. 3, 4, and 8. On this basis, we get a value of  $\Delta(2d_{3/2})=6.72$  MeV associated with the centroid  $E_{3/2}=2.19$  MeV of the two  $2d_{3/2}$  states, Nos. 4 and 8 in Sr<sup>89</sup>.

The only  $l_n=4$  transition aside from that of the ground state is seen to occur at  $E_x=7.426$  MeV in Sr<sup>88</sup>. This is assumed to be a member of the  $\{(1g_{7/2})_n \times (1g_{9/2})_n^{-1}\}_J$  multiplet. The rest were either too weak to be seen or lie outside the energy range of the experiment. Taking only this state, we estimate the value of  $\Delta(1g_{7/2})=7.4$  MeV.

A summary of these particle-hole state results for the  $\mathrm{Sr}^{87}(d,p)\mathrm{Sr}^{88}$  reaction, together with associated information from the  $\mathrm{Sr}^{88}(d,p)\mathrm{Sr}^{89}$  reaction,<sup>1</sup> is displayed

TABLE III. Sum of spectroscopic strengths from the  $Sr^{87}(d, p)Sr^{88}$ reaction. It is assumed here that  $l_n=0, 1, 2, and 3$  transitions transfer a neutron into the  $3s_{1/2}, 2p, 2d$ , and  $1f_{5/2}$  orbits. From the data, the separation of the two  $l_n=4$  transitions into the groups  $1g_{9/2}$  and  $1g_{7/2}$  appears obvious.

$\sum_{\alpha} \frac{(2J+1)}{(2J_0+1)} S_{ln,j}{}^{\alpha}$									
Experiment Shell model	$3s_{1/2}$ 2.2 2.0	$\leq_{0.16}^{2p}$	2d 8.72 10.0	${1 f_{5/2} \atop 0 \\ 0}$	$\frac{1g_{7/2}}{1.14}$ 8.0	1g <sub>9/2</sub> 1.20 1.0			

in Table II. The last column of the table lists the values of  $\Delta - E_x^{89}$  which, for the extreme weak-coupling case, should be a constant.

Table III gives the summed spectroscopic strengths for all classes of  $l_n$  transitions seen in the present work. Their physical interpretation is given through the following relationship<sup>15</sup>:

$$\sum_{\alpha} \frac{(2J+1)}{(2J_0+1)} S_{l_{n,j}\alpha}$$

= number of  $(l_n, j)$  neutron holes in target,

where the sum is taken over all final states for a given  $l_n$  and j. The expected values from the simple shell model are also tabulated for direct comparison.

It is clear from Table III that the  $3s_{1/2}$  and  $1g_{9/2}$ strengths seem completely accounted for from our data. However, it is also evident that some of the 2d and  $1g_{7/2}$  transitions have probably been missed and lie outside the energy range studied in this experiment. Two states, Nos. 24 and 33 at  $E_x=5.665$  and 6.237 MeV, respectively, had angular distributions that compared most closely to  $l_n=1$  DWBA curves (see Fig. 3). The upper limit entered in the 2p column is the sum of the two spectroscopic factors derived from these comparisons. If the assignments are correct, they would point to the presence of 2p holes in the Sr<sup>87</sup> target and, therefore, to a term in the ground-state wave function which is more complicated than just  $(1g_{9/2})_n^{-1}$ .

As a final remark, we note that, in addition to the stripping transitions, over 40% of the levels seen below 6.0 MeV had very weak yields and angular distributions which were dissimilar to the patterns predicted by the DWBA code for single-neutron transfers. These so-called nonstripping states undoubtedly involve different core-excited structure than the ones we have considered above and thus must be populated through weaker and higher-order reaction mechanisms.

### **B.** Comparison with Other Data on Sr<sup>88</sup>

The information on the level structure of Sr<sup>88</sup> presently available in the literature is summarized in Table IV. The listing is made in terms of the reaction mode

<sup>&</sup>lt;sup>15</sup> M. H. Macfarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).

Level No.	$\mathop{\mathrm{Sr}}^{87}_{E_x}(d,p)$	$Sr^{88a}$ $l_n$	$E_x^{(d,E)}$	${ m Ie^{3)^b} \over J^{\pi}}$	$E_x^{(\alpha,$	$lpha')^{\circ}_{J^{\pi}}$	Ex (e,	$(e')^{\mathrm{d}}_{J^{\pi}}$	$E_x$ (	$(p,p')^{\mathrm{e}}_{J^{\pi}}$	$E_x^{(d,d)}$	$J^{\pi}$
0 1 2 3 4 5	0 1.839 2.738 3.208 (3.590) 4.035	4 2 ns ns (ns) 2	0 1.84 3.21 3.48 3.64	$(0^+)$ $(2^+)$ $(2^+)$ $(1^+)$ $(3^+)$	0 1.84 2.74 3.21	0+ (2+) (3 <sup>-</sup> ) (2+)	0 1.84 2.74 4.0	0 2+ 3- (2+)	0 1.84 2.74 3.24 3.57 4.05	$0 \\ 2^+ \\ 3^- \\ 2^+ \\ (2^+) \\ (2^+, 4^+)$	0 1.835 2.74 3.20 3.52 3.61 4.02	0+ 2+ 3- 2+

TABLE IV. A summary comparison of the present  $Sr^{87}(d,p)Sr^{88}$  results with those from other sources. A discussion is given in the text.

<sup>a</sup> Present work.
<sup>b</sup> Reference 4.
<sup>c</sup> Reference 5.
<sup>d</sup> Reference 6.

Reference f Reference 8.

studied. A partial listing of levels from the present experiment is also included for direct comparison. Several examples of energy ambiguities are evident among the data. Many cases where levels have been recorded elsewhere, but not seen here, may be explained by the extremely selective character of the present (d,p) reaction. Any transition, other than direct singleparticle transitions, is strongly reduced in yield by the reaction mechanism and by the statistical factor of 10 arising from the  $J_0 = \frac{9}{2}$  target and, therefore, it may not have been discernible as a proton group.

In a previous work, Kavaloski et al.4 used the  $Y^{89}(d, He^3)Sr^{88}$  reaction to measure proton excited components in the lowest levels of Sr<sup>88</sup>. One of their conclusions was that the configurations  $\{(p_{1/2})_p (p_{3/2})_p^{-1}\}_{2^+}$ and  $\{(p_{1/2})_p(f_{5/2})_p^{-1}\}_{2^+}$  account for 80% of the first 2<sub>1+</sub> state at 1.839 MeV and 60% of the second  $2_{2}$ <sup>+</sup> state at 3.21 MeV. Since in the  $Sr^{87}(d,p)Sr^{88}$  reaction  $S_{l,j}$   $(2_1^+)=0.25$  measures directly the amplitude squared of the term  $\{(2d_{5/2})_n(1g_{9/2})_n^{-1}\}$  in the  $2_1^+$  wave function, we conclude that this term takes up a large fraction of the remaining strength unaccounted for in the  $(d, \text{He}^3)$ experiment. Furthermore, unlike the  $2_1^+$  case, the  $2_2^+$ state at 3.21 MeV is barely excited here, and we suggest that no particle-hole states involving a  $(g_{9/2})_n$  hole are included in the composition of its wave function.

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