for $^{27}Al(p,^7Be)$, which was measured by Furukawa *et al.*,[6] is shown for comparison with the $Si(p,^7Be)$ curve.

The only mechanism for producing 21-h ^{28}Mg by proton-induced reactions in silicon is $^{30}Si(p,3p)^{28}Mg$. A hint of peaks corresponding to 1350- and 1750-keV γ's, which follow decay of ^{28}Mg, were observed in spectra obtained from wafers for which E_p was 56 and 60 MeV. Comparison of these with more intense γ peaks from decay of 15-h ^{24}Na yields an estimate of $\lesssim 0.05$ mb for $Si(p,x)^{28}Mg$. This corresponds to a cross section of $\lesssim 1.5$ mb for $^{30}Si(p,3p)^{28}Mg$ at 56 to 60 MeV. Previous measurements at 130 to 400 MeV[5] yielded cross sections of 1.7–2.8 mb for $^{30}Si(p,3p)$.

The experimental data were compared with calculated excitation functions based on a theory of nuclear evaporation developed by Dostrovsky, Fraenkel, and Friedlander.[7] A Monte Carlo code written by Rogers[8] was used to perform the calculations with an IBM 360 computer.

Two parameters were varied in the calculations. One was parameter a which appears in the Weisskopf level-density[9] formula

$$W(E)=C\exp[2(aE)^{1/2}].$$

The other parameter that was varied is the nuclear radius parameter r_0. Values of a $\sim A/20$ have been found to give the best agreement between calculations and experimental data for targets in the mass region $45<A<75$.[7] This value was used for some of the calculations in this work. A detailed study of the effect of varying a was not made, but a value of $\sim A/7$ was also used to illustrate the effect of varying a in the calculations. Calculations were made for r_0 equal to 1.5 and 1.7 F. Thus, four excitation functions were calculated for each nuclide. The calculated excitation functions for ^{28}Al, ^{27}Mg, and ^{22}Na are shown in Figs. 2, 4, and 7, respectively.

The comparison of the calculated excitation functions with the data for ^{22}Na (Fig. 7) does not favor any set of parameters. In the case of ^{28}Al (Fig. 2) the data appear to agree better than the others with the calculated curve, for which $a=1.5$ and $r_0=1.7$. The most striking differences in the calculated excitation functions are observed for ^{27}Mg (Fig. 4). The curves for which $a=4.2$ show a maximum near 45 MeV and a minimum near 55 MeV; the curves for which $a=1.5$ do not show these features. The experimental data indicate better agreement with the curves for which $a=4.2$.

[6] M. Furukawa, S. Kume, and M. Ogawe, Nucl. Phys. **69**, 362 (1965).

[7] I. Dostrovsky, Z. Fraenkel, and G. Friedlander, Phys. Rev. **116**, 683 (1959).

[8] P. C. Rogers, Ph.D. thesis, Massachusetts Institute of Technology, 1962 (unpublished).

[9] V. F. Weisskopf, Phys. Rev. **52**, 295 (1937).

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Inelastic Scattering of 42-MeV Alpha Particles from K^{39} and V^{51}†

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The results of a study of the inelastic scattering of 42-MeV α particles from the odd nuclei K^{39} and V^{51} are compared with the predictions of the shell model and the weak-coupling collective model. The results of a variety of other experiments are correlated with the present results whenever possible in order to test the predictions of the two models. Some success is found for various states with each model, and, in particular, the 3.60- and 3.90-MeV states of K^{39} are assigned spins of $\frac{9}{2}^-$ and $\frac{7}{2}^-$ on the basis of the weak-coupling model, while spins of $\frac{3}{2}^-$ and $\frac{5}{2}^-$ are suggested for two states near 4.14 MeV. Good agreement with the data for V^{51} is found by using the shell-model prediction with the $L=2$ cross sections enhanced by a factor of 4.

I. INTRODUCTION

For odd nuclei with magic proton or neutron configurations, the level densities are such that several states may be resolved with the energy resolution available for α-particle scattering. In even nuclei the spectra are dominated by the strongly excited collective quadrupole and octupole states, but the models for odd nuclei may be more subtle. In the present work we shall be dealing mainly with two simple models—the collective vibrational weak-coupling model and the shell model with good seniority. The differing predictions of these models for the shapes and magnitudes of the angular distributions observed in inelastic

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α-particle scattering can provide information on the validity of the simple models and perhaps suggest some improvements to these.

One might expect the shell model to be valid for nuclei one nucleon removed from a double magic nucleus. Potassium-39 may be considered as a proton hole in Ca^{40} and hence its spectrum might be simply described by the shell model. A well-known collective octupole state is found at 3.73 MeV in Ca^{40}, so perhaps the weak-coupling model, with the proton hole coupled to this vibration, also might be valid. A more complicated spectrum is expected with three like nucleons outside a closed-shell nucleus. The spectrum of V^{51} has long been considered an example of the success of the seniority shell model,[1] although Ti^{50} and Cr^{52} both have states that might be called collective. Again, two models might be applied to this odd nucleus.

In the present work the results from the inelastic scattering of 42-MeV α particles from K^{39} and V^{51} will be compared to the predictions of the above models. Some preference is found for a shell-model interpretation of some levels, while the collective model is invoked to explain others.

II. DETAILS OF MODELS

The ground-state spin of K^{39} is $\frac{3}{2}^+$, as is expected for a $1d_{3/2}$ proton hole in doubly magic Ca^{40}. We might expect the following low-lying excited states from the shell model:

(1) The unpaired proton in the $1d_{3/2}$ orbital may be excited into the $1f_{7/2}$ or $2p_{3/2}$ orbitals.

(2) A proton in the filled $2s_{1/2}$ or $1d_{5/2}$ orbitals may be excited into the hole in the $1d_{3/2}$ orbital.

The description of the inelastic scattering to such shell-model states has recently been presented.[2–5] Only a brief review will be given here. An interaction (Gaussian, for example) is postulated between a projectile propagating via the optical-model distorted waves and some relevant bound nucleon (with harmonic-oscillator wave functions, for instance). The form factor entering the distorted-wave Born-approximation (DWBA) calculations is then written as

$$F_L(r)=4\pi V_G M_{LJ_iJ_f}\mathcal{G}_{Ll_il_f}(r).$$

The angular matrix element $M_{LJ_fJ_f}$ determines the values and weightings of the angular-momentum transfers L allowed in going from the ground state with spin J_i to the excited state with spin J_f. The radial matrix element $\mathcal{G}$ is calculated using the harmonic-oscillator wave functions for the bound nucleon.[4,5] For the present work we use a value of 2.00 F for a, the usual oscillator length parameter, while the range γ of the nucleon-projectile interaction is chosen to be 0.45 F^{-1}.[6] The strength V_G of this interaction will be determined by comparison of the predictions to the experimental results; the resulting strengths V_G will then be compared to the value obtained from free proton–α-particle scattering experiments.[6]

TABLE I. Angular matrix elements $M_{LJ_iJ_f}$ are shown for various shell-model transitions in K^{39} or V^{51}. The signs are not given.

Transition	$M_{LJ_iJ_f}$ 1	2	3	4	5
$(d_{3/2})^3\to(d_{3/2})^2(f_{7/2})$	···	···	0.199	···	···
$(d_{3/2})^3\to(d_{3/2})^2(p_{3/2})$	0.089	···	0.278	···	···
$(s_{1/2})^2(d_{3/2})^3\to(s_{1/2})(d_{3/2})^4$	···	0.399	···	···	···
$(d_{5/2})^6(d_{3/2})^3\to(d_{5/2})^5(d_{3/2})^4$	···	0.399	···	0.369	···
$(f_{7/2})^3\to(f_{7/2})_0{}^2(p_{3/2})$	···	0.154	···	0.085	···

For the promotion of the unpaired proton in K^{39} into the $f_{7/2}$ orbital, angular-momentum transfers of $L=3$ and $L=5$ are allowed, with the angular matrix elements shown in Table I. The radial matrix elements approach equality at large radii. For promotions into the $2p_{3/2}$ orbital, values of $L=1$ or $L=3$ are allowed, with the angular matrix elements found in Table I, which also contains the angular matrix elements for the promotions into the $d_{3/2}$ orbital from the deeper orbitals.

The predictions for excitation of the states of V^{51} formed from the proton $(f_{7/2})^3$ configuration have already been compared to the results for inelastic proton scattering.[2] This comparison allowed no firm conclusions to be drawn, and it is to be hoped that the more structured angular distributions found in α-particle scattering will provide a better comparison. In addition to these $(f_{7/2})^3$ states, we shall also look for a promotion of the unpaired proton into the $2p_{3/2}$ orbital. The angular matrix elements for this promotion are also listed in Table I.

In the collective vibrational model, the levels of an odd nucleus are formed by coupling the extra hole or particle to the vibrations of the neighboring even nucleus. The states so formed have spins

$$|L-j|\leq J\leq L+j,$$

where j is the angular momentum of the odd particle and L is the spin of the collective state. The energies E_J of these states often seem to obey a center-of-gravity rule:

$$(2j+1)(2L+1)E_L=\sum_J(2J+L)E_J,$$

[1] A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic Press Inc., New York, 1963).

[2] H. O. Funsten, N. R. Roberson, and E. Rost, Phys. Rev. **134**, B117 (1964).

[3] M. B. Johnson, L. W. Owen, and G. R. Satchler, Phys. Rev. **142**, 749 (1966); W. S. Gray, R. A. Kenefick, J. J. Kraushaar, and G. R. Satchler, *ibid.* **142**, 735 (1966).

[4] J. Alster, D. C. Shreve, and R. J. Peterson, Phys. Rev. **144**, 999 (1966).

[5] R. J. Peterson, Ph.D. thesis, University of Washington, 1966 (unpublished).

[6] S. Sack, L. C. Biedenharn, and G. Breit, Phys. Rev. **93**, 321 (1954).

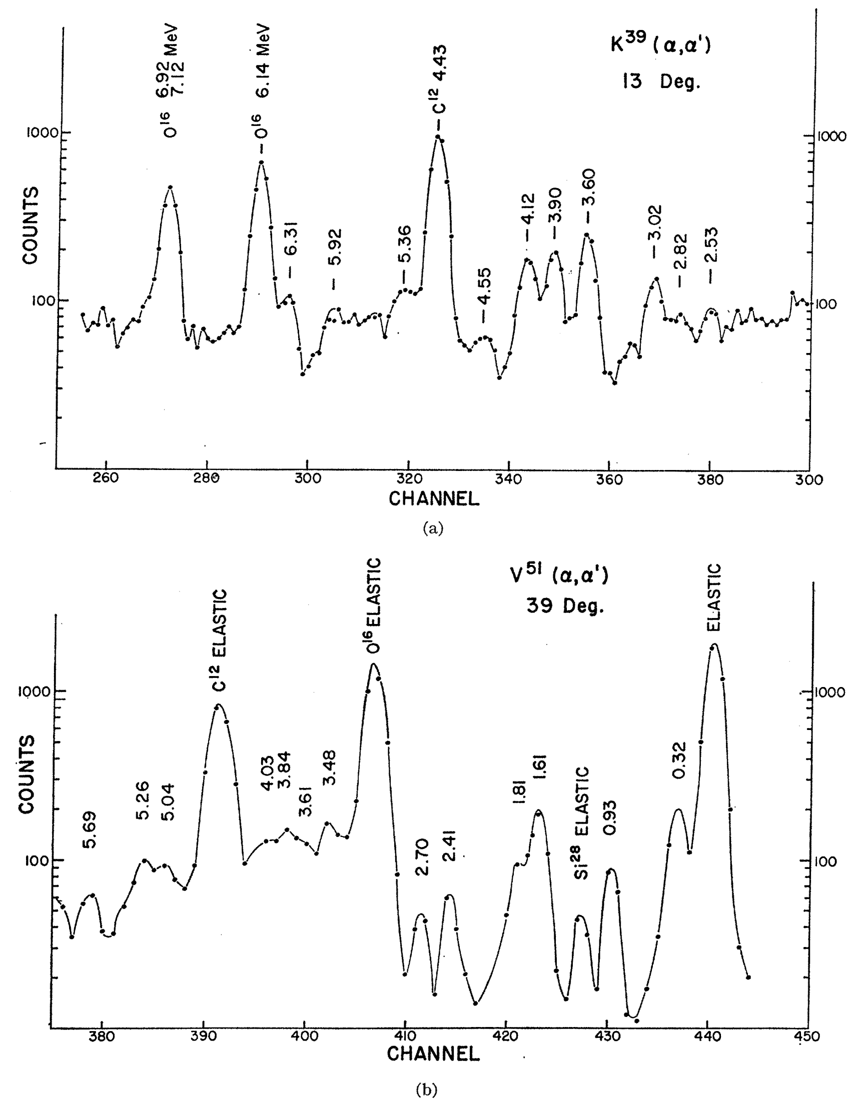

FIG. 1. (a) The spectrum of 42-MeV α particles inelastically scattered from a natural potassium target at an angle of 13°. The excited states of K^{39} are indicated. (b) The spectrum of 42-MeV α particles inelastically scattered from V^{51} at an angle of 39°. The excitation energies of all peaks that appeared consistently in the spectra are indicated, but angular distributions were obtained only for the low-lying states.

where E_L is the energy of the collective parent state. The angular distributions for populating these states all have the same shape, that of the angular distribution for populating the parent state, and hence are characterized by only one angular-momentum transfer and not all that are allowed by the angular momenta of the initial and final states. The strengths of the differential cross sections to the weak-coupling states are determined from

$$\sigma(J)=\frac{2J+1}{(2j+1)(2L+1)}\sigma(L),$$

while the sum of the cross sections $\sigma(J)$ is equal to that for the parent collective state.[4,7]

The angular distributions predicted for these models may then be obtained from a DWBA analysis with either the shell-model form factors described above or the usual collective form factor. The DWBA code JULIE was used for the present work.[8] The adiabatic model for Austern and Blair may also be used for the collective cross sections $\sigma(L)$.[9]

High-energy protons, being less strongly absorbed than α particles, make a more sensitive probe of the radial form factors, and differences in the shapes of the observed angular distributions allow one to examine the specific orbitals involved.[3] Here, however, it is not expected that the shapes of the angular distributions for a given value of the angular-momentum transfer will depend on the specific model used. We shall rely on determining the mixtures of angular-momentum transfers

[7] J. S. Blair, Argonne National Laboratory Report No. 6878, 1964 (unpublished).

[8] R. H. Bassel, R. M. Drisko, and G. R. Satchler, Oak Ridge National Laboratory Report No. ORNL-3240, 1962 (unpublished).

[9] N. Austern and J. S. Blair, Ann. Phys. (N. Y.) 33, 15 (1965).

TABLE II. This table exhibits the parameters derived from the elastic scattering of 42-MeV α particles from K^{39} and V^{51} that were used for the Austern-Blair and DWBA analyses of the inelastic scattering data. A volume imaginary optical-model well was used.

	Optical model and DWBA			
	r_0 (F)	V (MeV)	W (MeV)	a (F)
K^{39}	1.74	38	18	0.47
V^{51}	1.66	51	18	0.43
	Parametrized partial-wave analysis and Austern-Blair			
	l_A	Δl_A	δ	l_δ	Δl_δ
K^{39}	15.720	0.735	0.646	16.300	1.044
V^{51}	16.516	0.693	0.800	16.699	1.107

required to fit the data and on the relative and absolute magnitudes of the cross sections for our results.

III. EXPERIMENTAL ARRANGEMENT

The 42-MeV α-particle beam of the University of Washington cyclotron was energy-analyzed at the entrance to the scattering chamber, passed through thin metallic targets of natural potassium (93.1% K^{39}) or natural vanadium (99.8% V^{51}), and collected in a Faraday cup. Silicon solid-state detectors were used to measure the energies of the scattered particles. The energy resolution in the final spectra was typically 120–140 keV full width at half-maximum (FWHM).

Typical spectra are shown in Figs. 1(a) and 1(b). The K^{39} target contained 6.9% of K^{41}, which nucleus has many levels above 2 MeV.[10] These levels and the tail of the elastic scattering peak, large at the small angle chosen for the sample spectrum, contribute to the background. This target was prepared by evaporating metallic potassium onto a 50-μg/cm^2 carbon film inside the scattering chamber. The large amounts of carbon and oxygen seen in the spectrum are from this backing and from contaminants collected by the chemically active potassium. The self-supporting vanadium target was made by evaporating from chips of the metal on a tantalum filament. This target also contained appreciable carbon and oxygen contamination. No attempt was made to extract information from peaks obscured by these contaminants.

The normalization of the present data for K^{39} was accomplished as follows. For nuclei between calcium and nickel, where target thicknesses were measured or where the data were normalized by chemical-ratio methods,[5] it was observed that the ratio of the elastic differential cross sections to the Rutherford prediction near 18° is within 10% of 0.45. This is confirmed by optical-model and partial-wave-analysis predictions. The normalization of the present data was then determined by forcing the K^{39} elastic scattering data to fit this ratio. This is expected to be correct within ±10%. The thickness of the V^{51} target was determined by destroying and weighing it after the bombardment.

[10] H. A. Enge, W. H. Moore, and J. W. Kelley, Bull. Am. Phys. Soc. 3, 210 (1958).

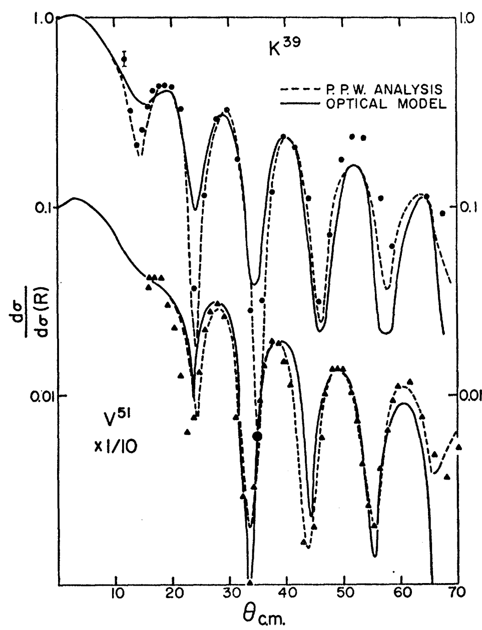

FIG. 2. The differential cross sections for the elastic scattering of 42-MeV α particles from K^{39} and V^{51} are exhibited as the ratio to the Rutherford prediction. The solid curves are the predictions of the optical model and the dashed curves those for a parametrized partial-wave analysis.

The energies of the excited states of K^{39} and V^{51} were determined by a calibration based on the levels of the C^{12} and O^{16} contaminants and on well-known levels of the target nuclei. These were chosen to be the 3.603-MeV state of K^{39} and the 0.930- and 1.611-MeV states of V^{51}. No discrepancies were found with published values for the energies of other states. For each new level found in the present work the average and the variance found for the excitation energies will be given. In Figs. 1(a) and 1(b) we show representative spectra with the energies of the excited states indicated.

IV. MODELS FOR ELASTIC AND INELASTIC SCATTERING

The elastic scattering data were fit with a four-parameter optical model, with parameters varied to fit the data. It is known that there are several families of parameters that will fit such data. The set with the shallowest real potential was used here. In Fig. 2 we exhibit the data for K^{39} and V^{51} and the optical-model fits with the parameters listed in Table II.

A parametrized partial-wave analysis was also used to fit the elastic scattering. The partial-wave amplitudes for nuclear scattering are parametrized as[11]

$$\eta_l = A_l \exp(2i\delta_l),$$
$$A_l = \{1+\exp[(l_0-l)/\Delta l]\}^{-1},$$
$$\delta_l = \delta_0\{1+\exp[(l-l_\delta)/\Delta l_\delta]\}^{-1}.$$

[11] J. A. McIntyre, K. H. Wang, and L. C. Becker, Phys. Rev. 117, 1337 (1960); J. Alster and H. E. Conzett, *ibid.* 139, B50 (1965).

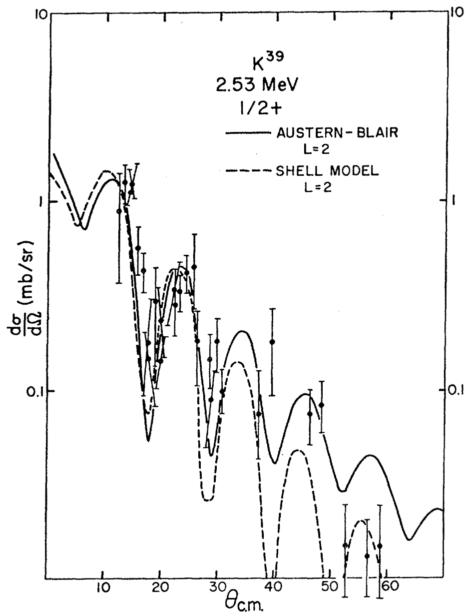

FIG. 3. The differential cross sections for the excitation of the 2.53-MeV ($\frac{1}{2}^+$) state of K^{39} are compared with the collective prediction of Austern and Blair (with $\delta_2=0.14$ F) and with the prediction for the promotion of a proton from the $2s_{1/2}$ orbital into the $1d_{3/2}$ orbital (with $V_G=27$ MeV).

The five parameters in this expression were found by a least-squares fitting to the data. The resulting parameters to give the fit shown in Fig. 2 are listed in Table II.

The DWBA code JULIE[8] was used to fit the observed angular distributions for inelastic scattering. Form factors for collective and for single-particle transitions were used. The collective form factor is

$$F_L(r)=-(2L+1)^{-1/2}(\beta_L R_0)(dU/dR),$$

where U is the complex optical potential and the deformation distance is βR_0. The radius R_0 is the midpoint of the optical-model well. In the collective model for a uniform distribution of nuclear charge, this value of βR_0 may also be used to compute the electromagnetic reduced matrix element

$$B(EL)\uparrow=(9/16\pi^2)Z^2(\beta R_c)^2R_c^{2L-2},$$

and we use $R_c=1.25A^{1/3}$ for the nuclear-charge radius. Note that the deformation distance is βR_c rather than βR_0, since the electromagnetic transition matrix element depends on the deformation of the charge and not on the nuclear radius. We use nuclear radii of $1.74A^{1/3}$ and $1.66A^{1/3}$ for K^{39} and V^{51}, respectively, and the ratios $(R_c/R_0)^2$ are then 0.52 and 0.57. It is interesting to compare the reduced matrix elements for α-particle, proton, and electromagnetic induced transitions, but we cannot expect these to be equal except perhaps for very collective transitions.

The shell-model form factors were generated as described in Sec. II. The DWBA prediction generated from these form factors was compared to the data in order to extract the strength factor V_G. These numbers may be compared to the value near 45 MeV expected from free proton-α scattering,[6] but the normalization is very dependent upon the values chosen for the range of the force and the oscillator well parameter for the bound nucleons.

We shall also use the approximation of the collective DWBA developed by Austern and Blair.[9] In this model the inelastic scattering amplitudes are generated from partial derivatives of the partial-wave coefficients for the elastic scattering. These coefficients are obtained from the parametrized partial-wave analysis discussed above. The Austern-Blair model makes two approximations to the DWBA amplitudes. The first is the neglect of the excitation energy of the nuclear state being populated. This might be expected to be good for states below 8 MeV or so, populated by the scattering of 42-MeV α particles. The second approximation ignores the difference in the angular momentum of the incoming and outgoing projectile. The important part of the transition occurs for projectiles with grazing incidence, about seventeen units in the present case. The transfer of two or three units to the nuclear excited state might then seem to have a negligible effect.

Both the DWBA and Austern-Blair predictions will be fitted to the data, and the normalization factors βR_0 will be extracted for each fit. This may be used to check the validity of the approximations made by Austern and Blair. Both models have been applied to the analysis of the inelastic scattering from even-even nuclei in the $f_{7/2}$ shell.[12]

V. RESULTS AND COMPARISON TO MODELS

A. Results for K^{39}

In this section we shall discuss the observed levels of K^{39}. Although K^{39} is a near neighbor of Ca^{40}, very little information is available for this nucleus. Previous results for the levels of K^{39} will be invoked in the course of the discussion to follow, and spin and model assignments will be made in several cases.

Several experiments that preferentially populate the proton hole states of K^{39} have been reported. These include the reactions $Ca^{40}(p,2p)$,[13] $Ca^{40}(d,He^3)$,[14–16] $Ca^{42}(p,\alpha)$,[17] $Ca^{40}(t,\alpha)$,[18] and $Ca^{40}(\gamma,p)$.[19] From these results it is found that the ground state of K^{39} is quite

[12] R. J. Peterson (to be published).
[13] G. Tibell, O. Sundberg, and U. Miklavzic, Phys. Letters **2**, 100 (1962); P. Newton, G. L. Salmon, and E. B. Clegg, Nucl. Phys. **82**, 499 (1966).
[14] B. Cujec, Phys. Rev. **128**, 2303 (1962).
[15] J. C. Hiebert, E. Newman, and R. H. Bassel, Oak Ridge National Laboratory Report No. ORNL-3800, 1965 (unpublished).
[16] J. C. Hiebert, E. Newman, and R. H. Bassel (to be published).
[17] J. A. Nolen and W. Gerace (private communication).
[18] S. Hinds and R. Middleton, Nucl. Phys. **84**, 651 (1966).
[19] S. A. E. Johansson and B. Forkman, Nucl. Phys. **36**, 141 (1962).

well described as a $d_{3/2}$ hole in the ground state of Ca^{40}. This wave function will be regarded as valid in all that follows.

The first excited state of K^{39} is found at 2.526 MeV.[20] The proton pickup reactions indicate that this state is formed from a $2s_{1/2}$ hole in Ca^{40}. This assignment is also suggested by the results from the $Ar^{38}(p,\gamma)$ reaction[21] and by studies of the photoproton reaction.[19] This state is only very weakly populated by the $Ar^{36}(\alpha,p)$ proton stripping reaction,[22] as would be expected for a hole state. For inelastic scattering, this 2.53-MeV state would be populated by an $L=2$ transition only, and indeed previous inelastic deuteron scattering studies do indicate a positive parity for this transition.[23]

The data from the present experiment are not very clean, since this 2.53-MeV peak is weakly excited and sits on a background of K^{41} levels. The angular distribution does fit an $L=2$ Austern-Blair prediction, however, as shown in Fig. 3. From this collective model the collective strength parameter $\delta_2=\beta_2 R$ would be equal to 0.14 ± 0.02 F. The over-all 10% uncertainty in the normalization of all of the differential cross sections has not been included in the uncertainty in this number, which is determined by the extreme credible fits of the theory to the data. From the collective DWBA prediction a value of $\delta_2=0.19\pm0.03$ F is obtained. This discrepancy with the Austern-Blair result is probably due to the approximations made in the latter model.[5] The observed value for $\delta_2=\beta_2 R$ is consistent with that determined from the electromagnetic transition, where a lifetime greater than 10^{-13} sec was measured.[24] It must be noted that this value of δ_2 is very small for a first quadrupole transition. For Ca^{42}, another possible candidate for the shell model, the deformation distance, measured with the Austern-Blair predicted fit, was 0.59 F.[12] If the statistical factors for the transitions are included, the transition strength in K^{39} is 56% of that for Ca^{42}.

The noncollective nature of this excitation suggests that a shell-model interpretation might perhaps be valid. A fairly good fit to the data was obtained, as shown in Fig. 3, using the shell-model form factor for the $L=2$ transition as described above. Any possible use of the differences at large angles between the collective and shell-model predictions was ruled out by the low quality of the data for these small cross sections. To normalize the shell-model prediction to the data, a value of V_G equal to 27 MeV was required. This is appreciably smaller than the value of 45 MeV to be expected from free proton–α-particle scattering[6] and is the lowest value found in the present series of experiments.[12] Some caution must be used in interpreting this

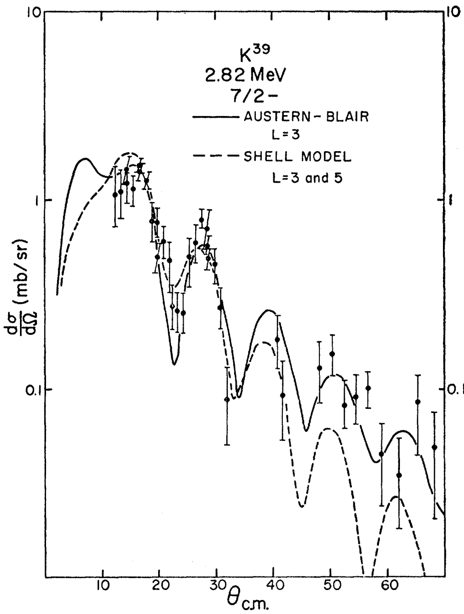

FIG. 4. The observed data for populating the 2.82-MeV state of K^{39} are compared with the predictions of the Austern-Blair collective model (with $\delta_3=0.19$ F) and with the DWBA shell-model prediction with a strength $V_G=58$ MeV. The shell-model prediction is a sum of $L=3$ and $L=5$ cross sections.

number, and it is to be noted that the normalization is very dependent upon the value of the oscillator length a, chosen to be equal to 2.00 F in the present series of experiments on nuclei between K^{39} and Fe^{54}. The results for V_G are valid as comparisons among the levels of any given nucleus, and the excitation of this 2.526-MeV state of K^{39} does seem to be anomalously weak.

The spin of the 2.526-MeV state certainly seems to be $\frac{1}{2}^+$, and the pickup reactions suggest that it consists mainly of a $2s_{1/2}$ hole in the ground state of Ca^{40}. The present work agrees with this for the most part, but the magnitude of the cross section is too small. Perhaps some small amount of collective quadrupole strength, maybe related to the 2^+ state of Ca^{40} at 3.90 MeV, is destructively interfering with this single-particle excitation.

The proton pickup reactions listed above weakly populate the 2.817-MeV state of K^{39}. The $Ca^{40}(d,He^3)$,[15,16] $Ca^{42}(p,\alpha)$,[17] and $Ca^{40}(t,\alpha)$ [18] angular distributions are consistent with the pickup of an $f_{7/2}$ proton. The results of the $Ar^{38}(p,\gamma)$ reaction indicate a spin not less than $\frac{7}{2}^-$,[21] while the $Ar^{36}(\alpha,p)$ reaction strongly populates this state,[22] as would be expected for stripping into the $f_{7/2}$ orbital. The $K^{39}(p,p')$ reaction is found to populate this state by an octupole transition,[25] while the (d,d') reaction indicates a negative-parity excitation.[23] If we normalize the relative (d,d') differential cross sections for the 3.60-MeV state to the present (α,α') cross sec-

[20] A. Sperduto and W. W. Buechner, Phys. Rev. **109**, 462 (1958).
[21] S. E. Arnell and K. C. Tripathe, Arkiv Fysik **26**, 485 (1964).
[22] R. B. Schwartz, J. W. Corbett, and W. W. Watson, Phys. Rev. **101**, 1370 (1956).
[23] E. W. Hamburger and L. H. Reber, Phys. Rev. **128**, 333 (1962).
[24] E. C. Booth and K. A. Wright, Nucl. Phys. **35**, 472 (1962).
[25] D. Newton, A. B. Clegg, and G. L. Salmon, Nucl. Phys. **73** 145 (1965).

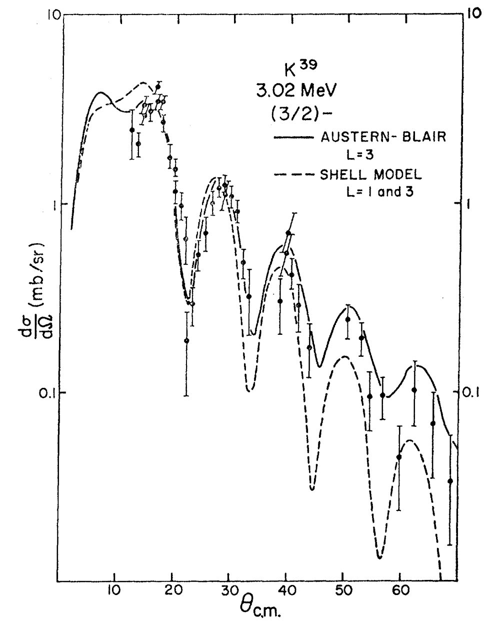

FIG. 5. The differential cross sections for the excitation of the 3.02-MeV state of K^{39} are compared with the collective octupole prediction of Austern and Blair (with $\delta_3=0.28$ F) and with the DWBA shell-model prediction with $V_G=94$ MeV. The shell-model prediction is a sum of $L=1$ and $L=3$ angular distributions.

tions for this state, the ratio of the (d,d') cross sections for the 2.82-MeV state provides a value of δ_3 equal to about 0.18 F.

The results of the present experiment for this 2.82-MeV state are seen in Fig. 4, where a comparison to the Austern-Blair prediction for a collective $L=3$ excitation is shown. A value of δ_3 equal to 0.19 ± 0.02 F is obtained from this fit, and a value of 0.29 F is obtained from the DWBA fit. These data are equally well fitted by the mixture of $L=3$ and $L=5$ curves predicted from the shell model for the excitation of the unpaired proton into the $f_{7/2}$ orbital. Once again we cannot choose one model over the other on the basis of the inadequate large-angle results. The projectile-proton strength determined by this fit is 58 MeV, a number in reasonable agreement with that obtained from the free proton–α-particle scattering.[6]

The spin of this 2.817-MeV state of K^{39} is determined to be $\frac{7}{2}^-$ by a variety of experiments, and the shell-model interpretation of this state as the $(d_{3/2})_0{}^2(f_{7/2})$ state is consistent with all of the reported data, including the present experiment.

The 3.021-MeV state of K^{39} has been tentatively assigned as due to the $(d_{3/2})_0{}^2(2p_{3/2})$ configuration.[15,16] A spin of $\frac{3}{2}^-$ is suggested from the results of the $Ar^{38}(p,\gamma)$ experiment,[21] but the $Ar^{36}(\alpha,p)$ stripping reaction did not populate this 3.02-MeV state strongly.[22] If this state were due to the $(d_{3/2})_0{}^2(2p_{3/2})$ configuration, it should be populated by this reaction. High-energy inelastic proton-scattering results indicate a state near 3.2 MeV with an octupole excitation,[25,26] while the inelastic deuteron-scattering experiment also indicated a negative parity,[23] with a value of δ_3 near 0.26 F from comparison to the 3.60-MeV cross sections. A shell-model interpretation of this inelastic scattering allows values of $L=1$ or $L=3$ (see Table I), so the above inelastic scattering reactions agree with the expected parity at least. The γ ray de-exciting this 3.021-MeV level proceeds directly to the ground state, consistent with a $\frac{3}{2}^-$ spin assignment.[27]

In Fig. 5 we show the differential cross sections measured for the scattering of 42-MeV α particles. Comparison to the Austern-Blair $L=3$ prediction determines a value of δ_3 equal to 0.28 ± 0.02 F, consistent with the results from deuteron scattering. In order to fit the sum of the $L=1$ and $L=3$ shell-model DWBA predictions, one requires a strength V_G of 94 MeV, about twice as large as expected. Moreover, the details of the predicted and observed angular distributions differ somewhat in the locations of the minima. The collective fit is slightly superior.

All of the scattering experiments indicate a negative parity for this 3.021-MeV state, but the assignment of spin $\frac{3}{2}$ is not completely firm, even from γ-ray correlation studies.[28] The assignment of the $(d_{3/2})_0{}^2(2p_{3/2})$ configuration for this state is not consistent with the present experiment. This configuration is mixed with

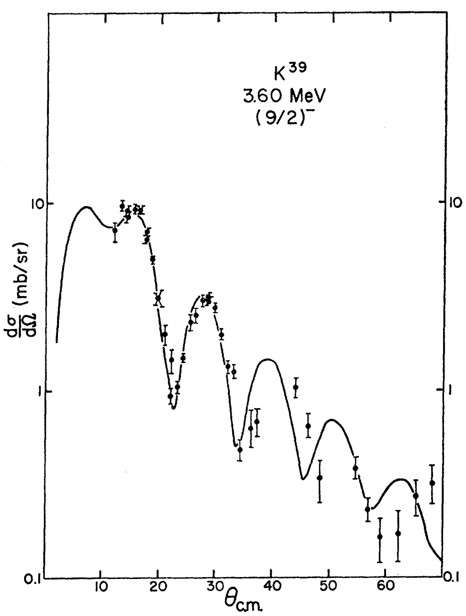

FIG. 6. The data for populating a state of K^{39} at 3.60 MeV are compared with the collective octupole prediction of Austern and Blair (with $\delta_3=0.45$ F). A spin assignment of $\frac{9}{2}^-$ is suggested from comparison to the weak-coupling model.

[26] J. C. Jacmart, M. Liu, R. A. Ricci, M. Rion, and C. Ruhla, Phys. Letters 8, 273 (1964).
[27] D. A. Lind and R. B. Day, Ann. Phys. (N. Y.) 12, 485 (1961).
[28] D. B. Reynolds and M. T. McEllistrem, Bull. Am. Phys. Soc. 11, 839 (1966).

another, perhaps the collective weak-coupling $\frac{3}{2}^-$ level to be discussed below.

The 3.603-MeV excited state of K^{39} is not reported from the $Ca^{40}(d,He^3)$ [15,16] or $Ca^{40}(p,2p)$ [13] reactions. In the inelastic scattering experiments, however, this is found to be the most strongly excited state. Hamburger and Reber reported this to be the strongest octupole-type transition seen in deuteron scattering from K^{39}.[23] This was also true for the high-energy (p,p') reaction.[25,26,29] A γ-ray branch from this state to the $\frac{7}{2}^-$ state at 2.82 MeV implies spins of $\frac{5}{2}$, $\frac{7}{2}$, or $\frac{9}{2}$ for this 3.603-MeV level,[25] while an angular-correlation study of the $(n,n'\gamma)$ reaction suggested a spin of $\frac{9}{2}$.[28]

In Fig. 6 we display the data from the present experiment and compare the results with the Austern-Blair prediction for an octupole transition. The fit is quite good. A value of δ_3 equal to 0.45 ± 0.02 F is found for this, the most strongly excited state seen in the present experiment; the DWBA fit provides a value of δ_3 equal to 0.72 F.

The parity of this 3.603-MeV state is certainly negative, and a pure $L=3$ transition is fully consistent with the data. The large value of δ_3 and the negative results of the pickup experiments indicate that this is a collective state, probably based on the well-known 3^- state at 3.73 MeV in Ca^{40}.[30,31] The spin is probably $\frac{9}{2}$, since this is the most strongly populated of the collective

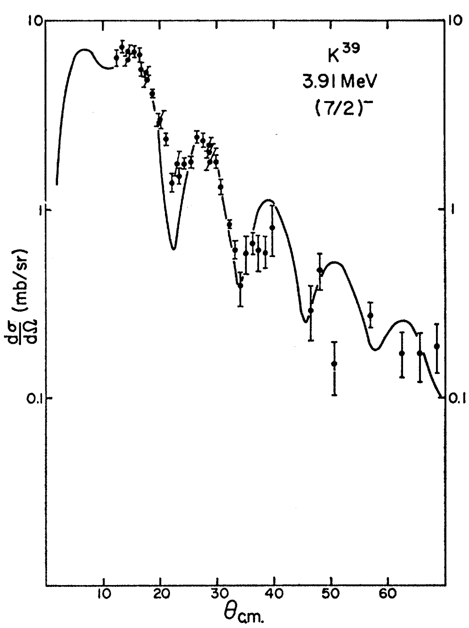

FIG. 7. The data for exciting a doublet in K^{39} at 3.91 MeV are compared with the collective octupole prediction of Austern and Blair (with $\delta_3=0.39$ F). One of the states is assigned a spin of $\frac{7}{2}^-$, while the other is expected to have positive parity.

[29] H. Tyren and Th. A. J. Maris, Nucl. Phys. **6**, 446 (1958).
[30] A. Springer and B. G. Harvey, Phys. Letters **14**, 116 (1965).
[31] E. P. Lippincott, Ph.D. thesis, Massachusetts Institute of Technology, 1966 (unpublished); E. P. Lippincott and A. M. Bernstein, Phys. Rev. **163**, 1170 (1967).

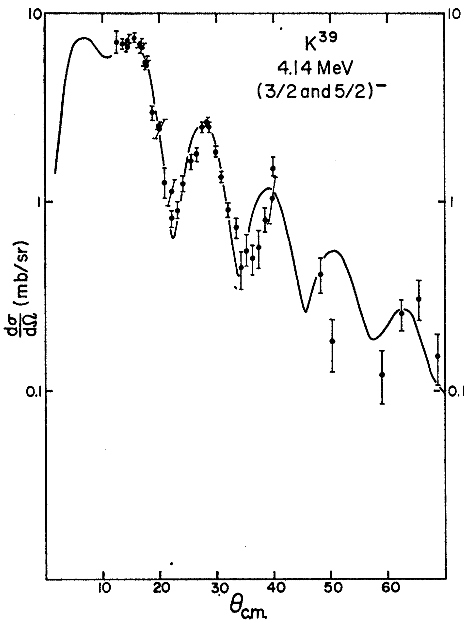

FIG. 8. The data for populating a triplet of states in K^{39} centered at 4.139 ± 0.014 MeV are compared with the collective octupole prediction of Austern and Blair (with $\delta_3=0.40$ F). It is suggested from comparison to the weak-coupling model that two of the states have spins $\frac{3}{2}^-$ and $\frac{5}{2}^-$.

states we see. Moreover, a spin of $\frac{9}{2}$ is possible from the γ decay branching[24,25] and is suggested by the $(n,n'\gamma)$ studies.[28]

One could make two possible weak preferences for a low-spin assignment, however. The first is from studies of the $(n,n'\gamma)$ reaction on potassium, where an increase in the intensity of the 2.52-MeV γ ray was observed at an incident neutron energy of 3.6 MeV.[27] This might indicate a decay branch from the 3.603-MeV state to the $\frac{1}{2}^+$ state at 2.526 MeV, suggesting a low spin. The 1.077-MeV γ ray is not seen, however. The second possible evidence for a low spin comes from the $Ar^{38}(p,\gamma)$ reaction, which proceeds through a $\frac{3}{2}^+$ capture state at 7.72 MeV. This state decays either to the 4.12-MeV state with a 3.60-MeV γ ray, and thence to the ground state, or it decays first to the 3.60-MeV state by emitting a 4.12-MeV γ ray.[21] In order to connect the $\frac{3}{2}^+$ capture and ground states, spins of $\frac{1}{2}$, $\frac{3}{2}$, or $\frac{5}{2}$ are needed for the intermediate state. We shall see below that it is more consistent for the 4.12-MeV state to be the intermediate level, hence still allowing a high spin for the 3.60-MeV state.

Two close states are found at 3.879 and 3.935 MeV.[20] The $Ca^{42}(p,\alpha)$ reaction is found to populate the 3.935-MeV level by an $l=3$ transition, looking much like the stronger transition to the 2.817-MeV $\frac{7}{2}^-$ state.[17] In the inelastic deuteron scattering an octupole transition was seen to this doublet, which could not be resolved.[23] A value of δ_3 equal to 0.30 F is obtained by comparison to the 3.60-MeV data.

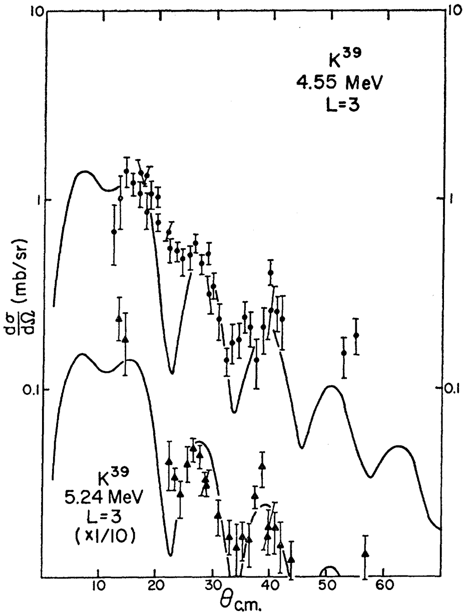

FIG. 9. The data for populating two states of K^{39}, at 4.55 $\pm$0.03 and 5.24$\pm$0.04 MeV, are compared with the collective octupole predictions of Austern and Blair (with values of δ_3 equal to 0.17 and 0.18 F, respectively). Some admixture of $L=5$ strength might improve the fit to the data for the 4.55-MeV state.

The present α-particle scattering experiment populates a peak centered at 3.911$\pm$0.018 MeV, noticeably wider than the 3.60-MeV peak. The angular distribution fits an $L=3$ curve quite well, as shown in Fig. 7, except for the shallow minima. The peak-to-valley ratio between the 15° and 23° data is only 5, whereas it is 10 for the 3.60- and 4.14-MeV angular distributions. A value of δ_3 equal to 0.39$\pm$0.02 F is obtained for the fit shown. Evidently, one member of the doublet is a strongly excited octupole state and the other has positive parity. The angular distribution to the latter is peaked where the $L=3$ data are at a minimum and hence tends to fill these minima.

A $\frac{7}{2}^-$ weak-coupling state would have cross sections 0.80 times those for the $\frac{9}{2}^-$ state. This 3.91-MeV state seems to meet this ratio. (See the summary of the weak-coupling collective interpretation below.) From the $Ca^{42}(p,\alpha)$ results and the present work we shall assign the 3.935-MeV state as the weak-coupling $\frac{7}{2}^-$ state.

There are three known close states at 4.078, 4.092, and 4.122 MeV.[20] From the $Ca^{40}(d,He^3)$ reaction, a state at 4.14 MeV was assigned to the $2s_{1/2}$ configuration,[15,16] while the $Ca^{42}(p,\alpha)$ reaction finds an angular distribution looking like $l=2$, suggesting a $d_{3/2}$ or $d_{5/2}$ component in the 4.122-MeV state.[17] The inelastic deuteron scattering to this triplet of levels found an octupole transition[23] with δ_3 equal to about 0.30 F, while the high-energy proton scattering to a peak at 4.0 MeV found a value of δ near 0.36 F.[25] This peak probably included both the 3.9-MeV doublet and the 4.1-MeV triplet. A γ decay branch to the 2.82-MeV $\frac{7}{2}^-$ state indicates a high-spin state somewhere among the five levels near 4.0 MeV.[25]

In Fig. 8 the differential cross sections for the scattering of 42-MeV α particles to a broad peak centered at 4.14$\pm$0.03 MeV are shown. The fit to the $L=3$ prediction of Austern and Blair is quite good, and, in particular, the minima are deep, indicating that no appreciable excitation of even-parity states is seen. A value of 0.40$\pm$0.02 F is found for the value of δ_3. Rather than discuss the weak-coupling interpretation of this state here, we shall proceed to a summary of the weak-coupling model for the 3.60-, 3.91-, and 4.14-MeV levels.

For a $d_{3/2}$ proton hole coupled to the 3^- vibrational state of Ca^{40}, we expect four states in K^{39}, with spins $\frac{9}{2}^-$, $\frac{7}{2}^-$, $\frac{5}{2}^-$, and $\frac{3}{2}^-$. There is some justification for assigning spins $\frac{9}{2}^-$ and $\frac{7}{2}^-$ to the 3.603- and 3.935-MeV states, respectively, as was discussed above. Let us suppose that the $\frac{5}{2}^-$ and $\frac{3}{2}^-$ members are among the three states near 4.1 MeV, the third perhaps being a very weakly excited positive-parity state, the existence of which is indicated from the proton pickup reactions.[17] The ratios of the cross sections to the weak-coupling states are predicted to be

$$\tfrac{9}{2}:\tfrac{7}{2}:\tfrac{5}{2}:\tfrac{3}{2}=10:8:6:4\,,$$

while, if the $\frac{5}{2}^-$ and $\frac{3}{2}^-$ states are unresolved, these become

$$\tfrac{9}{2}:\tfrac{7}{2}:\tfrac{5}{2}+\tfrac{3}{2}=10:8:10\,.$$

The observed ratios with the spins assumed above are

$$\tfrac{9}{2}:\tfrac{7}{2}:\tfrac{5}{2}+\tfrac{3}{2}=10:7.5:8.0\,,$$

quite close to the expected results. The center of gravity of these states is 3.87 MeV, compared to the octupole excitation energy of 3.73 MeV in the Ca^{40} core. If all of the differential cross sections to these three peaks are added, we obtain a value of δ_3 equal to 0.70$\pm$0.03 F from comparison to the adiabatic prediction of Austern and Blair. Furthermore, we note that the three individual angular distributions and the summed data have very similar shapes, agreeing better among themselves than they do with the predicted curve or with the data for populating the 2.82- and 3.02-MeV states. No data for the scattering of 42-MeV α particles to the 3.73-MeV 3^- state of Ca^{40} are available to compare with the shape and strength of the present angular distributions. We shall rely on the results of Springer and Harvey,[30] who scattered 51-MeV α particles from Ca^{40}, obtaining a value of 0.85 F for the strength δ_3 associated with the 3.73-MeV transition. The Austern-Blair description of the scattering was used, so we may compare this δ_3 value to the present results. In Sec. VI we shall also compare our DWBA results with those of Lippincott and Bernstein.[31] The weak-coupling interpretation comes close to agreeing with the data, but some difficulties are seen. The δ_3 strength for the summed data is 17% lower than the result of Springer and Harvey for Ca^{40}. The ratios of cross sections that we measure here for the proposed spin assignments in K^{39} do not agree in detail with the predictions of the weak-coupling model. Furthermore, the center of gravity is 0.14 MeV higher than we might expect. The weak-coupling model works about as well here as it ever does, but it does not contain the complete description. Section VI below will contain some possible improvements to the model.

Several states above these "weak-coupling" states provided angular distributions that were consistent and meaningful. However, the fits to these data are not good, and no spin or model assignments will be attempted.

At 4.55±0.03 MeV we excite a state with the angular distribution shown in Fig. 9. A negative parity is indicated by the observed oscillations, and, for comparison, an Austern-Blair $L=3$ curve is fitted to the data. Perhaps some admixture of an $L=5$ curve would help the small-angle fit, especially the lack of a sharp minimum near 22°. A value of δ_3 equal to 0.17±0.02 F is measured. Several states are known near here, at 4.472, 4.511, and 4.678 MeV,[20] and while we are presumably exciting mainly the 4.511-MeV state, the others may be contributing to the data.

We find two states of K^{39} at 5.24±0.04 and 5.36 ±0.04 MeV. The data for populating these states are shown in Figs. 9 and 10, where comparisons are made to the $L=3$ prediction of Austern and Blair. The fit to the 5.24-MeV data is fair, providing a value of δ_3 equal to 0.18±0.02 F. The fit to the 5.36-MeV data is far from satisfactory, but the fit and the value of δ_3 equal to 0.19±0.03 F will be commented on below. To ensure against confusion between these two close peaks, the sum of their cross sections was plotted and a good fit to an $L=3$ curve was found for the summed data.

If the 5.24-MeV state is populated by an $L=3$ transition, its spin is limited to $\frac{3}{2}^-$, $\frac{5}{2}^-$, $\frac{7}{2}^-$, or $\frac{9}{2}^-$, hardly a stringent restriction. For the 5.36-MeV state, however, we can compare the present results to those of Hiebert *et al.*,[15,16] who investigated the $Ca^{40}(d,He^3)$ reaction. From these results, it was claimed that much of the $1d_{5/2}$ hole strength of K^{39} was found at 5.32 MeV. The appropriate mixture of $L=2$ and $L=4$ angular distributions calculated with the shell-model form factors was compared to the α-particle scattering data in order to see if a fit could be obtained. The fit was quite poor, much inferior to that of the $L=3$ prediction, and the required strength V_G was only 22 MeV. A $\frac{5}{2}^+$ state here would have been populated more strongly than the state actually seen, and the parity assignment is contrary to that expected.

Beyond this energy, the background of weakly excited states begins to increase. One fairly strongly excited state was seen at 6.31±0.03 MeV. In Fig. 10 the data for scattering to this state are compared to the $L=3$ Austern-Blair prediction. The fit is adequate, with the parity being definitely negative. A value of δ_3 equal to 0.24±0.02 F is measured. Some octupole strength might indeed be expected near here, since a group of three octupole states is found in Ca^{40} with a center of gravity at 6.34 MeV.[30,31] Possibly this 6.31-MeV state of K^{39} is related to this second region of octupole strength in Ca^{40}.

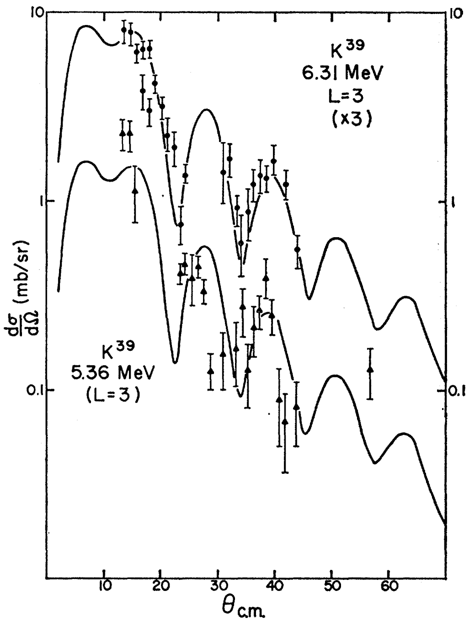

FIG. 10. The data for exciting two states of K^{39}, at 5.36±0.04 and 6.31±0.03 MeV, are compared with the collective octupole prediction of Austern and Blair (with values of δ_3 equal to 0.19 and 0.24 F, respectively).

The present experiment also revealed states of K^{39} at 5.54, 5.69, 5.92, 6.59, and 7.15 MeV, but no angular distributions were obtained and no comments will be made on these levels.

B. Results for V^{51}

Thirty-one levels of V^{51} are known below 4 MeV.[32] The spectrum of Fig. 16 shows that only the first few of these states are resolved in the present experiment. The spins and parities of some of these states are known, and it is also known that several of these states seem to be described by the proton $(f_{7/2})^3$ configuration.[33]

In the present section we shall compare the inelastic α-particle scattering data to the shell-model predictions and to the predictions of the weak-coupling vibrational model. Since the spin and parity assignments already exist for V^{51}, we shall use a somewhat different approach from that used for K^{39}.

The best average fit of the $L=2$ Austern-Blair prediction to the data for populating the $\frac{3}{2}^-$, $\frac{5}{2}^-$, $\frac{9}{2}^-$, and $\frac{11}{2}^-$ levels provided a collective quadrupole strength δ_2

[32] M. Mazari, W. W. Buechner, and A. Sperduto, Phys. Rev. **112**, 1691 (1958).
[33] J. Vervier, Phys. Letters **13**, 47 (1964).

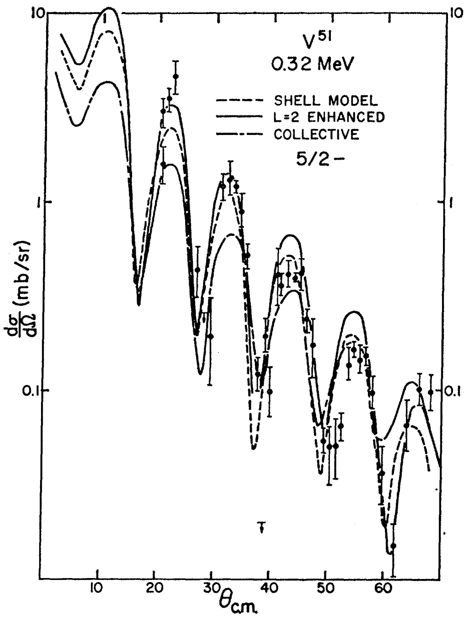

FIG. 11. The observed differential cross sections for exciting the 0.32-MeV ($\frac{5}{2}^-$) state of V^{51} are compared with the predictions of three models. The prediction of the shell model with good seniority is shown, with a strength V_G equal to 82 MeV for all four of the low excited states of V^{51}. The prediction with the $L=2$ component enhanced by a factor of 4 is shown, using a common strength equal to 48 MeV. The collective quadrupole weak-coupling prediction of Austern and Blair is also shown, with a value of δ_2 equal to 0.63 F.

equal to 0.63 F with a variance of 0.20 F. The weak-coupling prediction with the value of δ_2 is shown in Figs. 11–14, where the data are also exhibited. The fits of this prediction are adequate for the 0.93-MeV ($\frac{3}{2}^-$) and 1.61-MeV ($\frac{11}{2}^-$) data but are very poor for the 0.32-MeV ($\frac{5}{2}^-$) and 1.81-MeV ($\frac{9}{2}^-$) data. The average value of δ_2 is quite close to the Austern-Blair value of 0.61 F measured for the 1.43-MeV state of Cr^{52}.[12,34] An excited state with spin $\frac{7}{2}^-$ is also expected in the weak-coupling model. The 2.41-MeV state might be a candidate for this level, but the weak-coupling prediction is more than a factor of 2 higher than the data. In Fig. 16 we show the fit with a value of δ_2 equal to 0.18 F from the Austern-Blair fit and 0.31 F from the DWBA fit.

For states of the $(f_{7/2})^3$ proton configuration, the predicted angular distributions are sums of $L=2$, $L=4$, and $L=6$ contributions, with the angular matrix elements found in Ref. 2. The large amount of $L=4$ excitation for the $\frac{9}{2}^-$ state is in strong contrast to the weak-coupling prediction of a pure $L=2$ transition. Again an average fit to the first four states was used to determine an average strength, and a value of V_G equal to 82 MeV was found, with a variance of 5 MeV. The average fits derived from the shell model are shown in Figs. 11–14.

[34] J. R. Meriwether, I. Gabrielli, D. L. Hendrie, J. Mahoney, and B. G. Harvey, Phys. Rev. 146, 804 (1966).

Before the fits to the data are discussed in detail, one note of caution is necessary. The parameters used in the development of the inelastic scattering predictions are derived from the analysis of the elastic scattering.[11] For every low-lying state of V^{51} it is observed that the data decrease more rapidly with angle than does the Austern-Blair prediction. The parametrized partial-wave analysis fits the elastic scattering data well at back angles,[11] but the inelastic curves generated from the same set of parameters fail to follow the envelope of the observed oscillations. The optical-model fit to the elastic scattering data is only slightly below the parametrized partial-wave fit at large angles, and yet the DWBA predictions for the inelastic scattering are found to damp much faster, giving better agreement with the data. The inelastic data for V^{51} and Cr^{52} have the same envelope for their oscillations (see Fig. 17), demonstrating that the poor fit is anomalous, and probably due to the elastic scattering parameters. The fits at large angles are then much less relevant to our discussion than are the small-angle fits.

The general nature of the shell-model fits is not worse than the collective fits, and, in particular, we find better agreement with the observed ratio of magnitudes with the former model. It should be emphasized that only one free parameter giving the over-all magnitude is available in the fit to the four states. In Sec. VI an empirical enhancement of the $L=2$ components of the prediction will be seen to improve the shell-model fits a great deal.

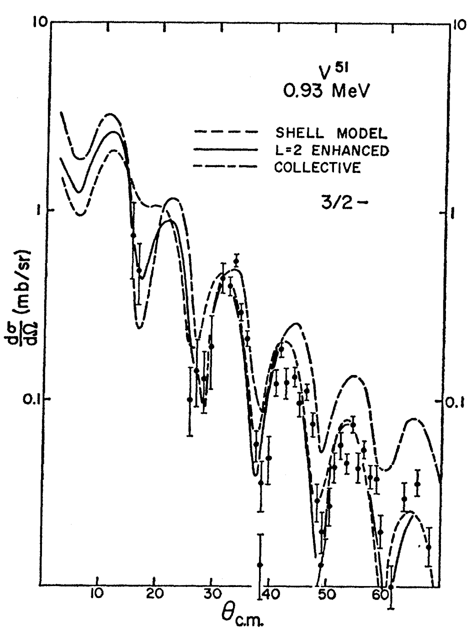

FIG. 12. The data for populating the 0.93-MeV ($\frac{3}{2}^-$) state of V^{51} are compared with the same three predictions as were described in the caption for Fig. 11. The same strength parameters were used for the predictions.

Nevertheless, the simple shell model does explain some obvious features of the data. In particular, the lack of a sharp minimum near 17° in the 1.81-MeV data is in agreement with what we expect for the $L=4$ component in the excitation of this state. Inelastic scattering events to the first excited state of Si^{28} can occur in the detector. These would make a peak near 1.8 MeV that would increase at small angles as the elastic scattering increases. This effect would fill in the 17° minimum for the 1.81-MeV state. This Si^{28} peak was not seen in experiments on even-even nuclei, where this region of the spectrum could be seen, and is presumably negligible in the present case.

The proton–α-particle interaction strength found for the states of V^{51} is about a factor of 2 larger than expected.[6] This is consistent with the results of α-particle scattering experiments to the first 2^+ states of the even nuclei with 20 protons or 28 neutrons.[12]

The shell-model $\frac{15}{2}^-$ state has been reported at 2.70 MeV.[33] The cross sections for a very small peak near this energy were smaller by a factor of 4 than the shell-model prediction with V_G equal to 84 MeV. The shape of the angular distribution was not in agreement with the expected sum of $L=4$ and $L=6$ curves.

Coulomb-excitation experiments have been used to determine the electromagnetic reduced matrix elements for the first four states of V^{51}.[35] We may use the collective DWBA best fits to the angular distributions to these states and use the formula of Sec. IV to obtain corresponding values. The Coulomb-excitation results

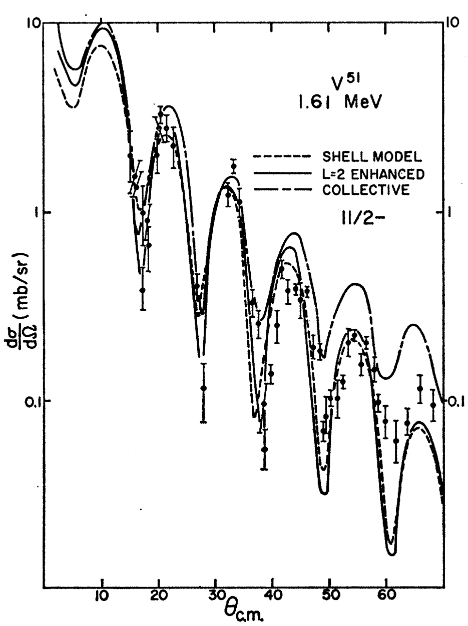

FIG. 13. The data for populating the 1.61-MeV ($\frac{11}{2}^-$) state of V^{51} are compared with the same three predictions as were described in the caption for Fig. 11. The same strength parameters were used for the predictions.

[35] I. Talmi, Phys. Letters **25B**, 313 (1967).

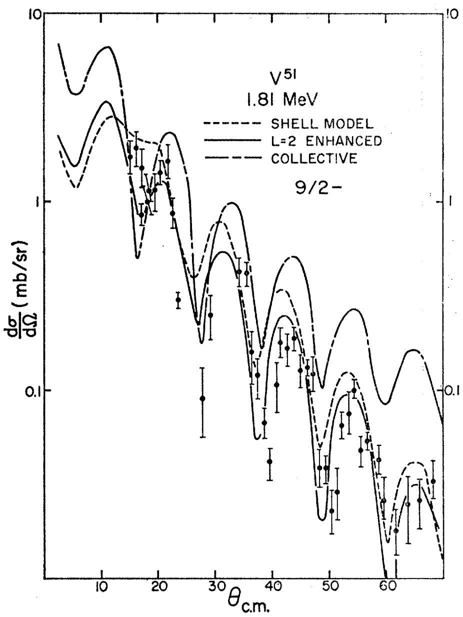

FIG. 14. The data for populating the 1.81-MeV ($\frac{9}{2}^-$) state of V^{51} are compared with the same three predictions as were described in the caption for Fig. 11. The same strength parameters were used for the predictions. Note the differences in the small-angle data and in the predictions for the 1.61- and 1.81-MeV states.

(in units of e^2F^4) are

$$\tfrac{3}{2}:\tfrac{5}{2}:\tfrac{9}{2}:\tfrac{11}{2}=27:92:22:90,$$

while the present results are

$$\tfrac{3}{2}:\tfrac{5}{2}:\tfrac{9}{2}:\tfrac{11}{2}=37:87:70:111.$$

The values calculated by Talmi, using the shell-model formalism with an effective charge of $1.61e$, are[35]

$$\tfrac{3}{2}:\tfrac{5}{2}:\tfrac{9}{2}:\tfrac{11}{2}=22:96:27:86.$$

The predictions of the weak-coupling collective model, normalized to $27e^2F^4$ for the excitation of the $\frac{3}{2}^-$ state, are

$$\tfrac{3}{2}:\tfrac{5}{2}:\tfrac{9}{2}:\tfrac{11}{2}=27:33:55:66.$$

Only the present results for the $\frac{9}{2}^-$ state do not agree with the Coulomb-excitation results. The excitation of the $\frac{9}{2}^-$ state proceeds with the largest contribution from $L=4$, which is not included in the Coulomb-excitation experiment or Talmi's calculation. The inelastic proton scattering results of Funsten, Roberson, and Rost[2] provide ratios of cross sections. When normalized to the value of $27e^2F^4$ for the $\frac{3}{2}^-$ state, the $B(E2)$'s from this work would be

$$\tfrac{3}{2}:\tfrac{5}{2}:\tfrac{9}{2}:\tfrac{11}{2}=27:102:40:75.$$

Again the $\frac{9}{2}^-$ state is too strong, presumably because of contributions from the $L=4$ terms.

The data of Meriwether *et al.*[34] were analyzed with the Austern-Blair model only and cannot be expected

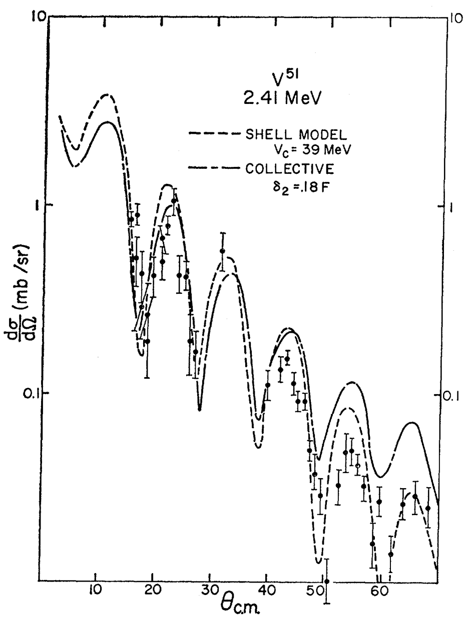

FIG. 15. The data for exciting the 2.41-MeV state of V^{51} are compared with the predictions for the promotion of a proton into the $2p_{3/2}$ orbital (with $V_G=39$ MeV) and with the collective quadrupole prediction of Austern and Blair.

to agree in detail with the present results. The values of $B(E2)\uparrow$ calculated from the results of Meriwether *et al.* are

$$\tfrac{3}{2}:\tfrac{5}{2}:\tfrac{9}{2}:\tfrac{11}{2}=19:50:20:50.$$

The 2.41-MeV state was rejected above as the $\frac{7}{2}^-$ member of the collective weak-coupling quintet of states. The (α,p) proton stripping reaction on Ti^{48} gave a large cross section to this 2.41-MeV state of V^{51} and an angular distribution consistent with the stripping of a $p_{3/2}$ proton.[36] An assignment of the configuration $(f_{7/2})_0{}^2(p_{3/2})$ for this state was suggested by γ-ray studies as well.[37] Accordingly, in Fig. 15 we show the shell-model prediction for this state compared to the data. The strength of the interaction required to fit the magnitude of this cross section is 39 ± 4 MeV, about as expected for a pure shell-model excitation. The shape of the fit is quite good, except for the expected slower falloff in the envelope of the oscillations. This experiment is consistent with a pure shell-model excitation of this state. The collective-model DWBA fit to this angular distribution gives a value of βR equal to 0.31, which corresponds to a $B(E2)\uparrow$ of $64e^2F^4$.

Beyond 2.41 MeV it was possible to analyze only two more states. The data for populating states of V^{51} at 3.32 ± 0.03 and 3.47 ± 0.04 MeV are shown in Fig. 16, where the comparison curve is the Austern-Blair $L=2$ prediction. The values of δ_2 are 0.25 and 0.27 F, respectively. It may also be noted that this collective prediction fits all of the observed maxima, in contrast to what was observed for the lower states. No statements of spin or model assignment are possible, but these states are close, in energy and transition strength, to the second 2^+ states commonly found in the neighboring even nuclei.

High-resolution studies of the inelastic scattering of 17.5-MeV protons from V^{51} show that many states beyond 3 MeV are excited quite strongly, but no meaningful angular distributions were obtained from the present work, even when peaks could be clearly seen above the background.

The data for the states of K^{39} and V^{51} have now been compared to the predictions for simple shell-model excitations and for collective vibrational excitations. In the next section some obvious empirical improvements to the models will be made and a better understanding of the data will perhaps be possible.

VI. REFINEMENTS TO MODELS

The octupole state of Ca^{40} at 3.73 MeV is formed from promotions of nucleons from the $2s$-$1d$ shell into the $1f$-$2p$ shell. Promotions out of the $d_{3/2}$ orbital are the most important.[38] In K^{39}, however, only seven nucleons are present in this orbital, and the matrix elements for an octupole promotion would be smaller by a factor of $\sqrt{\frac{7}{8}}$. The present Austern-Blair result of $\delta_3=0.70\pm0.03$ F for the summed weak-coupling cross sections, then, should be compared to 0.79 F instead of the result 0.85 F that was found for Ca^{40}.[30]

In Sec. V the states of K^{39} were labeled as either "shell-model" or "collective" states. In general, these states will be mixed, and we might better compare the total octupole transition strength in K^{39} to the result for Ca^{40}. For the sum of all of the cross sections discussed in the previous section except those for the 2.56- and 6.31-MeV levels we obtain (with the Austern-Blair theory) a value of δ_3 equal to 0.85 F. The eight contributing states have a center of gravity at 3.87 MeV, the same as was found for the original quartet of collective levels. The value of δ_3 alone is smaller than this, since the $L=1$ and $L=5$ components of the negative-parity excitations also contribute to the observed cross sections. Indeed, we note that the factor of $\frac{7}{8}$ times the sum of the strengths for the well-known states at 3.73 MeV (3^-) and 4.48 MeV (5^-) [30] in Ca^{40} gives a value of δ equal to 0.85 F, and that the center of gravity of these two states of Ca^{40} is at 3.84 MeV. The sum of the $\delta_3{}^2$ values obtained with the DWBA fits to the K^{39} data is equal to 1.88 F^2. The DWBA analysis of the scattering of 30-MeV α particles to the 3.73-MeV (3^-) and 4.48-MeV (5^-) states of Ca^{40} [31] provides values of $\delta_3{}^2$ equal to 1.85 F^2 and $\delta_5{}^2$ equal to 0.54 F^2. Approximately 80% of the Ca^{40} negative-parity strength is accounted for with this comparison.

[36] R. O. Ginaven, Ph.D. thesis, Massachusetts Institute of Technology, 1966 (unpublished).
[37] J. E. Schwäger, Phys. Rev. **121**, 569 (1961).
[38] V. Gillet and E. A. Sanderson, Nucl. Phys. **54**, 472 (1964).

The scattering of the collective octupole strength is not large and the present experiment seems to account for most of it. The mixing of the "collective" and "shell-model" states is not strong, but the weak-coupling picture is not complete.

The value of the collective octupole strength parameter δ_3 thus seems to be in reasonable agreement with the expected value. The differences between the expected and observed ratios among the weak-coupling states may be due to the mixing with the other states present. If we add the cross sections to the 2.82-MeV ($\frac{7}{2}^-$) state to those for exciting the state at 3.90 MeV, and the 3.02-MeV ($\frac{3}{2}^-$) cross sections to those for exciting the 4.14-MeV peak, we obtain the ratios

$$\tfrac{9}{2}:\tfrac{7}{2}:\tfrac{5}{2}+\tfrac{3}{2}=10.0:9.30:12.0.$$

We now have too much excitation of the $\frac{7}{2}$ and $\frac{3}{2}^-$ states, but again we are including any $L=1$ and $L=5$ components; there also remains an unknown contribution from the extra state under both the 3.90- and 4.14-MeV peaks. The two methods of evaluating the ratios of the weak-coupling cross sections bracket the expected result. A calculation of the mixing of the collective and shell-model configurations would be needed before further comments could be made on these results for K^{39}.[39]

In V^{51} the cross sections to the low-lying states are about a factor of 4 larger than the shell-model prediction. For the neighboring even nuclei, the first 2^+ state is also found to be excited four times as strongly as is predicted, but the first 4^+ state has approximately a shell-model cross section.[12] Accordingly, in V^{51} we shall then enhance the $L=2$ cross sections by a factor of 4 and leave the $L=4$ and $L=6$ components unchanged. In other words, we shall assume that the configuration mixing enhances the $L=2$ matrix elements to the same extent as is seen in the even nuclei.

This empirical enhancement of quadrupole shell-model transitions is familiar from observations of electromagnetic transitions and is ascribed to core polarization effects. Talmi[35] has used an effective charge equal to $1.61e$ to fit the Coulomb-excitation results for V^{51}. This corresponds to an enhancement by a factor of 2.6.

In Figs. 11–14 we show the resulting predictions compared to the data. A value of V_G equal to 48 MeV is used to fit the magnitudes of the data. The relative magnitudes of the fits are now better than the pure shell-model fits, and, moreover, the fits to the small-angle data for the 1.61- and 1.81-MeV states are improved. By enhancing the matrix element, the requisite strength of the proton–α-particle interaction is found to be reasonable. This value of V_G also gives a prediction that agrees with the magnitude of angular distribution to the 2.70-MeV ($\frac{15}{2}^-$) state.

[39] V. K. Thankapan and W. W. True, Phys. Rev. **137**, B793 (1965).

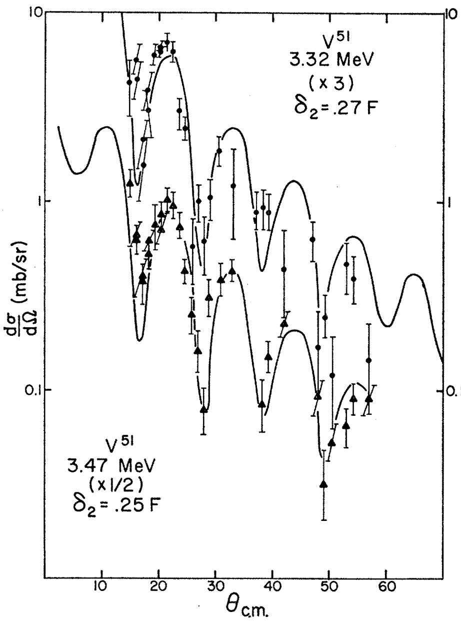

FIG. 16. The data for exciting two states of V^{51}, at 3.32 and 3.47 MeV, are compared with the collective quadrupole predictions of Austern and Blair.

Another way to present this empirical enhancement of the $L=2$ component in the cross sections for the low-lying states of V^{51} is to compare the sum of the cross sections to the 0.32-, 0.93-, 1.61-, and 1.81-MeV states of V^{51} to the sum of the cross sections to the states of Cr^{52} presumed to be due to the $(f_{7/2})^4$ configuration with seniority two. We shall add the cross sections to the 1.43-MeV (2^+), 2.37-MeV (4^+), and 2.77-MeV (4^+) states of Cr^{52}. The two 4^+ states share the $V=2$ 4^+ strength, while the cross section to the 6^+ state at 3.12 MeV is very small.[12] In Fig. 17 we compare the two sums. We find almost point-by-point agreement between the shapes of the two curves and a factor of 1.5 ± 0.15 between their magnitudes. The shell model with good seniority predicts a factor of 1.5 between these sums for the $L=2$ cross sections and a factor of 1.92 for the $L=4$ cross section, while any promotions into the $p_{3/2}$, $p_{1/2}$, or $f_{5/2}$ orbitals would give cross sections that scale as 1.33.

We might also compare the cross sections to the 2.41-MeV ($\frac{3}{2}^-$) state of V^{51} to the sum of the $L=2$ and $L=4$ members of the $(f_{7/2})_0{}^3(2p_{3/2})$ configuration in Cr^{52}. The 3.78-MeV (2^+) and 4.05-MeV (4^+) states seem to contain much of the strength of this configuration seen in the $Cr^{53}(p,d)Cr^{52}$ reaction.[40] We find that the V^{51} cross sections drop more rapidly with increasing angles than do the Cr^{52} summed cross sections. Also, the Cr^{52} sum is about 2.5 times as large as the V^{51} cross sections at small angles, compared to the expected factor of 1.33. Since no enhancement of the single-particle cross sec-

[40] C. A. Whitten, Ph.D. thesis, Princeton University, 1966 (unpublished).

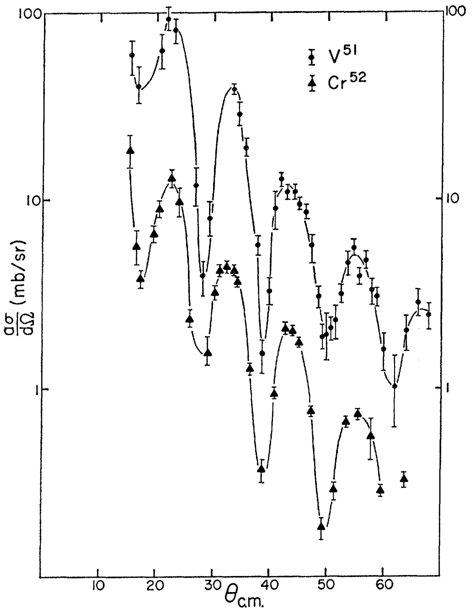

FIG. 17. The sums of the cross sections to the 0.32-, 0.93-, 1.61-, and 1.81-MeV states of V^{51} are compared with the sums of the cross sections to the 1.43-, 2.37-, and 2.77-MeV states of Cr^{52}. The lines serve only to connect the data points.

TABLE III. Results of the present experiment are summarized in this table. The excitation energy, spin (where available), and collective strength parameter δ_L are shown for each observed level. In K^{39} these δ's measure an octupole strength for all but the 2.53-MeV level, while all values of δ for V^{51} refer to quadrupole excitations. Where they are relevant, the shell-model strength parameters V_G are also exhibited.

	E (MeV)	I^π	δ_L Austern-Blair	δ_L DWBA	V_G (MeV)
K^{39}	2.53	$\frac{1}{2}^+$	0.14	0.19	27±5
	2.82	$\frac{7}{2}^-$	0.19	0.29	58±6
	3.02	$(\frac{3}{2})^-$	0.28	0.44	94±9
	3.60	$\frac{9}{2}^-$	0.45	0.72	
	3.91	$(\frac{7}{2})^-$	0.39	0.63	
	4.14	$(\frac{3}{2}^-$ and $\frac{5}{2}^-)$	0.40	0.65	
	4.55	$(\ )^-$	0.17	0.27	
	5.24	$(\ )^-$	0.18	0.30	
	5.36	$(\ )^-$	0.19	0.30	
	6.31	$(\ )^-$	0.24	0.36	
V^{51}	0.32	$\frac{5}{2}^-$	0.28	0.49	82
	0.93	$\frac{3}{2}^-$	0.18	0.32	82
	1.61	$\frac{11}{2}^-$	0.30	0.55	82
	1.81	$\frac{9}{2}^-$	0.26	0.44	82
	2.41	$\frac{3}{2}^-$	0.18	0.31	39±4
	3.32	$(\ )^-$	0.27	0.47	
	3.47	$(\ )^-$	0.25	0.44	

tions was needed for the 2.41-MeV state of V^{51} while the 3.78-MeV (2^+) state of Cr^{52} is anomalously strongly excited,[12] this is not surprising.

VII. SUMMARY AND CONCLUSIONS

Two rather extreme models of nuclear structure have been applied to the results of a study of the scattering of 42-MeV α particles from the odd nuclei K^{39} and V^{51}. The shell-model predictions agreed quite well with the data for exciting the 2.86-MeV ($\frac{7}{2}^-$) state of K^{39} but overestimated the magnitude of the cross section to the 2.53-MeV ($\frac{1}{2}^+$) state and underestimated the data for the 3.02-MeV ($\frac{3}{2}^+$) state. These three states were expected on other grounds to have a single-particle nature. The weak-coupling core-excitation prediction based on the octupole vibration of Ca^{40} gave consistent results for a further set of states in K^{39}. Tentative spins as well as model assignments were made for these states.

The quadrupole vibration weak-coupling prediction for the low-lying states of V^{51} did not agree at all well with the data, while the shell-model prediction gave magnitudes four times smaller than the data but did give approximately the correct ratio of cross sections.

A simple first-order correction to the weak-coupling model for K^{39} gave an improvement in the comparison of the strengths δ_3 between K^{39} and Ca^{40}, while adding up all of the possible octupole strength even overcompensated the difference between Ca^{40} and its daughter K^{39}. We conclude that even the uncorrected weak-coupling model works about as well as it ever does (with the spin assignments made with the aid of this model) and that the obvious correction improves the comparison between the model and the data.

For V^{51} no such simple correction was possible. The $L=2$ component of the shell-model prediction was multiplied by a factor of 4 as suggested from the analysis of the neighboring even nuclei, and a good fit to the data was obtained with this effective quadrupole transition operator. The cross sections to the 2.41-MeV state of V^{51} are found to be consistent with the interpretation of this state as an $(f_{7/2})_0{}^2(2p_{3/2})$ shell-model state.

In Table III, the results of the present work are summarized.

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