Capture and Elastic Scattering of Protons by ^{14}C†

JERRY D. HENDERSON,* EMMETT L. HUDSPETH, AND WILLIAM R. SMITH‡
Department of Physics, University of Texas, Austin, Texas 78712
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The reactions $^{14}C(p,\gamma)^{15}N$ and $^{14}C(p,p)^{14}C$ have been studied in the energy region $0.6<E_p<2.7$ MeV. Particular attention was directed to a level which is produced at $E_p=2.49$ MeV ($E_x=12.54$ MeV); this level is distinct from a frequently observed level at $E_x=12.51$ MeV. It is concluded on the basis of a dispersion-theory fit to the scattering data that the 12.54-MeV level has $T=\frac{3}{2}$, $J=\frac{5}{2}^+$, and that it is the analog of the first excited state in ^{15}C.

I. INTRODUCTION

THE $^{14}C(p,\gamma)^{15}N$ reaction in the region $0.3<E_p<2.3$ MeV has been studied by Bartholomew *et al.*[1] Of particular interest was a broad resonance at $E_p=1.5$ MeV ($E_x=11.61$ MeV in ^{15}N). This is a single-particle proton state of $J=\frac{1}{2}$ and is interpreted[2] as the isobaric analog of the ground state of ^{15}C and hence the first $T=\frac{3}{2}$ level of ^{15}N.

Further studies[3] of the $^{14}C(p,\gamma)^{15}N$ and $^{14}C(p,p)^{14}C$ reactions showed a previously unreported level at a bombarding energy near $E_p=2.5$ MeV, with a rather large proton reduced width and negligible neutron or α emission. Analysis indicated a D-state assignment for this level, and it appeared likely that this was the isobaric analog of the first excited state in ^{15}C, i.e., the second $T=\frac{3}{2}$ state for that isobar. The elastic scattering of protons by ^{14}C has also been reported by Harris and Armstrong.[4] They observed six anomalies in the region $1.0<E_p<2.7$ MeV and analyzed their data to obtain level assignments.

Additional experimental investigations of $^{14}C(p,\gamma)^{15}N$ and $^{14}C(p,p)^{14}C$ have now been completed in our laboratory and a dispersion-theory fit to the relative cross sections has been used to determine resonance parameters. We have placed particular emphasis on characterizing the state of ^{15}N which is formed near $E_p=2.5$ MeV and in establishing its identity as the second $T=\frac{3}{2}$ state of that nucleus.

II. EXPERIMENTAL DETAILS

Accelerator

The proton beam used in our studies was produced by the 3.5-MeV Van de Graaff generator at Balcones Research Center of the University of Texas. A 90° analyzing magnet and a suitable beam collimation system permit energy calibrations of ±0.02% relative to a known threshold; the maximum resolution obtainable under our experimental conditions is calculated to be 0.4%.

Targets

The isotope ^{14}C was obtained in the form of $BaCO_3$, with enrichments of 33 and 55%, from the Atomic Energy Commission, Oak Ridge, Tenn. The $BaCO_3$ was converted to acetylene by the New England Nuclear Corp., Boston, Mass.

Carbon targets for the study of $^{14}C(p,\gamma)^{15}N$ were prepared in our laboratory by polymerization of acetylene onto 0.010-in. tantalum backings by means of a high-frequency discharge, after the method described by Douglas *et al.*[5] The particular enriched target used in this phase of our work had a density of approximately 70 $\mu g/cm^2$, or a thickness of 10 keV for protons at 2 MeV. Target thickness was checked through the $^{13}C(p,\gamma)^{14}N$ reaction, which exhibits a very prominent resonance at a proton energy of 1.747 MeV and which has a natural width of only $\simeq$75 eV. The target thickness calculated from examination of the experimental width of this resonance agreed to within 15% of the thickness determined by weight measurements.

For elastic scattering studies of $^{14}C(p,p)^{14}C$, very thin self-supporting carbon foils were made, using a method described by Kashy *et al.*[6] Targets used in the scattering work were 55% enriched in ^{14}C and were approximately 30 $\mu g/cm^2$ in thickness.

Detectors

The γ-ray detector was a 3×3-in. NaI crystal. The γ-ray data for $^{14}C(p,\gamma)^{15}N$ were taken at 90° with respect to the bombarding beam.

Data on the elastic scattering of protons by ^{14}C were obtained by placing a self-supporting target in a 23-in. scattering chamber in which a diffused-junction silicon detector was mounted. The resolution of the counter was approximately 18 keV. This resolution allowed

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* Present address: Texas Nuclear Corporation, Austin, Tex.
‡ Present address: Physics Department, Trinity University, San Antonio, Tex.
[1] G. A. Bartholomew, F. Brown, H. E. Gove, A. E. Litherland, and E. F. Paul, Can. J. Phys. 33, 441 (1955).
[2] G. A. Bartholomew, A. E. Litherland, E. B. Paul, and H. E. Gove, Can. J. Phys. 34, 147 (1956).
[3] J. D. Henderson, Bull. Am. Phys. Soc. 6, 236 (1961); also abstract of more recent data by the present authors in *Proceedings of the International Nuclear Physics Conference* (Academic Press Inc., New York, 1967), p. 284.
[4] W. R. Harris and J. C. Armstrong, Bull. Am. Phys. Soc. 12, 516 (1967).
[5] R. A. Douglas, B. R. Gasten, and Ambuj Mukerji, Can. J. Phys. 34, 1097 (1956).
[6] E. Kashy, R. R. Perry, and J. R. Risser, Nucl. Instr. Methods 4, 167 (1959).

separation of the ^{12}C and ^{14}C scattering groups over a large range of proton energies at the back angles. At energies and angles where the ^{12}C and ^{14}C groups could not be resolved, the $^{12}C(p,p)^{12}C$ contribution was determined with a normal carbon target. Subtraction of the ^{12}C contribution was complicated by carbon buildup on the targets in spite of liquid-nitrogen trapping on all pumps. Although such an effect can be monitored by periodic checks at some reference energy, it tends to give rise to yield fluctuations which are outside statistics in that the buildup is not uniform. It is rather a function of the average beam density profile, thus making the yield sensitive to time variations in beam focus.

III. EXPERIMENTAL RESULTS

$^{14}C(p,\gamma)^{15}N$

Figure 1 shows the excitation curve for $^{14}C(p,\gamma)^{15}N$, taken with the detector bias set at (E_x-2 MeV); this is interpreted as essentially a ground-state decay curve. There is very close agreement with the Chalk River results[1] up to about 1.7 MeV, above which our data show almost none of the structure which they report. It is assumed that these experimenters were probably operating with a somewhat lower bias level and thus were observing strong neutron-emitting levels via the $^{127}I(n,\gamma)$ reaction in the NaI crystal. The triangles in Fig. 1 indicate the location of known neutron-emitting resonances[7] in the $^{14}C(p,n)^{14}N$ reaction; the representative pulse-height spectra in the figure show that the n, γ contribution in the crystal can be quite strong below $E_\gamma \simeq 8$ MeV. (Note, for example, the spectrum shown in the inset near the neutron-emitting level at 2.08 MeV.) The peak at 2.49 MeV in the figure is virtual, being the result of the sum peak of cascade γ rays through the 5.3-MeV doublet of ^{15}N (note associated spectrum); this conclusion was confirmed by varying the target-detector geometry.

We also obtained a $^{14}C(p,n)^{14}N$ excitation curve with a BF_3 detector. Comparison of our neutron curve (not shown) and the γ-ray excitation curve indicated strongly that a γ-emitting level of ^{15}N was formed at $E_p = 2.49$ MeV—very near the known neutron-emitting level[7] at $E_p = 2.46$ MeV, but definitely resolved from it. The strength of the γ rays from this level and the fact that it had not been seen in examinations of competing reactions suggested that it might be a single-particle proton state. We therefore undertook elastic scattering studies in order to learn more of the characteristics of this level.

$^{14}C(p,p)^{14}C$ Scattering

This reaction was investigated in the proton energy range from 0.76 to 2.8 MeV at center-of-mass (c.m.) angles of 165°, 141°, 125°, and 90°. The experimental results are shown as dots in Fig. 2. At lower bombarding energies at the back angles and at all energies at 90° it was impossible to resolve completely the ^{12}C and ^{14}C scattering peaks. At these energies therefore the $^{12}C(p,p)^{12}C$ yield was measured with a normal target and subtracted.

IV. THEORETICAL ANALYSIS AND DISCUSSION

Figure 2 shows five anomalies in the scattering of protons by ^{14}C; these appear at $E_p = 1.16$ and 1.31 MeV (both weak), and at 1.50, 2.08, and 2.49 MeV. Because of the nature of the targets, absolute cross sections could not be measured with sufficient accuracy for a phase-shift analysis. For this reason, a dispersion-theory fit to the relative cross sections has been used for determination of the resonance parameters. The necessary calculations were initiated by one of us

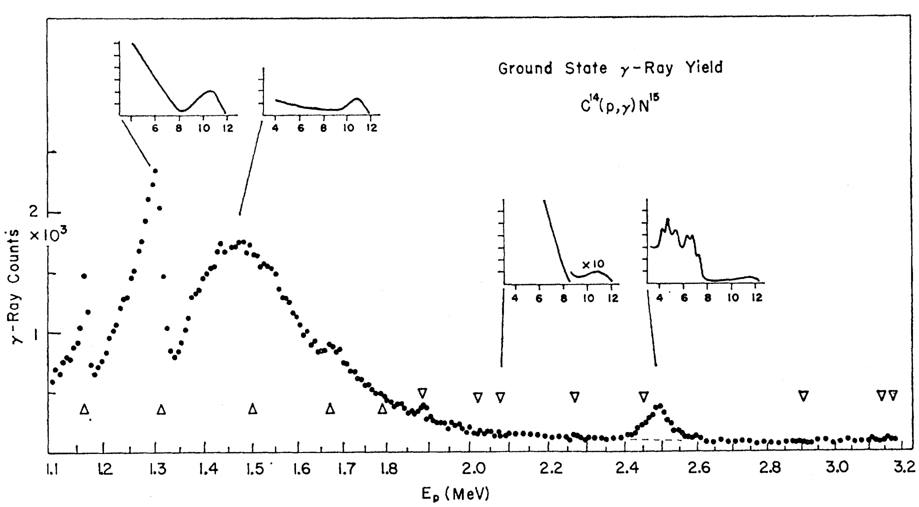

FIG. 1. Ground-state γ-ray yield for the $^{14}C(p,\gamma)^{15}N$ reaction. Triangles represent positions of known neutron-emitting levels in the $^{14}C(p,n)^{14}N$ reaction. Inserts show representative γ-ray spectra at the bombarding energies indicated.

[7] R. M. Sanders, Phys. Rev. 104, 1434 (1956).

(WRS) and carried out on the University of Texas CDC 1604 computer.

Mathematical Formulation

The properties of three of the resonances observed in the elastic scattering data (at $E_p=1.50$, 2.08, and 2.49 MeV) were determined by comparison with calculations made using the two-level, two-channel formulation of dispersion theory presented by Yagi.[8] According to this theory the differential elastic scattering cross section for protons incident on a spin-zero target has the form

$$\frac{d\sigma}{d\Omega}=\frac{1}{k^2}\left\{\left|\frac{-\eta}{2\sin^2\frac{1}{2}\theta}\exp(-i\eta\ln\sin^2\tfrac{1}{2}\theta)+\sum_l P_l(\cos\theta)\exp(i\omega_l)(2l+1)\sin\phi_l\exp(i\phi_l)+\sum_l P_l(\cos\theta)\{(l+1)R_l^{+}+lR_l^{-}\}\right|^2+\left|\sum_l \sin\theta P_l'(\cos\theta)\{R_l^{-}-R_l^{+}\}\right|^2\right\},$$

where $R_l^{\pm}=\exp\{i(\omega_l+2\phi_l)\}Y_l^{\pm}$.
The $(\pm)$ superscripts indicate that the angular momentum j of the proton has the value $j=l\pm\frac{1}{2}$. For two levels, 1 and 2, with the same spin and parity, where only two decay channels—p (proton channel) and α (other channel)—are open, $Y_l^{\pm}$ is given by

$$Y=(KM+LN)/(K^2+L^2)+i(LM-KN)/(K^2+L^2),$$

with

$$K=(E_1-E)(E_2-E)-T_1T_2+\{(T_{1p}T_{2p})^{1/2}\pm(T_{1\alpha}T_{2\alpha})^{1/2}\}^2,$$
$$L=\{T_1(E_2-E)+T_2(E_1-E)\},$$
$$M=\{T_{1p}(E_2-E)+T_{2p}(E_1-E)\},$$
$$N=\{(T_{1p}T_{2\alpha})^{1/2}\mp(T_{2p}T_{1\alpha})^{1/2}\}^2,$$

and

$$T=T_p+T_\alpha.$$

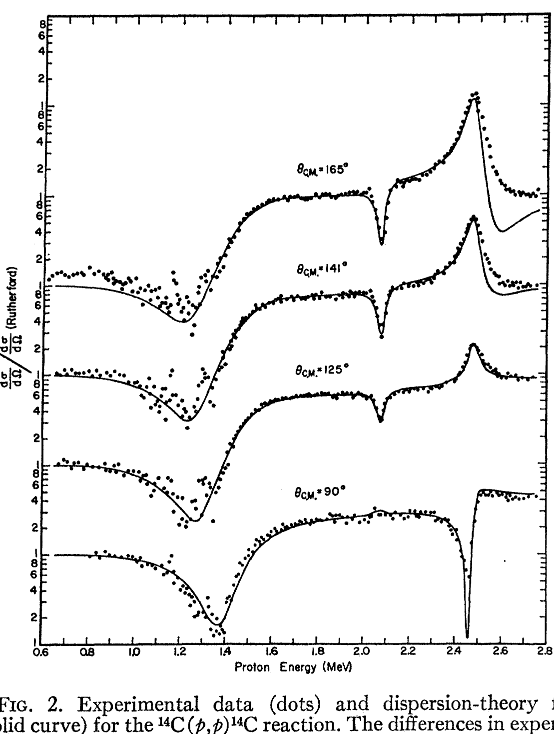

FIG. 2. Experimental data (dots) and dispersion-theory fit (solid curve) for the $^{14}C(p,p)^{14}C$ reaction. The differences in experimental data at low energies and the theoretical fit can be totally attributed to the uncertainty in subtracting the ^{12}C contribution in this region.

[8] K. Yagi, J. Phys. Soc. Japan 17, 604 (1962).

The choice of the upper or lower sign is determined by comparison with experiment. The theoretical meaning connected with this ambiguity in sign is discussed in Ref. 8.

The other symbols in the preceding expressions are now described. The symbol θ represents the c.m. scattering angle, $P_l(\cos\theta)$ is a Legendre polynomial and $P_l'(\cos\theta)$ its derivative with respect to $\cos\theta$,

$$k^2=2\mu E_{\mathrm{c.m.}}/\hbar^2,\quad \eta=Z_TZ_pe^2\mu/\hbar^2k,$$

$$T_i(E)=\left.\frac{\rho\gamma_i^2}{F_l^2(\rho)+G_l^2(\rho)}\right|_{\rho=ka},$$

$$\omega_0=0,\quad \omega_l=\omega_{l-1}+2\tan^{-1}(\eta/l),$$

$$\phi_l=-\tan^{-1}\left.\frac{F_l(\rho)}{G_l(\rho)}\right|_{\rho=ka},$$

$$E_i(E)=E_{0i}-T_i\left.\frac{F_l(\rho)}{G_l(\rho)}\right|_{\rho=ka}.$$

μ is the proton reduced mass, $E_{\mathrm{c.m.}}$ is the c.m. bombarding energy, Z_T and Z_p are the charge of the target nucleus and proton, respectively, γ_i^2 is the (energy-independent) reduced width of the ith resonance, F_l and G_l are, respectively, the regular and irregular Coulomb wave functions, and a is the interaction radius, here taken to be $1.45(A^{1/3}+1)$. The resonance energy E_{ki} of the ith level is defined as that energy satisfying the relationship

$$E_i(E_{ki})-E_{ki}=0.$$

The channel widths listed in Table I are evaluated at the resonance energies, i.e.,

$$\Gamma_i=\tfrac{1}{2}T_i(E_{ki}).$$

The usual form[9] of the single-level dispersion theory is obtained from the above by setting T_2, T_{2p}, $T_{2\alpha}$

[9] J. W. Olness, W. Haeberli, and H. W. Lewis, Phys. Rev. 112, 1702 (1958).

TABLE I. Resonance parameters deduced from experimental data on $^{14}C(p,p)^{14}C$ scattering.

E_R (MeV)	E_x[a] (MeV)	$J\pi$	l	Γ[b] (keV)	Γ_p/Γ	γ_p^2 (keV)	γ_p^2/γ_W^2[c]
1.50	11.62	$\frac{1}{2}+$	0	520	0.93	430	0.16
2.08	12.16	$\frac{3}{2}-$	1	54	0.34	20	0.0074
2.49	12.54	$\frac{5}{2}+$	2	60	1.0	180	0.066

[a] The threshold for formation of ^{15}N by $^{14}C+p$ is taken to be 10.214 MeV above the ground level of ^{15}N.
[b] The channel widths and reduced widths are in the c.m. system: estimated errors are $\pm$ 15%.
[c] γ_W^2 is the Wigner limit for the reduced width, expressed as $3\hbar^2/2\mu a^2$ =2.728 MeV.

equal to zero and changing the form of the result by means of complex-number identities. Thus

$$Y(\text{single level}) = (T_p/T)\sin\beta\exp(i\beta),$$

where $\beta=\tan^{-1}[T/(E_1-E)]$, and T is half the total channel width where now the number of open decay channels allowed is not restricted. Reference 8 also indicates how to treat two levels with identical spin and parity where more than two modes of decay are available.

Terms through $l=3$ were included in our analysis. With the large computer available it appeared more convenient to calculate the Coulomb wave functions directly rather than interpolate them from tables. The scheme employed is outlined elsewhere.[10]

Resonance Analysis

A theoretical fit to the scattering data is shown in Fig. 2. The resonance parameters used in the analysis are given in Table I. The analysis was simplified somewhat by the fact that the parameters of the resonances at 1.50 and 2.08 MeV have been reasonably well established by the study of other reactions. No attempt was made to fit the small anomalies at 1.16 and 1.31 MeV because of the poor statistics of the experimental data in this region. A discussion of the individual resonances follows:

1.50 MeV. This level has been studied by Bartholomew *et al.* by means of the $^{14}C(p,\gamma)^{15}N$ reaction.[2] Bartholomew *et al.* give a $J\pi$ assignment of $\frac{1}{2}+$ to this level and suggest that it is a single-particle state and the isobaric analog of the ground state of ^{15}C. The 1.50-MeV resonance parameters of Table I agree within the experimental error with those given in Ref. 1.

An anomalous rise in the low-energy 165° data is noted. This difference can be totally ascribed to the uncertainty previously mentioned in subtracting the ^{12}C contribution in this energy region.

2.08 MeV. The small size of the resonance at 90° indicates that l is odd. An assignment of $\frac{3}{2}-$ provides a good fit to the data. These data should definitely resolve the conflicting parity assignments given to this level from the $^{14}C(p,n)$ and $^{14}N(n,n)$ reactions.[2,11] Our assignment of $\frac{3}{2}-$ agrees with the results of Harris and Armstrong.[4]

2.49 MeV. This level has been observed in the $^{14}C(p,\gamma)^{15}N$ reaction but is not seen in competing reactions. All values of l and j for $l<3$ were tried for this resonance. The sample results shown in Fig. 2 of Ref. 8 agree qualitatively with our calculations and show conclusively that only $l=2$ is acceptable. Values of $J=\frac{3}{2}$ or $\frac{5}{2}$ give the proper anomalous shape but the latter value is preferred since it more nearly reproduces the heights of the experimental data at 141° and 165°. Discrepancies appear at these angles on the high-energy side of the resonance. An attempt to remove these discrepancies by including the effects of higher energy levels found in the $^{14}N(n,p)^{14}C$ and $^{14}N(n,\alpha)^{11}B$ studies by Gabbard *et al.*[12] was not successful.[13] The most probable explanation of the difficulty is that the phase shift does not conform to a dispersion-theory formulation. Such an effect has been noted in the analysis of the $^{12}C(p,p)^{12}C$ reaction by Reich *et al.*[14] It is also possible that at this energy optical-model, rather than hard-sphere, background scattering should be used.

This level lies at $E_x=12.54$ MeV in ^{15}N (see Table I), which is 0.92 MeV above the $\frac{1}{2}+$ level at $E_x=11.62$ MeV. It was pointed out above that the 11.62-MeV level is the first $T=\frac{3}{2}$ state in ^{15}N and is hence the analog of the ground state ($\frac{1}{2}+$) of ^{15}C. The first excited state of ^{15}C is 0.745 MeV above the ground state and has an assignment of $\frac{5}{2}+$. We should therefore expect that the second $T=\frac{3}{2}$ state in ^{15}N is $\sim$0.9 MeV above the first $T=\frac{3}{2}$ state, that it would exhibit a large reduced width for protons, and that its spin-parity assignment will be $\frac{5}{2}+$. All of these expectations are met by the state at 12.54 MeV.

Further support for this $T=\frac{3}{2}$ assignment[15] for the 12.54-MeV level in ^{15}N may be obtained from the $^{14}C(d,p)^{15}C$ stripping widths quoted by Macfarlane and French[16] from work by Moore.[17] The ratio of proton reduced widths which we obtain for the 11.62- and 12.54-MeV levels is 0.16/0.066, as shown in Table I.

[10] W. R. Smith, Oak Ridge National Laboratory Report No. ORNI-TM-1117, 1965 (unpublished).

[11] J. L. Fowler and C. H. Johnson, Phys. Rev. **98**, 728 (1955).

[12] F. Gabbard, H. Bichsel, and T. W. Bonner, Nucl. Phys. **14**, 277 (1959).

[13] A more complete account (private communication and report) of the work of Harris and Armstrong (Ref. 4) was received after this paper was submitted for publication. Their analysis was directed at levels in ^{15}N in the interval $11.29\leq E_x \leq 12.54$ MeV. We have concentrated primarily on the level at 12.54 MeV. For this level, they report a total width (c.m. system) of 80$\pm$20 keV, compared to our value of 60$\pm$10 keV (based on our p, γ observations). Their theoretical analysis also agrees best with $J\pi=\frac{5}{2}+$. They note that the "anomalous behavior of the cross section near $E_p=2.48$ MeV is not completely explained."

[14] C. W. Reich, G. C. Phillips, and J. L. Russell, Jr., Phys. Rev. **104**, 143 (1956).

[15] The argument presented here was called to our attention by Dr. Ian Mitchell of the Atomic Energy Research Establishment, Harwell, England. We are grateful for his comments and interest.

[16] M. H. Macfarlane and J. B. French, Rev. Mod. Phys. **32**, 567 (1960).

[17] W. E. Moore, thesis, University of Pittsburgh, 1959 (unpublished).

This agrees closely with the ratio of neutron reduced widths for the ^{15}C(g.s.) to ^{15}C(0.745 MeV), viz., 0.093/0.045. The ratio of neutron to proton reduced widths is about 0.6. The Clebsch-Gordan coefficients for coupling isospin would lead one to expect a value of 3 for this ratio, so that our experimental value disagrees by a factor of about 5. However, Macfarlane and French[16] have pointed out that the reduced width deduced from stripping data is "usually smaller by a factor of 4 or 5 than would be expected on the basis of some reasonable potential-well model of the nucleon transfer process, even in cases in which the overlap factor S should be close to unity." Thus the apparent "disagreement" by a factor of 5 may be regarded as additional empirical justification for the statement by Macfarlane and French.

We wish to make it clear that this level (produced at $E_p=2.49$ MeV) is distinct from the nearby $\frac{5}{2}^+$ level, which is excited at $E_p=2.46$ MeV, since there has been some confusion in the literature on this point. Sanders,[7] using earlier data which he cites, gave a $\frac{3}{2}^-$ assignment to the 2.46-MeV level, but later analysis of $^{11}B(\alpha,p)^{14}C$ at $E_\alpha=2.06$ MeV yields[18] a specific $\frac{5}{2}^+$ assignment and requires $T=\frac{1}{2}$.

We therefore conclude that there are two neighboring states in ^{15}N, at $E_x=12.51$ and 12.54 MeV, and that each of these is a $\frac{5}{2}^+$ state; however, the isospins are, respectively, $\frac{1}{2}$ and $\frac{3}{2}$.

[18] L. L. Lee, Jr., and J. P. Schiffer, Phys. Rev. **115**, 160 (1959)

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Ground State of Three Alpha Particles*

J. R. Fulco
University of California, Santa Barbara, Santa Barbara, California 93106

AND

D. Y. Wong
University of California, San Diego, La Jolla, California 92038

(Received 12 February 1968)

The Faddeev equation is applied to solve for the energy of three α particles using a local static two-body potential obtained by fitting scattering phase shifts. A single bound state is found with a binding energy of 2.79 MeV. Although this value is not very close to the ground-state energy of C^{12}, the existence of a 3-α bound state with a binding energy of several MeV indicates that the main structure of C^{12} is like a composite of three α particles. The effect of inelastic processes is estimated in a rough approximation by using a two-channel two-body potential. It is shown that they can easily increase the binding energy of the 3-α bound state by several MeV.

A THREE-α-PARTICLE model of C^{12} has been suggested by Harrington,[1] who solved the Faddeev equation with a separable (nonlocal) two-body potential obtained by fitting the s-wave α-α scattering length and effective range with the Coulomb effects removed.

Recently, a systematic method for solving the Faddeev equation with local potentials was presented by Ball and one of us (D. Y. W.).[2] Here we apply this method to investigate the possibility of 3-α bound states using the phenomenological α-α potential obtained by Darriulat *et al.*[3] by fitting the scattering phase shifts up to 120-MeV laboratory kinetic energy. This potential has the form

$$V(r)=U_1\{1+\exp[(r-r_1)/a_1]\}^{-1} -U_2\{1+\exp[(r-r_2)/a_2]\}^{-1} +4e^2/r+iW(r)\Theta(E_L-40\text{ MeV}). \quad (1)$$

We remark here that the Faddeev equation requires the knowledge of the two-body T matrix at energies below the threshold and therefore the imaginary part is absent. However, the Θ function is not analytic and the error in the continuation of the potential as a function of the energy can be a major source of uncertainty in the value of the three-body binding energy.

In this paper, we are addressing ourselves to the question of how closely the ground state of C^{12} can be described as a composite of three rigid α particles. Hence, we must use a static two-body potential such as the real part of that given by (1). Since the absorption term originates from other channels, such as (Li^7+p),

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[1] D. R. Harrington, Phys. Rev. **147**, 685 (1966).
[2] J. S. Ball and D. Y. Wong (to be published).
[3] P. Darriulat *et al.*, Phys. Rev. **137**, B315 (1965).