Transverse Magnetoresistance and Hall Effect in p-Type Germanium*

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The transverse magnetoresistance and Hall coefficient in high-resistivity p-type germanium were measured in pulsed magnetic fields up to 165 kG and at temperatures between 18 and 77°K. The magnetoresistance ratio had roughly the same field and temperature dependences for all four orientations measured; i.e., at fields above 100 kG and temperatures below 50°K it was proportional to $H^{0.8}T^{-1.3}$. A similar behavior has been observed in n-type germanium, and thus, these measurements do not reflect any difference in the structures of the germanium conduction and valence bands. The Hall coefficient increased with field at temperatures below 25°K. This is attributed to a decrease in the valence-band hole density due to a change in the ionization energy.

I. INTRODUCTION

MAGNETORESISTANCE measurements in p-type Ge have been primarily confined to the low-field, classical region with attention centered on the respective roles of light and heavy holes in the transport processes.[1–3] Measurements of the magnetoresistance at high fields and low temperatures have been made for a few other semiconductors; n-type Ge,[4–6] n-type InSb,[7,8] and PbTe[9]; but only one report of such measurements on p-type Ge is available.[10]

The "high-field" region is defined by the conditions $\omega_c\tau>1$, where ω_c is the cyclotron frequency, and τ is the carrier relaxation time, and $kT\ll\hbar\omega_c$. Under the first condition, carriers make complete orbits between collisions and their energies are quantized. Under the second condition, the levels become distinct. Thus the transport properties should begin to deviate from classical behavior in this region. The fields and temperatures for which these conditions are satisfied in p-type Ge are estimated as follows. The relaxation time is calculated from the conductivity mobility μ; for acoustic phonon scattering of heavy holes in Ge,

$$\mu\cong 3\times 10^{7}T^{-1.5}\ \text{cm}^2/\text{V sec},$$

where T is the temperature.[11] This expression is limited to temperatures below 100°K and gives the values of τ recorded in Table I. Bagguley *et al.*[12] found that the phonon-limited relaxation time was the same for light and heavy holes. Using the effective-mass ratios of $(m^*/m)_h=0.29$ and $(m^*/m)_l=0.042$ for heavy and light holes, respectively,[13] the field strengths necessary to satisfy the condition $\omega_c\tau=1$ can be calculated; these are given in Table I. The fields for which $\hbar\omega_c=kT$ are also given in Table I.

We have measured the magnetoresistance and Hall coefficient in fields 10–165 kG and at temperatures 18–77°K. Table I shows that this range of experimental conditions satisfies the high-field condition for both light and heavy holes and the quantum limit condition ($\hbar\omega_c>kT$) for light holes. For heavy holes, the quantum limit condition is achieved below 50°K.

II. EXPERIMENTAL PROCEDURE

A. Sample Preparation

All samples except one were cut from a single crystal of high-resistivity p-type Ge. The exception was taken from a lower-resistivity crystal. Slices approximately 0.8 mm thick were cut with a diamond saw, and the orientations determined by x-ray back-reflection Laue photographs. Bridge-shaped samples of the dimensions shown in Fig. 1 and, in a few cases, rod-shaped samples were cut from the slices with a dry-abrasive unit. The samples were labeled using the following notation. The directions of the field **H** and current **I** are given by [111] for **H** in the [111] direction, **I** in a random direction; [110] for **H** in the [110] direction, **I** in the

TABLE I. Fields required to approach the "high field" and "quantum limit" conditions in p-type Ge.

Temp (°K)	τ ($\times10^{-11}$ sec)	$\omega_c\tau=1$ (kG) Heavy holes	$\omega_c\tau=1$ (kG) Light holes	$\hbar\omega_c=kT$ (kG) Heavy holes	$\hbar\omega_c=kT$ (kG) Light holes
100	0.48	3.3	0.51	220	34
64	1.0	1.4	0.22	140	21
49	1.4	1.0	0.15	106	17
36	2.2	0.6	0.10	80	12
25	3.8	0.4	0.06	55	8.5

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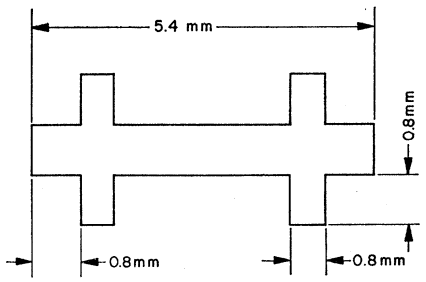

FIG. 1. Geometry and approximate dimensions of experimental samples.

[1$\bar{1}$0] direction; [110]′ for **H** in the [110] direction, **I** in the [001] direction; and [001] for **H** in the [001] direction, **I** in the [110] direction. The designation P refers to the doping; B or R identifies the sample as bridge- or rod-shaped. Samples of the same orientation are differentiated by the number at the end of the notation. The 10 samples studied are listed in Table II; the Hall coefficient in the extrinsic region for $H>20kG$ and the zero-field resistivity $\rho(0)$ at 77°K are given for each.

The samples were lapped with No. 600 silicon carbide grit and etched with CP4. Then the ends of the arms were lapped again, and leads of 8-mil copper wire were soldered on with tin-lead solder (50% of each) doped with 2% indium. The solder joints covered the ends of the arms of the bridge-shaped samples.

B. Apparatus

The magnetic field, obtained by discharging a 3000-V capacitor bank through a wire-wound solenoid, reached a maximum strength of 165 kG in 7 msec. A signal proportional to H produced the horizontal deflection of a Tektronix 535 oscilloscope. A constant current was maintained in the sample by a power supply connected through a large series resistor, and the voltage

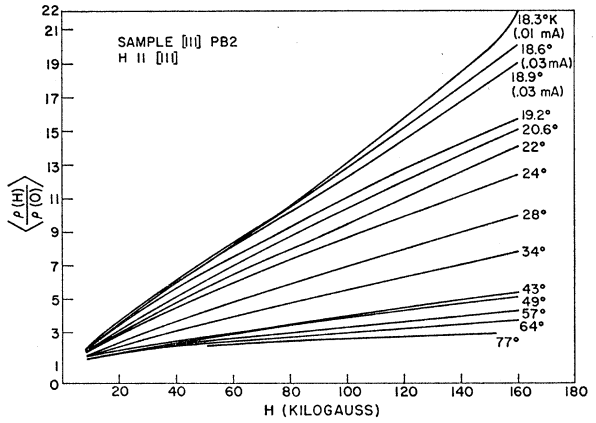

FIG. 2. Transverse magnetoresistance ratio as a function of magnetic-field strength for field in the [111] direction. Temperature is a parameter. The data have been averaged over both directions of current and field.

TABLE II. Identification of samples.

Sample	Field direction	Current direction	$R_H(T>30°K)$ $(H>20kG)$ ($cm^3\ C^{-1}$)	$\rho(0)$ at 77°K (Ω cm)
[111] PB1	[111]	Random	1.1×10^5	2.7
[111] PB2	[111]	Random	1.0×10^5	2.8
[111] PR1	[111]	Random	1.1×10^5	3.9
[111] PB5	[111]	Random	2.1×10^4	0.72
[001] PB1	[001]	[110]	0.93×10^5	2.7
[001] PB2	[001]	[110]	0.94×10^5	2.1
[110] PB1	[110]	[1$\bar{1}$0]	0.87×10^5	2.4
[110] PB2	[110]	[1$\bar{1}$0]	0.91×10^5	2.2
[110]′PB2	[110]	[001]	1.01×10^5	2.9
[110]′PB3	[110]	[001]		3.1

across the sample potential leads produced the vertical scope deflection. The resulting trace giving sample resistance as a function of H was photographed with a Polaroid Land camera. The same system was used for low-field measurements. The zero-field resistance was measured with a Rubicon Type-B Potentiometer.

The neck of a liquid-helium Dewar was inserted through the center of the solenoid. The sample, mounted on a wooden rod and placed in this neck, was cooled by helium gas evaporating from the Dewar bulb. This gas could be heated by a carbon resistor placed below the sample in the Dewar neck. The sample temperature was measured with a thermocouple of copper and gold alloyed with 2% cobalt. Sample temperatures between 15 and 75°K were attained with this system. A more detailed description of this entire apparatus is given by Love and Wei.[6]

III. TRANSVERSE MAGNETORESISTANCE

A. Results

The magnetoresistance ratio $\rho(H)/\rho(0)$ and the Hall coefficient R_H were measured at temperatures between 18 and 77°K in fields up to 165 kG. Typical values of $\rho(0)$ at various temperatures are given in Table III.

TABLE III. Zero-field resistivity at various temperatures (sample [111]PB1).

T (°K)	$\rho(0)$ (Ωcm)
77	2.7
60	2.0
50	1.7
40	1.4
35	1.3
30	1.2
28	1.2
26	1.2
24	1.2
22	1.3
20	1.6
19	1.8
18	2.4

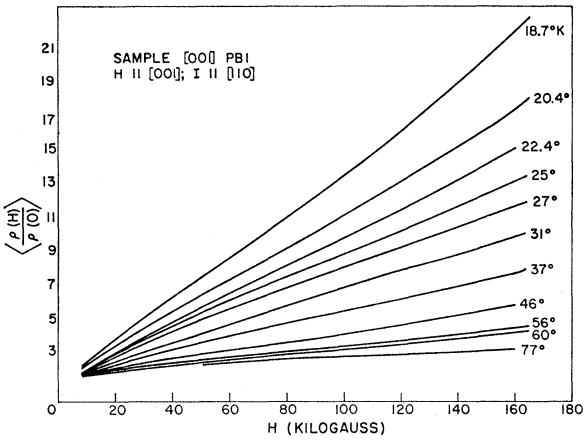

FIG. 3. Transverse magnetoresistance ratio as a function of magnetic-field strength for field in the [001] direction, current in the [110] direction. Temperature is a parameter. The data have been averaged over both directions of current and field.

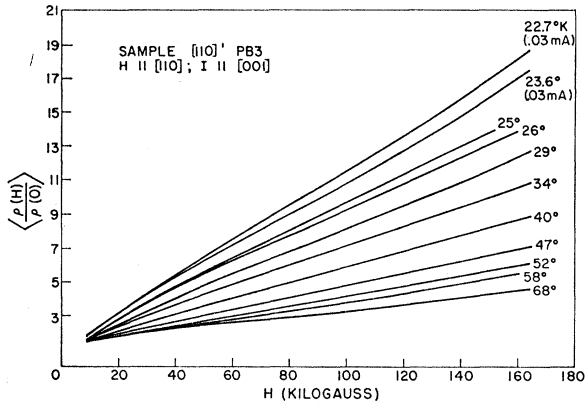

FIG. 5. Transverse magnetoresistance ratio as a function of magnetic-field strength for field in the [110] direction, current in the [001] direction. Temperature is a parameter. The data have been averaged over both directions of current and field.

Currents for which the samples were Ohmic at all fields were used. All quantities were measured under separate reversal of field and current. The variations under current reversal were typically 10%. The variations under field reversal were usually 20% but were as high as 30% in two samples and were roughly linear in H. Gradients in carrier concentration along the direction of current flow are probably responsible for the variations under field reversal. As discussed elsewhere, their effect can be eliminated by averaging over both directions of field.[4,14]

The data were averaged over both directions of field and current, and the averaged magnetoresistance ratios $\langle\rho(H)/\rho(0)\rangle$ for representative samples are given as functions of H in Figs. 2–5. The magnetoresistance ratio is roughly proportional to $H^{0.8}$ $T^{-1.3}$ for all four orientations.

To compare results obtained for different samples of the same orientation, $\langle\rho(150\text{ kG})/\rho(0)\rangle$ for all samples of each orientation are plotted as functions of temperature on the same graph (Figs. 6–9). The agreement between samples is very good; the maximum variation occurs for **H** in the [110] direction, **I** in the [001] direction, where $\langle\rho(150\text{ kG})/\rho(0)\rangle$ varies by 15% about the average. The agreement of sample [111]PB5 with the other samples with **H** in the [111] direction is significant because this sample was taken from a different crystal (see Table II) and has approximately five times more carriers than the other samples.

Stafeev and Tuchkevich[10] measured the transverse magnetoresistance in p-type samples having from 10^{15}–10^{16} carriers per cc, and found $\rho(H)/\rho(0)$ proportional to $H^{0.7}$ at 20 kG and temperatures between 50 and 100°K.

Herring[15] has suggested that the linearity of the transverse magnetoresistance results from random,

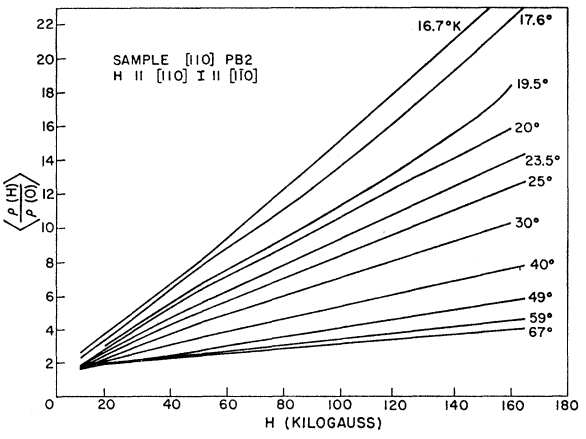

FIG. 4. Transverse magnetoresistance ratio as a function of magnetic-field strength for field in the [110] direction, current in the [1$\bar{1}$0] direction. Temperature is a parameter. The data have been averaged over both directions of current and field.

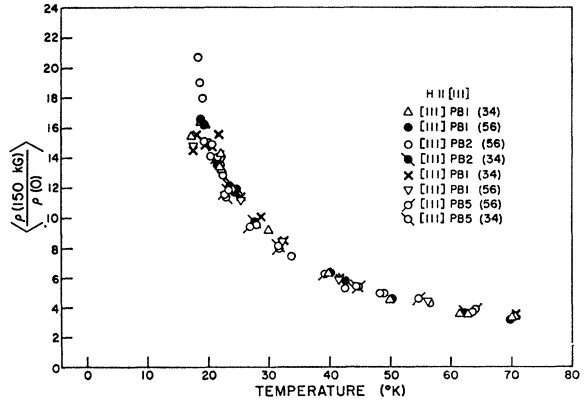

FIG. 6. Transverse magnetoresistance ratio at 150 kG as a function of temperature for all samples with **H** in the [111] direction. The numbers in parentheses identify the leads used.

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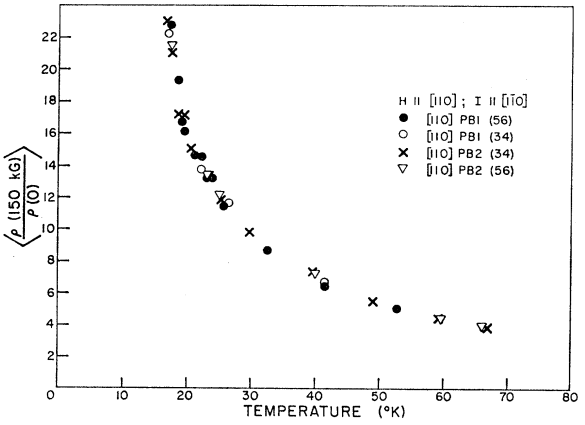

FIG. 7. Transverse magnetoresistance ratio at 150 kG as a function of temperature for all samples with **H** in the [110] direction and **I** in the [1$\bar{1}$0] direction. The numbers in parentheses identify the leads used.

small-scale inhomogeneities in the carrier concentration. Since [111]PB5 was taken from a different crystal than the other samples, it seems unlikely that it has the same spatial-distribution function for the impurities (i.e., the same inhomogeneities) as the other samples. However, Fig. 6 shows that its magnetoresistance agrees well with that of the other samples of the same orientation. This implies that random, small-scale inhomogeneties in carrier concentration are not the source of the linear magnetoresistance.

The size of contributions due to thermoelectric effects was estimated and found to be too small to influence these data.

IV. HALL COEFFICIENT

The Hall coefficient is independent of magnetic field and temperature for fields above 20 kG and temperatures between 30 and 77°K. The values for the different samples are given in Table II and (with the exception of [111]PB5) seem to fall into two groups. For **H** in the [111] direction, R_H is somewhat above the average value, and for **H** in the [001] direction and **H** in the [110] direction, **I** in the [1$\bar{1}$0] direction, R_H is somewhat below average. (For **H** in the [110] direction, **I** in the [001] direction, R_H was obtained for only one sample; no definite trend could be established.) The reason for this grouping is not clear. It may be due partly to errors in measurement of sample thickness which were only good to 10% and partly to small variations in carrier concentration. The samples for the two groups were taken from regions of the single crystal approximately 0.5 in. apart. Below 30°K, R_H increases with decreasing temperature. The probability of thermal release of holes from acceptor levels to the valence band decreases with decreasing temperature; thus the number of valence-band holes decreases (carrier freeze-out).

From 0 to 20 kG, R_H decreases as H increases. This behavior, shown in Fig. 10, was observed at 77 and 300°K using a dc magnet. Similar results were observed by Willardson *et al.*[2] and by Goldberg *et al.*,[3] and were interpreted using a two-carrier (light and heavy holes) model.

The Hall coefficient reaches a minimum at about 20 kG, and then at temperatures below 30°K increases with H as shown in Fig. 11. The effect becomes stronger with decreasing temperature and shows no apparent trend with orientation. This behavior, previously observed in n-type Ge[4,5] and n-type InSb,[16,17] is attributed to the magnetic freeze-out of carriers. The wave functions shrink and the ionization energy consequently increases in strong magnetic fields. Thus holes make transitions from the valence band to bound acceptor states and are trapped there if the temperature is low

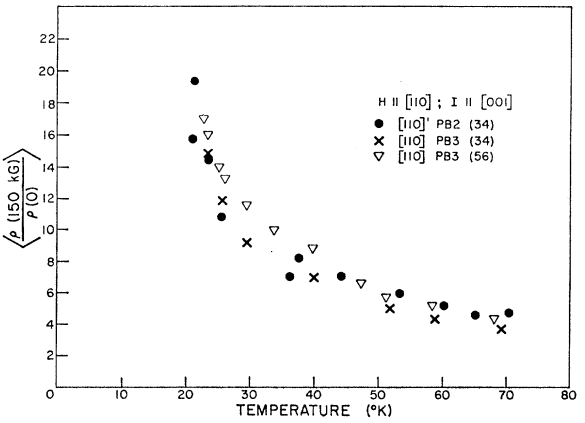

FIG. 8. Transverse magnetoresistance ratio at 150 kG as a function of temperature for all samples with **H** in the [110] direction and **I** in the [001] direction. The numbers in parentheses identify the leads used.

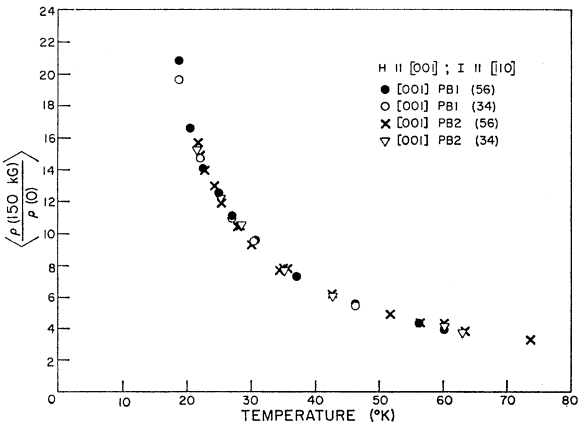

FIG. 9. Transverse magnetoresistance ratio at 150 kG as a function of temperature for all samples with **H** in the [001] direction and **I** in the [110] direction. The numbers in parentheses identify the leads used.

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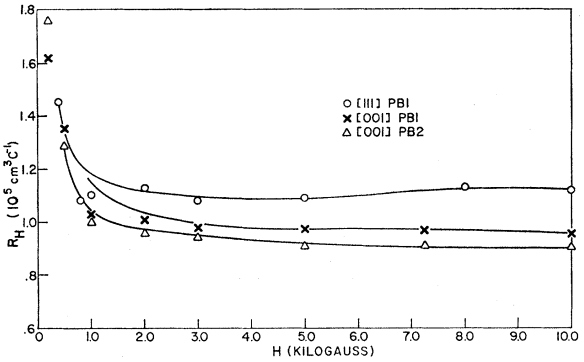

FIG. 10. Hall coefficient as a function of magnetic-field strength at 77°K.

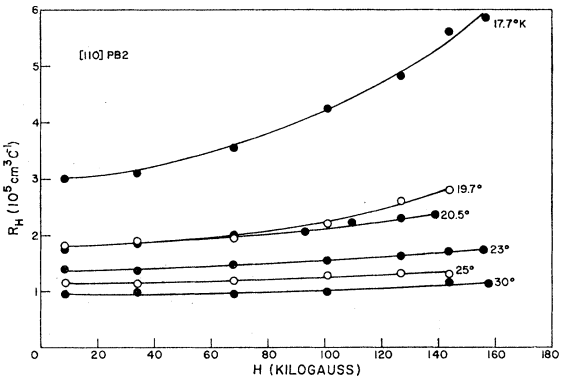

FIG. 11. Hall coefficient as a function of magnetic-field strength. Temperature is a parameter. Data have been averaged over both directions of current and field.

enough. In n-type Ge the freeze-out occurs at slightly lower temperatures than in p-type.[4] This is because the donor (Sb) levels in the n-type Ge have a slightly lower ionization energy (0.0096 eV) than the acceptor (Ga) levels in the p-type (0.0108eV).[18]

V. DISCUSSION

The transverse magnetoresistance in n-type Ge has also been found to be approximately linear in H in high magnetic fields[4,5]; its ratio to the magnetoresistance in p-type Ge is independent of temperature. This indicates that the measured magnetoresistance does not reflect any difference in the structures of the conduction and valence bands of Ge. The linear behavior has also been observed in n-type InSb[7,8] and in n-type PbTe.[9] However, the transverse magnetoresistance calculated for the extreme quantum limit does not have a linear H dependence.[19] The experimental results suggest that there may be a conduction mechanism or a magnetic field-induced scattering mechanism that has been overlooked in the past. This point of view is more creditable when one considers that theory is able to account in detail for results in the case of longitudinal magnetoresistance.[20]

ACKNOWLEDGMENTS

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