# Piezoresistance of *n*-Type Magnesium Stannide\*

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(Received 14 February 1968)

The piezoresistance tensor components for n-type Mg<sub>2</sub>Sn have been measured from 50-300°K. The piezoresistance effect in Mg<sub>2</sub>Sn is larger than the piezoresistance effect in germanium or silicon. From these measurements and the elastic constants reported by Davis et al., the elastoresistance tensor components were obtained and found to satisfy the conditions  $m_{11} = -2m_{12}$ ,  $m_{11}$  large, and  $m_{44}$  small, in the temperature range 50-200°K. These relationships confirm that n-type Mg<sub>2</sub>Sn is a many-valley semiconductor with constant-energy ellipsoids in the (100) directions. The fact that  $m_{11}$  and  $m_{12}$  were both linear in the temperature range 60-175°K indicates that intervalley scattering is unimportant in the extrinsic temperature region. The small value of the volume coefficient,  $\frac{1}{3}(m_{11}+2m_{12})$ , indicates that carrier mobility in *n*-type Mg<sub>2</sub>Sn is relatively insensitive to changes in sample volume. The deformation potential was determined from a combination of these piezoresistance results and Umeda's magnetoresistance data, and was found to satisfy the relation  $\Xi_u = \Xi_u^0(1+\alpha T)$ , where  $\Xi_u^0 = 10.1$  eV and  $\alpha = [(-4.3\pm0.6)\times10^{-4}]^{\circ}K^{-1}$ . High-stress piezoresistance measurements yielded a deformation potential of  $\Xi_u = 18$  eV and mobility anistropy of K = 2.65at 77.4°K. Umeda obtained K=3.51. A possible qualitative explanation of these differences can be given in terms of a change with stress of the position of the donor levels with respect to the conduction band.

## INTRODUCTION

AGNESIUM STANNIDE is a II-IV compound M semiconductor with the fluorite structure, and is a member of the  $Mg_2X$  family of compounds where X can be Si, Ge, Sn, or Pb. Recent experimental papers have discussed the electron structure,<sup>1-4</sup> phonon structure,<sup>5</sup> and transport properties<sup>6,7</sup> of Mg<sub>2</sub>Sn, and some theoretical studies<sup>8,9</sup> have also been made; but our knowledge of the basic characteristics of this interesting semiconductor is far from complete. This paper is concerned primarily with the electron structure of *n*-type Mg<sub>2</sub>Sn.

Piezoresistance studies of Mg<sub>2</sub>Si have shown this compound to be a many-valley semiconductor with ellipsoids in the (100) directions.<sup>10</sup> Umeda<sup>1</sup> has shown from magnetoresistance studies that Mg<sub>2</sub>Sn is also a many-valley semiconductor with ellipsoids in the (100)directions. By making piezoresistance measurements on Mg<sub>2</sub>Sn, we should be able to confirm Umeda's conclusions regarding the symmetry of the electron structure in Mg<sub>2</sub>Sn; and, in addition, by combining Umeda's

- <sup>6</sup> J. J. Martin and G. C. Danielson, Phys. Rev. 166, 879 (1968).
   <sup>7</sup> B. D. Lichter, J. Electrochem. Soc. 109, 819 (1962).
   <sup>8</sup> N. O. Folland and F. Bassani, J. Phys. Chem. Solids (to be
- published)

measurements and our piezoresistance measurements, we should be able to obtain the deformation potential. An independent determination of the deformation potential from high-stress piezoresistance measurements is also conceivable.

#### EXPERIMENTAL

Single-crystal ingots of Mg<sub>2</sub>Sn were grown by a variation of the Bridgman technique described by Morris et al.,<sup>11</sup> who grew Mg<sub>2</sub>Si crystals. In our method, the melt was passed slowly through the melting point twice, 20°C/h the first time and 5°C/h the second time. The first pass formed the compound while the second pass formed the single crystal. This method also provided some zone refining.

Samples were cut from the ingots with a wire saw and oriented to within 2° of the required orientation by a Laue back-reflection x-ray technique. Silicon carbide No. 600 grit suspended in kerosene was used as the cutting agent.

We found that Mg<sub>2</sub>Sn samples could not be lapped with a mechanical lapping wheel. The lapping wheel introduced strains in the samples which would, upon soldering, relax to produce microscopic cracks. The resistivities of these mechanically lapped crystals increased by a factor of 10-20 after they were soldered. However, the resistivities of samples, which were hand lapped on a plate glass with  $20-\mu$  aluminum oxide suspended in a lapping oil, remained the same before and after soldering. For the piezoresistance measurements, only samples which had the same resistivities before and after soldering were used. The samples were lapped to rectangular parallelepipeds with dimensions of 1.5×1.5×10 mm.

Indium electrical contacts were applied to the samples with an ultrasonic soldering iron. For the longitudinal

<sup>11</sup> R. G. Morris, R. D. Redin, and G. C. Danielson, Phys. Rev. 109, 1909 (1958).

<sup>\*</sup> Work was performed in the Ames Laboratory of the U.S. Atomic Energy Commission. Contribution No. 2255.

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&</sup>lt;sup>1</sup> T. Umeda, J. Phys. Soc. Japan 19, 2052 (1964).
<sup>2</sup> A. Stella, A. D. Brothers, R. H. Hopkins, and D. W. Lynch, Phys. Status Solidi 23, 697 (1967).
<sup>3</sup> L. D. Crossman and P. A. Temple, Bull. Am. Phys. Soc. 10, 1121 (1965); also, L. D. Crossman, Ph.D. thesis, Iowa State University, 1967 (unpublished).
<sup>4</sup> K. B. Kaiser and R. J. Kearney, Phys. Rev. 162, 712 (1967).
<sup>6</sup> L. C. Davis, W. B. Whitten, and G. C. Danielson, J. Phys. Chem. Solids 28, 439 (1967).
<sup>6</sup> I. J. Martin and G. C. Danielson, Phys. Rev. 166, 870 (1968).

<sup>&</sup>lt;sup>9</sup> P. M. Lee, Phys. Rev. 135, A1110 (1964). <sup>10</sup> W. B. Whitten and G. C. Danielson, in *Proceedings of the* Seventh International Conference on the Physics of Semiconductors, Paris, 1964, edited by M. Hulin (Dunod, Paris, 1964), Sec. 7, p. 537.

measurements, voltage probes were placed at least 2 mm from the ends of the sample to reduce spurious effects which could arise because of strain inhomogeneities near the ends of the sample. For transverse measurements, the long current electrodes had lengthto-width ratios of 4 or greater, and the electrodes were kept 2 mm from the ends of the samples. Since the resistivities of the longitudinal samples were the same before and after soldering, the current lines were distributed uniformly between the electrodes.

A detailed description of the apparatus is to be reported elsewhere.<sup>12</sup> The main innovation is that the uniaxial compressive stress was applied to the sample by a differential gas pressure across a nickel bellows. The advantage of this arrangement is that the rate at which pressure was applied to the sample and the magnitude of the pressure could be precisely controlled without disturbing the sample or its environment. Pressure was measured by a single-well mercury manometer and regulated by adjustment of two photocells mounted on the manometer tube. For the low-stress measurements, sample pressures up to  $5 \times 10^7$ dyn/cm<sup>2</sup> were used; for high-stress measurements, a continuous range of pressures up to  $2.4 \times 10^9$  dyn/cm<sup>2</sup> was available. After each change in pressure, sufficient time (30 sec) was allowed for the sample to regain



FIG. 1. Electrical resistivity of some piezoresistance samples of Mg<sub>2</sub>Sn.

<sup>12</sup> L. D. Crossman, P. A. Temple, and P. H. Sidles (to be published). Also, R. A. Temple, M. S. thesis, Iowa State University, 1965 (unpublished).



FIG. 2. Hall coefficient of some piezoresistance samples of Mg<sub>2</sub>Sn.

thermal equilibrium so that isothermal values of the piezoresistance would be obtained.

Resistivities were measured by a constant-current dc method. A constant current of 0.2-1.2 mA was obtained from fourteen 1.35-V mercury batteries in series with a decade resistance box. The unstrained



FIG. 3. Hall mobility of some piezoresistance samples of Mg<sub>2</sub>Sn. The  $-\frac{3}{2}$  slope indicates that acoustic mode scattering dominates in the extrinsic temperature region.



FIG. 4.  $\Delta \rho / \rho \chi$  as a function of temperature on oriented samples of  $Mg_2Sn. \pi_{11}$  is large and negative;  $\pi_{12}$  is half as large and positive;  $\frac{1}{3}(\pi_{11}+2\pi_{12}+2\pi_{44})$ , and hence  $\pi_{44}$ , is small in magnitude.

sample voltage was balanced out on a Biddle-Gray potentiometer and the output of the potentiometer connected to a Keithly  $m\mu V$  amplifier with the signal displayed on a strip chart recorder. A second potentiometer was introduced to calibrate the recorder. Voltage changes of 30 nV could be accurately measured. Measurements were made with forward and reversed currents to eliminate thermal effects.

At 77.4°K, the fractional change in resistivity  $\Delta \rho / \rho$ was measured as a function of stress X for each sample. The piezoresistance effect  $\Delta \rho / \rho X$  was linear for pressures up to  $10^8 \text{ dyn/cm}^2$ .

### SAMPLE CHARACTERISTICS

Figure 1 shows the resistivity data for the piezoresistance samples. The slope of these resistivity curves in the intrinsic region gives a room-temperature energy gap of 0.26 eV. Typical Hall coefficient data for several piezoresistance samples are shown in Fig. 2. Carrier concentrations for the piezoresistance samples varied

from  $3 \times 10^{16}$  to  $6 \times 10^{16}$  carriers/cm<sup>3</sup>. From a slope of  $\ln(RT)^{3/2}$  versus 1/T, an energy gap of 0.36 eV at 0°K was obtained. These energy-gap results are in agreement with earlier reported transport property measurements.<sup>13-15</sup> Figure 3 shows Hall mobility data for those samples shown in Fig. 2. The slope of nearly  $-\frac{3}{2}$ indicates that acoustic mode scattering is dominant in the extrinsic region.

### PIEZORESISTANCE

Figure 4 shows the results of measuring  $\Delta \rho / \rho \chi$  as a function of 1/T in the temperature range 50-300°K. All results were corrected for dimensional changes and the transverse results were also corrected for end effects as described by Smith.<sup>16</sup>

 <sup>&</sup>lt;sup>18</sup> R. F. Blunt, H. P. R. Frederikse, and W. R. Hosler, Phys. Rev. 100, 663 (1955).
 <sup>14</sup> W. D. Lawson, S. Nielsen, E. H. Putting, and V. Roberts, J. Electron. 1, 203 (1955).
 <sup>15</sup> U. Winkler, Helv. Phys. Acta 28, 633 (1955).
 <sup>16</sup> C. S. Smith, Phys. Rev. 94, 42 (1954).



FIG. 5. Average values of the piezoresistance tensor components at integral values of 1000/T. These were obtained by drawing a best-fit curve through the measured  $\Delta \rho / \rho \chi$  values. The intersections of the best-fit curves with integral values of 1000/T were averaged for each component. The component  $\pi_{44}$  was determined by combining the quantities  $\pi_{11}$ ,  $\pi_{12}$ , and  $\frac{1}{2}(\pi_{11}+\pi_{12}+\pi_{44})$ .

In Fig. 4,  $\pi_{11}$  is large and negative;  $\pi_{12}$  is about half as large and positive; while  $\frac{1}{3}(\pi_{11}+2\pi_{12}+2\pi_{44})$ , and hence  $\pi_{44}$ , is small in magnitude. The three piezo-



FIG. 6. Comparison of the observed components  $\frac{1}{3}(\pi_{11}+2\pi_{12}+2\pi_{44})$  and  $\frac{1}{2}(\pi_{11}+\pi_{12}-\pi_{44})$  with the values calculated with  $\pi_{11}$ ,  $\pi_{12}$ , and  $\pi_{44}$  obtained from Fig. 5. The good agreement indicated that the piezoresistance results were internally consistent.

resistance tensor components  $\pi_{11}$ ,  $\pi_{12}$ , and  $\pi_{44}$  are shown in Fig. 5 at integral values of 1000/T.  $\pi_{44}$  was determined from the three quantities  $\pi_{11}$ ,  $\pi_{12}$ , and  $\frac{1}{2}(\pi_{11}+\pi_{12}+\pi_{44})$ . The remaining two quantities,  $\frac{1}{2}(\pi_{11}+\pi_{12}-\pi_{44})$ and  $\frac{1}{3}(\pi_{11}+2\pi_{12}+2\pi_{44})$ , were used for an internal check on the other three. Figure 6 shows a comparison of the measured  $[100]_t$  and  $[110]_t$  quantities with the values calculated from the  $\pi_{11}$ ,  $\pi_{12}$ ,  $\pi_{44}$  given in Fig. 4. The good agreement shows that the piezoresistance results were internally consistent.

In the temperature range 50-200°K, the observed piezoresistance effect in Mg<sub>2</sub>Sn is very large relative to most semiconductors. It is a factor of 2 greater than the piezoresistance effect observed by Morin *et al.*<sup>17</sup> in either Si or Ge. It is about an order of magnitude greater than the piezoresistance effect in Mg<sub>2</sub>Si.

For a useful comparison with theory, the elastoresistance constants  $m_{11}$ ,  $m_{12}$ , and  $m_{44}$  are needed.  $\overline{\phantom{m_{17}}}$  F. J. Morin, T. H. Geballe, and C. Herring, Phys. Rev. 105, 525 (1957).



FIG. 7. Elastoresistance tensor components for Mg<sub>2</sub>Sn determined from the piezoresistance components in Fig. 5 and the elastic constants of Davis et al. (Ref. 5).

Theory<sup>18-20</sup> predicts that for a many-valley semiconductor with constant-energy ellipsoids in the  $\langle 100 \rangle$ directions, the elastoresistance constants would be given by

$$m_{11} = \frac{2}{3} \frac{\Xi_u}{kT} \frac{(1-K)}{(1+2K)},$$
(1)

$$m_{12} = -\frac{1}{2}m_{11}, \qquad (2)$$

$$m_{44}=0,$$
 (3)

where  $\Xi_u$  is the deformation potential and  $K = \mu_{\perp}/\mu_{\parallel}$ is the mobility anisotropy. Equations (1) and (2) also

predict that  $m_{11}$  and  $m_{12}$  should be linear with respect to 1/T if the mobility anisotropy and deformation potential are temperature-independent.

The elastoresistance constants were determined from the piezoresistance results in Fig. 5 and the elastic constants obtained by Davis et al.<sup>5</sup> The results of this calculation given in Fig. 7 show that  $m_{11}$  equals  $-2m_{12}$ and that  $m_{11}$  is large and  $m_{44}$  is small in the extrinsic region. We conclude that n-type Mg<sub>2</sub>Sn is a manyvalley semiconductor with constant-energy ellipsoids in the  $\langle 100 \rangle$  directions.

The elastoresistance constants vary nearly linearly with 1000/T. The small deviation from linearity can be attributed to the temperature dependence of the mobility anisotropy. Thus, not only do the magnitudes

 <sup>&</sup>lt;sup>18</sup> C. Herring, Bell System Tech. J. 34, 237 (1955).
 <sup>19</sup> C. Herring and E. Vogt, Phys. Rev. 101, 944 (1956).
 <sup>20</sup> R. W. Keyes, Solid State Phys. 11, 149 (1960).

of the elastoresistance constants for n-type Mg<sub>2</sub>Sn satisfy the necessary criteria for a (100) many-valley semiconductor, but these constants also have the correct temperature dependence.

The linearity of the elastoresistance constants can be used to estimate the importance of intervalley scattering in Mg<sub>2</sub>Sn. From the slope of the Hall mobility (Fig. 3) we know that acoustic mode scattering dominates in the extrinsic region, and we would like to know whether, at higher temperatures in the extrinsic region, the acoustic phonons acquire enough energy to cause intervalley scattering. Herring<sup>18</sup> and Keyes<sup>20</sup> point out that if intervalley scattering does become important, then the elastoresistance constants would decrease more rapidly than 1/T. But at still higher temperatures, the constants should again become linear with respect to 1/T. In Fig. 7,  $m_{12}$  shows no deviation from linearity in the temperature range 60-175°K. However,  $m_{11}$  shows a small deviation at  $T = 125^{\circ}$ K. From 145–180°K  $m_{11}$  again becomes linear. This small deviation could be explained in terms of intervalley scattering. However, the deviation is small and we conclude that intervalley scattering in Mg<sub>2</sub>Sn is unimportant in the extrinsic region.

At temperatures above 175°K there is a very rapid decrease in  $m_{11}$  and  $m_{12}$ . However, this rapid decrease cannot be attributed to intervalley scattering. Above 175°K, Mg<sub>2</sub>Sn enters the intrinsic region (Fig. 1). As a result, part of the conduction process is due to holes in the valence band. A comparison of our piezoresistance measurements with the piezoresistance measurements on p-type Mg<sub>2</sub>Sn by Kaiser and Kearney<sup>4</sup> reveal that  $m_{11}$  for p type is much less than  $m_{11}$  for n type. The effect of uniaxial pressure on hole conduction is much less than the effect of uniaxial pressure on electron conduction. Thus, at the higher temperatures where the crystal is in the intrinsic region, only a fraction of the total charge carriers are greatly affected by the application of pressure, and  $m_{11}$  and  $m_{12}$  would be expected to deviate from an extension of the linear region as shown in Fig. 7. At lower temperatures the deviation of  $m_{11}$  and  $m_{12}$  from linearity may result from the onset of impurity scattering. Hall-mobility measurements by Martin<sup>21</sup> on Mg<sub>2</sub>Sn samples down to 10°K indicate that ionized impurity scattering becomes important below 50°K.

To determine the effect of pressure on carrier mobility, the fundamental elastoresistance coefficients must be determined.<sup>22</sup> The volume coefficient  $\frac{1}{3}(m_{11})$  $+2m_{12}$ ) and the two shear coefficients  $m_{44}$  and  $\frac{1}{2}(m_{11})$  $(-m_{12})$  are shown in Fig. 8. The small value of  $\frac{1}{3}(m_{11})$  $+2m_{12}$ ) is understood because the volume coefficient gives the change in resistivity under an applied hydrostatic stress, and under hydrostatic stress all valleys are affected in the same manner so there is no electron transfer from one valley to another. It is true that hydrostatic stress will change the energy gap of a semiconductor and one might expect a change in resistance in the intrinsic temperature range. In the extrinsic region, however, the temperature is low enough to freeze out thermally induced carriers from the valence band, and the application of a small hydrostatic stress will not cause an appreciable change in the density of free electrons in any valley. Any change in resistivity with hydrostatic pressure arises from a change in the mobility and not from a change in the number of free electrons. Since the volume coefficient for n-type Mg<sub>2</sub>Sn is observed to be small, we can conclude that the change in mobility with change in volume is also small.

### **DEFORMATION POTENTIAL**

The deformation potential can be determined from piezoresistance measurements if the mobility anisotropy  $K = \mu_{\rm I}/\mu_{\rm II}$  is known. Umeda<sup>1</sup> has deduced the mobility anisotropy for Mg<sub>2</sub>Sn from magnetoresistance measurements [Fig. 9(b)]. These values of K and Eqs. (1) and (2) were used to determine the deformation potential  $\Xi_{u}$ . It was necessary to use both Eqs. (1) and (2) because both  $m_{11}$  and  $m_{12}$  depend on  $\pi_{11}$  and  $\pi_{12}$ . Hence, it cannot be said that the value for  $m_{11}$  is more reliable than the value for  $m_{12}$  or vice versa, even though it is true that the value of  $\pi_{11}$  is more reliable than the value of  $\pi_{12}$ . Figure 9(a) shows the result of combining both  $m_{11}$  and  $m_{12}$  with K to obtain the deformation potential. The dependence of  $\Xi_u$  on T can be represented by the relation  $\Xi_u = \Xi_u^0(1 + \alpha T)$ , where  $\Xi_u^0 = 10.1$  eV and  $\alpha = [(-4.3 \pm 0.6) \ 10^{-4}]^{\circ} K^{-1}$ . This temperature dependence is of the same order of magnitude as that found in n-type Si by Tufte and Stelzer,23 n-type Ge by Fritzsche,<sup>24</sup> and *n*-type AlSb by Ghaneker and Sladek.<sup>25</sup> It has the same sign as that for Ge but the opposite sign to that for Si and AlSb.

An attempt was also made to determine an independent value for the mobility anisotropy and deformation potential by extending the piezoresistance measurements to high stress. When a compressive uniaxial stress is applied in the [100] direction of a  $\langle 100 \rangle$  manyvalley semiconductor which has a positive deformation potential, the band edge of the parallel ellipsoids is lowered relative to the band edge of the perpendicular ellipsoids. Thus, electrons from [001] valleys empty into  $\lceil 100 \rceil$  valleys. If the stress is increased such that the separation in energy between the two valleys is much larger than kT, the upper valley will be effectively emptied and the piezoresistance effect will saturate.

<sup>&</sup>lt;sup>21</sup> J. J. Martin, Ph.D. thesis, Iowa State University, 1967 (unpublished).

<sup>&</sup>lt;sup>22</sup>J. M. Ziman, *Electrons and Phonons* (Oxford University Press, London, 1960).

D. N. Tufte and E. L. Stelzer, Phys. Rev. 133, A1705 (1964).
 H. Fritzsche, Phys. Rev. 115, 336 (1959).
 K. M. Ghanekar and R. J. Sladek, Phys. Rev. 146, 505 (1966).



FIG. 8. Elastoresistance coefficients of Mg<sub>2</sub>Sn. The volume coefficient,  $\frac{1}{3}(m_{11}+2m_{12})$ , and one of the shear coefficients  $m_{44}$  are small and temperature-independent in the extrinsic temperature region. The other shear coefficient  $\frac{1}{2}(m_{11}-m_{12})$  is large and decreases with decreasing temperature. These results suggest energy ellipsoids in the  $\langle 100 \rangle$  directions.

Aubrey *et al.*<sup>26</sup> have shown that at saturation the mobility anisotropy is given by

$$\rho_s/\rho_0 = \frac{1}{3}(1+2K), \qquad (4)$$

where  $\rho_s$  and  $\rho_0$  are the saturation and unstrained values of the resistivity. They assumed that under stress the scattering mechanisms and effective masses were unchanged, that intervalley scattering was negligible, that the carriers obeyed classical statistics, and that the number of carriers in the conduction band remained unchanged. The dependence of the resistivity  $\rho$  on the stress  $\chi$  can then be written as

$$\frac{\rho}{\rho_0} = \frac{(1+2K)[1+2\exp(-\beta X)]}{3[1+2K\exp(-\beta X)]},$$
(5)

where  $\beta$  is related to the deformation potential by

$$\beta = \Xi_u (S_{11} - S_{12}) / kT. \tag{6}$$

The  $S_{ij}$  are elastic compliance constants. Figures 10 and 11 show our results for high-stress piezoresistance measurements on *n*-type Mg<sub>2</sub>Sn at 77 and 50°K,

respectively. In Fig. 10 the solid line shows the expected result based on Eq. (5) with  $\Xi_u = 9.8$  eV and K = 3.51. The saturation value, 2.11, gives a mobility anisotropy of K = 2.65. The dashed curve shows a best fit of Eq. (5) to the experimental points with  $\Xi_u = 18$  eV and K = 2.65. In Fig. 11 the solid line shows the expected



FIG. 9. Temperature dependence of the deformation potential (a) as determined with the use of the mobility anisotropy (b) found by Umeda (Ref. 1).

<sup>&</sup>lt;sup>26</sup> J. E. Aubrey, W. Gubler, T. Henningsen, and S. H. Koenig, Phys. Rev. **130**, 1667 (1963).



FIG. 11. High-stress piezoresistance measurements on sample mosaic-1 at 50°K. The solid line indicates the expected result based on Eq. (5) with  $\Xi_u = 9.9$  eV and K = 3.21. The dashed curve shows the best fit to experimental data with  $\Xi_u = 18$  eV and K = 2.50.

result based on Eq. (5) with  $\Xi_u = 9.9$  eV and K = 3.21, while the dashed curve shows the best fit to experimental points with  $\Xi_u = 18$  eV and K = 2.50.

The samples labeled mosaic were taken from mosaicappearing polycrystalline ingots of Mg<sub>2</sub>Sn. The individual grains were misoriented from a common orientation by less than 3°. Single crystals broke at  $\chi = 12$  $\times 10^8$  dyn/cm<sup>2</sup>, while the mosaic crystals did not break until  $\chi = 24 \times 10^8$  dyn/cm<sup>2</sup>; and the breaking stress was independent of sample cross section. As a result, data are available for mosaic crystals, but not for single crystals, in the saturation region.

The deformation potential obtained from a combination of piezoresistance and magnetoresistance data  $(\Xi_u = 9.8 \text{ eV} \text{ at } 77^\circ \text{K})$  is believed to be a reliable value, and the value obtained from high-stress measurements



FIG. 10. High-stress piezoresistance measurements on Mg<sub>2</sub>Sn at 77°K. The solid line indicates the expected result based on Eq. (5) with  $\Xi_u=9.8$  eV and K=3.51. The dashed curve shows the best fit to experimental data with  $\Xi_u=18$  eV and K=2.65.

 $(\Xi_u=18 \text{ eV} \text{ at } 77^\circ \text{K})$  is believed to be in error. One reason for this belief is that in their thermal conductivity results, Martin and Danielson<sup>6</sup> found that an approximate value of  $\Xi_u=10.3 \text{ eV}$  at 10°K gave the best fit of their experimental values to theory. Their value for the deformation potential agrees very well with our value of 10.1 eV at 0°K but not at all well with a value of 18 eV.

Another indication that the deformation potential value 9.8 eV at 77°K is the more reliable value, is found in a comparison of our results with the piezo-resistance of silicon. Aubrey et al.,<sup>26</sup> obtained  $\Xi_u = 8.3$  eV for Si and Tufte and Stelzer<sup>23</sup> found K to be 5.0 for samples with a carrier concentration of 10<sup>16</sup> carriers/cm<sup>3</sup>. From the piezoresistance measurements of Morin et al.,<sup>17</sup> an approximate value for the elastoresistance constants of Si can be determined. Table I shows a comparison of the elastoresistance constant  $m_{11}$  for Mg<sub>2</sub>Sn. Even though the component  $\pi_{11}$  for Mg<sub>2</sub>Sn is larger than  $\pi_{11}$  for Si,  $m_{11}$  is approximately the same for both. If  $m_{11}$  is the same, Eq. (1) predicts that the factor  $A = \Xi_u(1-K)/(1+2K)$  should

TABLE I. Comparison of the elastoresistance tensor component  $m_{11}$  of Mg<sub>2</sub>Sn with  $m_{11}$  of Si at three different temperatures.

Т (°К)	<i>m</i> <sub>11</sub> from Si <sup>a</sup>	$m_{11}$ from Mg <sub>2</sub> Sn	
66.7	350	350	
80.0	303	297	
100.0	247	247	

<sup>a</sup> Determined from the piezoresistance data of Morin et al. (Ref. 17).

also be approximately the same. With the values 9.8 eV and 3.51, A = -3.0 for Mg<sub>2</sub>Sn, while with the values 8.3 eV and K = 5.0, A = -3.0 for Si. However, if we use 18 eV and 2.65 for Mg<sub>2</sub>Sn, A = -4.7 which is very different from A = -3.0.

Since we are led to the conclusion that the deformation potential of 9.8 eV at 77°K is the more reliable value for *n*-type Mg<sub>2</sub>Sn, the value 18 eV must be caused by some mechanism other than a transfer of electrons from one valley to another. One might question, for example, the assumption that neither the relaxation time nor the effective mass is a function of stress. Under large strains, the lattice constant of the crystal would be changed; and this change would alter the lattice vibrations, and thus the relaxation time. Also, strain inhomogeneities in the sample might change the relaxation time. Another possibility is that the energy ellipsoids could be warped, which would change the effective mass. These effects, however, are probably not the dominant factors leading to the discrepancies shown in Figs. 10 and 11.

Perhaps the most important factor, which could account for the observed discrepancy, is a shift of a valley band-edge point below a donor level.<sup>27</sup> It would then be possible to have a transfer of electrons not only from one valley to another, but also from a valley to a donor level. If there were such a transfer, the number of electrons in the conduction band would not remain constant, the theory would not apply, and we should not expect to get reliable values for the deformation potential and mobility anisotropy. The electrons transferred into the donor level would be lost to the conduction process, and the resistivity would increase more than expected when a stress is applied to the sample. Also, transfer of electrons to a donor level would produce saturation at a value of stress smaller than expected from the theory.

One might well ask, "Does this transfer of electrons from the upper valley to a donor level affect the validity of the low-stress piezoresistance results?" This question can be answered by looking at the  $\rho/\rho_0$  values at low stress. Equation (5) predicts that for  $\chi = 5 \times 10^7$  dyn/ cm<sup>2</sup> (pressure employed for the low-stress results)  $\rho/\rho_0$ should be 1.022. The ratio  $\rho/\rho_0$  for samples used to determine  $\pi_{11}$  was 1.025. Thus, it would appear that a transfer of electrons from the upper valley to a donor level did not affect the low-stress piezoresistance results.

### CONCLUSIONS

The piezoresistance tensor components  $\pi_{11}$ ,  $\pi_{12}$ , and  $\pi_{44}$  have been measured for *n*-type Mg<sub>2</sub>Sn in the temperature range 50–300°K. The component  $\pi_{11}$  is larger than that reported for most other semiconducting materials. A comparison of the experimentally observed quantities  $\frac{1}{2}(\pi_{11}+\pi_{12}-\pi_{44})$  and  $\frac{1}{3}(\pi_{11}+2\pi_{12}+2\pi_{44})$  with the values calculated from  $\pi_{11}$ ,  $\pi_{12}$ , and  $\pi_{44}$  showed that the piezoresistance measurements were internally consistent. The elastoresistance tensor components in the temperature range 50–200°K were found to satisfy the

condition  $m_{11} = -2m_{12}$  and the condition  $m_{11}$  large and  $m_{44}$  small. These conditions confirm that *n*-type Mg<sub>2</sub>Sn is a many-valley semiconductor with constant-energy ellipsoids in the  $\langle 100 \rangle$  directions. The fact that  $m_{11}$  and  $m_{12}$  were both linear in the temperature range 60–175°K indicates that intervalley scattering is unimportant in the extrinsic temperature region. Also, the small value of the volume coefficient,  $\frac{1}{3}(m_{11}+2m_{12})$ , indicates that carrier mobility in *n*-type Mg<sub>2</sub>Sn is relatively insensitive to changes in sample volume.

The deformation potential was determined from a combination of these piezoresistance results and the magnetoresistance data of Umeda<sup>4</sup> and found to satisfy the relation  $\Xi_u = \Xi_u^0(1 + \alpha T)$ , where  $\Xi_u^0 = 10.1$  eV and  $\alpha = [(-4.3 \pm 0.6) \ 10^{-4}]^{\circ}$ K<sup>-1</sup>. The temperature dependence  $\alpha$  is of the same order of magnitude as that found in silicon, germanium, and aluminum antimonide. It has the same sign as that found in germanium but the opposite sign to that observed in aluminum antimonide and silicon.

High-stress piezoresistance measurements yielded a deformation potential of  $\Xi_u = 18$  eV and a mobility anisotropy K=2.65 at 77.4°K. Umeda<sup>1</sup> obtained K=3.51. Strain-induced changes in the effective mass and relaxation times are possible mechanisms which could account for these discrepancies in mobility anisotropies and deformation potentials, but a more probable explanation may be the shift in energy of a valley bandedge point below a donor level since electrons will then be transferred not only between valleys, but also between valleys and donor levels.

To better understand the high-stress piezoresistance results, it would be desirable to make piezo-Hallmagnetoresistance measurements on Mg<sub>2</sub>Sn. If the Hall coefficient were measured as a function of stress, one might be able to determine the number of electrons in the conduction band as a function of stress. If magnetoresistance were measured as a function of stress, one might be able to determine the mobility anisotropy as a function of stress.

<sup>&</sup>lt;sup>27</sup> W. Kohn, Solid State Phys. 5, 258 (1957).