(1.0 ± 0.3) cm^{-1} for the exchange interaction of each Tb^{3+} ion with its four nearest neighbors in the a-b plane. A magnetic moment of $(9.0\pm0.6)\mu_B$ per ion at an angle of 35° from the a axis was obtained from the high-field Zeeman splitting.

Metamagnetic behavior produces anomalies in the Zeeman pattern at about 8 and 20 kOe. Good agreement was obtained between the experimentally determined exchange interactions, the positions of the metamagnetic anomalies, and the observed Néel point.

ACKNOWLEDGMENTS

The authors are grateful to W. Grodkiewicz for assistance in preparing the crystals. They also appreciate very much the critical reading of the manuscript by M. E. Lines and G. K. Wertheim.

PHYSICAL REVIEW VOLUME 171, NUMBER 2 10 JULY 1968

Absence of Magnetic Ordering in One and Two Dimensions in a Many-Band Model for Interacting Electrons in a Metal*

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(Received 8 September 1967; revised manuscript received 28 March 1968)

The Bogoliubov inequality is used to prove the absence of spontaneous magnetic ordering in one and two dimensions for a many-band model for electrons in a metal interacting via the Coulomb and the exchange interactions. The discussion follows closely Mermin and Wagner's recent proof of a similar theorem for the Heisenberg model of an interacting spin system.

USING an inequality due to Bogoliubov, Mermin and Wagner[1] have proved rigorously the absence of both ferromagnetism and antiferromagnetism in one- and two-dimensional spin systems described by the Heisenberg Hamiltonian. This theorem concerning the absence of magnetic ordering in one and two dimensions can be proved not only for systems described by the Heisenberg Hamiltonian, but for many others as well. Wegner[2] has shown that it applies to ferromagnetism in systems of locally interacting electrons and nuclei, and Hamilton[3] has shown that it applies to spin density waves in an electron gas. Much of the above work was stimulated by Hohenberg's[4] discussion of a similar theorem for one- and two-dimensional superfluids and superconductors.

Wegner proves the nonexistence of ferromagnetism in systems of electrons for all potentials V, describing the interaction of the electrons among themselves and with nuclei, that satisfy the condition

$$[C', V]=0, \tag{1}$$

where

$$C'=\sum_i \exp(i\mathbf{k}\cdot\mathbf{r}_i)\,(s_{ix}+is_{iy}) \tag{2}$$

and $\mathbf{r}_i$ and $\mathbf{s}_i$ are the position and spin of the ith electron. Potentials that are local (i.e., that do not contain the electron momenta $\mathbf{p}_i$) and do not contain the spin coordinates $\mathbf{s}_i$ satisfy this condition. Since V can contain a periodic potential, the proof applies, for example, to electrons in energy bands interacting via Coulomb forces.

The purpose of this paper is to demonstrate the absence of ferromagnetism and antiferromagnetism in systems of electrons described by the Hamiltonian (3), which is often used in practice[5] to discuss magnetic ordering in metals. Although the interaction, described by the terms H_C and H_{ex}, is usually assumed to represent approximately the effects of a full Coulomb, or a screened Coulomb interaction, it is not in general a local interaction. For this reason, a separate proof is required to establish that model Hamiltonians of the type (3) also do not admit magnetic ordering in one and two dimensions.

We consider systems described by the Hamiltonian

$$H=T+H_C+H_{ex}+H_Z, \tag{3}$$

where

$$T=\sum_{ll'r\sigma} T_r(\mathbf{l}-\mathbf{l}')\,c_{lr\sigma}{}^{\dagger}c_{l'r\sigma}, \tag{4}$$

$$H_C=\sum_{ll'\sigma\sigma'} K(ll')\,n_{l\sigma}n_{l'\sigma'}, \tag{5}$$

$$H_{ex}=-\sum_{l\neq l'} J(ll')\Big[\sum_\sigma n_{l\sigma}n_{l'\sigma}+2S_+(l)\,S_-(l')\Big], \tag{6}$$

$$H_Z=2h\sum_l \cos(\mathbf{q}\cdot\mathbf{l})\,S_Z(l). \tag{7}$$

The nuclei in the crystal are assumed to form a Bravais

* This work was supported in part by the National Research Council of Canada.

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lattice, $\mathbf{l}$ being the vector from the origin to a particular lattice site; r is the band index, σ $(=\pm\frac{1}{2})$ is the spin index and the notation $l\equiv(\mathbf{l}, r)$ has been used. The basic states in this description are the Wannier states $\psi_{lr}(\mathbf{r})$, which are related to the Bloch states by

$$\psi_{lr}(\mathbf{r})=N^{-1/2}\sum_{\mathbf{k}}\exp(-i\mathbf{k}\cdot\mathbf{l})\psi_{\mathbf{k}r}(\mathbf{r}),$$

and $c_{l\sigma}{}^{\dagger}$ and $c_{l\sigma}$ are the corresponding Fermi creation and annihilation operators. The spin operators have been defined in the usual way, i.e.,

$$S_+(l)=c_{l\uparrow}{}^{\dagger}c_{l\downarrow},\qquad S_-(l)=c_{l\downarrow}{}^{\dagger}c_{l\uparrow}$$

and

$$S_Z(l)=\tfrac{1}{2}(n_{l\uparrow}-n_{l\downarrow}). \tag{8}$$

T is the band energy, which includes the kinetic energy of the electrons and their interaction with a periodic potential, H_C and H_{ex} are the so-called Coulomb and exchange interactions between the electrons, and H_Z is the Zeeman interaction of the spins with a fictitious magnetic field of wave vector $\mathbf{q}$. For future use, we also introduce the spatial Fourier transforms

$$S_\alpha(\mathbf{k})=\sum_l\exp(-i\mathbf{k}\cdot\mathbf{l})\,S_\alpha(l) \tag{9}$$

of the spin operators (8) (S_α can be S_+, S_-, or S_Z).

The quantity we wish to compute is the spin magnetic moment

$$\bar{\sigma}=(1/N)\sum_l\cos\mathbf{q}\cdot\mathbf{l}\langle S_Z(l)\rangle \tag{10}$$

induced by the fictitious magnetic field. If the magnetic field can induce a magnetization $\bar{\sigma}$ that tends to a constant nonzero value when h tends to zero, the system will exhibit a spontaneous magnetization. If the induced magnetization tends to zero as h tends to zero, there will be no spontaneous magnetization. The case $\mathbf{q}=0$ of course corresponds to ferromagnetism, but the interaction (7) for arbitrary $\mathbf{q}$ probes for more complicated kinds of ordering.

Following Mermin and Wagner,[1] we make use of the Bogoliubov inequality

$$2kT\langle[C, A]\rangle^2\langle[[C, H], C^{\dagger}]\rangle^{-1}\leq\langle\{A, A^{\dagger}\}\rangle \tag{11}$$

to prove our result. For the operators C and A we use

$$C=S_-(-\mathbf{k}),\qquad A=\tfrac{1}{2}[S_+(\mathbf{k}+\mathbf{q})+S_+(\mathbf{k}-\mathbf{q})]. \tag{12}$$

The expectation values appearing in (11) are

$$\langle[[C, T], C^{\dagger}]\rangle=\sum_{\mathbf{l}_1\mathbf{l}_2r}\{\exp[i\mathbf{k}\cdot(\mathbf{l}_1-\mathbf{l}_2)]-1\}T_r(\mathbf{l}_1-\mathbf{l}_2)\times\langle c_{\mathbf{l}_1r\downarrow}{}^{\dagger}c_{\mathbf{l}_2r\downarrow}+c_{\mathbf{l}_2r\uparrow}{}^{\dagger}c_{\mathbf{l}_1r\uparrow}\rangle, \tag{13}$$

$$\langle[[C, H_C], C^{\dagger}]\rangle=0 \tag{14}$$

(in fact $[C, H_C]=0$),

$$\langle[[C, H_{\mathrm{ex}}], C^{\dagger}]\rangle=4\sum_{l_1l_2}J(l_1l_2)\{1-\exp[i\mathbf{k}\cdot(\mathbf{l}_2-\mathbf{l}_1)]\}\times\langle S_+(l_1)S_-(l_2)-2S_Z(l_1)S_Z(l_2)\rangle, \tag{15}$$

$$\langle[[C, H_Z], C^{\dagger}]\rangle=-4Nh\bar{\sigma}, \tag{16}$$

$$\langle[C, A]\rangle=-2N\bar{\sigma}. \tag{17}$$

Since each lattice site is a center of symmetry, the factors $[\exp(i\mathbf{k}\cdot(\mathbf{l}_1-\mathbf{l}_2))-1]$ in (13) and (15) can be replaced by $[\cos(\mathbf{k}\cdot(\mathbf{l}_1-\mathbf{l}_2))-1]$. Furthermore, it can be shown that

$$|\langle[[C, T], C^{\dagger}]\rangle|<Nak^2,$$
$$|\langle[[C, H_{\mathrm{ex}}], C^{\dagger}]\rangle|<Nbk^2, \tag{18}$$

where

$$a=2\sum_l|\mathbf{l}|^2|T_r(\mathbf{l})|$$

and

$$b=12\sum_{\mathbf{l}_1r_1r_2}|\mathbf{l}_1-\mathbf{l}_2|^2|J(l_1l_2)|. \tag{19}$$

It is of course essential that both expressions (19) should converge. The final result which we shall need is

$$\sum_{\mathbf{k}}\langle\{A, A^{\dagger}\}\rangle<2N^2n_b^2, \tag{20}$$

where n_b is the total number of bands, which we assume is finite. Now, putting the above results into the inequality (11), summing both sides over all values of $\mathbf{k}$ inside the Brillouin zone, we find

$$(2kT\bar{\sigma}^2/N)\sum_{\mathbf{k}}[(a+b)k^2+4|hs_Z|]^{-1}<n_b^2. \tag{21}$$

Since (21) is essentially the same as Eq. (11) of Mermin and Wagner,[1] we can immediately write their equations (1) and (2); i.e., for sufficiently small fields h,

$$|\bar{\sigma}|<(\mathrm{const}/T^{1/2})\,|\ln|h||^{-1/2},\qquad\text{(2 dimensions)} \tag{22}$$

$$|\bar{\sigma}|<(\mathrm{const}/T^{2/3})\,|h|^{1/3},\qquad\text{(1 dimension)}. \tag{23}$$

There is thus no spontaneous magnetization in one or two dimensions at any nonzero temperature T for interacting electron systems described by the many-band Hamiltonian (3). From (21), it is clear that no conclusion can be reached in three dimensions.

As an example, consider the simple model often used for the study of strong correlations between electrons in a single narrow band, for which the Hamiltonian is

$$H=\sum_{\mathbf{l}\mathbf{l}'\sigma}T(\mathbf{l}-\mathbf{l}')c_{l\sigma}{}^{\dagger}c_{l'\sigma}+U\sum_{\mathbf{l}}n_{l\uparrow}n_{l\downarrow}. \tag{24}$$

The few rigorous results that have been established concerning the properties of this Hamiltonian have been listed by Herring,[6] and the one pertinent to our discussion is that Lieb and Mattis[7] have shown that there can be no ferromagnetism for a linear chain of atoms with $T(1-1')$ nonzero only for nearest neighbors, but with an interaction energy that can be an arbitrary function of $n_{1\uparrow}+n_{1\downarrow}$. Since the Hamiltonian (24) is a particular case of our Hamiltonian (3), our conclusions, which cover more possibilities than those of Lieb and Mattis, apply to it. There is thus not only no ferromagnetism but no antiferromagnetism for both one and two dimensions; also the range of the hopping integral $T(1-1')$ is restricted only by the condition that the expression (19) for a converge. It is a trivial matter to show that our results also apply if the interaction is an arbitrary function of $n_{1\uparrow}+n_{1\downarrow}$, since $[C, n_{1\uparrow}+n_{1\downarrow}]=0$.

Although the above results have some intellectual interest, they are clearly inapplicable to real three-dimensional solids. However, as pointed out by Herring[6] in connection with the Lieb-Mattis theorems, many approximation schemes that have been applied to real solids can equally well be applied to one- and two-dimensional solids. If these approximation schemes predict the occurrence of spontaneous magnetization in one and two dimensions as well as in three dimensions for the Hamiltonian (3), the validity of these predictions in three dimensions should be clearly investigated more fully.

One of the authors (T.W.R.) would like to express his gratitude to the Department of Physics at the University of Toronto, and particularly to Professor J. Van Kranendonk, for their hospitality during his visit.

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PHYSICAL REVIEW VOLUME 171, NUMBER 2 10 JULY 1968

Theory of *s-d* Scattering in Dilute Magnetic Alloys with Spin-$\frac{1}{2}$ Impurities*

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(Received 7 June 1967; revised manuscript received 4 December 1967)

With the aid of a new Wick theorem for spin-$\frac{1}{2}$ operators, the properties of a single localized paramagnetic impurity in a metal are investigated, using diagrammatic techniques which are completely analogous to those of standard quantum field theory. Attention is directed at the high-temperature magnetic properties of this system. The present results include a new $\ln T$ term in the g shift for the impurity spin and for the electronic susceptibility. In the high-field limit, the former logarithmic result is replaced by the logarithm of the Zeeman energy. A high-order equation is also obtained for the resistivity by a selective resummation of a complete subseries of electron self-energy diagrams. In this approximation, the resistivity exhibits a maximum for both ferro- and antiferromagnetic coupling (but at different temperatures) as the characteristic temperature is approached. The "bound-state" behavior appears in this theory as an anomalous correlation between the two spin systems, and first occurs at the characteristic temperature when the external magnetic field is zero.

1. INTRODUCTION

THE model originally proposed by Kasuya[1] of a contact *s-d* exchange interaction between the conduction electrons and localized magnetic impurities in metals has led to many theoretical papers[2–9] which have gradually exposed several interesting properties lying behind its superficially simple structure. The divergent behavior in the resistivity in the case of isolated impurities was first found by Kondo,[3] who used standard perturbation theory to calculate the effect of the Pauli principle on the second-order intermediate states for the scattering probability of the conduction electrons. The sharpness of the Fermi surface gives rise to a $\ln T$ term in the resistivity; when this is combined with the usual lattice resistivity, the result is a resistance minimum in the case of antiferromagnetic coupling. This logarithmic temperature dependence implied that a divergence would occur in each term of higher order in perturbation theory as the temperature was decreased. Moreover, Nagaoka[5,6] pointed out that, even in third order, the lifetimes of the conduction elec-

* These results have arisen during the preparation of a Ph.D. thesis, submitted to the University of London.
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