Temperature Dependence of the Flux-Flow Resistance near H_{c2} in Type-II Superconductors*

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We have found that at nonzero temperatures the flux-flow resistivity in type-II superconducting foils is exponential in field near H_{c2}. The fraction of the normalized resistivity which can be represented by the exponential field dependence increases as the temperature approaches T_c. The quantity $\alpha=[(H/\rho_N)\times(d\rho_F/dH)]_{H=Hc2}$ increases monotonically with temperature, in qualitative agreement with the recent theory of Caroli and Maki, but quantitative comparison of the temperature dependence indicates substantial disagreement. The exponential dependence of the flux-flow resistivity on field suggests that overlap of fluxoids may be important in determining the resistivity at high fields. Alternatively, a recent theory of Clem's describes a thermal dissipation mechanism in addition to the electrical dissipation (Joule heat). Inclusion of this thermal dissipation yields results which are qualitatively suggestive of those which we present here.

INTRODUCTION

EARLIER measurements[1] of the flux-flow resistivity ρ_F in type-II superconducting foils have shown that at $T=0°K$, ρ_F is proportional to the magnetic field H, or

$$\rho_F/\rho_N=H/H_{c2}(0), \quad (1)$$

where ρ_N is the resistivity in the normal state and $H_{c2}(0)$ is the upper critical field appropriate to $T=0°K$. Kim *et al.*[1] have explained this relation by noting that the right side represents the fraction of the material occupied by the fluxoid cores, while Bardeen and Stephen[2] obtain the same result by means of a more detailed calculation.

For nonzero temperatures, ρ_F is linear in field only at low fields, showing a more rapid variation at higher fields so that $\rho_F=\rho_N$ at $H=H_{c2}(T)$. There have been recent theoretical attempts to account for the field dependence of ρ_F for $T_c>T>0°K$. In particular, Schmid[3] has obtained a time-dependent Ginzburg-Landau equation valid in the region of $T=T_c$, and calculated the slope of ρ_F/ρ_N at H_{c2}. Caroli and Maki[4] have extended the calculations to all temperatures and found that at H_{c2}

$$\rho_F/\rho_N=1-[4\kappa_1^2(0)/1.16[2\kappa_2^2(T)-1]][1-H/H_{c2}(T)]. \quad (2)$$

The temperature dependence of the flow resistivity thus derives from the temperature variation of $\kappa_2(T)$ and $H_{c2}(T)$. Schmid's results[3] are equivalent to Eq. (2) at $T=T_c$.

For comparison with experiment it is convenient to work with the dimensionless ratio

$$\alpha=[Hd(\rho_F/\rho_N)/dH]_{H=H_{c2}}, \quad (3)$$

where $\alpha=4\kappa_1^2(0)/1.16[2\kappa_2^2(T)-1]$ according to the Caroli-Maki theory.[4] Substitution for $\kappa_1(0)$ and $\kappa_2(T)$ shows that [for $2\kappa_2^2(T)\gg1$] α increases monotonically from 1.7 at $T=0$ to 2.5 at $T=T_c$.

Schmid,[3] using the data of Kim *et al.* on Nb–Ta alloys,[1] listed experimental values of α. We have not found it possible to verify the values given in the Schmid paper. There are two apparent difficulties in Schmid's procedure. First, it is experimentally difficult to get flux-flow curves very close to H_{c2} which are unaffected by sample heating. This is so even though the critical currents may be relatively small. Hence values of ρ_F determined by dc methods in this region are always questionable. Secondly, ρ_F is changing rapidly at high fields, so that the derivative value at a given field is difficult to specify without knowing the functional form of the variation.

We have reexamined the earlier data of Kim *et al.*[1] and have made additional measurements on Pb–Tl and In–Bi alloys with a range of κ values. We find important disagreements with the Caroli-Maki theory.[4] The field dependence of the flow resistivity at high fields suggests that it may be possible to account for the observed behavior by considering the overlap of fluxoids in the high-field region. Alternatively, Clem[5] has advanced a theory which describes an additional thermal-dissipation mechanism. The field dependence of this thermal dissipation qualitatively suggests the behavior we observe.

SAMPLE PREPARATION AND EXPERIMENTAL PROCEDURE

Our method of sample preparation has been described previously.[6] Alloy ingots were melted and mixed in

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[1] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **139**, A1163 (1965); A. R. Strnad, C. F. Hempstead, and Y. B. Kim, Phys. Rev. Letters **13**, 794 (1964).
[2] J. Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965).
[3] Albert Schmid, Phys. Kondensierten Materie **5**, 302 (1966).
[4] Christiane Caroli and Kazumi Maki, Phys. Rev. **164**, 591 (1967).
[5] John Clem, Phys. Rev. Letters **20**, 735 (1968); and (private communication).
[6] W. C. H. Joiner and G. E. Kuhl, Phys. Rev. **163**, 362 (1967).

TABLE I. T_c and κ values for our various samples. (a) Values estimated in this work. T_c values for the Pb–Tl samples were not measured directly, but were obtained by fitting the measured $H_{c2}(T)$ versus T data to the theoretical variation and extrapolating to $H_{c2}=0$. (b) T_c values estimated from the measurements of Claeson[a] in the Pb–Tl alloy system. (c) T_c and κ values estimated from the measurements of Bon Mardion *et al.*[b] on Pb–Tl alloys. (d) T_c and κ values estimated from measurements of Kinsel *et al.*[c] on In–Bi alloys.

Nominal composition	(a) T_c(°K)	(b) T_c(°K)	(c) T_c(°K)	(d) T_c(°K)	(a) κ	(c) κ	(d) κ
$Pb_{0.95}Tl_{0.05}$	7.05	7.05	7.10	···	1.35	1.4	···
$Pb_{0.60}Tl_{0.40}$	5.87	5.87	6.17	···	5.83	5.9	···
$Pb_{0.39}Tl_{0.61}$	4.57	4.56	4.95	···	4.85	5.6	···
$In_{0.98}Bi_{0.02}$	3.81	···	···	4.00	1.11	···	1.10
$In_{0.96}Bi_{0.04}$	4.42	···	···	4.22	···	···	1.46

[a] Reference 12.
[b] Reference 13.
[c] Reference 14.

Pyrex tubes under a vacuum of 10^{-6} Torr for periods exceeding 1 day. The starting materials were Cominco 69 Grade Pb, Tl, and In and A. D. Mackay 99.999% Bi. The molten ingots were air quenched, and the Pyrex tubes were etched off using boiling hydrofluoric acid. The alloy slugs were then flattened by compression in a hydraulic press.

Individual foils are prepared from a portion of the flattened slug by further compression between glass microscope slides. For the higher concentration alloys this requires a series of compressions for each foil, because the slides usually crack under the pressure. Usually the foils were cleaned with an electropolishing etch, but not necessarily after the last compression. The resulting foils were very smooth and shiny.

The foils were cut into rectangular strips with approximate dimensions of 1.5×0.140×0.005 in. They were then annealed in a vacuum of 10^{-6} Torr for 1 day within a few degrees of their melting point. This procedure produced samples with low critical currents,[7] and we did not find that longer annealing times reduced the critical currents to any measurable extent. We have never observed the "peak effect" reported by Swartz and Hart[8] in the Pb–Tl system, presumably indicating that the degree of perfection of the samples could be improved. We did notice some tarnishing of the sample surface while mounting the sample, and this may be the factor which sets the lower limit on our critical currents.

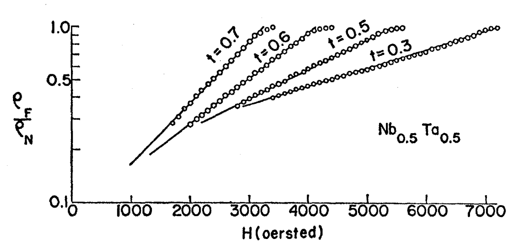

FIG. 1. Plot of $\log(\rho_F/\rho_N)$ versus H for $Nb_{0.5}Ta_{0.5}$ foil at different values of reduced temperature, $t=T/T_c$, using data of Kim *et al.* (Ref. 1).

[7] W. C. H. Joiner and G. E. Kuhl, Phys. Rev. **168**, 413 (1968).
[8] P. S. Swartz and H. R. Hart, Phys. Rev. **137**, A818 (1865).

The sample rig is a four-probe device fabricated from nylon and designed so that the sample received maximum exposure to the liquid-helium bath. The sample is mechanically clamped between thick copper current contacts at its ends and between copper potential leads 1.6 cm apart in the central portion. It is possible that these pressure contacts might cold-work the sample or that the differential rates of contraction of the sample and holder might strain the sample when the temperature is reduced to the liquid-helium range. Although we have no way of testing this directly, the current-voltage characteristics were quite linear at sufficiently high currents,[9] and thus the level of sample inhomogeneity would seem to have been low.

All flux-flow data were obtained in perpendicular magnetic fields. The samples were first aligned parallel to the field by searching for the position of a resistance minimum with fixed current and a fixed field just slightly larger than H_{c2}. The value of H_{c2} itself was determined at the position of the minimum by measuring the critical currents necessary to produce a 1-μV voltage drop across the sample for various fields. On a plot of $\log I_c$ versus H, the quantity H_{c2} was taken as the sharp discontinuity in the curve where the supercurrents switch from being totally surface currents to include bulk currents.[10] The uncertainty in this defini-

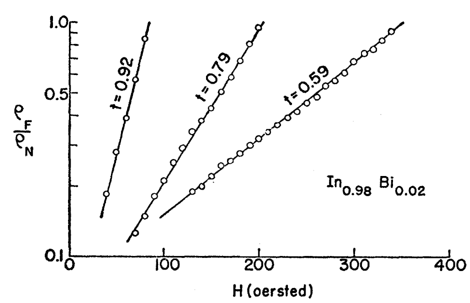

FIG. 2. Plot of $\log(\rho_F/\rho_N)$ versus H for $In_{0.98}Bi_{0.02}$ foil at various reduced temperatures, $t=T/T_c$.

[9] W. C. H. Joiner, Phys. Rev. Letters **19**, 895 (1967).
[10] R. V. Bellau, Proc. Phys. Soc. (London) **91**, 144 (1967).

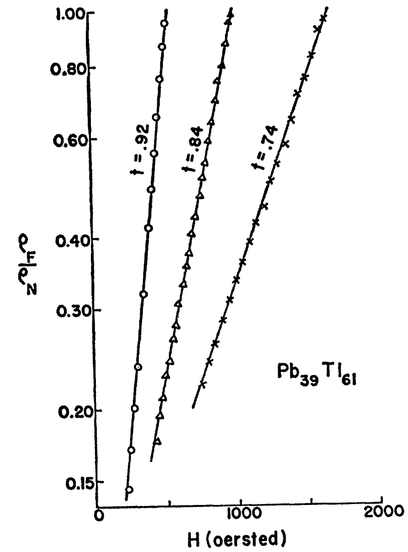

FIG. 3. Plot of $\log(\rho_F/\rho_N)$ versus H for $Pb_{0.39}Tl_{0.61}$ foil at various reduced temperatures, $t=T/T_c$. Note the general result that the fraction of ρ_F/ρ_N covered by the exponential plot increases as the reduced temperature increases.

tion of H_{c2} is determined by the width of the discontinuity, which was never more than a few oersted.

Current-voltage characteristics were obtained continuously on a Moseley Model 7000-A X-Y recorder. The sample voltage was amplified on a Keithley No. 148 Nanovoltmeter, and fed directly into the recorder. Voltage levels used on the various samples varied from a fraction of a millivolt to several millivolts. The voltage signal for the current axis was obtained from a $\frac{1}{10}$-Ω standard resistor in series with the sample.

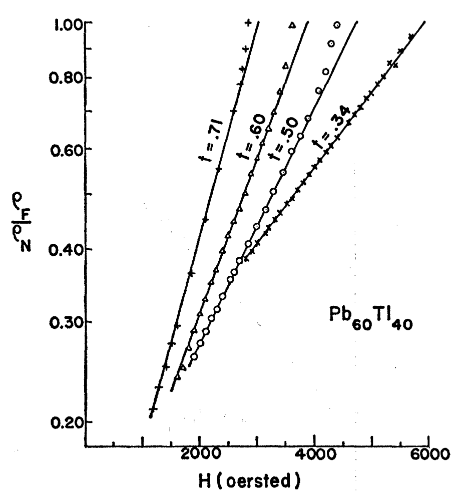

FIG. 4. Plot of $\log(\rho_F/\rho_N)$ versus H for $Pb_{0.60}Tl_{0.40}$ foil sample showing effects of sample heating. Values of (ρ_F/ρ_N) are too high in the vicinity of H_{c2} for $t=0.71$, 0.60, and 0.50. At $t=0.34$, below the λ point, the exponential holds almost to $\rho_F/\rho_N=1.00$, presumably because of the greater efficiency of the superfluid helium in removing heat. Other samples of this composition, but with lower critical currents, also followed the exponential more closely, even at temperatures above the λ point.

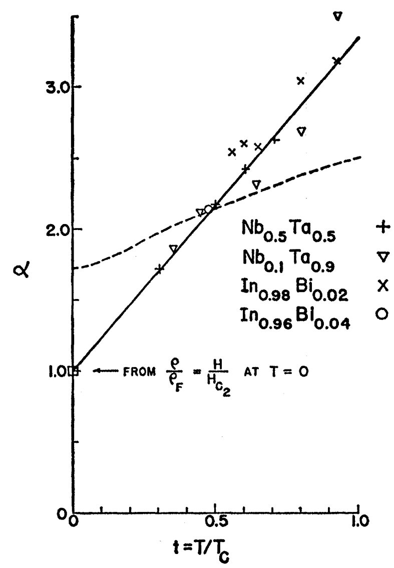

FIG. 5. The dimensionless parameter

$$\alpha=[(H/\rho_N)(d\rho_F/dH)]_{H=H_{c2}}$$

is plotted as a function of reduced temperature, $t=T/T_c$, for Nb–Ta and In–Bi alloys. The solid curve through the data points is drawn to conform to the requirement (Ref. 1) that $\rho_F/\rho_N=H/H_{c2}(0)$ at $t=0$. The dashed curve is that given by Caroli and Maki (Ref. 4) when $2\kappa_2^2(T)\gg1$. Although the data show the predicted monotonic increase of α with t, the disagreement between experiment and theory is large.

EXPERIMENTAL RESULTS AND DISCUSSION

The relevant parameters for our various samples are listed in Table I. We note that the samples represent a range of κ values extending from $\kappa=1.1$ to $\kappa=5.8$. The T_c values for the Pb–Tl samples listed in Table I were not measured directly but were determined by fitting the measured temperature variation

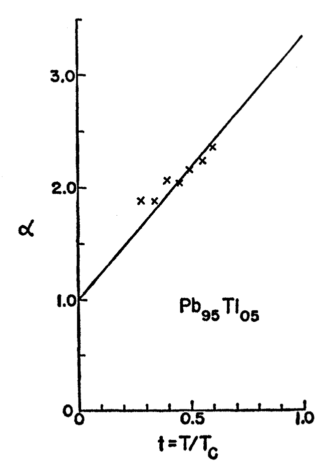

FIG. 6. The parameter α is plotted as a function of reduced temperature, $t=T/T_c$, for $Pb_{0.95}Tl_{0.05}$. The solid curve is the one drawn through the data of Fig. 5 and which satisfies the requirement that $\rho_F/\rho_N=H/H_{c2}(0)$ at $t=0$.

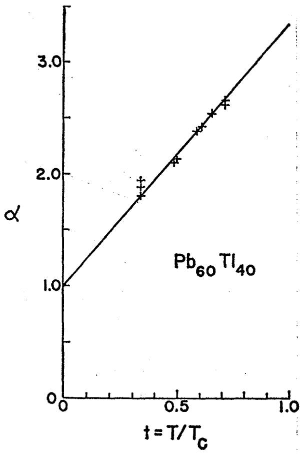

FIG. 7. The parameter α is plotted as a function of reduced temperature, $t=T/T_c$, for $Pb_{0.60}Tl_{0.40}$. The solid curve is that drawn through the data in Figs. 5 and 6 and which satisfies $\rho_F/\rho_N=H/H_{c2}(0)$ at $t=0$.

of H_{c2} to the temperature variation predicted by the theory, using Maki's temperature dependence for κ in the dirty limit.[11] Excellent agreement is obtained between these T_c values and T_c's determined in this alloy system by Claeson.[12] The agreement with T_c data obtained by Bon Mardion *et al.*[13] is not as good. Our estimated κ values for the $Pb_{0.95}Tl_{0.05}$ and $Pb_{0.60}Tl_{0.40}$ samples agree with the data of Bon Mardion *et al.*,[13] but for our $Pb_{0.39}Tl_{0.61}$ sample the κ value is about 10% lower. For our In-Bi samples, a comparison with data from Kinsel *et al.*[14] yields fair agreement.

Our analysis of the flux-flow results of Kim *et al.*[1] and our own data may be summarized as follows:

(1) At nonzero temperatures and at fields above which ρ_F is linear in H, the normalized flow resistivity can be represented by

$$\rho_F/\rho_N=e^{A(H-H_{c2})} \tag{4}$$

over a considerable range of resistivity. We show this in Fig. 1 for $Nb_{0.5}Ta_{0.5}$, where we have replotted the data of Kim *et al.*,[1] and in Figs. 2 and 3 for $In_{0.98}Bi_{0.02}$ and $Pb_{0.39}Tl_{0.61}$, using our own data. We draw special attention to the fact that the range of ρ_F/ρ_N for which the data can be fitted to Eq. (4) increases as the temperature approaches T_c.

By fitting our data to Eq. (4) it was also possible to distinguish points which were affected by sample heating. This is always a relevant consideration in flux-flow measurements. We have found that the flow resistivity is especially sensitive to heating in the immediate vicinity of H_{c2}, the specific field range where we wish to make a comparison with the Caroli-Maki theory.[4] When there is heating, the measured flow resistivity appears to have too high a value. We show an example of such heating in Fig. 4 for a $Pb_{0.60}Tl_{0.40}$ sample. Note the deviations from Eq. (4) near H_{c2}. Below the λ point ($t=0.34$) in Fig. 4, where the bath is more effective in dissipating the heat, the data more closely follow the exponential relationship. Reducing the critical currents in a sample also tends to eliminate this problem.

(2) Using Eq. (4), $\alpha=AH_{c2}$ at the extrapolated field where $\rho_F/\rho_N=1$, and we confirm the Caroli-Maki result[4] that α increases monotonically with increasing values of temperature. This can be seen in Fig. 5, where the experimentally determined values are plotted as a function of reduced temperature $t=T/T_c$, for the Nb–Ta and In–Bi alloys. In Figs. 6–8 we show the data for $Pb_{0.95}Tl_{0.05}$, $Pb_{0.60}Tl_{0.40}$, and $Pb_{0.39}Tl_{0.61}$. Despite the scatter in the data, it seems reasonable to conclude that α for all compositions (and hence for our various κ values) can be represented by a single curve linear in reduced temperature.

(3) Quantitative comparison with the Caroli-Maki results[4] [displayed as the dashed line in Fig. 5 for $2\kappa_2^2(T)\gg1$] shows that the measured values of α do not agree with the theory. We note in this regard that the Caroli-Maki value of α at $T=0°K$ is $\alpha(0)=1.7$, while according to Eq. (1) $\alpha(0)=1.0$.

We have already noted that we have independently determined H_{c2} in parallel fields by critical-current measurements. The transitions are in all cases quite sharp and H_{c2} is well defined. However, H_{c2} determined in this way is consistently about 5% lower than H_{c2} obtained by extrapolation of the exponential curves of the flux-flow resistivity. Essentially identical results are obtained for plated and unplated foils. We do not understand the origin of this discrepancy, which could conceivably account for a 5% adjustment in our ex-

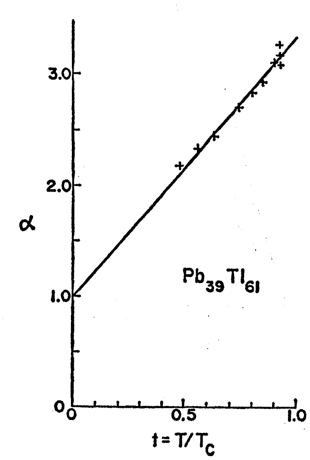

FIG. 8. The parameter α is plotted as a function of reduced temperature, $t=T/T_c$, for $Pb_{0.39}Tl_{0.61}$. The solid line also fits the data of Figs. 5–7 and also satisfies $\rho_F/\rho_N=H/H_{c2}(0)$ at $t=0$.

[11] Kazumi Maki, Physics **1**, 21 (1964).
[12] T. Claeson, Phys. Rev. **147**, 340 (1966).
[13] G. Bon Mardion, B. B. Goodman, and A. Lacaze, J. Phys. Chem. Solids **26**, 1143 (1965).
[14] T. Kinsel, E. A. Lynton, and B. Serin, Rev. Mod. Phys. **36**, 105 (1964).

perimental values of α. The disagreement with the Caroli-Maki theory far exceeds this possible adjustment.

In many samples we also found that above H_{c2} the flux-flow current-voltage curves did not pass exactly through the origin until we exceeded fields approximately equal to H_{c3}. That is, our samples continued to pass small supercurrents above H_{c2}, even in perpendicular fields. This is in contradiction to Hempstead and Kim,[15] who have reported that in perpendicular fields the full normal resistance is restored at H_{c2}. A natural explanation for our result might be that the foil edges, being parallel to the field, can support a surface current to H_{c3}. However copper plating the edges did not remove this effect, although it could be reduced by etching the sample surface. In general, the slope of the characteristic was essentially constant above H_{c2}, varying at most by a few percent. In certain samples the effect was barely detectable. We therefore do not believe that it contributes any appreciable error to the determination of α, at least insofar as our extrapolation procedure using Eq. (4) is valid. On the other hand, if one uses data just at H_{c2}, then even in the absence of sample heating there may still be a small error in the determination of the flux-flow resistivity.

The different functional dependence of the flow resistivity on field in the low- and high-field regions suggests that different mechanisms dominate in the two regions. At low fields where $\rho_F \sim H$ the incremental change in resistivity with field can be accounted for by the addition of independent fluxoids. At high fields the exponential dependence of the resistivity on field indicates that the incremental increase of resistivity with field is proportional to the existing resistivity value. This suggests that the additional dissipation may depend on the overlap of the existing fluxoid structure with those fluxoids added by the incremental increase of H.

As an alternative to this hypothesis, we note a recent calculation by Clem.[5] Clem points out the necessity for the existence of a temperature gradient across a moving fluxoid, and uses this to calculate an extra dissipation in addition to the electrical dissipation (Joule heat) considered previously.[1,2] This additional dissipation can be described by a thermal-viscosity coefficient η_{th}. The total viscosity coefficient is the algebraic sum of the separate electrical- and thermal-viscosity coefficients. At nonzero temperatures, η_{th} decreases with increasing field and becomes zero at $H = H_{c2}$. Since ρ_F is inversely proportional to η, this tends to reduce the value of ρ_F at low fields, but causes it to vary more rapidly than the linear variation of Caroli and Maki [Eq. (2)] near H_{c2}. Qualitatively, this is just the behavior we observe. However, there is not quantitative agreement between our α values and those based on the Clem theory.

Finally, we note that the Caroli-Maki calculations are strictly valid only at H_{c2}. Because of the various experimental difficulties we have enumerated, we have not been able to extend our measurements closer to H_{c2} than the fields represented by values of ρ_F/ρ_N of approximately 0.96–0.97, and still have confidence in the resistivity values. We believe that this constitutes the upper limit for unambiguous direct flux-flow measurements. However, the microwave-resistance measurements described by Rosenblum and Cardona[16] do not seem to have the same experimental limitations and might be usefully employed in examining this field region.

Note added in proof. Recent measurements [J. A. Cape and I. F. Silvera, Phys. Rev. Letters **20**, 326 (1968)] indicate that fluxoids can be depinned by the application of an rf magnetic field parallel to the sample surface. The resistivity is found to depend on the amplitude of the rf fields and for certain amplitudes the flux-flow curves become Ohmic and yield a resistivity versus dc field relation which is in substantial agreement with Caroli and Maki near T_c. We find it difficult to understand this result in view of the fact that the flux-flow resistivity is observed to be independent of pinning in the absence of rf fields. This would seem to imply that the pinning forces contribute to the dissipation, in contradiction to our present understanding of the dissipative processes.

ACKNOWLEDGMENTS

We have profited from discussions of Caroli and Maki's work with Professor B. Goodman. We are indebted to Dr. John Clem for his suggestions about the role of the thermal dissipation, and for a copy of his work prior to publication. Our work was made possible through the assistance of the University of Cincinnati Institute for Space Sciences and its NASA grant.

[15] C. F. Hempstead and Y. B. Kim, Phys. Rev. Letters **12**, 145 (1964).

[16] B. Rosenblum and M. Cardona, Phys. Rev. Letters **12**, 657 (1964).