scheme. Such processes may either introduce some slow time dependence of the metastable states considered or perhaps destroy the picture completely. By using ac fields with a characteristic time shorter than the relaxation time of the metastable states one might be able to see the influence of the sheath in the different regions of the κ-h_0 plane that we have considered. Also preparation of even more perfect sample surfaces would assist in providing a check on the adequacy of the present theory.

ACKNOWLEDGMENTS

The numerical integrations were performed on a GIER computer at the University of Copenhagen and on an IBM 7090 at NEUCC, Lundtofte, Denmark. We thank Professor H. Høјgaard Jensen for valuable suggestions and for a critical reading of the manuscript. Thanks are also due to Dr. J. P. McEvoy for information and discussion concerning his preliminary results.

PHYSICAL REVIEW VOLUME 171, NUMBER 2 10 JULY 1968

Complex Radio-Frequency Impedance of Type-II Superconductors*

George E. Possin† and Kenneth W. Shepard†
Department of Physics, Stanford University, Stanford, California
(Received 17 November 1967)

The complex ac impedance of a type-II superconductor in the intermediate state has been measured between 3 and 40 MHz. The results are compared with a model of vortices acted on by a pinning force and the Lorentz force. Also, the inertial inductance of the superelectrons has been measured at 10 MHz and is shown to be sufficiently large, for thin films, to provide a convenient measure of the penetration depth.

THE purpose of this paper is to point out that the motion of Abrikosov vortices can change not only the real but also the imaginary part of the complex ac impedance and to show that a simple model gives reasonably good agreement with experiment on thin Al films between 3 and 40 Mc/sec. It is also shown that the well-known inertial inductance of the superelectrons is not always negligible at low radio frequencies and that it provides a simple method for the measurement of the penetration depth in thin films.

In the last few years a considerable amount of evidence has demonstrated that most of the dissipation and hysteresis observed in type-II superconductors can be related to the motion of Abrikosov vortices.[1–4] The vortices are assumed to move under the influence of three forces: a Lorentz force $\mathbf{F}_L = c^{-1}\mathbf{J}\times\mathbf{\Phi}_0$, a structure-dependent pinning force $\mathbf{F}_p$, and a dissipative force $-\eta\mathbf{V}_L$. All forces are defined for a unit length of the vortex. Where possible we follow the notation of Kim *et al.*[1] The dissipation is thought to be due to the flow of normal currents in the core and surrounding region as discussed by Bardeen and Stephen.[4] The pinning force $\mathbf{F}_p$ is attributed to lattice defects.

The typical dc behavior for thin-film type-II superconductor to a normal field $H \gg H_{c_1}$ is shown in Fig. 1. This can be understood as follows. If α_c is the maximum value of the pinning force F_p, then for $F_L < \alpha_c$ the vortices do not move and the flow resistivity $\rho_f = 0$. For $F_L \gg \alpha_c$ the vortices will move with a velocity $V_L = F_L\eta^{-1}$, where for the moment we consider a defect-free sample (i.e., $\alpha_c = 0$). Therefore, we have

$$\rho_f = (B/\Phi_0)F_L V_L/J^2 = B\Phi_0/\eta c^2. \tag{1}$$

In a real sample we must consider the complex problem of scattering of vortices from the defects. At constant voltage this scattering leads to an additional dissipation as discussed by Yamafugi and Irie.[5] Kim *et al.*[1] have shown experimentally that for their samples this does not influence the slope of the V-I curve. We will assume that Eq. (1) holds if ρ_f is defined from the slope of the V-I curve.

We consider a single vortex in a potential well arising from the elastic displacement of the vortex relative to its pinning center. If the displacement is small we

---

* Work supported in part by the National Science Foundation, Army Research Projects Agency and the U.S. Office of Naval Research.

† National Science Foundation Fellow.

[1] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. **139**, A1163 (1965).

[2] Y. B. Kim, C. F. Hempstead, and A. R. Strnad, Phys. Rev. Letters **9**, 306 (1962).

[3] A. R. Strnad, C. F. Hempstead, and Y. B. Kim, Phys. Rev. Letters **13**, 794 (1964).

[4] J. Bardeen and M. J. Stephen, Phys. Rev. **140**, A1197 (1965).

[5] K. Yamafuji and F. Irie, Phys. Letters **25A**, 387 (1967).

assume the equation of motion

$$m\ddot{x}+\eta\dot{x}+kx=c^{-1}J\Phi_0, \quad (2)$$

where m is the effective mass per unit length of the vortex and kx is the first-order approximation to the pinning force. Setting $x(t)=\exp(i\omega t)$, we find

$$x=J\Phi_0/c[(k-m\omega^2)+i\omega\eta]. \quad (3)$$

The effective mass has been calculated by Suhl[6] and is so small that it is probably negligible for most materials even at microwave frequencies. The complex impedance Z for a sample of length l, width w, and thickness d, in a normal field B is then just the ratio of the complex power to the square of the current.

$$Z=(Bwld/\Phi_0)F_L\dot{x}/I^2$$
$$=(Bwld/\Phi_0)[(i\omega x)F_L/(wdJ)^2] \quad (4)$$

or

$$Z=R_\infty[\omega^2/(\omega^2+\omega_0^2)+i\omega\omega_0/(\omega^2+\omega_0^2)], \quad (5)$$

where

$$R_\infty=l/wd(B\Phi_0/c^2)\eta^{-1} \quad (6)$$

is the value of Z at $\omega\gg\omega_0$, and $\omega_0=\eta^{-1}k$. Comparison of (1), (4), and (5) shows that $R_\infty=R_f$, where $R_f=lw^{-1}d^{-1}\rho_f$ is the dc flux-flow resistance. Note that the reactive term is inductive. This represents the potential energy of the vortex in its potential well. The inductive impedance reaches its maximum value at $\omega=\omega_0$ where $\mathrm{Re}(Z)=\mathrm{Im}(Z)=\frac{1}{2}R_\infty$.

This model was first proposed by Gittleman and Rosenblum.[7] In their paper they ignore the imaginary part of the impedance. In a later paper[8] they correct this and their data on the magnitude of Z for foils of Pb–In and Nb–Ta fit the above model with ω_0 as a free parameter. In an experiment on ultrasonic attenuation in type-II superconductors, Shapira and Neuringer[9] have shown that if this complex impedance is included in an extension of the Alpher-Rubin theory of ultrasonic attenuation, then reasonable agreement with theory is obtained.

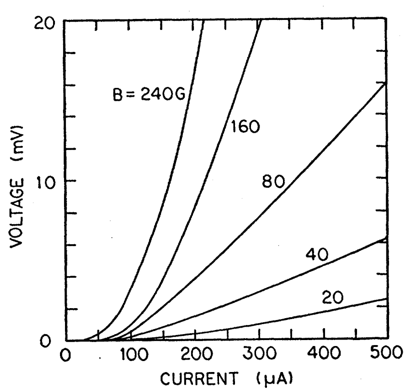

FIG. 1. I–V characteristics for an aluminum film 200 Å thick with $T_c=1.7$°K taken at $T=1.33$°K for various values of perpendicular magnetic field. The normal resistance of the film was 435 Ω. $H_{c2}=300$ G.

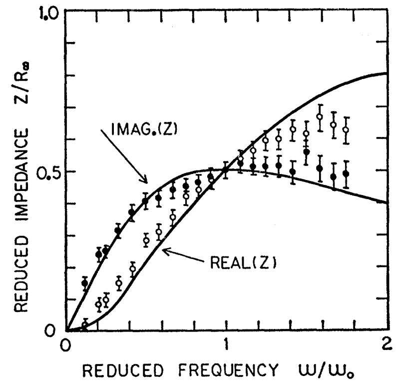

FIG. 2. Complex impedance versus frequency at $T=1.33$°K with a perpendicular magnetic field of 80 G. The solid lines are the real and imaginary parts of the impedance as given by Eq. (5); we have taken $\omega_0/2\pi=24$ Mc/sec and $R_\infty=38\,\Omega$ to obtain the best fit.

Our samples were thin (∼200 Å) films of aluminum evaporated from a tungsten filament at $p\sim5\times10^{-6}$ Torr onto glass microscope slides. After evaporation the films were scribed using the compound from a jeweler's lathe to move the substrate under a scribe which was an ordinary razor blade. The films were scribed in a zig-zag geometry to give a strip 0.8±0.01 mm in width and 11 cm in length. The scribe lines were sharp to about 2 μ. With better scribes such as sharpened steel needles, scribe lines which are sharp and parallel to about 1 μ can be produced on soft films such as aluminum or tin.

The low-temperature apparatus consisted of a stainless-steel He^4 Dewar in a He^4 bath. Using a booster-type diffusion pump, temperatures down to 0.9°K could be maintained. The temperature was stabilized electronically and measured using a carbon resistor calibrated against vapor pressure.

The sample was mounted at the end of a stainless-steel coaxial line and the impedance of the sample was calculated from the impedance at the top of the Dewar, measured by a General Radio type 916-A rf bridge. The detector was a communications receiver followed by a Princeton Applied Research lock-in detector and preceded by a low-noise cascode-type preamplifier which assured isolation of the sample from the receiver and improved the sensitivity of the detector to about 0.05 μV at 10 Mc/sec. A 10-Mc/sec fixed-frequency bridge was built and calibrated directly against the first bridge. It

[6] H. Suhl, Phys. Rev. Letters **14**, 227 (1965).

[7] Jonathan I. Gittleman and Bruce Rosenblum, Phys. Rev. Letters **16**, 734 (1966).

[8] Jonathan I. Gittleman and Bruce Rosenblum, in *Proceedings of the Conference on Low Temperterature Physics, Moscow, 1966* (Proczvodstrenno-Izdatelski, Kombinat, VINTI, Moscow, 1967).

[9] Y. Shapira and L. J. Neuringer, Phys. Rev. **154**, 375 (1967).

measured sample impedance in the range 0–30 Ω and had a sensitivity of about 0.1 Ω at 10-μA sample current, as compared to 0.5 Ω for the General Radio bridge. This enabled a more sensitive test of the field dependence at a fixed frequency.

The result for a typical sample is shown in Fig. 2. The transition temperature of 1.7°K is somewhat enhanced from the value for bulk aluminum. This effect has been observed by many investigators. The solid lines are the theoretical curves with $\omega_0/2\pi=$ 24 Mc/sec and $R_\infty=38$ Ω. R_f, which should equal R_∞, is 43±3 Ω as determined from the dc data for the same sample shown in Fig. 1. The geometric inductance is subtracted out by taking the zero point at $B\sim10^{-3}$ G and $T/T_c\sim0.5$. Both show roughly the predicted linear dependence.

In zero field the ac impedance consists of an inductive term due to the inertia of the superconducting electrons and a resistive term due to the dissipative flow of the normal electrons. This can be calculated from the two-fluid model, where the superconducting electrons obey the London equations. Assuming $\exp(i\omega t)$ time dependence,

$$i\omega(4\pi\lambda^2/c^2)\mathbf{J}_s=\mathbf{E}, \tag{7}$$

$$\mathbf{J}=\mathbf{J}_s+\mathbf{J}_n, \tag{8}$$

$$\mathbf{J}_n=(n_n/n)\sigma_n\mathbf{E}, \tag{9}$$

where $\mathbf{J}_n$ and $\mathbf{J}_s$ are the normal and superconducting current densities, λ is the penetration depth, n is the total electron density, and n_n is the normal electron density. Assuming the usual slab geometry we can solve the above equations for the impedance Z:

$$Z=V/I=l\,|\,\mathbf{E}\,|/wd\,|\,\mathbf{J}\,| \tag{10}$$

or

$$Z=(1/R+1/i\omega L)^{-1}\cong\omega^2L^2/R+iwL, \tag{11}$$

where we have assumed the usual case of $R_{\text{norm}}\gg\omega L(T)$ and we have taken

$$L=(l/wd)4\pi\lambda^2/c^2 \tag{12}$$

and

$$R=(l/wd)n/n_n\sigma_n. \tag{13}$$

The existence of the inductive term has been verified many times at microwave frequencies, most recently by the work of Gittleman *et al.*[10] What we wish to point out is that this term can dominate the field reactance and Re(Z) even at low radio frequencies. In particular, near T_c where $\lambda(T)\cong\lambda(0)T_c/4\Delta T$ we have seen an inertial inductance as large as 1 μH for 5-μ-wide strips of evaporated aluminum 50 Å thick and 3 cm long with $i_c\sim1$ μA. For some samples extrapolation from higher values of applied field indicated that the vortex contribution was negligible even in the Earth's field. However, in a few samples shielding to $B\sim1$ mG was not sufficient to eliminate the vortex contribution because η and k change rapidly near T_c. From the slope of a plot of inverse inertial inductance versus temperature the extrapolated value of the penetration depth at zero temperature can be calculated. For our aluminum samples we find 20% agreement with $\lambda=0.64\lambda_b(\xi_0/d)^{1/2}$,[11] where we take $\lambda_b=500$ Å as measured by Faber and Pippard[12] and $\xi_0=16\,000$ Å as calculated from the BCS theory, by Lynton.[13] The film thickness d was determined from the resistance change from room temperature to 4°K.

In conclusion, reasonable agreement is obtained between this elementary theory and our experimental results. However, ωL and R versus applied field are not perfectly linear and also the frequency dependence of Z shows definite deviations from theory. These deviations indicate the presence of more than one pinning strength. In summary, the observation of a complex impedance which is in agreement with the model of vortices undergoing heavily damped forced harmonic motion in pinning wells is further confirmation of the present description of vortex dynamics in type-II superconductors.

We are indebted to Professor W. A. Little for his guidance and for many helpful discussions throughout the experiment.

---

[10] J. Gittleman, B. Rosenblum, T. E. Seidel, and W. A. Wicklund, Phys. Rev. **137**, A527 (1965).

[11] P. G. de Gennes, *Superconductivity of Metals and Alloys* (W. A. Benjamin, Inc., New York, 1966), p. 225.

[12] T. E. Faber and A. B. Pippard, Proc. Roy. Soc. (London) **A231**, 336 (1955).

[13] E. A. Lynton, *Superconductivity* (John Wiley & Sons, Inc., New York, 1964), p. 65.