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Effect of Nonparabolicity on Light Scattering from Plasmas in Solids

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The effect of band nonparabolicity on the scattering of light from a plasma in a solid is discussed. Quasi-elastic scattering, which in an ordinary plasma is suppressed by Coulomb effects, can occur with a reasonable cross section in a nonparabolic plasma. Detailed calculations are performed for the cases of InSb and InAs. They indicate that the quasi-elastic scattering could be used to measure electron velocity distributions in these crystals. Magnetic field effects are discussed briefly, and an argument is given to indicate why the Landau-Raman and plasmon scattering do not mix appreciably.

I. INTRODUCTION

DURING the past two or three years, it has become apparent that the interaction of light with mobile electrons in crystals is a considerably more complicated phenomenon than its interaction with free, classical electrons. This difference is due to the complicated dynamics of electron motion in solids. The velocity of band electrons is generally a nonlinear function of their momenta. As a consequence, such electrons give rise to nonlinear optical processes that have no counterparts in a classical electron system. These effects have been studied both experimentally[1] and theoretically.[2] They are of interest as probes of electronic behavior in semiconducting crystals and as the basis for optical devices.[3]

To date, theoretical work concerning the nonlinear optical behavior of electrons in crystals has been concerned with the response of a single electron to the electromagnetic field. Collective effects have been discussed[4] within the framework of classical electron dynamics, but no attempt has been made to see how they are modified when the electrons have a nonparabolic energy-momentum relation. Our paper considers this question. We will see that nonparabolicity appreciably changes the collective response of electrons in solids. In particular, it gives rise to a new form of nearly elastic light scattering, which might be used to determine electron temperatures and velocity distributions in nonequilibrium situations. It also enables a light wave to couple to modes that are normally optically inactive.

To understand the novel features of light scattering from plasmas in crystals, we must briefly recall the analogous problem for a classical plasma. This problem has been extensively discussed,[5] and the results are now well known. In a classical plasma, light scattering is caused by electron density fluctuations. More specifically, the differential cross section for light scattering is proportional to the $(\mathbf{q}, \omega)$ Fourier component of the electron density-density correlation function. Here $\mathbf{q}$ is the difference of the wave vectors of the incident and scattered light waves, and ω is the corresponding frequency difference. Two quite different regimes are possible depending upon whether q is large, or small, compared to the characteristic wave vector q_D of the plasma (q_D is the Debye wave vector in a Maxwellian plasma, the Fermi-Thomas wave vector in a degenerate one). When $q \gg q_D$, collective effects are unimportant, and scattering is essentially caused by individual electrons in the plasma. The frequency of the scattered light is then very slightly shifted from that of the incident radiation because the electrons that cause the scattering are moving. Such scattering is often termed *quasi-elastic.* In quasi-elastic scattering $\omega/q \simeq v$, where v is the average velocity of electrons in the plasma. Its spectrum is a direct measure of the electron velocity distribution.

On the other hand, when $q \ll q_D$, light scattering is drastically modified by collective effects. The plasma now behaves as a continuum and quasi-elastic scattering is suppressed. Instead, one observes Raman scattering from collective modes. A single component plasma has one such mode, the plasma oscillation, with frequency $\omega_p^2 = 4\pi ne^2/m$.

In gas plasmas, the quantity (q/q_D) can be varied through unity. Both quasi-elastic[6] and Raman[7] scattering can be observed. The former has been used to

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[2] P. A. Wolff, Phys. Rev. Letters **16**, 225 (1966); Y. Yafet, Phys. Rev. **152**, 858 (1966); P. L. Kelley and G. B. Wright, Bull. Am. Phys. Soc. **11**, 812 (1966); J. W. McCaffrey, Jr., *ibid.* **12**, 890 (1967); P. A. Wolff and Gary A. Pearson, Phys. Rev. Letters **17**, 1015 (1966); B. Lax, W. Zawadski, and M. H. Weiler, *ibid.* **18**, 462 (1967).

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[5] J. P. Dougherty and D. T. Farley, Proc. Roy. Soc. (London) **A259**, 79 (1960); E. E. Salpeter, Phys. Rev. **120**, 1528 (1960); J. A. Fejer, Can. J. Phys. **38**, 1114 (1960); M. N. Rosenbluth and N. Rostoker, Phys. Fluids **5**, 776 (1962).

[6] W. E. R. Davies and S. A. Ramsden, Phys. Letters **8**, 179 (1964); H. J. Kunze, E. Fünfer, B. Kronast, and W. H. Kegel, *ibid.* **11**, 42 (1964); H. J. Kunze, Z. Naturforsch. **20a**, 801 (1965); A. W. DeSilva, D. E. Evans, and M. J. Forrest, Nature **203**, 1321 (1964).

[7] S. Ramsden and W. Davies, Phys. Rev. Letters **16**, 303 (1966); B. Kronast, H. Rohr, E. Glock, H. Zwicher, and E. Fünfer, *ibid.* **16**, 1082 (1966); S. Ramsden *et al.*, IEEE J. Quantum Electron. **2**, 267 (1966).

determine the velocity distribution of electrons in gas plasmas. In solid-state plasmas, however, light scattering experiments are invariably done in the collective regime $q \ll q_D$. Such experiments measure properties of the collective modes, but give no information about the velocity distribution. This, at least, is the case in a plasma of particles having a parabolic energy-momentum relation.

The situation is somewhat different in a plasma whose constituents have a nonparabolic energy-momentum relation. In such a medium a light wave can scatter from energy density fluctuations, as well as the usual density fluctuations. To see why this is so we consider a simple case, the Hamiltonian appropriate to n-type InSb or InAs. It is[8]

$$E(p) \simeq (p^2/2m^*) - (1/E_G)(p^2/2m^*)^2, \qquad (1)$$

where m^* is the effective mass and E_G the energy gap. When an electron with such a Hamiltonian is coupled to the electromagnetic field [via the replacement $\mathbf{p} \rightarrow \mathbf{p} - (e/c)\mathbf{A}$] a considerable variety of electron-photon interactions arise. Among them are terms of the form $(p^2/m^*E_G)A^2$ and $(\mathbf{p}\cdot\mathbf{A})^2/m^*E_G$. These interactions have no counterparts for a classical particle. They directly couple two photons to the momentum of the electron and, in a many-electron system, give rise to scattering from fluctuations in the energy-momentum tensor. Such fluctuations can occur without accompanying fluctuations in electron density. As a consequence, they are very much less affected by electron-electron interactions than are the density fluctuations. Quasi-elastic scattering from such fluctuations is *not* suppressed in the limit $q \ll q_D$. It could provide a new tool for studying velocity distributions in materials such as InSb or InAs.

The $(\mathbf{p}\cdot\mathbf{A})^2$ interaction is also[2] believed to be responsible for the Raman scattering of light from electrons in Landau levels (Landau-Raman scattering). In a many-electron system one can show that, in fact, this scattering is due to a collective mode of the electron gas—the Bernstein mode[9] at $\omega = 2\omega_c$ (ω_c = cyclotron frequency). This mode is nearly optically inactive in a classical plasma, but is activated by the band nonparabolicity. It is only very weakly coupled to the plasma oscillation.[9] This fact explains why, in experiments[1] on InAs, one observes no interaction between the plasmon mode and that at $\omega = 2\omega_c$.

In developing the theory of these subjects, we will first consider the problem of light scattering (in the absence of a magnetic field) from a plasma whose constituents have a rather general energy-momentum relation. The derivation of a formula for the scattering cross section is presented in Sec. II. As in the classical case, the cross section is proportional to the Fourier transform (with respect to space and time) of a correlation function for the unperturbed plasma. This function is evaluated in the random phase approximation (RPA).

A specific application of the formulas of Sec. II is made in Sec. III, where we consider the scattering of light from a plasma having an energy-momentum relation of the form given in Eq. (1). This is a good approximation in n-type semiconductors of the InSb type. The scattering formulas are evaluated for Fermi-Dirac and Maxwellian distributions. The Maxwellian case is of particular interest, since the calculation shows that light scattering might be used as a tool to determine electron temperatures in such plasmas.

Finally, in Sec. IV, we consider the problem of light scattering from a plasma in a magnetic field. The complete formulas are very complicated, so we only analyze the case in which $\mathbf{q} \perp \mathbf{B}$ (the applied magnetic field). In this geometry, we show that the Landau-Raman scattering at $\omega = 2\omega_c$ is due to the Bernstein mode. Its coupling to the plasmon can be determined from Bernstein's work, and is weak.

II. LIGHT SCATTERING CROSS SECTION

In this section we will discuss the problem of light scattering from a many-electron system described by the Hamiltonian

$$H = \sum_i [E(\mathbf{p}_i)] + \tfrac{1}{2} \sum_{i \neq j} [e^2/\epsilon_0 r_{ij}]. \qquad (2)$$

ϵ_0 is the static dielectric constant. The single-particle kinetic energy will not, in general, be of the classical form $E(p) = p^2/2m^*$. In Eq. (2) we are using an effective, one-band Hamiltonian to describe electron motion in semiconductors. Such an approach is valid when all photon energies ($\omega_0, \omega_1, \cdots$) are small compared to the energy-band gap.[10] These conditions are fairly well satisfied in many of the experiments referred to above. In the worst case (InSb pumped by a CO_2 laser) one expects corrections of order $(\hbar\omega_0/E_G)^2 \simeq 25\%$. A multiband treatment of the many-electron problem will be required to handle these finite-frequency effects. Some work has been done on this problem[11] but a complete treatment has not been presented. The main effect of working at finite frequency appears to be the replacement of the Thomson cross section $\sigma_T = (e^2/m^*c^2)^2$ by an enhanced cross section $\sigma = \sigma_T\{E_G^2/[E_G^2 - (\hbar\omega_0)^2]\}^2$. This effect will scale up all cross sections, but not appreciably change the frequency spectrum of the scattered light. Since we are mainly interested in the spectrum, we will ignore finite-frequency effects and work with the single-band Hamiltonian. The essential physical ideas we wish to discuss are well illustrated by this model, which is accurate in the limit $\hbar\omega_0/E_G \rightarrow 0$.

Let us now couple the many-electron system to the electromagnetic field. To do this we replace $\mathbf{p}_i$ by

[8] E. O. Kane, J. Phys. Chem. Solids **1**, 249 (1957).
[9] Ira B. Bernstein, Phys. Rev. **109**, 10 (1958).
[10] J. M. Luttinger and W. Kohn, Phys. Rev. **97**, 869 (1955).
[11] A. L. McWhorter and P. N. Argyres, Bull. Am. Phys. Soc. **12**, 102 (1967).

$\mathbf{p}_i-(e/c)\mathbf{A}_i$ in Eq. (2), where $\mathbf{A}_i=\mathbf{A}(\mathbf{r}_i)$ is the vector potential. Strictly speaking, one should symmetrize the resultant expression in $\mathbf{p}_i$ and $\mathbf{A}_i$, since these quantities do not commute. In the applications we will discuss, however, the wave vectors of the light waves are small, and this effect is unimportant. After replacing $\mathbf{p}_i$ by $\mathbf{p}_i-(e/c)\mathbf{A}_i$, we expand H in powers of $\mathbf{A}_i$:

$$H\simeq H_0+H_1+H_2+\cdots=\sum_i[E(\mathbf{p}_i)]+\tfrac{1}{2}\sum_{i\neq j}[e^2/\epsilon_0 r_{ij}]-(e/c)\sum_i[\mathbf{A}_i\cdot\partial E/\partial\mathbf{p}_i]+\tfrac{1}{2}(e/c)^2\sum_i[\mathbf{A}_i\cdot\partial^2E/\partial\mathbf{p}_i^2\cdot\mathbf{A}_i]+\cdots. \quad (3)$$

Here the electrons are linearly coupled to the field via H_1, and quadratically via H_2. For the classical case, it is well known[12] that the H_1 terms make an exceedingly small contribution to the light scattering cross section. The same argument applies when the bands are nonparabolic. Thus, to calculate the cross section, we may drop the H_1 term in Eq. (3) and consider only first-order transitions due to H_2.

For a process in which a light quantum scatters from state $(\mathbf{q}_0,\omega_0)$ to $(\mathbf{q}_1,\omega_1)$, and the electronic system goes from an initial state I to a final state F, the transition rate is

$$\mathcal{R}=(2\pi/\hbar)[(2\pi\hbar c^2)^2/\omega_0\omega_1]\langle I\,|\,\mathcal{H}_2^\dagger\,|\,F\rangle\times\langle F\,|\,\mathcal{H}_2\,|\,I\rangle\delta(E_I+\omega-E_F), \quad (4)$$

where $\mathbf{q}=\mathbf{q}_0-\mathbf{q}_1$, $\omega=\omega_0-\omega_1$,

$$\mathcal{H}_2=(e/c)^2\sum_i[\boldsymbol{\varepsilon}_0\cdot(\partial^2E/\partial\mathbf{p}_i\partial\mathbf{p}_i)\cdot\boldsymbol{\varepsilon}_1e^{i\mathbf{q}\cdot\mathbf{r}_i}] \quad (5)$$

is the electronic portion of H_2, and $\boldsymbol{\varepsilon}_0$ and $\boldsymbol{\varepsilon}_1$ are the polarization vectors.[13]

We may now manipulate Eq. (5) in the usual way[14] to express the cross section in terms of a correlation function. We sum over final states, average over initial, and use the representation of the δ function

$$\delta(x)=(1/2\pi)\int_{-\infty}^{\infty}e^{ixt}dt.$$

After some simple algebra we obtain the result

$$d^2\sigma/d\Omega d\omega=(\omega_1/\omega_0)\int_{-\infty}^{\infty}e^{i\omega t}\langle\mathcal{H}_2^\dagger(t)\mathcal{H}_2(0)\rangle(dt/2\pi), \quad (6)$$

where $\mathcal{H}_2=\mathcal{H}_2(0)$,

$$\mathcal{H}_2^\dagger(t)=e^{iH_0t}\mathcal{H}_2^\dagger(0)e^{-iH_0t}, \quad (7)$$

and the angular brackets indicate the thermal average of the enclosed operators. Equation (6) is our basic formula. In the case of a parabolic band

$$\mathcal{H}_2=(\boldsymbol{\varepsilon}_0\cdot\boldsymbol{\varepsilon}_1)(e^2/m^*c^2)\sum_i[e^{i\mathbf{q}\cdot\mathbf{r}_i}], \quad (8)$$

and is proportional to the Fourier transform of the electron density operator. The quantity $\langle\mathcal{H}_2^\dagger(t)\mathcal{H}_2(0)\rangle$ is then the familiar electron density-density correlation function. On the other hand, in nonparabolic situations $\mathcal{H}_2$ is a more complicated operator and more complicated correlation functions (such as that of energy density) play a role.

To evaluate the correlation function

$$J(t)=\langle\mathcal{H}_2^\dagger(t)\mathcal{H}_2(0)\rangle, \quad (9)$$

it is usually convenient to investigate the closely related Green's function:

$$G(t)=-i\theta(t)\langle[\mathcal{H}_2^\dagger(t),\mathcal{H}_2(0)]\rangle. \quad (10)$$

If the plasma is in thermal equilibrium the Fourier transforms of the two quantities are related by the fluctuation-dissipation theorem[15]

$$J(\omega)=2\{\mathrm{Im}[G(\omega)]\}/[1-e^{-\beta\omega}], \quad (11)$$

where $\beta^{-1}=kT$.[16]

To evaluate G we use the RPA. In second quantized notation

$$H_0=\sum_{\mathbf{k}}[E(k)c_{\mathbf{k}}^\dagger c_{\mathbf{k}}]+\tfrac{1}{2}\sum_{\mathbf{kk}',\mathbf{q}'}[c_{\mathbf{k}+\mathbf{q}'}^\dagger c_{\mathbf{k}'-\mathbf{q}'}^\dagger(4\pi e^2/\epsilon_0q'^2)c_{\mathbf{k}'}c_{\mathbf{k}}] \quad (12)$$

and

$$\mathcal{H}_2=\sum_{\mathbf{k}}[\mathfrak{F}(\mathbf{k},\mathbf{q})c_{\mathbf{k}+\mathbf{q}}^\dagger c_{\mathbf{k}}]. \quad (13)$$

The Green's function takes the form

$$G=-i\theta(t)\sum_{\mathbf{kl}}\{\mathfrak{F}^*(\mathbf{k},\mathbf{q})\mathfrak{F}(\mathbf{l},\mathbf{q})\langle[c_{\mathbf{k}-\mathbf{q}}^\dagger(t)c_{\mathbf{k}}(t),c_{\mathbf{l}}^\dagger(0)c_{\mathbf{l}-\mathbf{q}}(0)]\rangle\}. \quad (14)$$

The equation of motion for the quantity

$$\mathcal{G}(\mathbf{k},\mathbf{l},\mathbf{q})=-i\theta(t)\langle[c_{\mathbf{k}-\mathbf{q}}^\dagger(t)c_{\mathbf{k}}(t),c_{\mathbf{l}}^\dagger(0)c_{\mathbf{l}-\mathbf{q}}(0)]\rangle \quad (15)$$

is

$$i\partial\mathcal{G}/\partial t=\delta(t)\langle[c_{\mathbf{k}-\mathbf{q}}^\dagger c_{\mathbf{k}},c_{\mathbf{l}}^\dagger c_{\mathbf{l}-\mathbf{q}}]\rangle-i\theta(t)\langle[[c_{\mathbf{k}-\mathbf{q}}^\dagger(t)c_{\mathbf{k}}(t),H_0],c_{\mathbf{l}}^\dagger(0)c_{\mathbf{l}-\mathbf{q}}(0)]\rangle=\delta(t)\delta(\mathbf{k}-\mathbf{l})(n_{\mathbf{k}-\mathbf{q}}-n_{\mathbf{k}})$$
$$+[E(\mathbf{k})-E(\mathbf{k}-\mathbf{q})]\mathcal{G}-i\theta(t)\sum_{\mathbf{k}'\mathbf{q}'}\langle[c_{\mathbf{k}-\mathbf{q}}^\dagger(t)c_{\mathbf{k}'-\mathbf{q}'}^\dagger(t)(4\pi e^2/\epsilon_0q'^2)c_{\mathbf{k}'}(t)c_{\mathbf{k}-\mathbf{q}'}(t),c_{\mathbf{l}}^\dagger(0)c_{\mathbf{l}-\mathbf{q}}(0)]\rangle$$
$$+i\theta(t)\sum_{\mathbf{k}'\mathbf{q}'}\langle[c_{\mathbf{k}+\mathbf{q}'-\mathbf{q}}^\dagger(t)c_{\mathbf{k}'-\mathbf{q}'}^\dagger(t)(4\pi e^2/\epsilon_0q'^2)c_{\mathbf{k}'}(t)c_{\mathbf{k}}(t),c_{\mathbf{l}}^\dagger(0)c_{\mathbf{l}-\mathbf{q}}(0)]\rangle, \quad (16)$$

[12] P. M. Platzman and N. Tzoar, Phys. Rev. **136**, A11 (1964).
[13] In this treatment, we are ignoring any transverse currents induced in the electron gas by the perturbation H_2, which is longitudinal. Such currents can occur if the energy-momentum relation is sufficiently anisotropic. However, their effect is usually small. Moreover, they vanish in symmetric cases, such as that of InSb.
[14] L. Van Hove, Phys. Rev. **95**, 249 (1954).
[15] D. N. Zubarev, Usp. Fiz. Nauk **71**, 71 (1960) [English transl.: Soviet Phys.—Usp. **3**, 320 (1960)].
[16] Henceforth, we use units such that $\hbar=1$.

where

$$n_k = \langle c_k{}^\dagger c_k \rangle.$$

The essential approximation[17] of the RPA is to factor the last two terms of Eq. (16), retaining only those terms which force $\mathbf{q}=\mathbf{q}'$. These terms are large in the limit $q\to 0$ and must always be kept, even if the plasma is weakly coupled (which we assume), in the sense that the average potential energy of particles in it is small compared to their kinetic energy. In the RPA, Eq. (16) takes the form

$$\omega\mathcal{G}(\mathbf{k},\mathbf{l},\mathbf{q};\omega) = (1/2\pi)\delta(\mathbf{k}-\mathbf{l})(n_{k-q}-n_k) + [E(\mathbf{k})-E(\mathbf{k}-\mathbf{q})]\mathcal{G}(\mathbf{k},\mathbf{l},\mathbf{q};\omega) + (4\pi e^2/\epsilon_0 q^2)(n_{k-q}-n_k)\sum_{k'}[\mathcal{G}(\mathbf{k}',\mathbf{l},\mathbf{q};\omega)], \quad (17)$$

where $\mathcal{G}(\mathbf{k}, \mathbf{l}, \mathbf{q}; \omega)$ is the Fourier transform of $\mathcal{G}(\mathbf{k}, \mathbf{l}, \mathbf{q}; t)$, and ω has a small, positive imaginary part.

Equation (17) is a trivial integral equation whose solution is

$$\mathcal{G} = \frac{\delta(\mathbf{k}-\mathbf{l})(n_{k-q}-n_k)}{2\pi[\omega-E(\mathbf{k})+E(\mathbf{k}-\mathbf{q})]} + \left(\frac{4\pi e^2}{\epsilon_0 q^2}\right) \times \frac{(n_{k-q}-n_k)(n_{l-q}-n_l)}{2\pi[\omega-E(\mathbf{k})+E(\mathbf{k}-\mathbf{q})][\omega-E(\mathbf{l})+E(\mathbf{l}-\mathbf{q})]} \times[1+(4\pi e^2/\epsilon_0 q^2)\mathcal{L}_0(\mathbf{q},\omega)]^{-1}, \quad (18)$$

with

$$\mathcal{L}_0(\mathbf{q},\omega) = -\sum_k \left[\frac{n_{k-q}-n_k}{\omega-E(\mathbf{k})+E(\mathbf{k}-\mathbf{q})}\right]. \quad (19)$$

Using this result, we may now compute G from Eq. (14):

$$G = (2\pi)^{-1}\sum_l \left[\frac{|\mathcal{F}(\mathbf{l},\mathbf{q})|^2(n_{l-q}-n_l)}{\omega-E(\mathbf{l})+E(\mathbf{l}-\mathbf{q})}\right] + (2\pi)^{-1}(4\pi e^2/\epsilon_0 q^2) \times\left\{[1+(4\pi e^2/\epsilon_0 q^2)\mathcal{L}_0(\mathbf{q},\omega)]^{-1}\sum_k\left[\frac{\mathcal{F}^*(\mathbf{k},\mathbf{q})}{\omega-E(\mathbf{k})+E(\mathbf{k}-\mathbf{q})}\right] \times\sum_l\left[\frac{\mathcal{F}(\mathbf{l},\mathbf{q})}{\omega-E(\mathbf{l})+E(\mathbf{l}-\mathbf{q})}\right]\right\}. \quad (20)$$

As mentioned earlier, we are interested in the behavior of G for small q. In this limit, one may set $q=0$ in the functions $\mathcal{F}(\mathbf{k},\mathbf{q})$ and $\mathcal{F}^*(\mathbf{k},\mathbf{q})$. Also $\mathcal{F}^*(\mathbf{k},0)=\mathcal{F}(\mathbf{k},0)$. With these approximations, G may be rewritten in the rather simple form

$$G = -(2\pi)^{-1}\frac{\mathcal{L}_2(\mathbf{q},\omega)}{[1+(4\pi e^2/\epsilon_0 q^2)\mathcal{L}_0(\mathbf{q},\omega)]} + (2\pi)^{-1}\left(\frac{4\pi e^2}{\epsilon_0 q^2}\right)\frac{[\mathcal{L}_1{}^2(\mathbf{q},\omega)-\mathcal{L}_0(\mathbf{q},\omega)\mathcal{L}_2(\mathbf{q},\omega)]}{[1+(4\pi e^2/\epsilon_0 q^2)\mathcal{L}_0(\mathbf{q},\omega)]}, \quad (21)$$

[17] See, for example, David Pines and Philippe Nozieres, *The Theory of Quantum Liquids, I* (W. A. Benjamin, Inc., New York, 1966).

where we have defined two new functions

$$\mathcal{L}_1(\mathbf{q},\omega) = -\sum_k\left[\frac{\mathcal{F}(\mathbf{k})(n_{k-q}-n_k)}{\omega-E(\mathbf{k})+E(\mathbf{k}-\mathbf{q})}\right], \quad (22)$$

$$\mathcal{L}_2(\mathbf{q},\omega) = -\sum_k\left[\frac{\mathcal{F}^2(\mathbf{k})(n_{k-q}-n_k)}{\omega-E(\mathbf{k})+E(\mathbf{k}-\mathbf{q})}\right]. \quad (23)$$

Equation (21) is convenient for investigating the quasi-elastic scattering of light from the plasma. We are particularly interested in such scattering since it can be used to determine the velocity distribution. As was mentioned in the Introduction, quasi-elastic scattering from a classical plasma (parabolic E-versus-p relation) is strongly suppressed in the limit $q\ll q_D$. This statement may easily be verified from Eq. (21). In the parabolic case $\mathcal{F}=(\boldsymbol{\varepsilon}_0\cdot\boldsymbol{\varepsilon}_1)(e^2/m^*c^2)$, and $\mathcal{L}_1{}^2-\mathcal{L}_0\mathcal{L}_2=0$. The correlation function [see Eq. (11)] becomes (in the limit $kT\to 0$)

$$J(\omega) = (1/\pi)(\boldsymbol{\varepsilon}_0\cdot\boldsymbol{\varepsilon}_1)^2\left(\frac{e^2}{m^*c^2}\right)^2\frac{\mathrm{Im}(\mathcal{L}_0)\theta(\omega)}{|\epsilon(\mathbf{q},\omega)|^2}, \quad (24)$$

where the plasma dielectric constant

$$\epsilon(\mathbf{q},\omega) = [1+(4\pi e^2/\epsilon_0 q^2)\mathcal{L}_0(\mathbf{q},\omega)].$$

Equation (24) is well known. It indicates that the correlation function of a single-component classical plasma differs from that of a gas of noninteracting electrons by the factor $|\epsilon(\mathbf{q},\omega)|^{-2}$. In the quasi-elastic regime this factor produces an enormous reduction in the scattering cross section, since $|\epsilon|^{-2}\simeq(q/q_D)^4$ is typically 10^{-4}–10^{-6} in solid-state plasma experiments. As a consequence, quasi-elastic scattering from such plasmas is unobservable. The only scattering one can expect to see is Raman scattering from the plasma mode.

Now let us consider quasi-elastic scattering from a plasma having a nonparabolic energy-momentum relation. In this case the second term of Eq. (21) does not vanish, but is actually the dominant one in the limit $q\ll q_D$. This statement follows from the fact that, in the quasi-elastic range, $\epsilon=O(q_D{}^2/q^2)$. In this limit

$$\mathcal{G}\simeq(\mathcal{L}_1{}^2-\mathcal{L}_0\mathcal{L}_2)/2\pi\mathcal{L}_0 \quad (25)$$

and

$$J(\omega)\simeq\frac{\mathrm{Im}[\mathcal{L}_1{}^2\mathcal{L}_0{}^*-|\mathcal{L}_0|^2\mathcal{L}_2]}{\pi|\mathcal{L}_0|^2(1-e^{-\beta\omega})}. \quad (26)$$

It should be emphasized that this formula is only valid in the quasi-elastic range [where $\omega/q=O(v)$] and does not describe Raman scattering from the collective modes. It is accurate to order $(q/q_D)^2$. The crucial feature of Eq. (26) is the fact that $J(\omega)$ is *independent of* q and *finite* in the limit $(q/q_D)\to 0$. This is in striking contrast to the case of the classical plasma where $J(\omega)$ varies as $(q/q_D)^4$ in the quasi-elastic range.

In the next section we will evaluate Eq. (26) for a plasma having the energy-momentum relation of Eq. (1). Here we may anticipate these results to say that it is the nonparabolic terms in the expression for $\mathcal{H}_2$ that make the expression $(\mathcal{L}_1^2-\mathcal{L}_2\mathcal{L}_0)$ nonzero. As emphasized earlier, these terms couple light to the energy-momentum tensor of the plasma. It is these terms, and these alone, which give rise to a finite quasi-elastic cross section. This fact leads us to say that the quasi-elastic scattering is caused by fluctuations in the energy-momentum density.

III. EVALUATION OF THE CROSS SECTION

In the preceding section we have derived a fairly general formula [Eq. (26)] describing the quasi-elastic scattering of light from a plasma having a nonparabolic energy-momentum relation. We now wish to apply this result to a specific case, that of the band structure appropriate to n-type InSb and InAs. This case is an important one from several points of view. In the first place, the band structures of these materials are known to be strongly nonparabolic, and thus will give rise to considerable scattering of the sort that interests us here. Secondly, these materials are used in many solid-state plasma experiments.[18] It would be useful to have a general technique for studying electron velocity distributions in them. And, finally, these are materials in which we know from prior experience that light scattering from mobile electrons can be observed.[1]

The energy-momentum relation of conduction-band electrons in InSb or InAs has the form

$$E(p)\simeq(p^2/2m^*)-(1/E_G)(p^2/2m^*)^2+\cdots. \quad (27)$$

With such a Hamiltonian the electron–two-photon coupling term (H_2) is

$$H_2=\frac{e^2}{2m^*c^2}\sum_i\left\{A_i^2-\left(\frac{1}{E_G}\right)\left(\frac{p_i^2}{2m^*}\right)A_i^2-\left(\frac{1}{E_G}\right)A_i^2\left(\frac{p_i^2}{2m^*}\right)-\frac{2(\mathbf{p}_i\cdot\mathbf{A}_i)^2}{m^*E_G}\right\}. \quad (28)$$

For photon scattering from $(\mathbf{q}_0,\omega_0,\boldsymbol{\varepsilon}_0)$ to $(\mathbf{q}_1,\omega_1,\boldsymbol{\varepsilon}_1)$ the electronic portion of this operator becomes

$$\mathcal{H}_2=\frac{e^2}{m^*c^2}\sum_i\left\{\left[(\boldsymbol{\varepsilon}_0\cdot\boldsymbol{\varepsilon}_1)\left(1-\frac{2}{E_G}\left(\frac{p_i^2}{2m^*}\right)\right)-\frac{2}{m^*E_G}(\mathbf{p}_i\cdot\boldsymbol{\varepsilon}_0)(\mathbf{p}_i\cdot\boldsymbol{\varepsilon}_1)\right]e^{i\mathbf{q}\cdot\mathbf{r}_i}\right\}. \quad (29)$$

Here we have assumed that $\mathbf{p}_i$ and $\mathbf{A}_i$ commute. In second quantized notation

$$\mathcal{H}_2=\sum_{\mathbf{k}}[\mathfrak{F}(\mathbf{k})c_{\mathbf{k}}^\dagger c_{\mathbf{k}+\mathbf{q}}], \quad (30)$$

with

$$\mathfrak{F}(\mathbf{k})=\frac{e^2}{m^*c^2}\left[(\boldsymbol{\varepsilon}_0\cdot\boldsymbol{\varepsilon}_1)\left(1-\frac{k^2}{m^*E_G}\right)-\frac{2(\mathbf{k}\cdot\boldsymbol{\varepsilon}_0)(\mathbf{k}\cdot\boldsymbol{\varepsilon}_1)}{m^*E_G}\right]. \quad (31)$$

[18] Betsy Ancker-Johnson, *Semiconductors and Semimetals* (Academic Press Inc., New York, 1966), Vol. I.

We now return to Eq. (26), which determines the correlation function (and hence the scattering cross section) in the quasi-elastic regime. All quantities appearing in this formula are defined via Eqs. (19), (22), (23), and (31). The problem is that of evaluating the integrals that determine $\mathcal{L}_0$, $\mathcal{L}_1$, and $\mathcal{L}_2$. We will perform this computation in two cases—that of a Fermi-Dirac velocity distribution and a Maxwellian.

In the Fermi-Dirac case the expressions for $\mathcal{L}_\alpha$ are (for small q)

$$\mathcal{L}_\alpha\simeq\sum_{\mathbf{k}}\left[\frac{(\partial n/\partial E)(\mathbf{k}\cdot\mathbf{q}/m^*)\mathfrak{F}^\alpha(\mathbf{k})}{\omega-(\mathbf{k}\cdot\mathbf{q}/m^*)}\right]. \quad (32)$$

$\partial n/\partial E\simeq-\delta(E-E_F)$ in a Fermi distribution, so

$$\mathcal{L}_\alpha=-2m^*k_F\iint\frac{v_Fq\mu\mathfrak{F}^\alpha(k_F,\Omega)}{(\omega-qv_F\mu)}\frac{d\Omega}{(2\pi)^3}, \quad (33)$$

where μ is the cosine of the angle between q and k, and v_F is the Fermi velocity. The function $\mathfrak{F}$ is angularly dependent through its last term [see Eq. (31)], which is of the form

$$g_2(\Omega)\equiv-\left(\frac{2e^2}{m^*c^2}\right)\frac{(\mathbf{k}_F\cdot\boldsymbol{\varepsilon}_0)(\mathbf{k}_F\cdot\boldsymbol{\varepsilon}_1)}{m^*E_G}. \quad (34)$$

Other portions of $\mathfrak{F}$ are constant as far as the angular integrations in Eq. (33) are concerned. Consequently, they cancel when one calculates the combination $(\mathcal{L}_1^2-\mathcal{L}_0\mathcal{L}_2)$ appearing in Eq. (25). Only the contributions from g_2 remain, and they yield the result

$$(\mathcal{L}_1^2-\mathcal{L}_0\mathcal{L}_2)=\left(\frac{4e^2k_F}{m^*c^2E_G}\right)^2\left\{\left[\iint\frac{(\mathbf{k}_F\cdot\boldsymbol{\varepsilon}_0)(\mathbf{k}_F\cdot\boldsymbol{\varepsilon}_1)\mu d\mu d\varphi}{(\eta-\mu)(2\pi)^3}\right]^2-\left[\iint\frac{(\mathbf{k}_F\cdot\boldsymbol{\varepsilon}_0)^2(\mathbf{k}_F\cdot\boldsymbol{\varepsilon}_1)^2\mu d\mu d\varphi}{(\eta-\mu)(2\pi)^3}\right]\left[\iint\frac{\mu d\mu d\varphi}{(\eta-\mu)(2\pi)^3}\right]\right\}, \quad (35)$$

where $\eta=(\omega/qv_F)$.

The evaluation of the integrals appearing in this equation is straightforward, but very tedious. For most purposes, the important physical quantity is the sum of Eq. (35) over final polarizations, and its average over initial. This is the only quantity we will discuss. After doing the polarization sums and integrals appearing in Eq. (35), the result is

$$\langle\mathcal{L}_1^2-\mathcal{L}_0\mathcal{L}_2\rangle_{\text{polarization average}}=(e^2k_F^3/\pi^2E_Gm^*c^2)^2\times[P_0(\mu_s,\eta)Q^2(\eta)+P_1(\mu_s,\eta)Q(\eta)+P_2(\mu_s,\eta)], \quad (36)$$

where μ_s is the cosine of the scattering angle. P_0, P_1, and P_2 are real polynomials in the variables μ_s and η;

$$Q(\eta)=\int_{-1}^{1}\frac{\mu d\mu}{(\eta-\mu)}=\eta\ln\left(\frac{\eta+1}{\eta-1}\right)-2, \quad (37)$$

and is complex valued for $-1\leq\eta\leq1$. Also $\mathcal{L}_0(\mathbf{q},\omega)=[2m^*k_F/(2\pi)^3]Q(\eta)$. Hence

$$\mathrm{Im}(G)=\left(\frac{e^2}{m^*c^2}\right)^2\left(\frac{4k_F^5}{m^*E_G^2\pi}\right)\left[\frac{P_0|Q|^2-P_2}{|Q|^2}\right]\mathrm{Im}(Q). \quad (38)$$

The P_1 terms drops out of the expression for $\mathrm{Im}(G)$, and we are left with a formula involving P_0 and P_2. These polynomials turn out to be

$$P_0=\left[\left(\frac{-\mu_s^2}{64}+\frac{3\mu_s}{32}-\frac{9}{64}\right)+\left(\frac{\mu_s^2}{32}-\frac{7\mu_s}{16}+\frac{1}{32}\right)\eta^2 + \left(\frac{-\mu_s^2}{64}+\frac{11\mu_s}{32}+\frac{7}{64}\right)\eta^4\right] \quad (39)$$

and

$$P_2=\frac{4}{9}\left(\frac{\mu_s^2}{32}+\frac{3\mu_s}{16}+\frac{13}{32}\right). \quad (40)$$

One can show that $P_0\leq 0$ and $P_2\geq 0$ for $-1\leq\mu_s\leq 1$ and $0\leq\eta\leq 1$. Also, it should be noted that $P_0=0$ at $\eta=1$. Since $Q\to\infty$ as $\eta\to 1$, this means that the quasi-elastic cross section goes to zero as $\eta\to 1$.

We may now combine Eqs. (6), (11), (37), and (38) to obtain an expression for the quasi-elastic scattering cross section (per electron) in the Fermi-Dirac case. The result (at $T=0$) is

$$\frac{d^2\sigma}{d\Omega d\omega}=12\left(\frac{\omega_1}{\omega_0}\right)\left(\frac{e^2}{m^*c^2}\right)^2\left(\frac{E_F}{E_G}\right)^2\left(\frac{1}{E_F}\right) \times\left[-P_0(\mu_s,\eta)+\frac{P_2(\mu_s,\eta)}{|Q(\eta)|^2}\right]\eta\theta(1-\eta). \quad (41)$$

Here E_F is the Fermi energy. Once again it should be stressed that this formula is only correct in the quasi-elastic limit where $\omega/q=O(v_F)$. It does not describe scattering from plasmons, for which $\omega/q\gg v_F$. We will briefly discuss the effect of nonparabolicity on the plasmon scattering at the end of this section. This part of the problem is not particularly interesting because, in contrast to the quasi-elastic scattering, plasmon scattering is only slightly affected by nonparabolicity.

Equation (41) is a fairly complicated formula, involving both the energy transfer (η) and the scattering angle (μ_s). In experiments performed to date the scattering angle has been $\simeq 90°$. The energy distribution of quasi-elastic light predicted by Eq. (41) in this case is plotted in Fig. 1. The area under this line is of order

$$\frac{d\sigma}{d\Omega}\simeq\left(\frac{e^2}{m^*c^2}\right)^2\left(\frac{E_F}{E_G}\right)^2\left(\frac{\hbar q v_F}{E_F}\right). \quad (42)$$

In this expression the factor $(\hbar q v_F/E_F)$ is a result of the exclusion principle. The maximum energy that can be transferred to a single electron is $\hbar q v_F$, so only that fraction of the electrons within this energy range of the Fermi surface participate in the scattering. For typical experiments with a CO_2 laser in InSb or InAs, this factor is about 0.1. As a consequence, the total quasi-elastic cross section is quite small. We will see that the situation is considerably better in the nondegenerate plasma, where all electrons can participate in the scattering.

To calculate $J(\omega)$ for a Maxwellian distribution, we

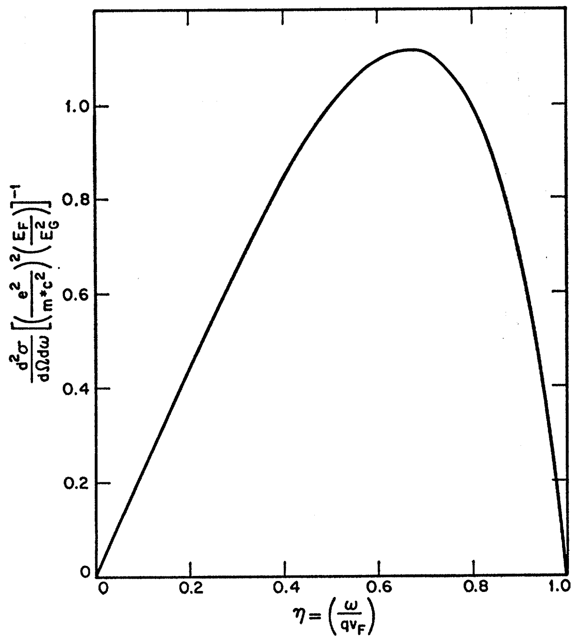

FIG. 1. Quasi-elastic spectrum for a degenerate plasma in n-type InSb.

again return to Eq. (25). Now the functions $\mathcal{L}_0$, $\mathcal{L}_1$, and $\mathcal{L}_2$ have the form

$$\mathcal{L}_\alpha=-\beta\int\frac{(\mathbf{k}\cdot\mathbf{q}/m^*)\,\mathcal{F}^\alpha(\mathbf{k})N(E)\,d^3\mathbf{k}}{\omega-(\mathbf{k}\cdot\mathbf{q}/m^*)}, \quad (43)$$

where $N(E)$ is the Maxwellian distribution. To evaluate these integrals we use cylindrical coordinates in velocity space. The calculation again is straightforward and the result, after summing over final polarizations and averaging over initial, is

$$\langle\mathcal{L}_1^2-\mathcal{L}_0\mathcal{L}_2\rangle_{\text{polarization average}}=\left(\frac{ne^2}{m^*c^2E_G}\right)^2 \times\{-R(\mu_s,\xi)[Z'(\xi)]^2+\text{linear terms in } Z'(\xi)+S(\mu_s)\}. \quad (44)$$

Here $\xi=(\omega/q)(m^*/2kT)^{1/2}$. $Z(\xi)$ is the plasma dispersion function[19] defined by

$$Z(\xi)=\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{\exp(-x^2)\,dx}{(x-\xi)} \quad (45)$$

for $\mathrm{Im}(\xi)>0$, and the analytic continuation of this for $\mathrm{Im}(\xi)\leq 0$. R and S are real polynomials in the variables ξ and μ_s:

$$R(\mu_s,\xi)=(\tfrac{13}{4}-\tfrac{3}{2}\mu_s+\tfrac{5}{4}\mu_s^2)+(1+\mu_s)\xi^2,$$
$$S(\mu_s)=(1+2\mu_s+\mu_s^2). \quad (46)$$

R and S are positive for all physical values of μ_s and ξ.

From Eqs. (6), (11), (25), and (44) we can now calculate the quasi-elastic cross section (per particle)

[19] Burton D. Fried and Samuel D. Conte, *The Plasma Dispersion Function* (Academic Press Inc., New York, 1961).

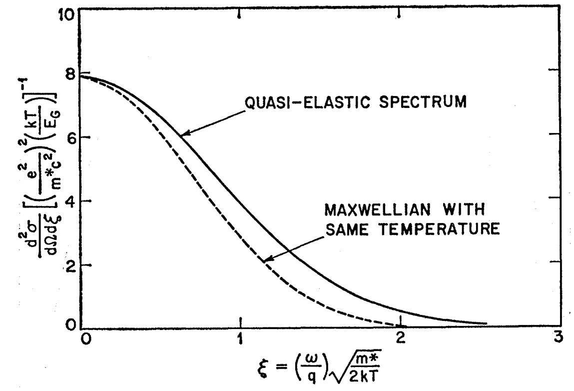

FIG. 2. Quasi-elastic spectrum for a Maxwellian plasma in n-type InSb. The corresponding Maxwellian distribution is indicated for comparison.

for a Maxwellian plasma. Linear terms in $Z'(\xi)$ drop out, and the result is

$$\frac{d^2\sigma}{d\Omega d\omega}=-\left(\frac{2}{\pi}\right)\left(\frac{e^2}{m^*c^2}\right)^2\left(\frac{\omega_1}{\omega_0}\right)\left(\frac{kT}{E_G}\right)^2 \times\left[R(\mu_s,\xi)+\frac{S(\mu_s)}{|Z'(\xi)|^2}\right]\frac{\mathrm{Im}[Z'(\xi)]}{\omega}. \quad (47)$$

For $\mu_s=0$, the frequency dependence of this formula is plotted in Fig. 2. It is important to realize that the width of this spectrum is a direct measure of electron temperature in the plasma. The width is slightly greater than that of a Maxwellian with the same temperature. The integrated cross section is about

$$\frac{d\sigma}{d\Omega}\simeq 8\left(\frac{e^2}{m^*c^2}\right)\left(\frac{\omega_1}{\omega_0}\right)\left(\frac{kT}{E_G}\right)^2. \quad (48)$$

For hot electrons ($T\gtrsim 100°K$) this cross section is greater than the plasmon cross section, which has been observed[1] in InAs. Thus, there would appear to be a distinct possibility of observing quasi-elastic scattering in such materials. Such an experiment would directly measure electron temperature in these semiconductors.

It remains to say a few words concerning the effect of nonparabolicity on the plasmon scattering. Such scattering corresponds to an entirely different limit of the variables q and ω, namely $\omega\gg qv_F$. Under these circumstances, the Green's function is most conveniently estimated from Eq. (21). In the limit $\omega\gg qv_F$ one can easily show that all three integrals ($\mathcal{L}_0$, $\mathcal{L}_1$, $\mathcal{L}_2$) are of order q^2. As a consequence, the total plasmon cross section is

$$\frac{d\sigma}{d\Omega}=\left(\frac{e^2}{m^*c^2}\right)^2\left(\frac{\omega_1}{\omega_0}\right)\left(\frac{q^2}{q_D{}^2}\right)(1+\delta), \quad (49)$$

where δ is a small correction, of order (E_F/E_G) or (kT/E_G), due to nonparabolicity. We see that nonparabolicity has essentially no effect on the plasmon scattering, whereas it completely changes the quasi-elastic scattering. Physically, this result is not hard to understand. The plasmon is an electron density fluctuation and will inevitably have a large electrostatic energy. This energy suppresses density fluctuations and accounts for the factor $(q/q_D)^2$ whether the plasma is parabolic or not. On the other hand, in the quasi-elastic regime, nonparabolicity couples light to other sorts of fluctuations which are uncharged and, therefore, not suppressed by Coulomb forces.

IV. LIGHT SCATTERING IN A MAGNETIC FIELD

In this section we will briefly discuss the scattering of light from a plasma subjected to a static magnetic field. These calculations are even more complicated than the preceding ones, and our presentation is far from complete. Our main aim is to understand the coupling between the Landau-Raman scattering (at $\omega=2\omega_c$) and the plasmon scattering. To simplify matters, we will omit spin-dependent terms from the electron-photon coupling H_2. Such terms are known to exist, and give rise to the spin-flip Raman scattering that has been observed in InSb and InAs.[20] However, one can see that they have a relatively small effect on orbital transitions, such as Landau-Raman scattering.[1] These transitions are our main concern in this section, so we are justified in dropping spin-dependent terms from H_2.

Equations (6), (9), (10), and (11) are the starting point for a derivation of the formula for the light scattering cross section in the presence of a magnetic field. To evaluate G, we again use the random phase approximation. The calculation closely parallels those given in the literature[21] for the case of a plasma of parabolic carriers in a magnetic field. It is also similar to that of Sec. II, and the final result is analogous to Eq. (21). We will present none of the details, but merely quote the final result, which is

$$G\cong-\mathcal{L}_M{}^{(2)}+\left(\frac{4\pi e^2}{q^2}\right)\frac{\mathcal{L}_M{}^{(1)}\tilde{\mathcal{L}}_M{}^{(1)}}{[1+(4\pi e^2/q^2)\mathcal{L}_M{}^{(0)}]}. \quad (50)$$

Here the functions $\mathcal{L}_M{}^{(0)}$, $\mathcal{L}_M{}^{(1)}$, $\tilde{\mathcal{L}}_M{}^{(1)}$, $\mathcal{L}_M{}^{(2)}$ are defined

[20] In a many-electron system, such scattering must be thought of as arising from spin density fluctuations in the electron gas. The approximation used in this paper (RPA) predicts that such fluctuations should be merely the sum of spin density fluctuations due to individual electrons, i.e., there is no spin-wave mode in the RPA. This conclusion may be altered when exchange forces between the electrons are taken into account. Paramagnetic spin waves have recently been observed in alkali metals [P. M. Platzman and P. A. Wolff, Phys. Rev. Letters **18**, 280 (1967); S. Schultz and G. Dunifer, *ibid.* **18**, 283 (1967)], and there is at least a possibility that they might also exist in degenerate semiconductors. If they do, spin-flip Raman scattering would be an ideal tool for studying their properties.

[21] N. David Mermin and Eric Canel, Ann. Phys. (N.Y.) **26**, 247 (1964); M. J. Stephen, Phys. Rev. **129**, 997 (1963).

as follows:

$$\mathfrak{L}_M^{(0)}(\mathbf{q},\omega)=-(m^*\omega_c)\sum_{n,n'}\int\frac{dk_z}{(2\pi)^2}\left\{\left|\int\varphi_n^*(x)e^{-iq_xx}\varphi_{n'}(x-q_ym^*\omega_c)\right|^2\left[\frac{f(n,k_z)-f(n',k_z+q_z)}{\omega+E(n,k_z)-E(n',k_z+q_z)}\right]\right\}, \tag{51}$$

$$\mathfrak{L}_M^{(1)}(\mathbf{q},\omega)=-(m^*\omega_c)\sum_{n,n'}\int\frac{dk_z}{(2\pi)^2}\left\{\left[\int\varphi_n^*(x)\mathcal{H}_2\varphi_{n'}(x-q_ym^*\omega_c)\right]\right.$$
$$\left.\times\frac{[\int\varphi_{n'}^*(x-q_ym^*\omega_c)e^{iq_xx}\varphi_n(x)][f(n,k_z)-f(n',k_z+q_z)]}{[\omega+E(n,k_z)-E(n',k_z+q_z)]}\right\}, \tag{52}$$

$$\tilde{\mathfrak{L}}_M^{(1)}(\mathbf{q},\omega)=-(m^*\omega_c)\sum_{n,n'}\int\frac{dk_z}{(2\pi)^2}\left\{\left[\int\varphi_n^*(x)e^{-iq_xx}\varphi_{n'}(x-q_ym^*\omega_c)\right]\right.$$
$$\left.\times\frac{[\int\varphi_{n'}^*(x-q_ym^*\omega_c)\mathcal{H}_2^\dagger\varphi_n(x)][f(n,k_z)-f(n',k_z+q_z)]}{[\omega+E(n,k_z)-E(n',k_z+q_z)]}\right\}, \tag{53}$$

$$\mathfrak{L}_M^{(2)}=-(m^*\omega_c)\sum_{n,n'}\int\frac{dk_z}{(2\pi)^2}\left\{\left|\int\varphi_n^*(x)\mathcal{H}_2\varphi_{n'}(x-q_ym^*\omega_c)\right|^2\left[\frac{f(n,k_z)-f(n',k_z+q_z)}{\omega+E(n,k_z)-E(n',k_z+q_z)}\right]\right\}. \tag{54}$$

In these formulas the φ_n's are Landau-level wave functions, $E(n,k_z)$ is the energy of an electron in the nth Landau level with momentum k_z in the field direction, and ω_c is the cyclotron frequency. We have used the gauge $A=(0,Bx,0)$ in which, with an appropriate change of variables, $\mathcal{H}_2$ can be written as a function of x, p_x, and k_z (independent of k_y). f is the sum of the spin-up and spin-down Fermi functions. Spin enters the problem in this relatively trivial way only because we have ignored the spin dependence of $\mathcal{H}_2$. Equations (51)–(54) are the natural generalizations of Eqs. (19), (22), and (23) of Sec. II. $\mathfrak{L}_M^{(0)}$ defined by Eq. (51) is essentially the same as the function L° of Mermin and Canel[21] [their Eq. (2.27)].

Equations (50) and (54) are exceedingly complicated. They simplify considerably, however, in the special case in which $\mathbf{q}$ is perpendicular to the applied magnetic field $\mathbf{B}$. In particular, to lowest order in the band nonparabolicity, one may ignore the small effects of nonparabolicity in the energy denominators of Eqs. (51)–(54). The functions $\mathfrak{L}_M^{(\alpha)}$ then take the form

$$\mathfrak{L}_M^{(\alpha)}\simeq\sum_{\nu=-\infty}^{\infty}\left[\frac{l_M^{(\alpha)}(\nu)}{\omega+\nu\omega_c}\right]. \tag{55}$$

These functions are simple, in the sense that their only singularities are a set of discrete poles at frequencies $\omega=\nu\omega_c$.

We now calculate Im(G) (which determines the correlation function) in this approximation. From Eq. (50) we have

$$G=-\mathfrak{L}_M^{(2)}+\left(\frac{4\pi e^2}{q^2}\right)\left[\frac{\mathfrak{L}_M^{(1)}\tilde{\mathfrak{L}}_M^{(1)}}{1+(4\pi e^2/q^2)\mathfrak{L}_M^{(0)}}\right]$$
$$=-\mathfrak{L}_M^{(2)}+\left(\frac{4\pi e^2}{q^2}\right)\left[\frac{\mathfrak{L}_M^{(1)}\mathfrak{L}_M^{*(1)}}{1+(4\pi e^2/q^2)\mathfrak{L}_M^{(0)}}\right]$$
$$-i\pi\left(\frac{4\pi e^2}{q^2}\right)\frac{\mathfrak{L}_M^{(1)}\sum_\nu[\tilde{l}_M^{(1)}(\nu)\delta(\omega+\nu\omega_c)]}{[1+(4\pi e^2/q^2)\mathfrak{L}_M^{(0)}]}. \tag{56}$$

From Eq. (55) we see that $\mathfrak{L}_M^{(0)}$ diverges as

$$[l_M^{(0)}(\nu)/(\omega+\nu\omega_c)]$$

at the point $\omega+\nu\omega_c=0$. Hence, in the last term of Eq. (56), only terms in which this divergence is canceled by a corresponding pole of $\mathfrak{L}_M^{(1)}$ survive. We have

$$G=-\mathfrak{L}_M^{(2)}-i\pi\sum_\nu\left[\frac{l_M^{(1)}(\nu)\tilde{l}_M^{(1)}(\nu)\delta(\omega+\nu\omega_c)}{l_M^{(0)}(\nu)}\right]$$
$$+\left(\frac{4\pi e^2}{q^2}\right)\frac{|\mathfrak{L}_M^{(1)}|^2}{[1+(4\pi e^2/q^2)\mathfrak{L}_M^{(0)}]}. \tag{57}$$

Finally,

$$\mathrm{Im}(G)=\pi\sum_\nu\left\{\left[\frac{l_M^{(0)}(\nu)l_M^{(2)}(\nu)-l_M^{(1)}(\nu)\tilde{l}_M^{(1)}(\nu)}{l_M^{(0)}(\nu)}\right]\right.$$
$$\left.\times\delta(\omega+\nu\omega_c)\right\}+(4\pi e^2/q^2)\,|\mathfrak{L}_M^{(1)}|^2$$
$$\times\,\mathrm{Im}\left[\frac{1}{1+(4\pi e^2/q^2)\mathfrak{L}_M^{(0)}}\right]. \tag{58}$$

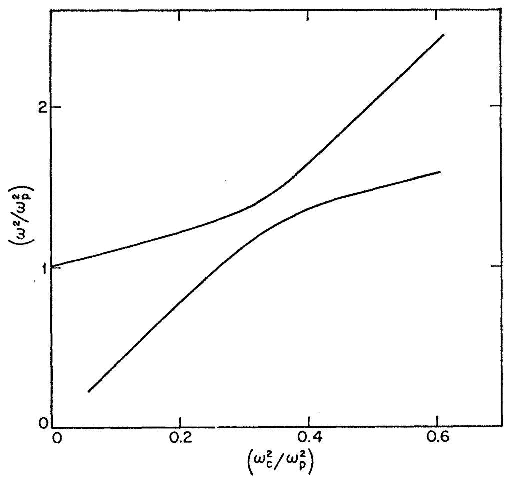

FIG. 3. Magnetic field variation of plasma and Bernstein modes in the geometry $\mathbf{q}\perp\mathbf{B}$. These curves are for the case

$$3q^2\langle v_\perp^2\rangle/\omega_p^2=0.01.$$

A straightforward calculation of matrix elements shows that, to lowest order in q^2, the combination

$$[l_M^{(0)}(\nu)l_M^{(2)}(\nu)-l_M^{(1)}(\nu)\bar{l}_M^{(1)}(\nu)]$$

vanishes for $\nu=\pm 2$. Thus, for $\omega\simeq 2\omega_c$,

$$\mathrm{Im}G\simeq(4\pi e^2/q^2)\,|\,\mathcal{L}_M^{(1)}\,|^2\,\mathrm{Im}\left[\frac{1}{1+(4\pi e^2/q^2)\mathcal{L}_M^{(0)}}\right]. \tag{59}$$

This formula is a very interesting one because it shows that near $\omega=2\omega_c$ ImG is proportional to Im$(1/\epsilon)$, where $\epsilon=[1+(4\pi e^2/q^2)\mathcal{L}_M^{(0)}]$. The zeros of $\epsilon(q,\omega)$ determine the collective modes of the plasma. Equation (59) indicates that these modes, rather than single-particle excitations, are responsible for the scattering. In the $\mathbf{q}\perp\mathbf{B}$ geometry there is a plasma mode at frequency $(\omega_p^2+\omega_c^2)^{1/2}$, and the Bernstein[9] modes at frequencies $\omega=2\omega_c$, $3\omega_c\cdots$. The Landau-Raman scattering at $\omega=2\omega_c$ is, in reality therefore, scattering from the Bernstein mode (at least for $\mathbf{q}\perp\mathbf{B}$). Light is coupled to this mode by band nonparabolicity. This coupling is much stronger than that which occurs in a classical plasma, via density fluctuations, which are of order q^4 in Bernstein modes. The strength (per particle) of the nonparabolicity induced scattering from Bernstein modes is the same as that one estimates[2] for a single carrier.

The plasma mode and the Bernstein mode at $2\omega_c$ cross (as a function of magnetic field) when $\omega_p^2=3\omega_c^2$. This behavior is illustrated in Fig. 3. At the crossover point the modes mix, but weakly in the limit $q/q_D\ll 1$. The dispersion relation (ignoring modes at $3\omega_c$, $4\omega_c$, etc.) can be written in the form

$$[\omega^2-(\omega_p^2+\omega_c^2)][\omega^2-4\omega_c^2]-3q^2\omega_p^2\langle v_\perp^2\rangle=0. \tag{60}$$

This is Bernstein's result,[9] which was derived for a Maxwellian plasma, but it can also be shown to be correct in the Fermi-Dirac case. The mode splitting at the cross-over point is $\frac{3}{2}q(\langle v_\perp^2\rangle)^{1/2}$. In the experiments referred to above,[1] this splitting is small. These experiments, unfortunately, were not performed in the relatively simple $\mathbf{q}\perp\mathbf{B}$ geometry and our analysis does not apply directly to them. Nevertheless, it is not surprising that the measurements appear to indicate that the plasma mode and Bernstein mode pass through one another, without interaction. A quite refined experiment would be required to detect the small splitting (typical of the solid-state plasma case) indicated in Fig. 3. The interaction between the two modes is weak because they are of quite a different character; the plasmon is a perturbation of total electron density, whereas the Bernstein mode is mainly an angular distortion of the distribution function in velocity space which, in the limit $q\to 0$, is a Y_2^2 spherical harmonic. The two are nearly orthogonal to one another.

{*Note added in proof.* Quasi-elastic scattering has recently been observed by Mooradian [Phys. Rev. Letters **20**, 1102 (1968)] in several semiconductors. For the case he studies most carefully (n-type GaAs) the intensity of the quasi-elastic scattering is considerably larger than that predicted by our theory.

A. L. McWhorter has pointed out that the nonparabolicity induced scattering can only be thought of as arising from energy density fluctuations when the energy-momentum has the form of Eq. (1). In more general cases, other sorts of fluctuations will be involved.

The author wishes to thank Dr. A. L. McWhorter and Dr. A. Mooradian for correspondence and several stimulating discussions.}

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