Natural Zeros of Regge Residues†

H. F. Jones and M. D. Scadron*
Physics Department, Imperial College, London, SW 7, England
(Received 13 February 1968)

We discuss the method of Reggeization of invariant amplitudes with particular reference to the kinematic zeros of the residue functions at nonsense points in J and at $t=0$. The zeros which occur naturally at these points yield in the first case the sense-choosing solution, and in the second case a well-defined scheme, covariant evasion, for satisfying the $t=0$ conditions on helicity amplitudes. Factorization is automatically satisfied in our method.

The problems of Reggeizing processes with high spin[1] and/or unequal masses[2] have been the subject of much discussion in the recent literature.[3] An approach to these problems which deals with invariant amplitudes rather than helicity amplitudes has been proposed.[4] In this article we discuss the kinematic zeros of the Regge residues of this approach. We begin by reviewing briefly the method of Reggeizing invariant amplitudes.

The method stems from the remark that the appropriate (t-channel) angular functions, in terms of which the covariant M function is to be expanded, can be obtained from the exchange of "particles" of spin J and mass $\sqrt{t}$. The sum of all such exchanges weighted with arbitrary coefficients then has the form of a partial-wave expansion of the M function, which can subsequently be Reggeized.

To calculate the contributions to the process $p+q\to p'+q'$ from such exchange graphs one needs the spin-J projection operator[5] $\mathcal{P}^J{}_{\beta_1\cdots\beta_J;\alpha_1\cdots\alpha_J}(\Delta)$ along with partial contractions such as

$$\mathcal{P}^J{}_{\beta;}(P,Q;\Delta)\equiv P_{\beta_2}\ldots P_{\beta_J}\mathcal{P}^J{}_{\beta\beta_2\cdots\beta_J;\alpha_1\cdots\alpha_J}(\Delta)Q_{\alpha_1}\ldots Q_{\alpha_J}.$$

Here $P=\frac{1}{2}(p+p')$, $Q=\frac{1}{2}(q+q')$, and $\Delta=p'-p=q-q'$.

Thus for spinless external particles the t-channel contribution to the amplitude is proportional to

$$\mathcal{P}^J(P,-Q;\Delta)\equiv P_\beta\mathcal{P}^J{}_{\beta;}(P,-Q;\Delta)=c_J\mathcal{P}_J,\qquad(1)$$

where

$$c_J=\frac{2^J(J!)^2}{(2J)!}=\frac{\sqrt{\pi}}{2^J}\frac{\Gamma(J+1)}{\Gamma(J+\frac{1}{2})}\qquad(2)$$

and $\mathcal{P}_J$ is the *solid* spherical harmonic $\mathcal{P}_J=(PQ)^JP_J(z_t)$.

The contributions to the M function for processes with spin involve projection operators with free labels such as $\mathcal{P}^J{}_{\beta;}$ or $\mathcal{P}^J{}_{\beta;\alpha}$, which can be obtained from Eq. (1) by a differential technique[6] and have been tabulated in Ref. 7.

For example, in πN scattering only normal parity exchange $P=(-1)^J$ can occur, and the contribution is

$$g(g_1P_\beta+g_2\gamma_\beta)\mathcal{P}^J{}_{\beta;}(P,-Q;\Delta)=gc_J[g_1\mathcal{P}_J+g_2J^{-1}(\gamma\cdot Q\mathcal{P}_J'-mP^2\mathcal{P}_{J-1}')]\qquad(3)$$

which, with a suitable identification of gg_1 and gg_2, gives the usual partial-wave expansion[8] for the invariant amplitudes A and B.

In general, the appropriate spin-J angular functions are obtained by summing all possible exchanges $\mathcal{C}(P):\mathcal{P}^J(P;-Q;\Delta):\mathcal{C}(-Q)$, where the $\mathcal{C}$'s are the coupling functions of Ref. 7.

The partial-wave expansion can then be Reggeized by the prescription $\mathcal{P}_J(z_t)\to\mathcal{P}_\alpha(-z_t)\times\frac{1}{2}(1\pm e^{-i\pi\alpha})/\sin\pi\alpha$, coupling constants becoming Regge residues, functions of $\Delta^2\equiv t$.

For the exchange of a single Regge pole the amplitude thus obtained will be automatically factorized. It will explicitly contain the correct threshold factor $(PQ)^\alpha$, and the explicit separation of c_α from the vertex residues yields the very simple asymptotic behavior $c_\alpha\mathcal{P}_\alpha\sim\nu^\alpha$ *exactly* for large $\nu\equiv P\cdot Q=\frac{1}{4}(s-u)$.

The remaining factorized residues $g(\alpha(t))$ will then be free of kinematic singularities in $\alpha(t)$ and, even for high external spins, can be chosen[7] to be free of kinematic singularities in t. They may, however, have kinematic zeros in J and in t, and it is to these that we now turn our attention. Because of our explicit factorization we discuss each vertex separately.

(*i*) *Kinematic zeros in J—nonsense couplings*. For integral[9] $J>s_1+s_2$ the number of couplings of a spin-J particle to spins s_1 and s_2 is[7] $(2s_1+1)(2s_2+1)$. However, for integral $J<s_1+s_2$ the number of couplings is reduced by $\mathcal{J}(\mathcal{J}+1)$, where $\mathcal{J}=s_1+s_2-J$. This reduction is achieved in the Regge amplitude by making appropriate couplings vanish at integral nonsense values of J. A well-defined prescription for the "appropriate" couplings is as follows: *Those couplings which involve more*

† The research reported in this document has been sponsored in part by the Air Force Office of Scientific Research OAR through the European Office Aerospace Research, U. S. Air Force.
* National Science Foundation Postdoctoral Fellow.
[1] E. Leader, Phys. Rev. **166**, 1599 (1968).
[2] D. Z. Freedman and J. M. Wang, Phys. Rev. Letters **17**, 569 (1966); Phys. Rev. **153**, 1596 (1967).
[3] A rather complete review is given by L. Bertocchi, CERN Report No. TH 835, 1967 (unpublished).
[4] H. F. Jones and M. D. Scadron, Nucl. Phys. **B4**, 267 (1968).
[5] This is a traceless symmetric tensor in both initial and final indices satisfying $\Delta_\beta\mathcal{P}^J{}_{\beta\ldots}=\mathcal{P}^J{}_{\alpha\ldots}\Delta_\alpha=0$.
[6] C. Zemach, Phys. Rev. **140**, B97 (1965).
[7] M. D. Scadron, Phys. Rev. **165**, 1640 (1968).
[8] V. Singh, Phys. Rev. **129**, 1889 (1963).
[9] We limit ourselves here to boson Regge poles. Similar remarks apply to fermions.

labels of the projection operator than are available at a nonsense point vanish there.[10]

Thus, for example, the residue $g_2^{(\alpha)}$ of the NN-α vertex [Eq. (3)] will vanish at $\alpha=0$, this being the usual[11] explanation of the dip in πN charge exchange. The coupling g_1 is *not* constrained to vanish at this point, since $g_1 P_\beta \mathcal{P}^J{}_{\beta;}$ can be written as $g_1\mathcal{P}^J$, a form not involving any labels.

As another example, consider the process $\pi N \to \omega N$, where by the conventional Regge analysis[12] a dip is predicted when $\alpha_\rho=0$. This can be seen very simply in our method, since the $\pi\omega\rho$ vertex has the form $g\epsilon_{\mu\alpha\sigma\tau}Q^\sigma\Delta^\tau$. Since this involves one propagator label it must vanish at $\alpha_\rho=0$.

Finally, all couplings will have zeros at negative integers where the coupling cannot exist at all. This will prevent "ghost" poles arising at negative-integer right-signature points[13] and will give dips at negative-integer wrong-signature points.

(*ii*) *Kinematic zeros in t—evasion.* At $t=0$ the Regge pole simulates a massless particle, which, for any $J>0$, has only two spin states. Correspondingly, the number of couplings is reduced to $2(2s_{\min}+1)$ for $s_1\neq s_2$ and to $4s+1$ for $s_1=s_2=s$. Again this reduction is achieved by making appropriate couplings vanish or become related at $t=0$. A well-defined scheme which always ensures the correct number of couplings is that the *couplings are constrained to be gauge invariant at $t=0$.* That is to say, we require that[14] $\Delta_\beta \mathcal{C}_\beta(P)=0$.

The rate at which (combinations of) couplings go to zero is determined by demanding that no terms in $1/\Delta^2$, which would be indeterminate at $t=0$, should survive when we make the replacements $V_\beta \to V_\beta - V\cdot\Delta\Delta_\beta/\Delta^2$, $g_{\mu\beta}\to g_{\mu\beta}-\Delta_\mu\Delta_\beta/\Delta^2$ for the vectors and tensors occurring in $\mathcal{C}_\beta(P)$.[15]

The conditions on couplings thus obtained turn out to be sufficient in the equal-mass case to ensure that invariant amplitudes remain finite at $t=0$, so that the dispersion procedure proposed in (4) is no longer necessary. For unequal masses one will still have to disperse or introduce daughter trajectories, but by the introduction of the concept of "reduced" couplings[15] we have separated the problems of high spin and unequal masses in a well-defined way.

As a specific example, consider NN scattering. Here the coupling of the A_1 trajectory, $f_2\gamma_5\gamma_\beta$, is the only one which is not by itself gauge invariant. Thus we expect f_2 to vanish like t at $t=0$. Since the A_1 trajectory alone gives terms in $1/t$,[16] giving a contribution $f_2^2(\nu\mathcal{P}_\alpha'' + 4m^2t^{-1}\mathcal{P}_\alpha')$ to the pseudoscalar amplitude A_P, this ensures that all invariant amplitudes are finite at $t=0$, i.e., that the forward scattering conditions are satisfied. In fact our condition might seem too strong, as $f_2^2\sim t^2$, but it is indeed necessary that $f_2\sim t$, as can be seen, for example, from a detailed analysis of $\gamma N\to\pi N$.[17]

As another example consider the normal $\pi\rho$-α vertex.[17] The reduced coupling is $g_1Q_\mu Q_\beta+g_2g_{\mu\beta}$, so that the gauge condition on the residues at $t=0$ is $Q\cdot\Delta g_1(t) - 2g_2(t)\sim t$.

The "gauge invariance" requirement thus provides a well-defined scheme of evasion[1]—covariant evasion—which is factorized from the beginning. We have arrived at an evasive solution because we have required each Regge pole *separately* to have the correct number of couplings at $t=0$. As this gives rise to a very natural scheme, we would incline to those solutions[18] of the np charge-exchange problem and others which do not involve a conspiring pion trajectory.[19]

Conspiratorial solutions can also be constructed in our method, although they do not arise very naturally and correspond to a weakening of one or other of our assumptions. Thus class-II (A_1-like) conspiracies arise when the leading Regge pole does not obey the gauge condition, while the leading Regge poles of a class-III (parity-doubling) conspiracy have singular residues with square-root branch points at $t=0$.

Finally, we remark that calculations of cross sections are surprisingly easy if one works with the covariant Regge amplitude as a whole rather than splitting it up into contributions to invariant amplitudes. Thus, it is easy to see that, to leading order, cross-section contributions from each Regge pole decouple into a product of vertex factors.[20] Moreover, the interference terms between different normality exchanges can easily be seen to be suppressed by at least one power in s.

Thus, the cross section will be an incoherent sum of individual factorized Regge contributions.

[10] This corresponds to the "choosing-sense" solutions. In our formalism it is hard to see how the "choosing-nonsense" solution could arise.

[11] F. Arbab and C. B. Chiu, Phys. Rev. **147**, 1045 (1966).

[12] L. L. Wang, Phys. Letters **16**, 756 (1966).

[13] To avoid ghosts at $\alpha=0$ for even-signature trajectories one will have to invoke extra (dynamical) zeros which cannot be derived from the above arguments. A zero in the sense coupling would correspond to the "noncompensating" mechanism (see Ref. 3).

[14] Such "mass-shell gauge invariance" has been discussed in detail by S. Weinberg, Phys. Rev. **135**, B1049 (1964).

[15] For unequal masses, where $\Delta\cdot P\neq 0$, this criterion must be applied only to the *reduced* coupling $\mathcal{C}^R(P)$ defined by separating off all common factors P_β,

$$\mathcal{C}_{\beta_1\cdots\beta_J}(P)=\mathcal{C}^R{}_{\beta_1\cdots\beta_r}(P)\quad P_{\beta_{r+1}}\cdots P_{\beta_J}.$$

[16] L. Durand, III, Phys. Rev. Letters **18**, 58 (1967).

[17] F. Gault (private communication).

[18] K. Huang and I. J. Muzinich, Phys. Rev. **164**, 1726 (1967).

[19] F. Arbab and J. W. Dash, Phys. Rev. **163**, 1603 (1967); R. J. N. Phillips, Nucl. Phys. **B2**, 394 (1967).

[20] For example, in πN scattering, Eq. (3), ρ exchange gives $s^{2-2\alpha}d\sigma/dt\propto[(mg_1+g_2)^2-\frac{1}{4}tg_1^2]$, while in NN scattering it contributes the square of this factor.