Pion Conspiracy and Pion Residue from the Finite-Energy Photoproduction Sum Rule*

D. P. Roy and Shu-Yuan Chu
Physics Department, University of California, Riverside, California
(Received 2 February 1968)

The pion residue is obtained as an integral over the relevant photoproduction amplitude following the usual prescription of the finite-energy sum rule. The latter, studied as a function of t, reveals the following interesting features of the pion residue. It is nonzero at $t=0$ (thus showing pion conspiracy), but vanishes around $-1.5\mu^2$, as required by high-energy data. The magnitude of the residue at $t=0$ also agrees extremely well with that obtained from the high-energy forward photoproduction cross section.

The recent Regge analysis of high-energy pion-photoproduction data by Drell and Sullivan[1] and by Frautschi and Jones[2] strongly suggests a nonzero pion residue at $t=0$, which requires pion conspiracy. On the other hand, Ball, Frazer, and Jacob[3] have obtained a Regge fit to the high-energy forward peak in the photoproduction cross section by requiring the pion residue to vanish at $-1.5\mu^2$. A zero in pion residue near $t=0$ has also been suggested by Mandelstam,[4] on the basis of conspiracy and partially conserved axial-vector current (PCAC), and by Arbab and Dash[5] from a Regge fit to the charge-exchange n-p scattering. In order to study these behaviors of the pion residue, we construct a t-channel amplitude that would couple only to the pion trajectory. The pion contribution is then obtained in terms of a finite integral over the corresponding photoproduction amplitude by the usual method of using a finite-energy sum rule.[6] This has the obvious advantage over the high-energy cross-section fits, in that the latter always contain the contribution of the conspirator along with that of the pion, so that the latter can be disentangled only by assuming a model for the conspirator.[7]

Following closely the notation of Zweig,[8] we pick up the combination of invariant amplitudes, $(A_1^{(-)}+tA_2^{(-)})$, where the superscript $(-)$ means that the t-channel isospin is 1 and the G parity is negative (isovector photon). Following Ref. 2, we have

$$A_1^{(-)}+tA_2^{(-)}=-[2\sqrt{2}M/(\mu^2-t)]\bar{f}^t, \qquad (1)$$

where μ and M are the pion and nucleon mass and $\bar{f}^t$ is the kinematic-singularity-free, definite spin-parity helicity amplitude, which couples in this case to the nucleon-antinucleon singlet state $|\frac{1}{2}\frac{1}{2}\rangle-|-\frac{1}{2}-\frac{1}{2}\rangle$. Therefore, we must have $C=-P=(-1)^J$. This, together with the isospin and G-parity requirement, allows only pion exchange. Then, on absorbing certain innocent factors into the reduced residue function $\gamma(t)$, we get the usual Regge contribution:

$$A_1^{(-)}+tA_2^{(-)}=\alpha\left(\frac{1+e^{-i\pi\alpha}}{\sin\pi\alpha}\right)\frac{\gamma(t)}{\mu^2-t}\left(\frac{\nu}{\nu_0}\right)^{\alpha-1}, \qquad (2)$$

where

$$\nu=(s-u)/4M=k_L+(t-\mu^2/4M). \qquad (3)$$

Here, k_L is the lab photon momentum and ν_0 is a scale factor, which we choose for convenience to be 1 GeV. Then, using the odd crossing property of our amplitude[8] under $\nu\to-\nu$, we get the following finite-energy sum rule:

$$S_0=\frac{1}{4M}eg\frac{\mu^2+t}{\mu^2-t}\frac{1}{\pi}\int_{\nu_{\rm th}}^{N}\mathrm{Im}(A_1^{(-)}+tA_2^{(-)})d\nu$$
$$=\frac{1}{\pi}\frac{\gamma(t)}{\mu^2-t}\left(\frac{N}{\nu_0}\right)^{\alpha(t)}\nu_0. \qquad (4)$$

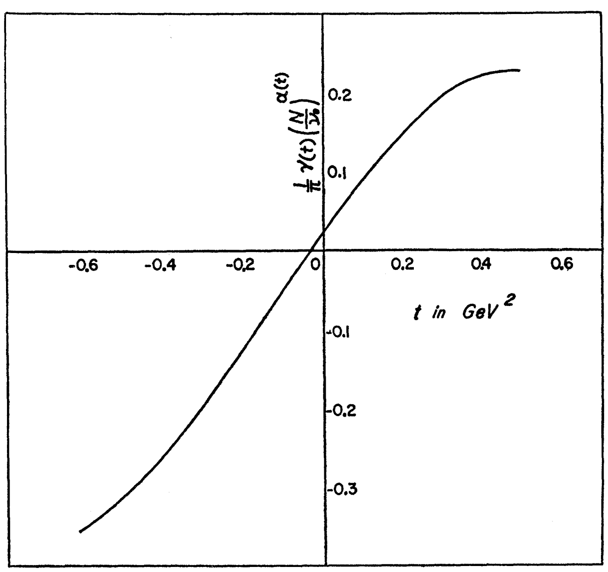

Fig. 1. Pion contribution, as given by the finite-energy sum rule, as a function of t.

* Work supported in part by the Atomic Energy Commission under Contract No. AEC AT(11-1)34P107A.

[1] S. Drell and J. Sullivan, Phys. Rev. Letters **19**, 268 (1967).
[2] S. Frautschi and L. Jones, Phys. Rev. **163**, 1820 (1967).
[3] J. S. Ball, W. R. Frazer, and M. Jacob, Phys. Rev. Letters **20**, 518 (1968).
[4] S. Mandelstam, talk at Irvine Conference on πN Scattering, 1967 (unpublished).
[5] F. Arbab and J. Dash, Phys. Rev. **163**, 1603 (1967).
[6] K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 625 (1967); R. Dolen, D. Horn, and C. Schmid, Phys. Rev. Letters **19**, 402 (1967).
[7] We wish to emphasize this distinction between the photoproduction case and that of charge-exchange π-p scattering, where ρ contributions obtained from high-energy data are more reliable.
[8] G. Zweig, Nuovo Cimento **32**, 6 89 (1964).

TABLE I. Nucleon Born term and resonance contributions to $(1/\pi)\gamma(0)(N/\nu_0)^{\alpha(0)}$; $\nu_0=1$ GeV.

Resonances	$N(940)$	$N^*(1236)$	$N^*(1471)$	$N^*(1519)$	$N^*(1561)$	$N^*(1652)$	$N^*(1672)$
Multipole	Born term	E_{1+} M_{1+}	E_{1-} M_{1-}	E_{2-} M_{2-}	E_{0+}	E_{2+} M_{2+}	E_{3-} M_{3-}
Contribution (10^{-2} GeV)	1.85	0.662	0.049	0.034	−0.254	−0.004	−0.011

The first term above is the nucleon pole contribution, which contains the well-known kinematic pion pole, arising from gauge invariance. We take $e^2/4\pi=1/137$ and $g^2/4\pi=14$. To evaluate the integral, the amplitude is expanded in terms of the electric and magnetic multipoles.[8] These multipoles have been empirically obtained by Walker[9] in terms of Breit-Wigner poles plus nonresonant parts, up to a lab momentum of 1.2 GeV/c. The nonresonant parts are indeed very small. Our $A^{(-)}$ amplitudes are connected to the amplitudes A^+ and A^- corresponding to π^+ and π^- photoproduction by[10]

$$A^{(-)}=-(1/2\sqrt{2})(A^++A^-). \quad (5)$$

We evaluate the continuum contribution, using Walker's multipole parameters. The upper limit N of the integration corresponds to $k_L=1.2$ GeV/c.

We have plotted in Fig. 1 the quantity

$$\left[\frac{1}{4M}eg(\mu^2+t)-(\mu^2-t)\int_{\nu_{\rm th}}^{N}\mathrm{Im}(A_1{}^{(-)}+tA_2{}^{(-)})d\nu\right]\Big/\nu_0=\frac{1}{\pi}\gamma(t)\left(\frac{N}{\nu_0}\right)^{\alpha(t)}. \quad (6)$$

Near $t=0$, this essentially gives us the residue function. We note the following interesting features: First, the residue does not vanish at $t=0$, thus showing that the pion chooses conspiracy.[11] It, however, falls steeply and vanishes at $t=-0.03$ (GeV/c)^2, as required by high-energy data.[3] Finally the magnitude of the residue at $t=0$ [$\gamma(0)=0.023\pi$] agrees very well with that obtained from the fit to the high-energy forward cross section,[3] which gives $\gamma(0)=0.026\pi$. This agreement, we believe, is very significant because the high-energy forward cross section receives a contribution not from the pion but from its conspirator, so that the pion residue is obtained only via the conspiracy equation. In other words, the conspirator residue calculated from our pion residue $\gamma(0)$, using a conspiracy equation, would quantitatively account for the high-energy forward cross section.

In order to assess the accuracy of our results, we list in Table I the Breit-Wigner pole contributions to Eq. (6) at $t=0$, together with the nucleon Born term. We note that the dominant contributions are from the nucleon and the $N^*(1236)$, and that the two are additive. The higher resonances are seen not to affect the net contribution to any significant extent. Our sum rule would then evidently be insensitive to variations in the higher-resonance parameters and the cutoff value N. We have, in fact, repeated the calculation by changing the integration limit to incident momenta of 1 and 1.5 GeV/c and found very little deviation.[12] Thus the question, whether Regge asymptotic behavior has set in around the energy region where we cut off the integral, should not affect the residue estimate in any significant way. Equally insignificant should be the difficulty associated with the partial-wave expansion at nonzero values of t since, for the range of t of interest to us, the unphysical region is a very small nonresonant region near threshold. Further, we see that the continuum contribution is dominated by the $N^*(1236)$ resonance. The higher resonances reduce this contribution by about one-third only. Thus it is highly unlikely that a possible variation in their parameters or the cutoff would flip the sign of the net continuum contribution. Moreover, the integral is seen to change very little in this small interval of t, the major variation coming from the (μ^2-t) factor. Thus the net cancellation can occur only beyond $-\mu^2$, where the Born term flips sign. In fact, varying the integral between zero and about twice its present magnitude would make the zero-residue point vary between $-\mu^2$ and $-2\mu^2$, which we consider to be safe limits. In this respect we favor the parametrization of Ref. 3 over that of Ref. 5, which requires a zero at $-0.85\mu^2$. Finally, we wish to remark that, in the present model, one can see phenomenologically [see Eq. (6)] how the zero-residue point moves to $t=0$ as we approach the soft-pion limit.[13]

It is a pleasure to express our gratitude to Professor R. L. Walker for allowing us to use his preliminary data. We are also grateful to Dr. B. R. Desai, Dr. P. E. Kaus, and Dr. H. K. Shepard for helpful discussions.

[9] R. L. Walker (private communication).

[10] The sign of the A^+ term here is opposite to that given in Ref. 8, because of a slightly different isospin convention used by Walker.

[11] This part of the calculation was earlier done by M. B. Halpern, Phys. Rev. **160**, 1441 (1967). Unfortunately, he had started by postulating a nonconspiring pion and was led to rather implausible conclusions. Besides, his estimates are quantitatively inaccurate.

[12] This is in contrast to the second-moment sum rule S_2, which receives a negligible contribution from N and $N^*(1238)$, and is dominated by higher-resonance contributions of alternating sign. With our set of parameters, this would give a small negative slope for the pion trajectory, but the result is extremely sensitive to these higher-resonance parameters as well as the value of the cutoff. We feel that more precise data over a wider energy range will be very helpful in this regard.

[13] *Note added in proof.* Due to a programming error, the Born term has been underestimated by a factor M^2 (=0.88). Thus in Table I the Born term should read 2.10 instead of 1.85. However, this has a negligible effect on the position of the zero-residue point, and increases the $\gamma(0)$ estimate ($=0.023\pi$) by only 10% (thus bringing it embarrassingly close to the high-energy estimate of 0.026π). We are grateful to Professor K. V. Vasavada for pointing out this error.