PHYSICAL REVIEW VOLUME 171, NUMBER 5 25 JULY 1968

Intermediate Bosons and the Electromagnetic Field*

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(Received 4 March 1968)

Possible symmetries between the (hypothetical) charged intermediate-boson field $W_\nu^\pm$ and the derivative of the electromagnetic field $\partial F_{\mu\nu}/\partial x_\nu$ are investigated. We assume that (a) the total electromagnetic current operator $\mathcal{J}_\nu^\gamma = e_0^{-1}(\partial F_{\mu\nu}/\partial x_\nu)$ is proportional to a neutral member W_ν^0 of the intermediate-boson fields, (b) all hadron mass differences between different members of the same isospin multiplet consist of finite $O(e^2)$ terms but no $O(f^2)$ terms, and (c) all known leptonic, semileptonic, and $\Delta S \neq 0$ nonleptonic weak processes are transmitted by a single $W_\nu^\pm$ field, where f, e, and e_0 are, respectively, the semiweak coupling constant, the renormalized charge, and the unrenormalized charge. The simplest model compatible with (a), (b), and (c) is found to be one consisting of six intermediate bosons, which may be regarded as forming an SU_3 triplet and its Hermitian conjugate. Assumption (a) also implies finite radiative corrections for other processes such as weak decays, charge renormalization, etc. The unrenormalized charge e_0 is shown to be finite, bounded by the inequality $1 \leq (e_0/e) \leq \sqrt{2}$. A lower limit of f^2 is established. Neglecting higher-order corrections, one finds this lower limit of f^2 to be $\frac{1}{8}e^2\sec^2\theta$ and, in this limit, $e_0 = \sqrt{2}e$ and $m_W/m_N \cong \alpha^{-1}$, where α is the fine structure constant, m_W and m_N are, respectively, the $W^\pm$ mass and the nucleon mass, and θ is the Cabibbo angle. The same model can also lead, in a reasonably natural way, to CP nonconservation.

1. INTRODUCTION

THAT there exists a similarity between the weak interaction and the electromagnetic interaction has been noted ever since the early formulation of the vector theory of β decay by Fermi[1]; this similarity has prompted many authors to consider the possibility[2] that the weak interaction might also be transmitted through an intermediate boson $W^\pm$, in analogy with the photon γ. However, there are important differences between the (hypothetical) $W^\pm$ and the (real) γ; for example, even the degrees of freedom of these two fields are completely different. At a given momentum, $W^\pm$, being massive, has three spin states, but the zero-mass γ has only two.

In a recent paper,[3] hereafter referred to as I, it was pointed out that one should seek possible symmetries between the intermediate-boson field $W_\nu^\pm$ and the total electromagnetic current $\mathcal{J}_\nu^\gamma$, i.e., between $W_\nu^\pm$ and the derivative of the electromagnetic field $(\partial F_{\mu\nu}/\partial x_\nu)$ rather than between $W_\nu^\pm$ and the electromagnetic 4-potential A_ν. In I, the total electromagnetic current operator $\mathcal{J}_\nu^\gamma(x)$ is assumed to be proportional to the field operator $W_\nu^0(x)$ of an appropriate neutral member of the intermediate-boson fields. From such a field-current identity,[4] one can readily establish the finiteness of all second-order electromagnetic corrections. The same conclusions can also be extended to all higher powers of e^2, at least for a system consisting of only spin-$\frac{1}{2}$ and spin-0 particles. As emphasized in I, these radiative correction calculations are quite insensitive to the details of the intermediate-boson models, provided that the general field-current identity between $\mathcal{J}_\nu^\gamma$ and W_ν^0 holds.

In order to determine the detailed symmetry, or broken symmetry, between the charged intermediate-boson field and the derivative of the electromagnetic field, additional assumptions have to be introduced. Only then can the corresponding structure of the intermediate-boson system become definite. To be specific, in this paper we assume the following three conditions:

(a) The total electromagnetic current operator $\mathcal{J}_\nu^\gamma(x)$ is proportional to a neutral intermediate-boson field $W_\nu^0(x)$.

(b) There is no $O(f^2)$ mass splitting between different hadrons of the same isospin multiplet, where f is the semiweak coupling constant, related to the Fermi constant G_F and the $W^\pm$ mass m_W by

$$f^2/m_W^2 = 2^{-1/2}G_F. \tag{1.1}$$

This assumption removes the usual $O(f^2)$ infinities in such mass differences.[5] Therefore, all these mass differences are of the form

$$\delta m = [\text{finite } O(e^2) \text{ term}] + O(f^4, f^2e^2, e^4), \tag{1.2}$$

where δm can be either the mass difference δm_π between π^+ and π^0, or δm_K between K^+ and K^0, or δm_N between p and n, etc.

(c) All known leptonic, semileptonic, and the

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* Research supported in part by the U. S. Atomic Energy Commission.

[1] E. Fermi, Z. Physik **88**, 161 (1934).

[2] For an early discussion on the possible existence of such an intermediate-boson field universally coupled to all hadrons and leptons, see T. D. Lee, M. Rosenbluth, and C. N. Yang, Phys. Rev. **75**, 905 (1949).

[3] T. D. Lee, Phys. Rev. **168**, 1714 (1968).

[4] It should be emphasized that the concept of field-current identity implies the observed electromagnetic and weak interaction current operators $\mathcal{J}_\nu^\gamma$ and $\mathcal{J}_\nu^{wk}$ to be directly regarded as basic field variables; i.e., $\mathcal{J}_\nu^\gamma$, $\mathcal{J}_\nu^{wk}$, and their derivatives satisfy the appropriate canonical commutation relations of spin-1 fields. By definition, these observed current operators have finite matrix elements between physical states; therefore, they clearly represent the renormalized fields. One could, and perhaps should, directly use $\mathcal{J}_\nu^\gamma$ and $\mathcal{J}_\nu^{wk}$ without ever introducing W_ν^0 and $W_\nu^\pm$; it is mainly for reasons of pure convention that the notations W_ν^0 and $W_\nu^\pm$ are adopted in this paper.

[5] Condition (b) can be replaced by a weaker condition (b′) which simply requires the absence of all infinite $O(f^2)$ terms in δm. Since all powers of $(e/f)^2$ will be kept in our discussions, (b) and (b′) become closely related.

strangeness-nonconserving ($\Delta S\neq 0$) nonleptonic weak processes are transmitted by a single $W^{\pm}$.

As will be discussed in Sec. 2, the simplest model compatible with these three conditions is one in which there are six intermediate bosons, forming an SU_3 triplet

$$\begin{pmatrix} W^+ \\ W_N{}^0 \\ W_S{}^0 \end{pmatrix} \tag{1.3}$$

and its Hermitian conjugate

$$\begin{pmatrix} W^- \\ \bar{W}_N{}^0 \\ \bar{W}_S{}^0 \end{pmatrix}. \tag{1.4}$$

In this model, the total electromagnetic current operator $\mathcal{J}_\nu{}^\gamma$ becomes

$$\mathcal{J}_\nu{}^\gamma(x) = -(m_W{}^2/f_0)W_\nu{}^0(x), \tag{1.5}$$

where $W_\nu{}^0$ is related to the components of $W_N{}^0$ and $\bar{W}_N{}^0$ by

$$W_\nu{}^0 = -2^{-1/2}i(W_N{}^0 - \bar{W}_N{}^0)_\nu \tag{1.6}$$

and f_0 is a constant related to f and the Cabibbo angle[6] θ by

$$f_0 = (\tfrac{3}{2})^{1/2} f \cos\theta. \tag{1.7}$$

In Sec. 3, we study the general form of the total Lagrangian which includes all interactions: weak, electromagnetic, and strong. The usual canonical commutation rules imply that $\mathcal{J}_\nu{}^\gamma$ satisfy the following field algebra:

$$[\mathcal{J}_4{}^\gamma(\mathbf{r},t), \mathcal{J}_4{}^\gamma(\mathbf{r}',t)] = [\mathcal{J}_i{}^\gamma(\mathbf{r},t), \mathcal{J}_j{}^\gamma(\mathbf{r}',t)] = 0, \tag{1.8}$$

$$[\mathcal{J}_4{}^\gamma(\mathbf{r},t), \mathcal{J}_j{}^\gamma(\mathbf{r}',t)] = \lambda\nabla_j\delta^3(\mathbf{r}-\mathbf{r}'), \tag{1.9}$$

and

$$[(\partial/\partial t)\mathcal{J}_j{}^\gamma(\mathbf{r},t) - i\nabla_j\mathcal{J}_4{}^\gamma(\mathbf{r},t), \mathcal{J}_k{}^\gamma(\mathbf{r}',t)] = -i\lambda'\delta_{jk}\delta^3(\mathbf{r}-\mathbf{r}'), \tag{1.10}$$

where λ and λ' are c numbers. These relations together with the conservation law

$$(\partial/\partial x_\mu)\mathcal{J}_\mu{}^\gamma = 0 \tag{1.11}$$

require that the operator

$$\left[\frac{\partial}{\partial t}\mathcal{J}_\mu{}^\gamma(\mathbf{r},t),\ \mathcal{J}_\mu{}^\gamma(\mathbf{r}',t)\right] - \langle \mathrm{vac}|\left[\frac{\partial}{\partial t}\mathcal{J}_\mu{}^\gamma(\mathbf{r},t),\ \mathcal{J}_\mu{}^\gamma(\mathbf{r}',t)\right]|\mathrm{vac}\rangle = 0. \tag{1.12}$$

Thus, by using the general method developed by Bjorken,[7] one proves the finiteness of all second-order electromagnetic corrections.

In the present case, unlike those discussed in I, the magnetic moment of $W^{\pm}$ can be simply

$$(2m_W)^{-1}e. \tag{1.13}$$

Consequently, the second-order electromagnetic mass difference between $W^{\pm}$ and W^0 also becomes finite.[8] As will be shown in Sec. 4, these commutation relations (1.8)–(1.10), and therefore also (1.12), can be valid to *all* powers of e^2 and f^2 [while in I, Eq. (1.10) is valid only to the lowest power of f^2].

In Sec. 5, the photon propagator is analyzed by closely following the general considerations given in I. It is shown that, in the Landau gauge, the unrenormalized photon propagator $D_{\mu\nu}{}^\gamma$ becomes

$$-i(e/e_0)^2q^{-2}(\delta_{\mu\nu} - q^{-2}q_\mu q_\nu), \quad \text{as} \quad q^2 \to 0$$

and

$$-iq^{-2}(\delta_{\mu\nu} - q^{-2}q_\mu q_\nu), \quad \text{as} \quad q^2 \to \infty \tag{1.14}$$

where e_0 and e are, respectively, the unrenormalized and the renormalized charge and q_μ is the 4-momentum of the photon. The unrenormalized charge e_0 is found to be finite, bounded by the inequality

$$1 \leqq e_0/e \leqq \sqrt{2}. \tag{1.15}$$

Furthermore, the semiweak coupling constant f^2 satisfies the inequality

$$f_0{}^2 \geqq 4e^2 m_W{}^4 \int M^{-4}\sigma_W(M^2)dM^2, \tag{1.16}$$

where f_0 is related to f by (1.7) and $\sigma_W(M^2)$ is the spectral function of the $W_\nu{}^0$ propagator. If one neglects $O(e^2)$, $O(f^2)$, and all higher-order corrections, but keeps all powers of $(e/f)^2$, then the inequality (1.16) becomes simply

$$f_0{}^2 \geqq 2e^2, \tag{1.17}$$

and in the lower limit $f_0{}^2 = 2e^2$ one has

$$e_0{}^2 = 2e^2,$$

$$f^2 = \tfrac{4}{3}e^2 \sec^2\theta,$$

and

$$m_W = 4(\tfrac{1}{3}\pi\alpha/G_F)^{1/2}2^{1/4}\sec\theta \cong m_N/\alpha,$$

where α is the fine-structure constant. These results suggest that f^2 may be of the same order of magnitude as e^2. Therefore, the $O(f^2)$ terms in the hadronic mass-shift calculations should be discussed on the same basis as the $O(e^2)$ terms, in accordance with the above condition (b).

It is natural to identify the SU_3 triplet $(W^+, W_N{}^0, W_S{}^0)$ as the CP conjugate of $(W^-, \bar{W}_N{}^0, \bar{W}_S{}^0)$. The decomposition of $W_N{}^0$ and $\bar{W}_N{}^0$ into their Hermitian components

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[6] N. Cabibbo, Phys. Letters **10**, 513 (1963); M. Gell-Mann and M. Lévy, Nuovo Cimento **16**, 705 (1960).

[7] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

[8] Details will be given in a separate paper by T. D. Lee and K. Y. Ng (to be published).

$W^{0\prime}$ and W^0,

$$(W_N{}^0)_\lambda = 2^{-1/2}(W_\lambda{}^{0\prime} + iW_\lambda{}^0)$$

and $\qquad$ (1.18)

$$(\bar{W}_N{}^0)_\lambda = 2^{-1/2}(W_\lambda{}^{0\prime} - iW_\lambda{}^0),$$

then leads to opposite CP values for W^0 and $W^{0\prime}$; i.e.,

$$CP(W^0) = -CP(W^{0\prime}). \tag{1.19}$$

Yet, as will be shown in Sec. 2, in order to satisfy (b), both W^0 and $W^{0\prime}$ are coupled to the two $S=0$ neutral members of the *same* hadronic vector current, which requires

$$CP(W^0) = +CP(W^{0\prime}). \tag{1.20}$$

Such a mismatch implies CP nonconservation. As will be discussed in Sec. 6, since $W_\mu{}^0$ is the source of the electromagnetic field, the magnitude of such CP-nonconservation amplitudes becomes related to the fine-structure constant α. Another interesting feature is that the CP-violating amplitudes in radiative weak decays may become bigger than those in the corresponding nonradiative weak decays by a factor e^{-1}.

Some further experimental consequences of such heavy intermediate bosons are briefly discussed in Sec. 7.

In Appendix A, the general solution of various possible models consistent with the three conditions (a), (b), and (c) is given. For clarity of presentation, most of our discussions in the text are given explicitly only for the simplest model. It will become clear, however, that many of our conclusions are also valid for other models.

2. MODEL OF INTERMEDIATE-BOSON FIELDS

From (c), it follows that the interaction between $W^\pm$ and the known hadrons and leptons is of the form[9]

$$-fJ_\lambda W_\lambda{}^- - fJ_\lambda{}^* W_\lambda{}^+, \tag{2.1}$$

where

$$J_i{}^* = J_i{}^\dagger, \qquad W_i{}^+ = (W_i{}^-)^\dagger,$$
$$J_4{}^* = -J_4{}^\dagger, \quad W_4{}^+ = -(W_4{}^-)^\dagger, \tag{2.2}$$

and the superscript dagger denotes Hermitian conjugation. All known leptonic, semileptonic, and the $\Delta S \neq 0$ nonleptonic weak processes are described by the second-order "effective" Lagrangian density

$$\mathfrak{L}_{\text{eff}}(x) = f^2 \int J_\lambda(x) D_{\lambda\mu}(x-x') J_\mu{}^*(x') d^4x', \tag{2.3}$$

where $D_{\lambda\mu}(x-x')$ is the covariant propagator of $W^\pm$ and J_λ can be decomposed into a sum of the lepton part $j_\lambda{}^{\text{wk}}(l)$ and the hadron part $j_\lambda{}^{\text{wk}}(h)$:

$$J_\lambda = j_\lambda{}^{\text{wk}}(l) + j_\lambda{}^{\text{wk}}(h). \tag{2.4}$$

[9] Throughout the paper, all repeated indices are to be summed over. The Greek subscript λ denotes the space-time index; $\lambda = i$ (or j, or k) refers to the space component, $\lambda = 4$ is the time component, and $x_4 = it$.

In terms of the lepton fields $\psi_l(x)$ and $\psi_{\nu_l}(x)$, one has the familiar expression

$$j_\lambda{}^{\text{wk}}(l) = -i \sum_{l=e,\mu} \psi_l{}^\dagger \gamma_4 \gamma_\lambda (1+\gamma_5) \psi_{\nu_l}, \tag{2.5}$$

where $\gamma_1, \gamma_2, \cdots, \gamma_5$ are the usual five anticommuting Hermitian Dirac matrices. The hadron part $j_\lambda{}^{\text{wk}}(h)$ consists of the usual $\Delta S=0$ and $\Delta S \neq 0$ vector and axial-vector currents.

In the following, for simplicity, we assume that the field-current identity concept can also be extended to these hadron currents.[10,11] We may then regard $j_\lambda{}^{\text{wk}}(h)$ as a basic field variable and write

$$j_\lambda{}^{\text{wk}}(h) = -2^{1/2}(m_\rho{}^2/g_\rho)\{\cos\theta\,(\rho_\lambda{}^+ + a_\lambda{}^+) + \sin\theta[(K_V{}^+)_\lambda + (K_A{}^+)_\lambda]\}, \tag{2.6}$$

where, for pure convenience, $j_\lambda{}^{\text{wk}}(h)$ is resolved to a linear combination of $\rho_\lambda{}^+(x)$, $a_\lambda{}^+(x)$, $K_{V\lambda}{}^+(x)$, and $K_{A\lambda}{}^+(x)$ which may, respectively, be referred to as the field operators of ρ^+, $A_1{}^+$, $K_V{}^{*+}$, and $K_A{}^{*+}$ mesons, m_ρ is the mass of the neutral ρ meson, and g_ρ is the ρ coupling constant, $g_\rho{}^2/4\pi \cong 2.5$.

Both conditions (a) and (b) require the presence of neutral intermediate-boson fields.[12] To be consistent with (c), the second-order effects of these neutral boson fields on the hadronic system are restricted only to $\Delta S=0$ nonleptonic weak processes, of which very little is known at present.

The general solution of various possible models that satisfy (a), (b), and (c) is given in Appendix A. In this section, we only discuss the simplest one. As will be shown in Appendix A, the intermediate-boson system forms an SU_3 triplet (1.3) and its Hermitian conjugate (1.4). The weak-interaction Lagrangian can be written as a sum of three terms:

$$\mathfrak{L}_{\text{weak}} = \mathfrak{L}_{W-l} + \mathfrak{L}_{W-h} + \mathfrak{L}_{W-h}{}'. \tag{2.7}$$

The W-lepton part is

$$\mathfrak{L}_{W-l} = f[W_\lambda{}^- \sum i\psi_l{}^\dagger \gamma_4 \gamma_\lambda (1+\gamma_5) \psi_{\nu_l} + \text{H.c.}] + f_0 W_\lambda{}^0 \sum i\psi_l{}^\dagger \gamma_4 \gamma_\lambda \psi_l, \tag{2.8}$$

where the sum extends over $l=e$ and μ. The W-hadron

[10] N. M. Kroll, T. D. Lee, and Bruno Zumino, Phys. Rev. **157** 1376 (1967). The field-current identity concept gives, in the language of the local field theory, a precise formulation of the vector-dominance idea discussed by Y. Nambu, Phys. Rev. **106**, 1366 (1957); W. R. Frazer and J. R. Fulco, *ibid.* **117**, 1603 (1960); J. J. Sakurai, Ann. Phys. (N.Y.) **11**, 1 (1960); and M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).

[11] T. D. Lee and Bruno Zumino, Phys. Rev. **163**, 1667 (1967).

[12] In the literature, quite often neutral intermediate-boson fields are introduced in order to satisfy the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule, or the octet dominance, which is a different motivation from the present one. [See T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960); B. d'Espagnat, Phys. Letters **7**, 209 (1963); M. L. Good, L. Michel, and E. deRafael, Phys. Rev. **151**, 1194 (1966)]. In the present case, the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule is not satisfied; instead one has the simple (current $\times$ current) form (2.3).

parts are given by

$$\mathcal{L}_{W-h}=f\cos\theta\,(m_\rho^2/g_\rho)2^{1/2}\{(\rho_\lambda^+ + a_\lambda^+)W_\lambda^- + (\rho_\lambda^- + a_\lambda^-)\times W_\lambda^+ + [(\tfrac{3}{4})^{1/2}(3^{-1/2}\rho_\lambda^0 - v_\lambda^0) + a_\lambda^0](W_\lambda^0)' + (\tfrac{3}{4})^{1/2}(\rho_\lambda^0 + 3^{-1/2}v_\lambda^0)W_\lambda^0\} \quad (2.9)$$

and

$$\mathcal{L}_{W-h}'=f\sin\theta\,(m_\rho^2/g_\rho)2^{1/2}[(K_V^+ + K_A^+)_\lambda W_\lambda^- + (K_V^0 + K_A^0)_\lambda(\bar{W}_S^0)_\lambda] + \text{H.c.}, \quad (2.10)$$

where W_λ^0 and $(W_\lambda^0)'$ are the two Hermitian fields related to W_N^0 and $\bar{W}_N^0$ by (1.18). In the above expressions, we have extended the field-current identity to *all* hadron currents, where, just as in (2.6), ρ_λ, a_λ, $(K_V)_\lambda$, and $(K_A)_\lambda$ are, respectively, the $I=1$ vector, $I=1$ axial-vector, $I=\frac{1}{2}$ vector, and $I=\frac{1}{2}$ axial-vector strongly interacting meson fields (or the corresponding hadron currents), and the superscripts $+$, $-$, and 0 denote their charges. The v_λ^0 field is the eighth member of the same strongly interacting vector meson octet field. Because of the SU_3 breaking interaction, v_λ^0 is a linear combination of the ϕ^0 and ω^0 meson fields.

The electromagnetic interaction will be introduced in Sec. 3. As we shall see, the W_λ^0 field will turn out to be the source of the electromagnetic field; i.e., (1.5) is satisfied and f_0 is given by (1.7). The free intermediate-boson system, of course, is assumed to satisfy the SU_3 symmetry (or, more generally, the SO_6 symmetry). The electromagnetic interaction singles out the W_λ^0 field and thereby breaks the SU_3 symmetry. The weak interaction (2.7) also violates the SU_3 symmetry.

It is interesting to observe that $\mathcal{L}_{W-h}'$ preserves a subgroup of SU_2 symmetry by regarding

$$\begin{pmatrix}K_A^+\\K_A^0\end{pmatrix},\quad\begin{pmatrix}K_V^+\\K_V^0\end{pmatrix},\quad\text{and}\quad\begin{pmatrix}W^+\\W_S^0\end{pmatrix} \quad (2.11)$$

as SU_2 doublets. Similarly, $\mathcal{L}_{W-h}$ also preserves a (different) SU_2 symmetry, provided that one identifies

$$\begin{bmatrix}a^+\\a^0\\a^-\end{bmatrix},\quad\begin{bmatrix}\rho^+\\\frac{1}{2}(\rho^0-3^{1/2}v^0)\\\rho^-\end{bmatrix},\quad\text{and}\quad\begin{bmatrix}W^+\\W^{0\prime}\\W^-\end{bmatrix} \quad (2.12)$$

as SU_2 triplets and

$$\tfrac{1}{2}(3^{1/2}\rho_0+v^0)\quad\text{and}\quad W^0 \quad (2.13)$$

as SU_2 singlets.

To evaluate the $O(f^2)$ hadronic mass shifts, there are three terms due, respectively, to the products $\mathcal{L}_{W-h}'\mathcal{L}_{W-h}'$, $\mathcal{L}_{W-h}\mathcal{L}_{W-h}'$, and $\mathcal{L}_{W-h}\mathcal{L}_{W-h}$. The first term $\mathcal{L}_{W-h}'\mathcal{L}_{W-h}'$ conserves the *usual* isospin. The second term $\mathcal{L}_{W-h}\mathcal{L}_{W-h}'$ violates the strangeness conservation; therefore, it does not contribute. In the third term there is no interference between the vector and axial-vector fields because of parity conservation; thus, $\mathcal{L}_{W-h}\mathcal{L}_{W-h}$ only gives two terms which are, respectively, proportional to

$$2\rho^+\rho^- + \tfrac{1}{4}(\rho^0-3^{1/2}v^0)^2 + \tfrac{1}{4}(3^{1/2}\rho^0+v^0)^2 = 2\rho^+\rho^- + \rho^0\rho^0 + v^0v^0 \quad (2.14)$$

and

$$2a^+a^- + a^0a^0, \quad (2.15)$$

and both terms conserve the *usual* isospin. Condition (b) is then satisfied. From the explicit form of $\mathcal{L}_{\text{weak}}$, one sees that condition (c) is also satisfied.

In the above discussion, only the isospin transformation properties of the hadron currents are used. Thus the validity of the intermediate-boson model is clearly independent of whether these hadron currents $j_\lambda^{\text{wk}}(h)$ and $j_\lambda^\gamma(h)$ are, or are not, basic field variables.

3. TOTAL LAGRANGIAN

To introduce the electromagnetic field A_μ, it is convenient to separate out the W_λ^0-dependent part in $\mathcal{L}_{\text{weak}}$. Equation (2.7) can be written as

$$\mathcal{L}_{\text{weak}} = f_0 W_\lambda^0[(m_\rho^2/g_\rho)(\rho_\lambda^0+3^{-1/2}v_\lambda^0) + i\sum_l \psi_l^\dagger\gamma_4\gamma_\lambda\psi_l] + \mathcal{L}_{\text{weak}}', \quad (3.1)$$

where $\mathcal{L}_{\text{weak}}'$ is independent of W_λ^0. Let us define

$$\hat{\phi}_\lambda^0 = (\tfrac{3}{4})^{1/2}(\rho_\lambda^0+3^{-1/2}v_\lambda^0), \quad (3.2)$$

$$\hat{W}_\lambda^0 = W_\lambda^0 + (e_0/f_0)A_\lambda, \quad (3.3)$$

and

$$\phi_\lambda^0 = \hat{\phi}_\lambda^0 - (f_0/g_0)\hat{W}_\lambda^0, \quad (3.4)$$

where e_0 is the unrenormalized charge of the electron ($e_0<0$), f_0 and g_0 are both *finite* coupling constants, given by (1.7) and, for reasons which will become clear,

$$g_0 = g_\rho(\tfrac{3}{4})^{1/2}. \quad (3.5)$$

The total Lagrangian of the entire system is assumed to be of the form

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{2}(m_W W_\nu^0)^2 - \tfrac{1}{4}(1+\eta)(\hat{W}_{\mu\nu}^0)^2 - \tfrac{1}{2}(m_\rho\phi_\nu^0)^2 + \mathcal{L}_h(\hat{\phi}_{\mu\nu}^0, D_\nu\psi_h, \psi_h) + \mathcal{L}_l(\psi_l, \partial_\nu\psi_l) + \mathcal{L}_W(W_\lambda^\pm, \partial_\nu W_\lambda^\pm) + \mathcal{L}_{\text{weak}}', \quad (3.6)$$

where η is a constant[3] connected with the renormalization problem of W_λ^0, ψ_h refers to all hadron fields except $\hat{\phi}_\lambda^0$,

$$\hat{W}_{\mu\nu}^0 = \frac{\partial}{\partial x_\mu}\hat{W}_\nu^0 - \frac{\partial}{\partial x_\nu}\hat{W}_\mu^0, \quad (3.7)$$

$$\hat{\phi}_{\mu\nu}^0 = \frac{\partial}{\partial x_\mu}\hat{\phi}_\nu^0 - \frac{\partial}{\partial x_\nu}\hat{\phi}_\mu^0, \quad (3.8)$$

$$F_{\mu\nu} = \frac{\partial}{\partial x_\mu}A_\nu - \frac{\partial}{\partial x_\nu}A_\mu, \quad (3.9)$$

$$D_\nu\psi_h = (\partial/\partial x_\nu + ig_0Q_h\hat{\phi}_\nu^0)\psi_h, \quad (3.10)$$

$$\partial_\nu\psi_l = (\partial/\partial x_\nu - if_0\hat{W}_\nu^0)\psi_l, \quad (l=e^-\text{ or }\mu^-) \quad (3.11)$$

$$\partial_\nu W_\lambda^\pm = (\partial/\partial x_\nu \pm if_0\hat{W}_\nu^0)W_\lambda^\pm, \quad (3.12)$$

and Q_h is the charge of ψ_h. For clarity, we do not explicitly exhibit the ν_l dependence in $\mathcal{L}_l$, nor the dependence on W_S^0, $\bar{W}_S^0$, and $W^{0\prime}$ in $\mathcal{L}_W$.

The usual strong interaction is given by the $e_0=f_0=0$ limit of

$$-\tfrac{1}{2}m_\rho^2(\phi_\nu^0)^2+\mathcal{L}_h(\hat{\phi}_{\mu\nu}^0,D_\nu\psi_h,\psi_h). \tag{3.13}$$

As shown in Ref. 11, the requirement of the field conservation law $\partial\phi_\nu^0/\partial x_\nu=0$ determines the general structure of the strong-interaction Lagrangian which, in turn, leads to the appropriate non-Abelian field algebra[13] satisfied by $\rho^\pm$, ρ^0, v^0, etc.

The charge lepton part of $\mathcal{L}_l$ has the usual form

$$-\sum_l \psi_l^\dagger\gamma_4(\gamma_\mu\partial_\mu+m_l^0)\psi_l, \tag{3.14}$$

where m_l^0 is the unrenormalized mass. The $W^\pm$ part of $\mathcal{L}_W$ is given by

$$-\tfrac{1}{4}(1+\eta)(G_{\mu\nu}^+G_{\mu\nu}^-+G_{\mu\nu}^-G_{\mu\nu}^+) -\tfrac{1}{2}m_W^2(W_\mu^+W_\mu^-+W_\mu^-W_\mu^+), \tag{3.15}$$

where, in accordance with (3.12),

$$G_{\mu\nu}^\pm=\partial_\mu W_\nu^\pm-\partial_\nu W_\mu^\pm. \tag{3.16}$$

From (3.13), it follows that the zeroth-order (therefore, $\eta=0$) magnetic moment of $W^\pm$ is

$$\mu_W=(e/2m_W)\times(\mathrm{spin})_W.$$

As a result, the $O(e^2)$ mass renormalization of $W^\pm$ will turn out to be finite.[8]

Under the local electromagnetic gauge transformation

$$\psi\to\exp[ie_0Q\Lambda(x)]\psi, \tag{3.17}$$

where ψ denotes all charged fields ψ_h, ψ_l, and $W_\mu^\pm$, and Q is the appropriate charge, the neutral fields ϕ_λ^0, W_λ^0, and A_λ transform according to

$$\phi_\lambda^0\to\phi_\lambda^0,\qquad W_\lambda^0\to W_\lambda^0, \tag{3.18}$$

and

$$A_\lambda^0\to A_\lambda^0-\partial\Lambda/\partial x_\lambda.$$

Therefore, one has

$$\hat{\phi}_\lambda^0\to\hat{\phi}_\lambda^0-(e_0/g_0)\partial\Lambda/\partial x_\lambda$$

and

$$\hat{W}_\lambda^0\to\hat{W}_\lambda^0-(e_0/f_0)\partial\Lambda/\partial x_\lambda. \tag{3.19}$$

The gauge invariance of the total Lagrangian (3.6) is then obvious.

We note that if the gauge-independent field ϕ_λ^0 is regarded as an independent hadron field variable, then in the total Lagrangian (3.6) all W^0 hadron couplings are through the gauge-covariant derivatives $D_\nu\psi_h$ and $\hat{\phi}_{\mu\nu}^0$. On the other hand, one may choose, instead of ϕ_λ^0, the gauge-dependent field $\hat{\phi}_\lambda^0$ as an independent variable. In the latter case, the W^0 hadron couplings are completely contained in the mass term $-\tfrac{1}{2}(m_\rho\phi_\lambda^0)^2$. By using (3.3) and (3.4), one finds

$$\begin{aligned}-\tfrac{1}{2}(m_\rho\phi_\lambda^0)^2=&f_0W_\lambda^0(m_\rho^2/g_0)\hat{\phi}_\lambda^0\\ &+e_0A_\lambda(m_\rho^2/g_0)\hat{\phi}_\lambda^0-e_0f_0(m_\rho^2/g_0^2)W_\lambda^0A_\lambda\\ &-\tfrac{1}{2}m_\rho^2[(\hat{\phi}_\lambda^0)^2+(f_0W_\lambda^0/g_0)^2+(e_0A_\lambda/g_0)^2]\end{aligned}$$

in which, on account of (3.2) and (3.5), the first term

$$f_0W_\lambda^0(m_\rho^2/g_0)\hat{\phi}_\lambda^0=f_0W_\lambda^0(m_\rho^2/g_\rho)(\rho_\lambda^0+3^{-1/2}v_\lambda^0)$$

is the same W^0 hadron coupling term given by $\mathcal{L}_{\mathrm{weak}}$ (3.1).

The equations of motion for A_ν, W_ν^0, and $W_\nu^\pm$ are given by

$$\partial F_{\mu\nu}/\partial x_\mu=-e_0(m_W^2/f_0)W_\nu^0, \tag{3.20}$$

$$(1+\eta)\frac{\partial}{\partial x_\mu}\hat{W}_{\mu\nu}^0-m_W^2W_\nu^0 =f_0[j_\nu^\gamma(l)+j_\nu^\gamma(h)+j_\nu^\gamma(W)], \tag{3.21}$$

and

$$(1+\eta)\frac{\partial}{\partial x_\mu}G_{\mu\nu}^+-m_W^2W_\nu^+=f[j_\nu^{\mathrm{wk}}(l)+j_\nu^{\mathrm{wk}}(h)], \tag{3.22}$$

where

$$j_\nu^\gamma(l)=-i\psi_e^\dagger\gamma_4\gamma_\nu\psi_e-i\psi_\mu^\dagger\gamma_4\gamma_\nu\psi_\mu, \tag{3.23}$$

$$j_\nu^\gamma(h)=-(m_\rho^2/g_0)\phi_\nu^0, \tag{3.24}$$

$$j_\nu^\gamma(W)=i\tfrac{1}{2}(1+\eta)(W_\mu^-G_{\mu\nu}^++G_{\mu\nu}^+W_\mu^--G_{\mu\nu}^-W_\mu^+ -W_\mu^+G_{\mu\nu}^-), \tag{3.25}$$

and $j_\nu^{\mathrm{wk}}(l)$ and $j_\nu^{\mathrm{wk}}(h)$ are, respectively, given by (2.5) and (2.6).

The equations of motion for other fields can be readily obtained by using (3.6). For example, ϕ_ν^0 satisfies

$$\frac{\partial}{\partial x_\mu}\left(-\frac{\partial\mathcal{L}_h}{\partial\hat{\phi}_{\mu\nu}^0}+\frac{\partial\mathcal{L}_h}{\partial\hat{\phi}_{\nu\mu}^0}\right)-m_\rho^2\phi_\nu^0=g_0S_\nu, \tag{3.26}$$

where

$$S_\nu=-i\sum_h\frac{\partial\mathcal{L}_h}{\partial D_\nu\psi_h}Q_h\psi_h \tag{3.27}$$

and the sum extends over all ψ_h.

From (3.20) and (3.21) one sees that the total charge operator Q is given by

$$Q=i\int\frac{m_W^2}{f_0}W_4^0d^3r =-i\int[j_4^\gamma(l)+j_4^\gamma(h)+j_4^\gamma(W)]d^3r. \tag{3.28}$$

By equating

$$-i\int j_4^\gamma(h)d^3r=I_z+\tfrac{1}{2}Y, \tag{3.29}$$

[13] T. D. Lee, S. Weinberg, and Bruno Zumino, Phys. Rev. Letters 18, 1029 (1967).

one finds the familiar relations

$$i\int \frac{m_\rho^2}{g_\rho}\rho_4{}^0 d^3r = I_z, \qquad (3.30)$$

$$i\int \frac{m_\rho^2}{g_\rho}v_4{}^0 d^3r = (\tfrac{3}{4})^{1/2}Y, \qquad (3.31)$$

where I_z and Y are, respectively, the z-component isospin and the hypercharge, and, similarly to (3.2), the gauge-independent field $\phi_\lambda{}^0$ is related to $\rho_\lambda{}^0$ and $v_\lambda{}^0$ by

$$\phi_\lambda{}^0 = (\tfrac{3}{4})^{1/2}(\rho_\lambda{}^0 + 3^{1/2}v_\lambda{}^0). \qquad (3.32)$$

4. FIELD ALGEBRA

To quantize the Lagrangian (3.6), it is convenient to use the Coulomb gauge and choose

$$A_j{}^{\text{tr}},\ \hat{W}_j{}^0,\ \hat{\phi}_j{}^0,\ W_j{}^\pm,\ \psi_h,\ \psi_l$$

as the generalized coordinates; their conjugate momenta are, respectively,

$$P(A_j{}^{\text{tr}}) = -E_j{}^{\text{tr}}, \qquad (4.1)$$

$$P(\hat{W}_j{}^0) = i\hat{W}_{4j}{}^0(1+\eta) = (1+\eta)\left(\frac{\partial}{\partial t}\hat{W}_j{}^0 - i\nabla_j\hat{W}_4{}^0\right), \quad (4.2)$$

$$P(\hat{\phi}_j{}^0) = i(\partial\mathcal{L}_h/\partial\hat{\phi}_{j4}{}^0) - i(\partial\mathcal{L}_h/\partial\hat{\phi}_{4j}{}^0), \qquad (4.3)$$

$$P(W_j{}^\pm) = i(1+\eta)G_{4j}{}^\mp, \qquad (4.4)$$

$$P(h) = -i(\partial\mathcal{L}_h/\partial D_4\psi_h), \qquad (4.5)$$

and

$$P(l) = i\psi_l{}^\dagger, \qquad (4.6)$$

where $A_j{}^{\text{tr}}$ is the transverse part of the vector potential, which satisfies

$$\nabla_j A_j{}^{\text{tr}} = 0, \qquad (4.7)$$

and $E_j{}^{\text{tr}} = -\partial A_j{}^{\text{tr}}/\partial t$ is the transverse part of the electric field.

By using the equations of motion (3.21) and (3.26), one finds

$$\phi_4{}^0 = m_\rho{}^{-2}[i\nabla_j P(\hat{\phi}_j{}^0) - g_0\sum_h P(h)Q_h\psi_h] \qquad (4.8)$$

and

$$W_4{}^0 = m_W{}^{-2}\{i\nabla_j P(\hat{W}_j{}^0) - f_0[j_4{}^\gamma(h) + j_4{}^\gamma(l) + j_4{}^\gamma(W)]\}, \qquad (4.9)$$

where

$$j_4{}^\gamma(h) = -(m_W{}^2/g_0)\phi_4{}^0,$$
$$j_4{}^\gamma(l) = -P(e)\psi_e - P(\mu)\psi_\mu,$$

and

$$j_4{}^\gamma(W) = P(W_j{}^+)W_j{}^+ - P(W_j{}^-)W_j{}^-.$$

As a result of the usual canonical commutation relations, the total electromagnetic current operator

$$\mathcal{J}_\nu{}^\gamma = -(m_W{}^2/f_0)W_\nu{}^0$$

satisfies the field algebra (1.8)–(1.10) to *all* powers of e^2 and f^2; i.e.,

$$[\mathcal{J}_4{}^\gamma(\mathbf{r},t), \mathcal{J}_4{}^\gamma(\mathbf{r}',t)] = [\mathcal{J}_j{}^\gamma(\mathbf{r},t), \mathcal{J}_k{}^\gamma(\mathbf{r}',t)] = 0,$$
$$[\mathcal{J}_4{}^\gamma(\mathbf{r},t), \mathcal{J}_j{}^\gamma(\mathbf{r}',t)] = \lambda\nabla_j\delta^3(\mathbf{r}-\mathbf{r}'),$$

and

$$[(\partial/\partial t)\mathcal{J}_j{}^\gamma(\mathbf{r},t) - i\nabla_j\mathcal{J}_4{}^\gamma(\mathbf{r},t), \mathcal{J}_k{}^\gamma(\mathbf{r}',t)] = -i\lambda'\delta_{jk}\delta^3(\mathbf{r}-\mathbf{r}'),$$

where the constants λ and λ' are given by

$$\lambda = m_W{}^2/f_0{}^2 = \tfrac{2}{3}\sqrt{2}(G_F\cos^2\theta)^{-1}, \qquad (4.10)$$

$$\lambda' = \lambda m_0{}^2, \qquad (4.11)$$

and

$$m_0{}^2 = m_W{}^2[(1+\eta)^{-1} + (e_0/f_0)^2]. \qquad (4.12)$$

As shown in I, m_0 may be regarded as the mechanical mass of $W_\lambda{}^0$.

The hadron currents $j_\lambda{}^{\text{wk}}(h), j_\lambda{}^\gamma(h)$, etc., or the corresponding fields $\rho_\lambda{}^\pm(x)$, $\rho_\lambda{}^0(x)$, etc., can satisfy the usual SU_2, or chiral $SU_2\times SU_2$, or chiral $SU_3\times SU_3$ field algebra; these algebraic relations have already been discussed in the literature.[11,13]

It should be pointed out that the validity of the equal-time commutation relations (1.8)–(1.10) satisfied by the total electromagnetic current operator $\mathcal{J}_\nu{}^\gamma(x)$ is, however, independent of whether the hadron currents $j_\nu{}^{\text{wk}}(h)$ and $j_\nu{}^\gamma(h)$, are, or are not, basic field variables; i.e., whether these hadron current operators satisfy the non-Abelian field algebra, or the corresponding current algebra.[14]

5. PROPAGATORS AND INEQUALITIES

Let $\sigma_W(M^2)$ be the spectral function of W^0, defined by

$$\langle\text{vac}|[W_\mu{}^0(x), W_\nu{}^0(0)]|\text{vac}\rangle = \int \sigma_W(M^2)\left[\delta_{\mu\nu} - M^{-2}\frac{\partial^2}{\partial x_\mu\partial x_\nu}\right]\Delta_M(x)dM^2, \quad (5.1)$$

where

$$\Delta_M(x) = -i(2\pi)^{-3}\int(\mathbf{q}^2+M^2)^{-1/2}[\sin(\mathbf{q}^2+M^2)^{1/2}t] \times\exp(i\mathbf{q}\cdot\mathbf{r})d^3\mathbf{q}. \quad (5.2)$$

From the canonical commutation relations, one finds that,[3,15] to all powers of e^2 and f^2,

$$\int_0^\infty M^{-2}\sigma_W dM^2 = m_W{}^{-2} \qquad (5.3)$$

and

$$\int_0^\infty \sigma_W dM^2 = (m_0/m_W)^2. \qquad (5.4)$$

[14] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Physics **1**, 63 (1964).

[15] K. Johnson, Nucl. Phys. **25**, 435 (1961).

The covariant W^0 propagator $D_{\mu\nu}{}^W$ is defined to be the sum of all connected Feynman graphs with two external W^0 lines. In terms of σ_W, $D_{\mu\nu}{}^W$ is given by the familiar expression

$$D_{\mu\nu}{}^W(q)=-i\int_0^\infty \frac{dM^2\sigma_W(M^2)}{q^2+M^2-i\epsilon}\left(\delta_{\mu\nu}+\frac{q_\mu q_\nu}{M^2}\right), \quad (5.5)$$

where q_μ is the 4-momentum carried by the external W^0 line and ϵ is a positive infinitesimal. Since the sum rule (5.3) implies

$$D_{\mu\nu}{}^W(q)=-im_W{}^{-2}\delta_{\mu\nu} \quad \text{at} \quad q_\mu=0,$$

where m_W is the physical $W^\pm$ mass, $D_{\mu\nu}{}^W(q)$ is well defined at $q_\mu=0$. Thus, as discussed in I, there is no wave-function renormalization necessary for W^0.

The unrenormalized photon propagator $D_{\mu\nu}{}^\gamma$ is defined to be the sum of all connected Feynman graphs with two external photon lines. By following the general method developed in I, it can be shown that (see Appendix B) to all powers of e^2 and f^2,

$$D_{\mu\nu}{}^\gamma(q)=\frac{-i}{q^2}\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right) \times\left[1-\left(\frac{e_0}{f_0}\right)^2 m_W{}^4\int_0^\infty \frac{dM^2\sigma_W(M^2)}{M^2(q^2+M^2-i\epsilon)}\right], \quad (5.6)$$

where, for convenience, we have chosen the Landau gauge. The asymptotic behavior of the unrenormalized photon propagator is given by

$$\begin{aligned} D_{\mu\nu}{}^\gamma(q) &\to -i(e/e_0)^2 q^{-2}(\delta_{\mu\nu}-q^{-2}q_\mu q_\nu) \quad \text{as} \quad q^2\to 0 \\ &\to -iq^{-2}(\delta_{\mu\nu}-q^{-2}q_\mu q_\nu) \quad \text{as} \quad q^2\to\infty. \quad (5.7)\end{aligned}$$

By taking the $q^2\to 0$ limit of (5.6), one finds that the unrenormalized charge e_0 and the renormalized charge e are related by

$$\frac{e^2}{e_0{}^2}=1-\left(\frac{e_0}{f_0}\right)^2 m_W{}^4\int_0^\infty M^{-4}\sigma_W dM^2. \quad (5.8)$$

As will be shown in Appendix B, the following inequalities can then be established:

$$1\leqq\left(\frac{e_0}{e}\right)^2\leqq 2 \quad (5.9)$$

and

$$f_0{}^2\geqq 4e^2 m_W{}^4\int_0^\infty M^{-4}\sigma_W dM^2. \quad (5.10)$$

These inequalities are valid to all powers of e^2 and f^2; they are stronger inequalities than similar ones obtained in I. We find that not only is e_0 finite; its value is limited to between e and $\sqrt{2}e$.

The renormalized propagator $(D_{\mu\nu}{}^\gamma)_{\text{ren}}$ is, by definition, related to the unrenormalized propagator $D_{\mu\nu}{}^\gamma$ by

$$e^2(D_{\mu\nu}{}^\gamma)_{\text{ren}}=e_0{}^2 D_{\mu\nu}{}^\gamma. \quad (5.11)$$

Thus,

$$\begin{aligned}(D_{\mu\nu}{}^\gamma)_{\text{ren}} &\to -iq^{-2}(\delta_{\mu\nu}-q^{-2}q_\mu q_\nu) \quad \text{as} \quad q^2\to 0 \\ &\to -i(e_0/e)^2 q^{-2}(\delta_{\mu\nu}-q^{-2}q_\mu q_\nu) \\ &\qquad \text{as} \quad q^2\to\infty. \quad (5.12)\end{aligned}$$

Since the asymptotic behavior of $(D_{\mu\nu}{}^\gamma)_{\text{ren}}$ can, in principle, be experimentally observed, so should be e_0/e.

The renormalized electromagnetic field $(F_{\mu\nu})_{\text{ren}}$ is related to the unrenormalized electromagnetic field $F_{\mu\nu}$ by

$$e(F_{\mu\nu})_{\text{ren}}=e_0 F_{\mu\nu}. \quad (5.13)$$

Thus, in terms of $(F_{\mu\nu})_{\text{ren}}$, the Maxwell equation becomes

$$\frac{\partial}{\partial x_\mu}(F_{\mu\nu})_{\text{ren}}=e\left(\frac{e_0}{e}\right)^2 \mathcal{J}_\nu{}^\gamma, \quad (5.14)$$

where $\mathcal{J}_\nu{}^\gamma$ is the total electromagnetic current operator. The total charge operator Q is given by

$$Q=-i\int \mathcal{J}_4{}^\gamma d^3r \quad (5.15)$$

and

$$\langle e^-|Q|e^-\rangle=-1, \quad \langle p|Q|p\rangle=+1, \text{ etc.}$$

Therefore, $\mathcal{J}_\nu{}^\gamma$ has well-defined matrix elements between the various physical states. We emphasize that in order to have finite matrix elements for the renormalized field $(F_{\mu\nu})_{\text{ren}}$ between any two physical states, one *must* require the finitude of e_0/e; otherwise, the Maxwell equation (5.14) is not mathematically defined.[11]

The inequality (5.10) sets a lower limit on the semiweak coupling constant f_0, and therefore also on f. As $M^2\to 0$, it can be shown[16] that $\sigma_W\to cM^{10}$ where c is a constant. As $M^2\to\infty$, because of the sum rule (5.3), $\sigma_W\to 0$. Thus the integral $\int_0^\infty M^{-4}dM^2\sigma_W$ must exist. The precise value of

$$m_W{}^4\int_0^\infty M^{-4}\sigma_W dM^2 \quad (5.16)$$

is not known, especially since the integral

$$\int \sigma_W dM^2 \quad (5.17)$$

may diverge, corresponding to a possibly infinite mechanical W mass m_0.

[16] As $M^2\to 0$, $\sigma_W(M^2)$ is determined by the product of the phase space of 3γ, with a total energy M in its rest frame, times $|\langle \text{vac}|W^0|3\gamma\rangle|^2$. The former is given by the integral of

$$\prod_{i=1}^3 d^3\mathbf{k}_i\delta^3(\sum \mathbf{k}_i)\delta(\sum|\mathbf{k}_i|-M),$$

and it is proportional to M^5; the latter is proportional to $(M^{5/2}m_W{}^{-2})^2=M^5 m_W{}^{-4}$, where the factor $m_W{}^{-2}$ is due to the propagator of W^0 which satisfies $D_{\mu\nu}{}^W(q=0)=-im_W{}^{-2}\delta_{\mu\nu}$.

In the usual power-series expansion, we would regard

$$\eta = O(f^2), \tag{5.18}$$

even though, as shown in I, this $O(f^2)$ term does explicitly contain divergent integrals. Nevertheless, we may assume the power-series divergences to be simply manifestations of the presence of possible terms such as $f^2 \ln f^2$, or $f^{3/2}$, etc., in the theory. For example, if η is indeed $O(f^2 \ln f^2)$, then in a formal power-series expansion the coefficient of f^2 would naturally be infinite. Therefore, for small values of e^2 and f^2, such an assumption enables us to keep all powers of $(e/f)^2$, but neglect all $O(e^2)$ and $O(f^2)$ terms *including* η. The sum rule (5.4) then becomes

$$\int \sigma_W dM^2 = \left(\frac{\bar{m}_W}{m_W}\right)^2 = 1 + \left(\frac{e_0}{f_0}\right)^2, \tag{5.19}$$

where, as discussed in I, $\bar{m}_W$ is the physical mass of W^0 and m_W that of $W^\pm$. In the same approximation,

$$\sigma_W = (\bar{m}_W/m_W)^2 \delta(M^2 - \bar{m}_W^2), \tag{5.20}$$

and therefore both sum rules (5.3) and (5.19) are satisfied. Consequently, (5.10) becomes

$$f_0^2 \geqq 4e^2[1 + (e_0/f_0)^2]^{-1}, \tag{5.21}$$

which, as shown in Appendix B, can be reduced to

$$f_0^2 \geqq 2e^2. \tag{5.22}$$

Furthermore, at its lower limit, one has

$$f_0^2 = e_0^2 = 2e^2. \tag{5.23}$$

By using (1.1) and (1.7), we find, to the same approximation,

$$f^2 \geqq \tfrac{4}{3} e^2 \sec^2\theta \tag{5.24}$$

and

$$m_W \geqq 4(\tfrac{1}{3}\pi\alpha/G_F)^{1/2} 2^{1/4} \sec\theta, \tag{5.25}$$

where θ is the Cabibbo angle and α is the fine-structure constant. It is interesting to observe that numerically

$$4(\tfrac{1}{3}\pi\alpha/G_F)^{1/2} 2^{1/4} \sec\theta \cong 137 m_N.$$

Thus the lower limit of m_W is

$$m_W \cong \alpha^{-1} m_N. \tag{5.26}$$

The inequality (5.10) suggests at least that f^2 may be of the same order of magnitude as e^2. Therefore, the electromagnetic interaction should be studied in close connection with the weak interaction, and the $O(f^2)$ terms in the mass shifts should be treated on the same basis as the $O(e^2)$ terms, in accordance with our basic condition (b).

From the general Lagrangian (3.6), one sees that the electromagnetic field A_μ is coupled to the charged particles only through $\hat{W}_\mu^0$. Following the discussions given in I, it is convenient to define an *effective* photon propagator $\mathfrak{D}_{\mu\nu}{}^\gamma$ [see Eq. (4.5) in I]. In terms of Feynman graphs, $\mathfrak{D}_{\mu\nu}{}^\gamma$ represents the sum of products of propagators

$$\mathfrak{D}_{\mu\nu}{}^\gamma = (\hat{W}_\mu{}^0 - \gamma - \hat{W}_\nu{}^0) + (\hat{W}_\mu{}^0 - \gamma - \hat{W}_\lambda{}^0 - \gamma - \hat{W}_\nu{}^0) + \cdots.$$

Neglecting higher orders in e^2 and f^2, but keeping all orders in $(e/f)^2$, one finds

$$\mathfrak{D}_{\mu\nu}{}^\gamma = \frac{-i m_W{}^2 \bar{m}_W{}^2}{q^2(q^2+m_W{}^2)(q^2+\bar{m}_W{}^2)} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right), \tag{5.27}$$

where $\bar{m}_W$ is given by (5.19). As $q^2 \to \infty$,

$$\mathfrak{D}_{\mu\nu}{}^\gamma \to -iq^{-6} m_W{}^2 \bar{m}_W{}^2 (\delta_{\mu\nu} - q^{-2} q_\mu q_\nu), \tag{5.28}$$

in contrast to the corresponding asymptotic behavior of the photon propagator $D_{\mu\nu}{}^\gamma$, or $(D_{\mu\nu}{}^\gamma)_{\text{ren}}$, as given by (5.7) and (5.12). This q^{-6} behavior of the effective photon propagator provides the necessary convergence factor for obtaining finite second- and higher-order radiative corrections.

6. *CP* ASYMMETRY

It is natural to assume that the SU_3 triplet $(W^+, W_N{}^0, W_S{}^0)$ should be related to its Hermitian conjugate $(W^-, \bar{W}_N{}^0, \bar{W}_S{}^0)$ by a *CP* transformation. Under such a *CP* transformation, one has

$$\begin{bmatrix} W^+ \\ W_N{}^0 \\ W_S{}^0 \end{bmatrix} \to \begin{bmatrix} W^- \\ \bar{W}_N{}^0 \\ \bar{W}_S{}^0 \end{bmatrix}. \tag{6.1}$$

The decomposition (1.18) of $W_N{}^0$ into its Hermitian components W^0 and $W^{0\prime}$ leads to

$$CP(W^0) = -CP(W^{0\prime}). \tag{6.2}$$

On the other hand, according to (2.9), both W^0 and $W^{0\prime}$ are coupled to ρ^0 and v^0, which requires a different *CP* assignment, called *CP*′, where

$$CP'(W^0) = +CP'(W^{0\prime}). \tag{6.3}$$

The mismatch

$$CP \neq CP' \tag{6.4}$$

implies *CP* asymmetry.

This mismatch can also be stated in a slightly different form. The couplings between W^0 and the known particles are completely determined by the electromagnetic interaction $\mathfrak{L}_\gamma$ and the field-current identity between $\mathfrak{J}_\nu{}^\gamma$ and $W_\nu{}^0$. Thus, through $\mathfrak{L}_\gamma$, the transformation properties of W^0 under the charge-conjugation operation C_γ and the corresponding space-inversion operation P_γ are given by[17]

$$C_\gamma W_j{}^0 C_\gamma{}^{-1} = -W_j{}^0 \tag{6.5}$$

and

$$P_\gamma W_j{}^0 P_\gamma{}^{-1} = -W_j{}^0. \tag{6.6}$$

[17] For a general discussion of C_γ, P_γ, and the related time reversal T_γ, see T. D. Lee and G. C. Wick, Phys. Rev. **148**, 1385 (1966).

One may regard the mismatch as simply

$$CP \neq C_\gamma P_\gamma. \tag{6.7}$$

For W^0, $C_\gamma P_\gamma = CP' = +1$; i.e.,

$$C_\gamma P_\gamma W_j^0 (C_\gamma P_\gamma)^{-1} = +W_j^0. \tag{6.8}$$

Such a mismatch between CP and $C_\gamma P_\gamma$ can arise if

$$CPW_j^0(CP)^{-1} = -W_j^0. \tag{6.9}$$

We emphasize that the identification of W_N^0 to be the CP conjugate of $\bar{W}_N^0$, or W_j^0 to have $CP=-1$, must be implied by some *interactions*, since both assignments (6.2) and (6.3), or (6.8) and (6.9), are consistent with the free Lagrangian.

$$\begin{aligned} -\tfrac{1}{2}(1+\eta)(\bar{W}_N^0)_{\mu\nu}(W_N^0)_{\mu\nu} - m_W^2(\bar{W}_N^0)_\mu(W_N^0)_\mu \\ = -\tfrac{1}{4}(1+\eta)[(W_{\mu\nu}^{0\prime})^2 + (W_{\mu\nu}^0)^2] \\ -\tfrac{1}{2}m_W^2[(W_\mu^{0\prime})^2 + (W_\mu^0)^2], \end{aligned} \tag{6.10}$$

where

$$W_{\mu\nu} = (\partial/\partial x_\mu)W_\nu - (\partial/\partial x_\nu)W_\mu \tag{6.11}$$

and

$$W = W^0, \quad \text{or} \quad W^{0\prime}, \quad \text{or} \quad W_N^0, \quad \text{or} \quad \bar{W}_N^0.$$

To necessitate the CP assignment (6.2), or (6.9), we may include in the total Lagrangian (3.6) any general function

$$F(W^+, W_N^0, W_S^0;\ W^-, \bar{W}_N^0, \bar{W}_S^0) \tag{6.12}$$

provided that (i) F is invariant under CP, (ii) F is not invariant under $C_\gamma P_\gamma$, or CP', and (iii) F preserves the field-current identity

$$\mathcal{J}_\nu^\gamma = -(m_W^2/f_0)W_\nu^0. \tag{6.13}$$

Otherwise, the function F can be of an arbitrary form; it may also depend on other field variables ψ_h, ψ_l, etc.

As a simple example, let us consider the quadratic function

$$F = \kappa f^2[(\bar{W}_S^0)_{\mu\nu}(W_N^0)_{\mu\nu} + (\bar{W}_N^0)_{\mu\nu}(W_S^0)_{\mu\nu}], \tag{6.14}$$

where κ is a constant, and $(W_S^0)_{\mu\nu}$ and $(\bar{W}_S^0)_{\mu\nu}$ are both given by (6.11) in which W denotes W_S^0 and $\bar{W}_S^0$, respectively. Of course, in order to preserve the field-current identity (6.13), we must replace in (6.14)

$$W_\lambda^0 \quad \text{by} \quad \hat{W}_\lambda^0 = W_\lambda^0 + (e_0/f_0)A_\lambda. \tag{6.15}$$

It follows from $\mathcal{L}_{W-h}'$, (2.10), that W_S^0 is the CP conjugate of $\bar{W}_S^0$; consequently, (6.14) implies W_N^0 to be also the CP conjugate of $\bar{W}_N^0$, and it leads to the CP assignment (6.9), in conflict with $C_\gamma P_\gamma$.

The mismatch between CP and $C_\gamma P_\gamma$ necessitates CP asymmetry. Since W^0 is the source of the electromagnetic field, such a CP-violating amplitude becomes related to the fine-structure constant α. This is illustrated in Fig. 1. The CP-violating amplitude in the nonradiative nonleptonic $\Delta S \neq 0$ decays, such as K^0 (or K^0) $\to 2\pi$, becomes proportional to either

$$\kappa f^2 e^2 \quad \text{or} \quad \kappa f^4, \tag{6.16}$$

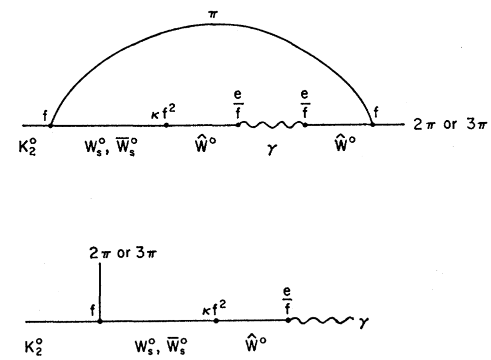

FIG. 1. Examples of CP-nonconserving transitions. [F is given by (6.14)].

and it exists in both $|\Delta \mathbf{I}| = \frac{1}{2}$ and $\frac{3}{2}$ channels. Because of (6.15), there will be CP-violating amplitudes proportional to

$$\kappa f^2 e \tag{6.17}$$

in the corresponding radiative weak decays, such as

$$K^0 \text{ (or } \bar{K}^0) \to 2\pi\gamma. \tag{6.18}$$

A characteristic feature of such a mismatch is that the CP-violating amplitudes in nonradiative decays may be smaller than those in the corresponding radiative decays by a factor e. To the same lowest powers of f^2 and e^2, there is *no CP* asymmetry in the usual semileptonic and leptonic weak processes, all of which involve neutrinos, and, furthermore, the neutron does *not* have an electric dipole moment.

As another example of F, we may consider the cubic function [with the replacement (6.15)]

$$F = i\lambda f^3\left[(W_N^0)_{\alpha\beta}(W_S^0)_\mu \frac{\partial}{\partial x_\mu}(W_S^0)_{\alpha\beta} - \text{H.c.}\right], \tag{6.19}$$

where λ is a constant. Under CP, $\nabla_j \to -\nabla_j$; and the intermediate-boson fields transform according to (6.1). Therefore, F is CP-invariant, and it requires W^0 to satisfy the CP assignment (6.9), in conflict with $C_\gamma P_\gamma$. To the lowest order, the resulting CP-violations in a hadronic system occur only in $\Delta S = \pm 2$ processes, and the CP-violating amplitudes are proportional to $\lambda f^4 e^2$, λf^6. Such a function F would lead to a special model of the "superweak" interaction.[18]

In these order-of-magnitude estimations, we have tacitly assumed that, to lowest order, each power f^2 should be accompanied by a multiplicative factor $(m_N/m_W)^2$. These factors can be due to the W propagator, or they can be explicitly included in the coefficients κ and λ.

Note that (6.14) and (6.19) are only the two simplest

[18] L. Wolfenstein, Phys. Rev. Letters **13**, 562 (1964); T. D. Lee and L. Wolfenstein, Phys. Rev. **138**, 1490 (1965).

examples[19] of a large variety of possible function F. A systematic study of the general case lies outside the scope of the present paper, and it will be given elsewhere. In the same connection, we may recall that the general question of fourth-order terms $O(f^4,f^2e^2,e^4)$ is also not analyzed here. The magnitudes of these terms depend not only on the explicit powers of f^2 and e^2, but also on the powers of $(m_N/m_W)^2$. These higher-order terms determine the K_1^0-K_2^0 mass difference; from the present view, depending on the model, they may also be relevant to the problem of CP nonconservation.

7. PRODUCTION AND DECAY

In the present theory, the mass of the intermediate boson is unusually heavy. At present, they can only be produced in cosmic radiation. In a very high energy, say, $p+p$ collision with a sufficiently large momentum transfer, the fraction of W production can be crudely estimated to be about

$$f^2 \sim O(e^2). \tag{7.1}$$

These intermediate bosons are of extremely short lifetimes; they can decay into both leptons and hadrons. The decay rates into leptons are given by

$$r_0 = \text{rate}(W^0 \to \mu^+ + \mu^-) \cong \text{rate}(W^0 \to e^+ + e^-) = (12\pi)^{-1} f_0{}^2 \bar{m}_W \tag{7.2}$$

and

$$r_+ = \text{rate}(W^+ \to \mu^+ + \nu_\mu) \cong \text{rate}(W^+ \to e^+ + \nu_e) = G_F m_W{}^3 (6\pi\sqrt{2})^{-1}, \tag{7.3}$$

where m_W and $\bar{m}_W$ are, respectively, the $W^\pm$ mass and the W^0 mass. If we take the lower limit (5.23)

$$f_0{}^2 = 2e^2, \quad \bar{m}_W = \sqrt{2} m_W$$

and the approximate value

$$m_W = \alpha^{-1} m_N,$$

then these lepton decay rates become

$$r_0 = \tfrac{2}{3}\sqrt{2} m_N \cong 1.4 \times 10^{24} \text{ sec}^{-1} \tag{7.4}$$

and

$$r_+/r_0 = G_F m_N{}^2 / 8\pi\alpha^3 \cong 1.0. \tag{7.5}$$

These intermediate bosons, if they exist, may give important contributions to the very-high-energy muons, such as those recently observed in cosmic radiation.[20]

ACKNOWLEDGMENTS

I wish to thank Professor L. A. Radicati and Professor G. C. Wick for discussion.

[19] In both examples, F is proportional to either f^2 or f^3. It is clear that the inclusion of such a weak interaction F is consistent with our basic condition (b), nor does it change our conclusions about the finiteness of other radiative corrections. So far as the CP-asymmetry problem is concerned, F may very well also contain strong-interaction terms between the W's. However, in such a case, one must reopen the question of whether the radiative corrections to, say, weak decays are, or are not, finite.

[20] H. E. Bergeson, J. W. Keuffel, M. O. Larson, E. R. Martin, and G. W. Mason, Phys. Rev. Letters **19**, 1487 (1967).

APPENDIX A

In this Appendix, we give the general solution of the various possible models which satisfy the three conditions (a), (b), and (c). The hadronic part of the weak interaction $\mathcal{L}_{\text{weak}}(h)$ is expected to be a linear function of the eight vector and the eight axial-vector hadron currents, or their corresponding strongly interacting meson fields $v_\mu{}^\alpha$ and $a_\mu{}^\alpha$ where $\alpha = 1, 2, \cdots, 8$. We may write

$$\mathcal{L}_{\text{weak}}(h) = \sum C_V{}^\alpha V_\mu{}^\alpha(x) v_\mu{}^\alpha(x) + \sum C_A{}^\alpha A_\mu{}^\alpha(x) a_\mu{}^\alpha(x), \tag{A1}$$

where $C_V{}^1 V_\mu{}^1(x)$, $C_A{}^1 A_\mu{}^1(x)$, $C_V{}^2 V_\mu{}^2(x)$, $\cdots$, $C_A{}^8 A_\mu{}^8(x)$ represent the appropriate cofactors of such a linear functional dependence. These functions $C_V{}^\alpha V_\mu{}^\alpha(x)$ and $C_A{}^\alpha A_\mu{}^\alpha(x)$ will turn out to be linear functions of the appropriate intermediate-boson fields. For conveneince, we choose $V_\mu{}^\alpha(x)$ and $A_\mu{}^\alpha(x)$ to have the same normalization so that, e.g., for the free fields, the commutator

$$[V_\mu{}^\alpha(x), V_\nu{}^\alpha(0)] = [A_\mu{}^\alpha(x), A_\nu{}^\alpha(0)] \tag{A2}$$

is independent of α.

To find the various possible solutions of the intermediate-boson system, the simplest method is to adopt the general idea of the schizon scheme.[21] We first regard these 16 Hermitian fields $V_\mu{}^\alpha$ and $A_\mu{}^\alpha$ as all independent, and then introduce linear relations between these fields to reduce the total number of independent fields.

Following the general schizon idea, we require that, in the absence of these linear relations, the weak interactions between $V_\mu{}^\alpha$, $A_\mu{}^\alpha$, and the hadrons conserve both isospin and parity. The isospin symmetry implies that (b) is satisfied. Our problem is to study all possible linear relations between these fields so that the resulting model becomes compatible with (a), (b), and (c).

The isospin symmetry requires

$$\begin{aligned} &C_V{}^1 = C_V{}^2 = C_V{}^3, \\ &C_A{}^1 = C_A{}^2 = C_A{}^3, \\ &C_V{}^4 = C_V{}^5 = C_V{}^6 = C_V{}^7, \\ \text{and} \quad &C_A{}^4 = C_A{}^5 = C_A{}^6 = C_A{}^7, \end{aligned} \tag{A3}$$

where the SU_3 supercript α follows the usual convention with $\alpha = 1, 2, 3$ denoting the isosvector components, and $\alpha = 4, 5, 6, 7$ denoting the hypercharge $Y = \pm 1$ components.

Condition (c) requires that the charged $W^\pm$ field is given by

$$\begin{aligned} W_\lambda{}^+ &= 2^{-1/2}(V_\lambda{}^1 - iV_\lambda{}^2) = 2^{-1/2}(V_\lambda{}^4 - iV_\lambda{}^5) \\ &= 2^{-1/2}(A_\lambda{}^1 - iA_\lambda{}^2) = 2^{-1/2}(A_\lambda{}^4 - iA_\lambda{}^5). \end{aligned} \tag{A4}$$

The $O(f^2)$ mass correction does not contain any term

[21] T. D. Lee and C. N. Yang, Phys. Rev. **119**, 1410 (1960).

which violates either parity-conservation or strangeness-conservation; thus (b) remains valid. Condition (a) requires that the source of the electromagnetic field is, apart from a multiplicative constant,

$$W_\lambda^0=(\tfrac{3}{4})^{1/2}(V_\lambda^3+3^{-1/2}V_\lambda^8). \tag{A5}$$

The Lagrangian density (A1) becomes

$$\mathcal{L}_{\mathrm{weak}}(h)=\mathcal{L}_{W-h}+\mathcal{L}_{W-h}', \tag{A6}$$

$$\begin{aligned}\mathcal{L}_{W-h}=C[(\rho_\lambda^++a_\lambda^+)W_\lambda^-+(\rho_\lambda^-+a_\lambda^-)W_\lambda^+\\ +a_\lambda^0A_\lambda^3+(\tfrac{3}{4})^{1/2}(\rho_\lambda^0+3^{-1/2}v_\lambda^8)W_\lambda^0\\ +(\tfrac{3}{4})^{1/2}(3^{-1/2}\rho_\lambda^0-v_\lambda^8)W_\lambda^{0\prime}]+C_A^8a_\lambda^8A_\lambda^8\end{aligned} \tag{A7}$$

and

$$\begin{aligned}\mathcal{L}_{W-h}'=C'[(K_V^++K_A^+)_\lambda W_\lambda^-+2^{-1/2}(K_V^0)_\lambda\\ \times(V_\lambda^6+iV_\lambda^7)+2^{-1/2}(K_A^0)_\lambda\\ \times(A_\lambda^6+iA_\lambda^7)+\mathrm{H.c.}],\end{aligned} \tag{A8}$$

where

$$C,C',C_A^8 \text{ are constants} \tag{A9}$$

and

$$(W_\lambda^0)'=(\tfrac{3}{4})^{1/2}(3^{-1/2}V_\lambda^3-V_\lambda^8). \tag{A10}$$

ρ_λ, $(K_V)_\lambda$, and v_λ^8 are, respectively, the $I=1$, $I=\frac{1}{2}$, and $I=0$ components of the vector hadron octet current (in the text, $v_\lambda^8=v_\lambda^0$). The $I=\frac{1}{2}$ component of the axial-vector hadron octet current is $(K_A)_\lambda$; its $I=1$ components are $a_\lambda^\pm$ and a_λ^0, and its eighth ($I=0$) component is a_λ^8.

Thus the maximum number of independent (Hermitian) fields is 10: W_λ^+, W_λ^-, W_λ^0, $W_\lambda^{0\prime}$, A_λ^3, A_λ^8, V_λ^6, V_λ^7, A_λ^6, and A_λ^7. It can be readily verified that all three conditions (a), (b), and (c) are satisfied. These 10 fields can be further reduced by imposing any number of the following identities:

$$A_\lambda^8=0, \qquad (\text{or, } C_A^8=0) \tag{A11}$$

$$W_\lambda^{0\prime}=A_\lambda^3, \tag{A12}$$

$$V_\lambda^6=A_\lambda^6, \tag{A13}$$

and

$$V_\lambda^7=A_\lambda^7 \tag{A14}$$

which are all consistent with (a), (b), and (c). The simplest model discussed in the text is one in which all of these four identities (A11)-(A14) are used. The constants C and C' are given by

$$C=2^{1/2}f\cos\theta\,(m_\rho^2/g_\rho)$$

and

$$C'=2^{1/2}f\sin\theta\,(m_\rho^2/g_\rho).$$

Equations (A7) and (A8) then reduce to (2.9) and (2.10), respectively, where

$$(W_S^0)_\lambda=2^{-1/2}(V_\lambda^6-iV_\lambda^7)=2^{-1/2}(A_\lambda^6-iA_\lambda^7)$$

and

$$(\bar{W}_S^0)_\lambda=2^{-1/2}(V_\lambda^6+iV_\lambda^7)=2^{-1/2}(A_\lambda^6+iA_\lambda^7).$$

APPENDIX B

To derive (5.6), we follow the same method given in Appendix C of I. By using the transformation [Eq. (C1) of I]

$$A_\mu'=N^{-1/2}A_\mu+N^{1/2}(1+\eta)(e_0/f_0)W_\mu^0, \tag{B1}$$

where

$$N=[1+(1+\eta)(e_0/f_0)^2]^{-1}, \tag{B2}$$

the Lagrangian (3.6) becomes

$$\mathcal{L}=\mathcal{L}_0+\mathcal{L}_1, \tag{B3}$$

where

$$\mathcal{L}_0=-\tfrac{1}{4}F_{\mu\nu}'^2-\tfrac{1}{4}(1+\eta)N(W_{\mu\nu}^0)^2-\tfrac{1}{2}m_W^2(W_\nu^0)^2, \tag{B4}$$

$$\begin{aligned}\mathcal{L}_1=-\tfrac{1}{2}(m_\rho\phi_\nu^0)^2+\mathcal{L}_h(\phi_{\mu\nu}^0,D_\nu\psi_h,\psi_h)+\mathcal{L}_l(\psi_l,\partial_\nu\psi_l)\\ +\mathcal{L}_W(W_\lambda^\pm,\partial_\nu W_\lambda^\pm)+\mathcal{L}_{\mathrm{weak}}',\end{aligned} \tag{B5}$$

$$W_{\mu\nu}^0=\frac{\partial}{\partial x_\mu}W_\nu^0-\frac{\partial}{\partial x_\nu}W_\mu^0, \tag{B6}$$

and

$$F_{\mu\nu}'=\frac{\partial}{\partial x_\mu}A_\nu'-\frac{\partial}{\partial x_\nu}A_\mu'. \tag{B7}$$

Let ψ denote *all* fields, except A_μ' and W_μ^0; its covariant derivative $\partial_\nu\psi$ is defined by

$$\partial_\nu\psi=(\partial/\partial x_\nu+if_0Q_\psi\hat{W}_\nu^0)\psi, \tag{B8}$$

where Q_ψ is its charge and

$$\hat{W}_\nu^0=W_\nu^0+(e_0/f_0)A_\nu=NW_\nu^0+(e_0/f_0)N^{1/2}A_\nu'. \tag{B9}$$

For $\psi=\psi_l$ and $W_\lambda^\pm$, this definition of $\partial_\nu\psi$ is identical to that given by (3.11) and (3.12). For $\psi=\psi_h$, one finds that, on account of (3.10), $\partial_\nu\psi_h$ is related to $D_\nu\psi_h$ by

$$D_\nu\psi_h=\partial_\nu\psi_h+ig_0Q_h\phi_\nu^0\psi_h. \tag{B10}$$

Thus $\mathcal{L}_1$ is of the form

$$\mathcal{L}_1=\mathcal{L}_1(\psi,\partial_\nu\psi,\hat{W}_{\mu\nu}^0), \tag{B11}$$

where

$$\hat{W}_{\mu\nu}^0=\frac{\partial}{\partial x_\mu}\hat{W}_\nu^0-\frac{\partial}{\partial x_\nu}\hat{W}_\mu^0. \tag{B12}$$

We shall consider the perturbation series by using $\mathcal{L}_0$ as the zeroth-order Lagrangian and $\mathcal{L}_1$ as the perturbation. The zeroth-order W_ν^0 propagator and A_ν' propagator are given by

$$\begin{aligned}\Delta_{\mu\nu}^W(q)=-i[(1+\eta)Nq^2+m_W^2]^{-1}\\ \times[\delta_{\mu\nu}+m_W^{-2}(1+\eta)Nq_\mu q_\nu]\end{aligned} \tag{B13}$$

and

$$\Delta_{\mu\nu}^A(q)=-iq^{-2}(\delta_{\mu\nu}+\lambda q^{-2}q_\mu q_\nu), \tag{B14}$$

where q_μ is the 4-momentum and λ depends on the gauge.

Let $D_{\mu\nu}^{WW}(q)$, $D_{\mu\nu}^{AA}(q)$, $D_{\mu\nu}^{AW}(q)$, and $D_{\mu\nu}^{WA}(q)$ be, respectively, the sum of all Feynman propagator graphs in which the (initial, final) fields are $(W_\mu{}^0,W_\nu{}^0)$, (A_μ',A_ν') $(A_\mu',W_\nu{}^0)$, and $(W_\mu{}^0,A_\nu')$. These are, by definition, covariant functions and $D_{\mu\nu}{}^{WW}(q)$ is the same $D_{\mu\nu}{}^W(q)$ given by (5.5). We have

$$D_{\mu\nu}{}^{WW}(q)=D_{\mu\nu}{}^W(q)$$
$$=-i\int\frac{\sigma_W}{q^2+M^2-i\epsilon}\left(\delta_{\mu\nu}+\frac{q_\mu q_\nu}{M^2}\right)dM^2. \quad \text{(B15)}$$

The interaction between $W_\nu{}^0$, A_ν', and ψ depends only on the combination

$$f_0\hat{W}_\nu{}^0=f_0'W_\nu{}^0+e_0'A_\nu', \quad \text{(B16)}$$

where

$$f_0'=Nf_0,\quad e_0'=N^{1/2}e_0. \quad \text{(B17)}$$

There is a one-to-one correspondence between the set of all Feynman graphs in $D_{\mu\nu}{}^{WW}$ and those in $D_{\mu\nu}{}^{AA}$; to each Feynman graph in $D_{\mu\nu}{}^{WW}$ there is a corresponding graph in $D_{\mu\nu}{}^{AA}$ which differs *only* in the external lines. Thus one finds the identity

$$D_{\mu\nu}{}^{AA}=\Delta_{\mu\nu}{}^A+(e_0'/f_0')^2\Delta_{\mu a}{}^A(\Delta^W)^{-1}{}_{ab}\times(D_{bc}{}^{WW}-\Delta_{bc}{}^W)(\Delta^W)^{-1}{}_{cd}\Delta_{d\nu}{}^A, \quad \text{(B18)}$$

where $(\Delta^W)^{-1}{}_{\mu\nu}$ is the inverse of $\Delta_{\mu\nu}{}^W$, given by

$$(\Delta^W)^{-1}{}_{\mu\nu}=i[(1+\eta)N(q^2\delta_{\mu\nu}-q_\mu q_\nu)+m_W{}^2\delta_{\mu\nu}]. \quad \text{(B19)}$$

Similarly, one can establish

$$D_{\mu\nu}{}^{AW}=(e_0'/f_0')\Delta_{\mu a}{}^A(\Delta^W)^{-1}{}_{ab}(D_{b\nu}{}^{WW}-\Delta_{b\nu}{}^W) \quad \text{(B20)}$$

and

$$D_{\mu\nu}{}^{WA}=(e_0'/f_0')(D_{\mu a}{}^{WW}-\Delta_{\mu a}{}^W)(\Delta^W)^{-1}{}_{ab}\Delta_{b\nu}{}^A. \quad \text{(B21)}$$

By using (B15) and the sum rules (5.3) and (5.4), we obtain

$$D_{\mu\nu}{}^{AA}=\frac{-i}{q^2}\delta_{\mu\nu}\left[1-\left(\frac{e_0'}{f_0'}\right)^2m_W{}^4\int\frac{\sigma_W}{q^2+M^2-i\epsilon}\times\left(\frac{1}{M^2}-\frac{1}{m_0{}^2}\right)\left(1+\frac{q^2}{m_0{}^2}\right)dM^2\right]+O(q_\mu q_\nu) \quad \text{(B22)}$$

and

$$D_{\mu\nu}{}^{AW}=D_{\mu\nu}{}^{WA}=-im_W{}^2\left(\frac{e_0'}{f_0'}\right)\delta_{\mu\nu}\int\frac{\sigma_W}{q^2+M^2-i\epsilon}\times\left(\frac{1}{m_0{}^2}-\frac{1}{M^2}\right)dM^2+O(q_\mu q_\nu), \quad \text{(B23)}$$

where m_0 is given by (4.12) and $O(q_\mu q_\nu)$ denotes the gauge-dependent part.

The photon propagator $D_{\mu\nu}{}^\gamma$, used in the text, refers to the propagator of A_ν. By using (B1), we find

$$D_{\mu\nu}{}^\gamma=ND_{\mu\nu}{}^{AA}-N^{3/2}(1+\eta)(e_0/f_0)(D_{\mu\nu}{}^{AW}+D_{\mu\nu}{}^{WA})+N^2(1+\eta)^2(e_0/f_0)^2D_{\mu\nu}{}^{WW}. \quad \text{(B24)}$$

These expressions enable one to express $D_{\mu\nu}{}^\gamma$ in terms of $D_{\mu\nu}{}^{WW}$ or its spectral function σ_W. The result is (5.6).

From (5.8), it follows that

$$e_0{}^2=e^2\xi^{-1}[1-(1-2\xi)^{1/2}], \quad \text{(B25)}$$

where

$$\xi=2\left(\frac{e^2}{f_0{}^2}\right)m_W{}^4\int M^{-4}\sigma_W dM^2. \quad \text{(B26)}$$

As $e^2\to0$ at fixed $f_0{}^2$, $\xi\to0$ and therefore

$$e_0{}^2\to e^2.$$

This limit requires us to choose in (B25) the negative branch $-(1-2\xi)^{1/2}$, instead of the positive one. Since $e_0{}^2$ is real and positive, we must have

$$\xi\leqq\tfrac{1}{2},$$

i.e.,

$$f_0{}^2\geqq4e^2m_W{}^4\int M^{-4}\sigma_W dM^2. \quad \text{(B27)}$$

As ξ varies from 0 to $\frac{1}{2}$, the ratio $(e_0/e)^2$ varies from 1 to 2. Thus we find

$$1\leqq(e_0/e)^2\leqq2. \quad \text{(B28)}$$

Both inequalities (5.9) and (5.10) are then proved.

If we neglect $O(e^2)$ and $O(f^2)$ terms, but keep all powers of (e/f^2), the inequality (B27) becomes (5.21); i.e.,

$$f_0{}^2\geqq4e^2[1+(e_0/f_0)^2]^{-1}. \quad \text{(B29)}$$

At the lower limit, ξ is $\frac{1}{2}$ and therefore

$$e_0{}^2=2e^2. \quad \text{(B30)}$$

Because of (B28), the inequality (B29) implies

$$f_0{}^2\geqq4e^2[1+2(e/f_0)^2]^{-1}$$

or simply

$$f_0{}^2\geqq2e^2, \quad \text{(B31)}$$

which is (5.22).