$\Delta I=\frac{1}{2}$ Rule in Current-Current Models of CP Violation

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Current-current models of CP violation of the types discussed by Glashow and Alles are applied to the decay $K\rightarrow 2\pi$, using current algebra and partially conserved axial-vector current. It is found that the CP-violating amplitude may violate the $\Delta I=\frac{1}{2}$ rule even though the CP-conserving amplitude does not. It is not, however, possible to predict the ratio $K_L\rightarrow\pi^0\pi^0$ to $K_L\rightarrow\pi^+\pi^-$ in a model of this type.

I. INTRODUCTION

EXPERIMENTS on the CP-violating decay $K^0\rightarrow 2\pi$,[1] specifically, the rate of $K_L{}^0\rightarrow\pi^0\pi^0$,[2] indicate that the CP-violating amplitude may also strongly violate the $\Delta I=\frac{1}{2}$ rule. In this paper, we wish to explore the possibility of violating the $\Delta I=\frac{1}{2}$ rule in models of CP violation.

We assume CP violation to occur in the weak Hamiltonian, so that we may write

$$\mathcal{H}_w=\mathcal{H}_w{}^+ + \mathcal{H}_w{}^-, \quad \text{with} \quad CP\mathcal{H}_w{}^\pm(CP)^{-1}=\pm\mathcal{H}_w{}^\pm,$$

where CP is defined by the strong interactions. In Sec. II, we examine the relation between the quantities which might be calculated from particular models of $\mathcal{H}_w{}^-$ and the phenomenological parameters ϵ and ϵ' which can be experimentally determined.

We then turn our attention to models in which both $\mathcal{H}_w{}^+$ and $\mathcal{H}_w{}^-$ have a current-current form, meaning that they are constructed from linear combinations of equal-time anticommutators of the usual vector and axial-vector currents,[3] so that they are vulnerable to analysis using the techniques of current algebra and partially conserved axial-vector current (PCAC).[4] For the usual CP-conserving interaction, it has been shown by several authors that a $\Delta I=\frac{1}{2}$ rule is predicted for the nonleptonic K decays.[5] In Sec. III, using Weinberg's techniques,[6] we rederive this result, but in a form which clearly reveals the isospin structure of the decay amplitude.

Section IV examines the particular structure of the usual weak Hamiltonian that yields the $\Delta I=\frac{1}{2}$ rule, as well as exacting isotopic spin predictions for more general current-current forms. Finally, in Sec. V, we discuss specific current-current models of CP violation which have been proposed by Glashow and Alles.[7,8]

II. GENERAL CONSIDERATIONS ON MODELS OF CP VIOLATION

We can construct two linear combinations of $|K^0\rangle$ and $|\bar{K}^0\rangle$ (call these $|K_+\rangle$ and $|K_-\rangle$) which are respectively even and odd under CP.[9] If $\langle 2\pi;I=I'|$ represents the two-pion standing-wave state of total isospin I', we define the following parameters:

$$\begin{aligned}\langle 2\pi;I=0|\mathcal{H}_w{}^+(0)|K_+\rangle&=A_0,\\ \langle 2\pi;I=2|\mathcal{H}_w{}^+(0)|K_+\rangle&=\beta A_0,\\ \langle 2\pi;I=0|\mathcal{H}_w{}^-(0)|K_-\rangle&=i\alpha A_0,\\ \langle 2\pi;I=2|\mathcal{H}_w{}^-(0)|K_-\rangle&=i\alpha\chi A_0,\end{aligned}\tag{1}$$

where A_0, α, β, and χ are real by CPT. In addition to the CP-violating quantities α, χ, we need one additional parameter defined by

$$\langle K_-|M|K_+\rangle=-im',$$

where m' is real by CPT, M being the conventional "mass matrix."[10] In terms of matrix elements of the weak Hamiltonian, we find

$$-im'=P\sum_n\frac{\langle K_-|\mathcal{H}_w|n\rangle\langle n|\mathcal{H}_w|K_+\rangle}{E_K-E_n},\tag{2}$$

where n runs over all possible intermediate states.

It should be emphasized that the phase convention employed is not the conventional one of Wu and Yang.[11] Instead, we use $\mathcal{H}_w{}^+$ to define our phase convention. It is, of course, possible to express the Wu-Yang quantities ϵ and ϵ' in terms of our parameters, and in Appendix I it is shown that

$$\epsilon\cong e^{i\delta_0}(\sqrt{\tfrac{1}{2}})(-m'/\Delta m+\alpha),$$

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[1] C. R. Christenson *et al.*, Phys. Rev. Letters **13**, 138 (1964).

[2] J. W. Cronin *et al.*, Phys. Rev. Letters **18**, 25 (1967); J. M. Gaillard *et al.*, *ibid.* **18**, 20 (1967). Recent reports (T. Kamae, invited paper at Chicago meeting of APS, 1968) have cast doubt on the conclusions of these references. We shall use the published results in specific numerical calculations here, but the theory is concerned with the general question of the $\Delta I=\frac{1}{2}$ rule in CP-violating models.

[3] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[4] See, for example, M. Suzuki, Phys. Rev. Letters **15**, 986 (1965); H. Abarbanel, Phys. Rev. **153**, 1547 (1967).

[5] M. Suzuki, Phys. Rev. **144**, 1154 (1966); D. K. Elias and J. C. Taylor, Nuovo Cimento **44**, 518 (1966); W. Alles and R. Jengo, *ibid.* **42**, 417 (1966).

[6] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966).

[7] S. Glashow, Phys. Rev. Letters **14**, 35 (1964).

[8] W. Alles, Phys. Letters **15**, 348 (1965).

[9] Section II closely follows a talk by L. Wolfenstein, presented to the Seminar on the Problems of CP Violation, Moscow, 1968 (unpublished).

[10] M, Γ are defined by T. D. Lee and C. S. Wu, Ann. Rev. Nucl. Sci. **16**, 471 (1966), as 2×2 Hermitian matrices such that $id\psi/dt=(M-\frac{1}{2}i\Gamma)\psi$, where ψ is a two-component object describing the K^0, $\bar{K}^0$ system.

[11] T. T. Wu and C. N. Yang, Phys. Rev. Letters **13**, 380 (1964).

with

$$\delta_s=\tan^{-1}[-2\Delta m/\Delta\gamma]\cong\tfrac{1}{4}\pi,\quad \epsilon'\cong(\sqrt{\tfrac{1}{2}})\alpha\chi e^{i(\delta_2-\delta_0+\frac{1}{2}\pi)},\quad (3)$$

where $\Delta m=m_1-m_2$ is the mass difference, $\Delta\gamma=\gamma_1-\gamma_2$ is the difference in widths, and we have assumed that $\beta\ll 1$. These numbers ϵ and ϵ' can be calculated from the experimental values of $|\eta_{+-}|$, $|\eta_{00}|$, and φ_{+-}. As shown by Wolfenstein,[12] two solutions are possible, but the ambiguity may be resolved by charge asymmetry experiments.[13]

We ask what a theory of CP violation is expected to yield. There is a class of theories in which α is zero. This category includes the superweak interaction and other models in which $\mathcal{H}_w^-$ does not contribute to the parity-violating nonleptonic decay amplitudes. Such theories are, of course, excluded if the experimental results of Ref. 2 are correct, although recent reports have cast doubt on their validity. We do not propose to discuss this class of models. Instead, we consider models in which CP violation and P violation can occur simultaneously, so that α is nonzero. Then, of the three CP-violating parameters α, χ, and m', only χ can be accurately predicted in the models which we discuss. Since only virtual states are involved in the summation in Eq. (2), m' cannot be accurately calculated. The parameter α will depend in general on some as yet unknown phase angle or some other parameter ζ which measures the relative magnitude of the CP violation. Then, given a theory, we may be able to calculate χ but must fit α and m' to the empirical values of ϵ and ϵ'. We are thus given little predictive power for $K_L\to 2\pi$ in that we are unable to calculate the branching ratio $\Gamma(K_L\to\pi^0\pi^0)/\Gamma(K_L\to\pi^+\pi^-)$.

Unfortunately, as mentioned earlier, the experimental situation with respect to $|\eta_{00}|$ is unclear. If, however, the two published results, which require $|\epsilon|$ and $|\epsilon'|$ to be of the same order of magnitude, are accepted, then it is generally argued that theories which yield $\chi\ll 1$ (the CP-violating decay amplitude for $K\to 2\pi$ satisfies a $\Delta I=\frac{1}{2}$ rule on the mass shell) or $\chi\gg 1$ (the CP-violating decay amplitude satisfies a $\Delta I>\frac{1}{2}$ rule on the mass shell) are excluded. We wish to emphasize that, while this is reasonable, it is not required. Thus, even if $\chi\ll 1$, we could choose $\alpha\gg|\epsilon'|$ and $m'/(\alpha\Delta m)=1+O(\chi)$, whereas if $\chi\gg 1$, we could choose $\alpha\ll|\epsilon'|$ and $m'/(\alpha\Delta m)=O(\chi)\gg 1$. Our inability to quantitatively evaluate sums over virtual states prohibits ruling these cases out. In order to resolve this uncertainty, we require an independent evaluation of either m' or ζ, as could result from an analysis of experimental evidence for CP violation in some other process, such as $K\to 3\pi$. Until this is found, we can at best state that, accepting the published values for $|\eta_{00}|$, theories which yield $\chi\sim O(1)$ are favored.[14]

III. GENERAL FORMALISM AND CP-CONSERVING DECAYS

In order to discuss $K\to 2\pi$, we employ Weinberg's method[6] of taking both pions off the mass shell,[15] although the fact that $\langle\pi|\mathcal{H}_w(0)|K\rangle$ is not physical dictates one slight change. Reducing the pions in the usual way and using the definition[16]

$$\boldsymbol{\varphi}_\pi(x)=(1/iF_\pi m_\pi^2)\partial_\mu\mathbf{A}^\mu(x),$$

we have

$$(4\omega_q\omega_p)^{1/2}\langle\pi_q{}^{a'}\pi_p{}^{b'}|\mathcal{H}_w(0)|K_k{}^n\rangle=\frac{(-q^2+m_\pi^2)(-p^2+m_\pi^2)}{F_\pi^2 m_\pi^4}\int d^4x d^4y\, e^{iq\cdot x+ip\cdot y}\langle 0|T(\partial_\mu A_a{}^\mu(x)\partial_\nu A_b{}^\nu(y)\mathcal{H}_w(0))|K_k{}^n\rangle,$$

where (a',b',n) are isospin indices, (q,p,k) are the appropriate momenta, and $\partial_\mu A_a{}^\mu(x)=\hat{e}^{a'}\cdot\partial_\mu\mathbf{A}^\mu(x)$. Now Weinberg showed that

$$\begin{aligned}
T(\partial_\mu A_a{}^\mu(x)\partial_\nu A_b{}^\nu(y)\mathcal{H}_w(0))=&\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}T(A_a{}^\mu(x)A_b{}^\nu(y)\mathcal{H}_w(0))-\delta(x^0-y^0)T([A_b{}^0(y),\partial_\mu A_a{}^\mu(x)]\mathcal{H}_w(0))\\
&-\tfrac{1}{2}\delta(x^0-y^0)T\left\{\left(\frac{\partial}{\partial x^\nu}+\frac{\partial}{\partial y^\nu}\right)[A_a{}^0(x),A_b{}^\nu(y)]\mathcal{H}_w(0)\right\}+\frac{1}{2}\left(\frac{\partial}{\partial x^\nu}-\frac{\partial}{\partial y^\nu}\right)\delta(x^0-y^0)T([A_a{}^0(x),A_b{}^\nu(y)]\mathcal{H}_w(0))\\
&-\delta(y^0)T(\partial_\mu A_a{}^\mu(x)[A_b{}^0(y),\mathcal{H}_w(0)])-\delta(x^0)T(\partial_\nu A_b{}^\nu(y)[A_a{}^0(x),\mathcal{H}_w(0)])\\
&-\tfrac{1}{2}\delta(x^0)\delta(y^0)[A_a{}^0(x),[A_b{}^0(y),\mathcal{H}_w(0)]]-\tfrac{1}{2}\delta(x^0)\delta(y^0)[A_b{}^0(y),[A_a{}^0(x),\mathcal{H}_w(0)]].\quad (4)
\end{aligned}$$

Now the first term, being of second order in the momenta, vanishes in the limit $(q,p)\to 0$, and the third vanishes because of the conserved vector current (CVC) hypothesis. The second is the well-known σ commutator and is

[12] L. Wolfenstein, Nuovo Cimento **42**, 17 (1966).

[13] D. Dorfan *et al.*, Phys. Rev. Letters **19**, 987 (1967); S. Bennett *et al.*, *ibid.* **19**, 993 (1967).

[14] If it turns out that $|\eta_{00}|\neq\sim 2|\eta_{+-}|$, our specific conclusions may have to be somewhat modified. If, for example, $|\eta_{00}|\cong|\eta_{+-}|$, then the charge asymmetry experiments suggest a solution with $|\epsilon'|\ll|\epsilon|$. In this case, arguments as in Sec. II would favor theories with $\chi\ll 1$, although another possible solution would be $\chi\sim O(1)$, $\alpha\simeq\sqrt{2}|\epsilon'|$, $m'/(\alpha\Delta m)\simeq|\epsilon/\epsilon'|\gg 1$.

[15] Note that this involves a violation of total energy momentum conservation in the unphysical region. We shall reinstate this principle at a later stage.

[16] In the quark model we use $A_i{}^\mu(x)=\bar{\psi}(x)\frac{1}{2}\lambda_i\gamma^\mu\gamma^5\psi(x)$, so that our Goldberger-Treiman relation is $F_\pi=-iMg_A/g_r$.

assumed negligible.[17] Weinberg treated these last five terms and obtained fair agreement with K_{e4} decays. In his case, however, he could use experimental results for $\langle\pi|\mathcal{J}^\nu(0)|K\rangle$, whereas, for our calculation, we note that

$$\delta(y^0)T(\partial_\mu A_a{}^\mu(x)[A_b{}^0(y),\mathcal{H}_w(0)])=(\partial/\partial x^\mu)\delta(y^0)T(A_a{}^\mu(x)[A_b{}^0(y),\mathcal{H}_w(0)])-\delta(x^0)\delta(y^0)[A_a{}^0(x),[A_b{}^0(y),\mathcal{H}_w(0)]].$$

The first term contains a pion pole, but vanishes in the soft-pion limit. Thus we have, finally,

$$\begin{aligned}T(\partial_\mu A_a{}^\mu(x)\partial_\nu A_b{}^\nu(y)\mathcal{H}_w(0))\\ =(\text{terms which vanish in soft-pion limit})+\tfrac{1}{2}(\partial/\partial x^\nu-\partial/\partial y^\nu)\delta(x^0-y^0)T([A_a{}^0(x),A_b{}^\nu(y)]\mathcal{H}_w(0))\\ +\tfrac{1}{2}\delta(x^0)\delta(y^0)[A_a{}^0(x),[A_b{}^0(y),\mathcal{H}_w(0)]]+\tfrac{1}{2}\delta(x^0)\delta(y^0)[A_b{}^0(y),[A_a{}^0(x),\mathcal{H}_w(0)]].\end{aligned}$$

Defining

$$F_a{}^5(t)\equiv\int d^3x\,A_a{}^0(\mathbf{x},t),$$

we find

$$\begin{aligned}(4\omega_q\omega_p)^{1/2}\langle\pi_q{}^{a'}\pi_p{}^{b'}|\mathcal{H}_w(0)|K_k{}^n\rangle \xrightarrow[q,p\to 0]{} (1/2F_\pi{}^2)\Big[\langle 0|([F_a{}^5(0),[F_b{}^5(0),\mathcal{H}_w(0)]]+[F_b{}^5(0),[F_a{}^5(0),\mathcal{H}_w(0)]])|K_k{}^n\rangle\\ -i(q-p)_\nu\int d^4x\, e^{i(q+p)\cdot x}i\epsilon_{abc}\langle 0|T(V_c{}^\nu(x)\mathcal{H}_w(0))|K_k{}^n\rangle\Big],\end{aligned} \quad (5)$$

where the last term was kept because in the soft-pion limit, it has a kaon pole term in which the K first interacts with the vector current, propagates as a K, and then disappears through $\mathcal{H}_w(0)$ into the vacuum. Such a term is zeroth order in (q,p) and must be retained in this approximation. This term yields

$$\epsilon_{abc}(q-p)_\nu\left[\langle 0|\mathcal{H}_w(0)|K^m\rangle\frac{i}{(k-p-q)^2-M_K{}^2+i\epsilon}(\tfrac{1}{2}\tau^c)_{mn}(2k-p-q)^\nu\right]\cong\tfrac{1}{2}i\epsilon_{abc}\frac{k\cdot(p-q)}{k\cdot(p+q)}\langle 0|\mathcal{H}_w(0)|K^m\rangle\tau_{mn}{}^c. \quad (6)$$

We have gone as far as we can without assuming a specific form for $\mathcal{H}_w(0)$. Forgetting about CP violation momentarily, we assume that CP-conserving decays are given by the usual current-current interaction:

$$\mathcal{H}_w(x)=(G_V/\sqrt{2})\tfrac{1}{2}\{\mathcal{J}^\nu(x),\mathcal{J}_\nu{}^\dagger(x)\},$$

where the curly brackets denote an anticommutator, $G_V=1.0\times10^{-5}\,M_p{}^{-2}$ is the usual vector coupling constant, and, according to Cabibbo,

$$\mathcal{J}^\nu(x)=\cos\theta\,[V_{\pi^-}{}^\nu(x)+A_{\pi^-}{}^\nu(x)]+\sin\theta\,[V_{K^-}{}^\nu(x)+A_{K^-}{}^\nu(x)]+j^\nu{}_{\text{leptonic}}.$$

Now it is well known that[18]

$$[F_a{}^5(0),\mathcal{H}_w(0)]=[F_a(0),\mathcal{H}_w(0)], \quad (7)$$

where $F_a(0)=\int d^3x\,V_a{}^0(\mathbf{x},0)$ is by the CVC hypothesis just the ath component of the isospin operator. Now instead of commuting F_a with $\mathcal{H}_w$, just let it operate on the vacuum and kaon, respectively, yielding

$$\begin{aligned}\langle 0|[F_a(0),[F_b(0),\mathcal{H}_w(0)]]|K^n\rangle=(\langle 0|F_a(0))[F_b(0),\mathcal{H}_w(0)]|K^n\rangle-\langle 0|[F_b(0),\mathcal{H}_w(0)](F_a(0)|K^n\rangle)\\ =-\tfrac{1}{2}\tau_{ln}{}^a\langle 0|[F_b(0),\mathcal{H}_w(0)]|K^l\rangle=\tfrac{1}{4}(\tau^b\tau^a)_{mn}\langle 0|\mathcal{H}_w(0)|K^m\rangle.\end{aligned}$$

Thus, finally,[19]

$$(4\omega_q\omega_p)^{1/2}\langle\pi_q{}^{a'}\pi_p{}^{b'}|\mathcal{H}_w(0)|K_k{}^n\rangle\xrightarrow[q,p\to 0]{}\frac{1}{4F_\pi{}^2}\left[\delta^{ab}+i\frac{k\cdot(p-q)}{k\cdot(p+q)}\epsilon^{abc}\tau^c\right]_{mn}\langle 0|\mathcal{H}_w(0)|K^m\rangle. \quad (8)$$

Since $\tau^a\tau^b=\delta^{ab}+i\epsilon^{abc}\tau^c$, while $\tau^b\tau^a=\delta^{ab}-i\epsilon^{abc}\tau^c$, the above result reproduces the fact that the limit as $(q,p)\to 0$ depends on the order in which the limits are taken[20]:

$$\begin{aligned}(4\omega_q\omega_p)^{1/2}\langle\pi_q{}^{a'}\pi_p{}^{b'}|\mathcal{H}_w(0)|K_k{}^n\rangle\xrightarrow[q\to 0,\text{ then }p\to 0]{}(1/4F_\pi{}^2)(\tau^a\tau^b)_{mn}\langle 0|\mathcal{H}_w(0)|K^m\rangle\\ \xrightarrow[p\to 0,\text{ then }q\to 0]{}(1/4F_\pi{}^2)(\tau^b\tau^a)_{mn}\langle 0|\mathcal{H}_w(0)|K^m\rangle.\end{aligned} \quad (9)$$

[17] This is in line with Weinberg's work and is justified in Ref. 6.
[18] W. Alles and R. Jengo, Ref. 5.
[19] This form is equivalent to the result for $K\to 2\pi$ obtained by H. Abarbanel, Ref. 4.
[20] Such behavior was also seen in the treatment of K_{e4} by C. G. Callan and S. B. Treiman, Phys. Rev. Letters **16**, 153 (1966), and was subsequently shown by Weinberg in Ref. 6 to be due to a similar pole term.

Thus, it yields the well-known Nambu-Hara relations for $K\to 2\pi$,[21] which have been shown by Sakurai to follow from a vector-meson-pole model,[22] and also by Schechter to follow from a certain type of $SU(3)$ symmetry breaking.[23]

$$\begin{aligned}&\langle\pi^+\pi^0;\,q(\pi^+)=0|\mathcal{H}_w(0)|K^+\rangle\\&\quad=-\langle\pi^+\pi^0;\,q(\pi^0)=0|\mathcal{H}_w(0)|K^+\rangle=\langle\pi^+\pi^-;\,q(\pi^+)=0|\mathcal{H}_w(0)|K_1{}^0\rangle=\langle\pi^+\pi^-;\,q(\pi^-)=0|\mathcal{H}_w(0)|K_1{}^0\rangle\\&\quad=(1/2F_\pi)\langle\pi^+|\mathcal{H}_w(0)|K^+\rangle,\\&\qquad\langle\pi^+|\mathcal{H}_w(0)|K^+\rangle=-(1/F_\pi)\langle 0|[F_{\pi^-}{}^5(0),\mathcal{H}_w(0)]|K^+\rangle=(\sqrt{\tfrac{1}{2}})(1/F_\pi)\langle 0|\mathcal{H}_w(0)|K^0\rangle.\end{aligned}\tag{10}$$

Let us now reinstate momentum conservation and use PCAC, which suggests that the results on the mass shell are not very different from those obtained in the soft-pion limit. We then have

$$(4\omega_q\omega_p)^{1/2}\langle\pi_q{}^{a'}\pi_p{}^{b'}|\mathcal{H}_w(0)|K_k{}^n\rangle\simeq\frac{1}{4F_\pi{}^2}\left[\delta^{ab}-i\frac{m_{a'}{}^2-m_{b'}{}^2}{M_K{}^2}\epsilon^{abc}\tau^c\right]_{mn}\times\langle 0|\mathcal{H}_w(0)|K^m\rangle.\tag{11}$$

In the $SU(2)$ limit, $m_{a'}{}^2=m_{b'}{}^2$ and only the δ^{ab} term can contribute. This represents the $I=0$ state of the 2π system and thus we have the $\Delta I=\frac{1}{2}$ rule, even though $\mathcal{H}_w(0)$ contains both $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ terms. This is just a result shown by several authors using different methods.[5] We shall examine this feature in more detail in Sec. IV.

If we use the observed pion masses, then we find

$$\left|\frac{\langle\pi^+\pi^0|\mathcal{H}_w(0)|K^+\rangle}{\langle\pi^+\pi^-|\mathcal{H}_w(0)|K_1{}^0\rangle}\right|=\frac{m_+{}^2-m_0{}^2}{M_K{}^2},$$

which differs by a factor of 2 from the Nambu-Hara or Schechter expression, obtained from $SU(3)$ considerations. Such a relation has also been discussed by Okubo, Marshak, and Mathur.[24] For the remainder of this paper, we shall assume the $SU(2)$ limit, however, so that we need not worry about the contribution of the pole term.

IV. WHY $\Delta I=\frac{1}{2}$

One of the most interesting predictions discussed in the previous section is that of the $\Delta I=\frac{1}{2}$ rule for the usual current-current Hamiltonian. It is of interest to see just why such a result is obtained. We deal here with current-current structures, defined as linear combinations of anticommutators of the usual vector and axial currents. We shall be discussing only the strangeness-changing combinations, so that these forms can only have isospin $\frac{1}{2}$ or $\frac{3}{2}$. Since, in the previous section we have related the amplitude for $K\to 2\pi$ to the matrix element between K and the vacuum, one might naively think that the $\Delta I=\frac{3}{2}$ part is always therefore suppressed. To see what is involved, however, suppose we resolve a particular current-current structure $\mathcal{H}_w$ into $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ parts:

$$\mathcal{H}_w=\alpha\mathcal{H}_w{}^{1/2}+\beta\mathcal{H}_w{}^{3/2}.$$

Now a double commutator with $F_{a,b}{}^5$, where for the case of interest (a,b) are isospin indices involved in neutral K decay, must yield again a current-current form with the same parity, strangeness, and I_3 quantum numbers. That is, we must have just a new linear combination of $\mathcal{H}_w{}^{1/2}$ and $\mathcal{H}_w{}^{3/2}$. If $\mathcal{H}_w{}^{1/2}\equiv\mathcal{H}_w{}^1$ and $\mathcal{H}_w{}^{3/2}\equiv\mathcal{H}_w{}^2$, we can define a 2×2 matrix $C_{ab}{}^{ij}$ such that

$$[F_a{}^5,[F_b{}^5,\mathcal{H}_w{}^i]]=\sum_{j=1}^{2}C_{ab}{}^{ij}\mathcal{H}_w{}^j.$$

Then it is clear that a necessary and sufficient condition for a $\Delta I=\frac{1}{2}$ rule is that $(C_{ba}+C_{ab})^{12}=0$, for, if

$$(C_{ba}+C_{ab})=\begin{pmatrix}\gamma & 0\\ \epsilon & \delta\end{pmatrix},$$

we have

$$\begin{aligned}&(4\omega_q\omega_p)^{1/2}\langle\pi_q{}^{a'}\pi_p{}^{b'}|\alpha\mathcal{H}_w{}^1+\beta\mathcal{H}_w{}^2|K_k{}^n\rangle\\&\xrightarrow[q,p\to 0]{}(2F_\pi{}^2)^{-1}\langle 0|[\alpha\gamma\mathcal{H}_w{}^1+(\epsilon\alpha+\delta\beta)\mathcal{H}_w{}^2]|K^n\rangle\\&=(1/2F_\pi{}^2)\langle 0|\alpha\gamma\mathcal{H}_w{}^1|K^n\rangle\\&\xleftarrow[q,p\to 0]{}\langle\pi_q{}^{a'}\pi_p{}^{b'}|\alpha\mathcal{H}_w{}^1|K_k{}^n\rangle(4\omega_q\omega_p)^{1/2}.\end{aligned}\tag{12}$$

Similarly, the condition for a $\Delta I=\frac{3}{2}$ rule is that $(C_{ba}+C_{ab})^{11}=0$.

For the usual current-current interaction, we get a $\Delta I=\frac{1}{2}$ rule, since we can replace $F_{a,b}{}^5$ by $F_{a,b}$, which are just isospin operators. Now the commutation of an isospin operator with an isotensor operator of rank I cannot change the value of the rank, so that C_{ab} and C_{ba} are separately diagonal and we predict a $\Delta I=\frac{1}{2}$ rule. Since we shall need the results in a later section, let us see how this applies to the case of a current-current parity-violating, $\Delta S=-1$ interaction. We can define then two types of $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ operators:

$$\begin{aligned}L^{1/2}&=(\sqrt{\tfrac{2}{3}})\{V_{\pi^+}{}^\nu,A_{K^-}{}^\nu\}-(\sqrt{\tfrac{1}{3}})\{V_{\pi^0}{}^\nu,A_{\bar{K}^0}{}^\nu\},\\L^{3/2}&=(\sqrt{\tfrac{1}{3}})\{V_{\pi^+}{}^\nu,A_{K^-}{}^\nu\}+(\sqrt{\tfrac{2}{3}})\{V_{\pi^0}{}^\nu,A_{\bar{K}^0}{}^\nu\},\\K^{1/2}&=(\sqrt{\tfrac{2}{3}})\{V_{K^-}{}^\nu,A_{\pi^+}{}^\nu\}-(\sqrt{\tfrac{1}{3}})\{V_{\bar{K}^0}{}^\nu,A_{\pi^0}{}^\nu\},\\K^{3/2}&=(\sqrt{\tfrac{1}{3}})\{V_{K^-}{}^\nu,A_{\pi^+}{}^\nu\}+(\sqrt{\tfrac{2}{3}})\{V_{\bar{K}^0}{}^\nu,A_{\pi^0}{}^\nu\},\end{aligned}\tag{13}$$

where L^i can be constructed from K^i by merely inter-

[21] Y. Nambu and Y. Hara, Phys. Rev. Letters **16**, 875 (1966).
[22] J. J. Sakurai, Phys. Rev. **156**, 1508 (1967).
[23] J. Schechter, Phys. Rev. **161**, 1660 (1967).
[24] S. Okubo, R. Marshak, and V. Mathur, Phys. Rev. Letters **19**, 407 (1967).

changing V and A. Under double commutation with $F_{a,b}{}^5$, any of the above operators goes into a linear combination of itself and the other three. Thus, in the basis, 4×4 C matrices are needed. If, however, we use the linear combinations (K^i+L^i) or (K^i-L^i), which are respectively even or odd under the interchange $V\leftrightarrow A$, it is shown in Appendix II that double commutation with $F_{a,b}{}^5$ cannot change this symmetry, and thus we have then two types of 2×2 matrices—$C_{ab}{}^{(+)ij}$ or $C_{ab}{}^{(-)ij}$. Explicit evaluation yields

$$2C_{00}{}^{(+)}=\begin{pmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{pmatrix},\quad C_{+-}{}^{(+)}+C_{-+}{}^{(+)}=\begin{pmatrix}\frac{1}{2} & 0\\ 0 & \frac{7}{2}\end{pmatrix},$$

$$2C_{00}{}^{(-)}=\begin{pmatrix}19/6 & 4\sqrt{2}/3\\ 4\sqrt{2}/3 & 11/6\end{pmatrix},\tag{14}$$

$$C_{+-}{}^{(-)}+C_{-+}{}^{(-)}=\begin{pmatrix}19/6 & -2\sqrt{2}/3\\ -2\sqrt{2}/3 & 5/6\end{pmatrix}.$$

Thus the symmetric combination will yield a $\Delta I=\frac{1}{2}$ rule, whereas the antisymmetric will not. This is clear from the fact that for any such symmetric combination

$$[F_a{}^5,[F_b{}^5,\{\text{Sym.}\}]]=[F_a,[F_b,\{\text{Sym.}\}]],$$

as shown in Appendix II. The symmetric combination appears, of course, in the usual current-current Hamiltonian, while the antisymmetric combination appears in certain models of CP violation to be discussed in Sec. V.

V. SPECIFIC MODELS OF *CP* VIOLATION

We examine in this section two particular models of CP violation which have been formulated within the current-current model, to see whether they can account for the experimental results. Since $\mathcal{H}_w\propto\{\mathcal{J}^\nu,\mathcal{J}_\nu{}^\dagger\}$, any current such that $CP\mathcal{J}^\nu(CP)^{-1}\neq e^{i\alpha}\mathcal{J}_\nu{}^\dagger$, with α an arbitrary phase, is suitable. One such model is that of Glashow,[7] who merely inserts arbitrary phases on the axial currents:

$$\mathcal{J}^\nu(x)=\cos\theta\,[V_{\pi^-}{}^\nu(x)+A_{\pi^-}{}^\nu(x)e^{i\varphi}]+\sin\theta\,[V_{K^-}{}^\nu(x)+A_{K^-}{}^\nu(x)e^{i\psi}]+j^\nu{}_{\text{leptonic}}.$$

This is the most general way to add phases, since the over-all phase, as well as the relative phase between $\Delta S=0$ and $\Delta S\neq 0$ parts, is unobservable. Using trigonometry, we find

$$\begin{aligned}
&\mathcal{H}_w{}^+(P\text{-violating},\,\Delta S=-1)\\
&=(G_V/2\sqrt{2})\cos\theta\sin\theta[\cos(\tfrac{1}{2}(\varphi+\psi))\cos(\tfrac{1}{2}(\varphi-\psi))(\{A_{\pi^+}{}^\nu,V_{K^-}{}^\nu\}+\{V_{\pi^+}{}^\nu,A_{K^-}{}^\nu\})\\
&\qquad+\sin(\tfrac{1}{2}(\varphi+\psi))\sin(\tfrac{1}{2}(\varphi-\psi))(\{V_{\pi^+}{}^\nu,A_{K^-}{}^\nu\}-\{A_{\pi^+}{}^\nu,V_{K^-}{}^\nu\})];\\
&\mathcal{H}_w{}^-(P\text{-violating},\,\Delta S=-1)\\
&=i(G_V/2\sqrt{2})\cos\theta\sin\theta[-\cos(\tfrac{1}{2}(\varphi+\psi))\sin(\tfrac{1}{2}(\varphi-\psi))(\{A_{\pi^+}{}^\nu,V_{K^-}{}^\nu\}+\{V_{\pi^+}{}^\nu,A_{K^-}{}^\nu\})\\
&\qquad+\cos(\tfrac{1}{2}(\varphi-\psi))\sin(\tfrac{1}{2}(\varphi+\psi))(\{V_{\pi^+}{}^\nu,A_{K^-}{}^\nu\}-\{A_{\pi^+}{}^\nu,V_{K^-}{}^\nu\})].
\end{aligned}\tag{15}$$

In order to evaluate χ for this theory, we may use the results of Eq. (14). Since $\{V_{\pi^+}{}^\nu,A_{K^-}{}^\nu\}=(\sqrt{\tfrac{2}{3}})L^{1/2}+(\sqrt{\tfrac{1}{3}})L^{3/2}$, we find

$$\begin{aligned}
R&=\frac{\langle\pi^+\pi^-|\mathcal{H}_w{}^-|K_-\rangle}{\langle\pi^0\pi^0|\mathcal{H}_w{}^-|K_-\rangle}\\
&=\frac{\cos(\frac{1}{2}(\varphi+\psi))\sin(\frac{1}{2}(\varphi-\psi))\Gamma_+\operatorname{Tr}(C_{+-}{}^{(+)}+C_{-+}{}^{(+)})M+\cos(\frac{1}{2}(\varphi-\psi))\sin(\frac{1}{2}(\varphi+\psi))\Gamma_-\operatorname{Tr}(C_{+-}{}^{(-)}+C_{-+}{}^{(-)})M}{\cos(\frac{1}{2}(\varphi+\psi))\sin(\frac{1}{2}(\varphi-\psi))\Gamma_+\operatorname{Tr}2C_{00}{}^{(+)}M+\cos(\frac{1}{2}(\varphi-\psi))\sin(\frac{1}{2}(\varphi+\psi))\Gamma_-\operatorname{Tr}2C_{00}{}^{(-)}M},
\end{aligned}$$

where M is the 2×2 matrix

$$(\sqrt{\tfrac{1}{3}})\begin{pmatrix}\sqrt{2} & 0\\ 1 & 0\end{pmatrix}$$

and

$$\Gamma_\pm=\langle 0|(\{A_{\pi^+}{}^\nu,V_{K^-}{}^\nu\}\pm\{V_{\pi^+}{}^\nu,A_{K^-}{}^\nu\})|K^0\rangle.$$

Carrying out the traces and expanding the sine and cosine terms to first order in ψ, φ yields

$$R=\frac{\Gamma_+(\varphi-\psi)+5\Gamma_-(\varphi+\psi)}{\Gamma_+(\varphi-\psi)+9\Gamma_-(\varphi+\psi)}.\tag{16}$$

Using Clebsch-Gordan coefficients, we find

$$\chi=-(\sqrt{\tfrac{1}{2}})\frac{1-R}{\frac{1}{2}+R}=-(\sqrt{\tfrac{1}{2}})\frac{8\rho t}{3+19\rho t},\tag{17}$$

where we have defined $\rho\equiv(\varphi+\psi)/(\varphi-\psi)$ and $t\equiv\Gamma_-/\Gamma_+$. The function $\chi(\rho t)$ is plotted in Fig. 1. In the same way we find that

$$\alpha\chi=+\tfrac{2}{3}\sqrt{2}(\varphi+\psi)t.\tag{18}$$

We discuss two cases which are of particular interest. The first is that of $\rho=0$ or $\varphi=-\psi$, which yields $\chi=0$ and gives the same results for $K_L\to 2\pi$ as the super-

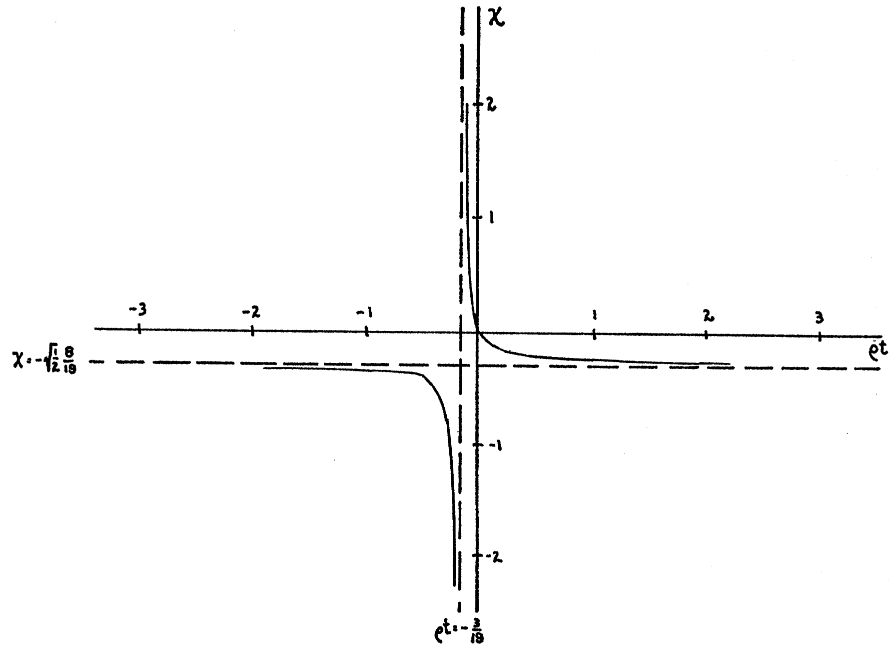

FIG. 1. The function $\chi(\rho t)$ defined in Eq. (17).

weak model.[25] In fact, from Eq. (15) we see that $\mathcal{H}_w^-$ (P-violating, $\Delta S=-1$) differs from the usual $\mathcal{H}_w^+$ (P-violating, $\Delta S=-1$) by only a factor $i\tan\varphi$. This case may be distinguished from the superweak model in that it predicts CP violation in $K\to 3\pi$.

The second case is that of $1/\rho=0$ or $\varphi=\psi$, which yields $\chi=-(\sqrt{\frac{1}{2}})\times 8/19$. This choice is of interest in that the total weak current is connected to its strangeness-conserving part by just an $SU(3)$ rotation, as in Cabibbo's original proposal based upon universality. Another feature is that no strangeness-changing, P-conserving, CP-violating effects are predicted. For this case, we have fitted α and m' to the values of ϵ and ϵ' given by Truong[26] and list the results in Table I as a function of φ_{+-}, since this angle is not well determined. The data on the charge asymmetry have been used only to rule out the small ϵ solution. For each value of φ_{+-} we have two solutions, corresponding to the ambiguity of π in $\delta_2-\delta_0$.

In order to determine a specific value for the phase angle φ, we note that

$$\langle\pi^0\pi^0|\mathcal{H}_w^-|K_-\rangle=-i9\varphi t\langle\pi^0\pi^0|\mathcal{H}_w^+|K_+\rangle$$
$$=i\alpha A_0[-(\sqrt{\tfrac{1}{3}})+(\sqrt{\tfrac{2}{3}})\chi]=-i\alpha A_0(\sqrt{\tfrac{1}{3}})\times 27/19.$$

Since $\langle\pi^0\pi^0|\mathcal{H}_w^+|K_+\rangle=-(\sqrt{\frac{1}{3}})A_0$, we find $\varphi=-3\alpha/19t$.[27] Now we expect that $|t|\sim O(1)$ and an explicit calculation, using the $SU(3)$ sum rules[28] and the convergent intermediate vector-boson model of Glashow Schnitzer, and Weinberg,[29] yields

$$t\cong-2.05.$$

Thus we have

$$\varphi\cong\alpha/13.0. \tag{19}$$

In order to decide whether $\varphi=\psi$ is the correct model, we must seek evidence for CP-violating, P-conserving processes such as $K_+\to 3\pi^0$.

A similar model is that of Alles,[30] who proposes that

$$\mathcal{J}^\nu(x)=\cos\theta[V_{\pi^-}{}^\nu(x)+A_{\pi^-}{}^\nu(x)]$$
$$+\sin\theta[V_{K^-}{}^\nu(x)+A_{K^-}{}^\nu(x)]$$
$$+i\lambda\{\cos\theta'[V_{\pi^-}{}^\nu(x)-A_{\pi^-}{}^\nu(x)]$$
$$+\sin\theta'[V_{K^-}{}^\nu(x)-A_{K^-}{}^\nu(x)]\}.$$

TABLE I. Values of α and m' resulting from the ϵ and ϵ' of Truong,[a] as a function of φ_{+-} and for each of the two possible values of $\delta_2-\delta_0$. $\eta_{00}=2.0$; ϵ, ϵ', $|\eta_{00}|$, and α are measured in terms of $|\eta_{+-}|$.

φ_{+-}	$\lvert\epsilon\rvert$	$\lvert\epsilon'\rvert$	$\delta_2-\delta_0$	α	$m'/\Delta m$
0°	0.93	0.74	−154°	−3.5	−4.8
			26°	+3.5	+2.2
30°	1.29	+0.41	173°	−1.9	−3.7
			−7°	+1.9	+0.1
45°	1.33	+0.33	135°	−1.5	−3.4
			45°	+1.5	−0.4
60°	1.29	+0.41	97°	−1.9	−3.7
			−83°	+1.9	+0.1
90°	0.93	+0.74	64°	−3.5	−4.8
			−116°	+3.5	+2.2

[a] Reference 26.

[25] L. Wolfenstein, Phys. Rev. Letters **13**, 562 (1964).

[26] T. Truong, talks presented at the Symposium on the Present Status of CP Violation, Argonne, Ill., 1967 (unpublished).

[27] We see from Eq. (18) that, independent of the value of ρ, $(\varphi+\psi)$ is just twice the magnitude of φ obtained from Table I and Eq. (19).

[28] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 139 (1967).

[29] S. L. Glashow, H. J. Schnitzer, and S. Weinberg, Phys. Rev. Letters **19**, 205 (1967).

[30] Reference 8. Alles chose the value $\theta=\theta'$, but this is not favored, since it would predict $\epsilon'=0$.

We then find that

$$\mathcal{H}_w^-(P\text{-violating}, \Delta S=-1) = i(G_V/2\sqrt{2})\lambda(\cos\theta' \sin\theta+\cos\theta \sin\theta') \times[\{V_{\pi^+}{}^\nu, A_{K^-\nu}\}-\{A_{\pi^+}{}^\nu, V_{K^-\nu}\}]. \quad (20)$$

This Hamiltonian is thus dynamically equivalent to that for the case $\varphi=\psi$ in Glashow's model, since both involve just the combination

$$(\{V_{\pi^+}{}^\nu, A_{K^-\nu}\}-\{A_{\pi^+}{}^\nu, V_{K^-\nu}\})$$

which is antisymmetric under the interchange $V\leftrightarrow A$. Thus Alles's model yields $\chi=-(\sqrt{\frac{1}{2}})\times 8/19$ and is "favored." In order to determine λ and θ', it is necessary to look for other manifestations of CP violations, such as the parity-conserving decays, where the choice $\theta=\theta'$ would predict $\langle 3\pi^0|\mathcal{H}_w|K_+\rangle=0$.

We conclude then that models of CP violation within a current-current framework are able to account for the experimental situation in $K_L\to 2\pi$, and that the employment of the current algebra enables one to evaluate previously unknown parameters in these theories. Before leaving this section, we remark that one other model of CP violation has been treated in this way, this being the model of Zacharisen and Zweig.[31] One can show that in this theory $\chi=-(\sqrt{\frac{1}{2}})\frac{1}{2}$,[32] making it also a "favored" one.

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APPENDIX I

We take the view that we append experimental π-π phase shifts onto the current-algebra results.[33] One can show that in our representation the mixing parameter δ is given by[34]

$$\delta=-\frac{\Gamma_{+-}+i2M_{+-}}{\Delta\gamma+i2\Delta m}, \quad (21)$$

where $\Delta m=m_1-m_2$ is the mass difference, $\Delta\gamma=\gamma_1-\gamma_2$ is the difference in widths, and Γ is the Hermitian "decay matrix" whose matrix element is given by

$$\tfrac{1}{2}\Gamma_{+-}=\pi\sum_n \langle K_+|\mathcal{H}_w|n\rangle\langle n|\mathcal{H}_w|K_-\rangle\delta(E_K-E_n).$$

δ as defined above is such that

$$|K_1{}^0\rangle\cong|K_+\rangle+\delta|K_-\rangle, \quad |K_2{}^0\rangle\cong|K_-\rangle+\delta|K_+\rangle.$$

Using Clebsch-Gordan coefficients and Eqs. (1), we find

$$\begin{aligned}
\text{Amp}(K_1{}^0\to\pi^+\pi^-)&=-(\sqrt{\tfrac{1}{3}})A_0e^{i\delta_0}-(\sqrt{\tfrac{1}{6}})\beta A_0e^{i\delta_2}+O(\delta\alpha),\\
\text{Amp}(K_2{}^0\to\pi^+\pi^-)&=-(\sqrt{\tfrac{1}{3}})i\alpha A_0e^{i\delta_0}-(\sqrt{\tfrac{1}{6}})i\alpha\chi A_0e^{i\delta_2}-(\sqrt{\tfrac{1}{3}})\delta A_0e^{i\delta_0}+O(\delta\beta),\\
\text{Amp}(K_1{}^0\to\pi^0\pi^0)&=-(\sqrt{\tfrac{1}{3}})A_0e^{i\delta_0}+2(\sqrt{\tfrac{1}{6}})\beta A_0e^{i\delta_2}+O(\delta\alpha),\\
\text{Amp}(K_2{}^0\to\pi^0\pi^0)&=-(\sqrt{\tfrac{1}{3}})i\alpha A_0e^{i\delta_0}+2(\sqrt{\tfrac{1}{6}})i\alpha\chi A_0e^{i\delta_2}-(\sqrt{\tfrac{1}{3}})\delta A_0e^{i\delta_0}+O(\delta\beta),\\
\eta_{+-}&=\frac{\text{Amp}(K_2{}^0\to\pi^+\pi^-)}{\text{Amp}(K_1{}^0\to\pi^+\pi^-)}=\epsilon+\epsilon'=i\alpha+\delta+(\sqrt{\tfrac{1}{2}})\alpha\chi e^{i(\delta_2-\delta_0+\pi/2)}+O(\beta\delta),\\
\eta_{00}&=\frac{\text{Amp}(K_2{}^0\to\pi^0\pi^0)}{\text{Amp}(K_1{}^0\to\pi^0\pi^0)}=\epsilon-2\epsilon'=i\alpha+\delta-2(\sqrt{\tfrac{1}{2}})\alpha\chi e^{i(\delta_2-\delta_0+\pi/2)}+O(\beta\delta).
\end{aligned} \quad (22)$$

Clearly, $\epsilon'=(\sqrt{\frac{1}{2}})\alpha\chi e^{i(\delta_2-\delta_0+\pi/2)}$, while $\epsilon=\delta+i\alpha$. We find then

$$(\Delta\gamma+i2\Delta m)\epsilon=[-\Gamma_{+-}-i2M_{+-}+i\alpha\Delta\gamma-2\alpha\Delta m].$$

We write specifically the contribution from the 2π $I=0$ state. Then

$$\Gamma_{+-}=\Phi A_0\times i\alpha A_0-i\times(\text{Remainder}),$$

where Φ is some phase-space factor. But we also have $\Delta\gamma\cong\gamma_1\cong\Phi A_0\times A_0$, where Φ is this same phase-space factor. Thus [using $\Delta\gamma+i2\Delta m\cong\Delta\gamma(1-i)$ $=-\sqrt{2}2\Delta m\; e^{-i\pi/4}$],

$$\epsilon=[2m'-2\alpha\Delta m+i\text{ Remainder}]/(-2\sqrt{2}\Delta m e^{-i\pi/4}) = e^{i\pi/4}(\sqrt{\tfrac{1}{2}})(-m'/\Delta m+\alpha-i\text{ Remainder}/2\Delta m). \quad (23)$$

If we now use the argument of Wolfenstein that $\arg\epsilon\cong\frac{1}{4}\pi$,[9] we may neglect (Remainder/$2\Delta m$) and the result follows.

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[31] F. Zachariasen and G. Zweig, Phys. Rev. Letters 14, 794 (1965).

[32] This corresponds to the value $R=\frac{1}{2}$ derived by F. Zachariasen and J. C. Pati (unpublished).

[33] The current algebra seems to show that the $\pi\pi$ phase shifts vanish in the unphysical limit of $q(\pi^{a'})$, $q(\pi^{b'})\to 0$. Since the experimental phase shifts are rather large, as shown in Table I, it is improper to ignore them. We instead assume that as we continue into the physical region, the magnitude of the matrix element remains unchanged, but that it acquires the appropriate experimental strong-interaction phase shift.

[34] This is derived in the K^0, $\bar{K}^0$ representation in Ref. 7.

APPENDIX II

We consider operators $\mathcal{O}^{\pm} = \{V_1, A_2\} \pm \{V_2, A_1\}$ which are

$$\begin{pmatrix} \text{even} \\ \text{odd} \end{pmatrix}$$

under interchanges of V and A. It is shown here that

(1) $[F_a{}^5,[F_b{}^5,\mathcal{O}^+]] = [F_a,[F_b,\mathcal{O}^+]]$;

(2) double commutation with $F_a{}^5$, $F_b{}^5$ does not change the symmetry under V, A interchange.

Both results follow from the identity

$$\begin{aligned}
&[F_a{}^5,[F_b{}^5,\mathcal{O}^{\pm}]] \\
&= (\{[F_a{}^5,[F_b{}^5,V_1]],A_2\} \pm \{V_2,[F_a{}^5,[F_b{}^5,A_1]]\}) \\
&+ (\{V_1,[F_a{}^5,[F_b{}^5,A_2]]\} \pm \{[F_a{}^5,[F_b{}^5,V_2]],A_1\}) \\
&+ (\{[F_a{}^5,V_1],[F_b{}^5,A_2]\} \pm \{[F_a{}^5,A_1],[F_b{}^5,V_2]\}) \\
&+ (\{[F_b{}^5,V_1],[F_a{}^5,A_2]\} \pm \{[F_b{}^5,A_1],[F_a{}^5,V_2]\}). \quad (24)
\end{aligned}$$

That the symmetry property is retained is clear from inspection. To see property (1), note that we may replace F^5 by F in the double-commutation terms on the right-hand side, since

$$[F_a{}^5,[F_b{}^5,\mathcal{O}]] = [F_a,[F_b,\mathcal{O}]], \quad \mathcal{O} = A \text{ or } V.$$

For the single-commutation terms, since

$$[F_a{}^5,V] = [F_a,A], \quad [F_a{}^5,A] = [F_a,V],$$

replacement of F^5 by F must be accompanied by interchange of V and A, under which the symmetric case (+) will just transform into the same terms with F^5 replaced by F, but the antisymmetric case (−) transforms into minus the terms with F^5 replaced by F. Thus property (1) holds for $\mathcal{O}^+$ but not for $\mathcal{O}^-$.

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Radiative Corrections to $K_{e3}{}^0$ Decays and the $\Delta I = \frac{1}{2}$ Rule

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The radiative corrections to the Dalitz plot in $K_{e3}{}^0$ decays are calculated assuming a phenomenological weak K-π vertex and using perturbation theory. The answer depends logarithmically on a cutoff, as is the case for nuclear β decay. An interesting feature of these decays is that they offer a means of measuring the q^2 dependence of the form factors $f_{\pm}(q^2)$. The radiative corrections contribute an additional energy dependence which cannot be separated experimentally. It is found that the radiative corrections are considerable, i.e., greater than 3% in absolute magnitude, over a large portion of the Dalitz plot, and are not particularly sensitive to a reasonable choice of cutoff. The corrections to the lepton and pion spectra, and the decay rate are also given. A comparison with previous results for $K_{e3}{}^0$ reveals that the $K_{e3}{}^+$ correction is of the same order of magnitude but everywhere more positive. In particular, the ratio of the decay rates $\Gamma(K_{e3}{}^0)/\Gamma(K_{e3}{}^+)$, which is equal to 2 according to the $\Delta I = \frac{1}{2}$ rule, must be modified by a factor $(1+\delta)$ due to the radiative corrections. It is found that $\delta \simeq 1\frac{1}{4}\%$ and is independent of the cutoff.

I. INTRODUCTION

THE subject of this paper is the estimate of the radiative corrections to the three-body leptonic decays of neutral kaons, $K_{l3}{}^0$ for short. The same approach is adopted as that used in previous papers[1,2] concerned with the radiative corrections to $K_{l3}{}^{\pm}$. These estimates are relevant for several reasons. Recent experimental interest in these decays will result in measurements of sufficient precision to be sensitive to radiative corrections.[3] Thus, in a measurement of the energy dependence of the phenomenological form factors $f_{\pm}(q^2)$, one must allow for an additional, unavoidable q^2 dependence due to radiative effects. Moreover, electromagnetic interactions do not conserve isospin; therefore, the radiative corrections will modify predictions based upon isospin selection rules, such as the $\Delta I = \frac{1}{2}$ rule, which relate the experimentally measured decay rates of charged and neutral kaons. Finally, the presence of an electromagnetic final-state interaction [Fig. 1(b)] in the $K_{l3}{}^0$ (but not $K_{l3}{}^+$) radiative correction gives rise to an apparent violation of time-reversal invariance in the measurement of the transverse lepton polarization.[4] The numerical results presented in this paper depend upon the overwhelming simplification which results from neglecting the lepton mass (in particular, this excludes lepton polarization). Thus, the numerical results apply only to $K_{e3}{}^0$.

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[2] E. S. Ginsberg, Phys. Rev. **162**, 1570 (1967).

[3] Princeton Conference on K mesons, Princeton, N. J., 1967 (unpublished).

[4] N. Byers, S. W. MacDowell, and C. N. Yang, in *Proceedings of the Seminar in High-Energy Physics and Elementary Particles, Trieste, 1965* (International Atomic Energy Agency, Vienna, 1965), p. 953.