Born term, while the negative terms differ by two units of charge and, since these are strangeness-changing currents, two units of strangeness. (We assume the validity of the $\Delta Q=\Delta S$ rule for such processes.) For K_{e3} decay the $S=0$ contribution includes many channels and has a contribution from the A_1 and any other abnormal-parity multipion resonances or Regge recurrences, while the $S=-2$ bosonic states include no known resonances. Thus we argue that the former terms can be expected to dominate over the latter, giving a net negative contribution to Eq. (3) from the sum, $\sum_{n\neq\pi^+}$.

To summarize, we obtain Eq. (2) from an Ademollo-Gatto theorem for which we can argue with some plausibility that the dispersion integral contributes with a particular sign. This argument is based on the fact that we can relate the correction term to a difference of squared terms, where the terms of strangeness zero, which include many channels and any abnormal-parity, $S=0$, multipion resonances such as the A_1, contribute with one sign while the terms of the opposite sign have the quantum numbers $B=0$, $Y=-2$ and include no known resonances. In such an integral it is very likely that the former terms dominate over the latter, and thus we infer the sign of the dispersion contribution. Apart from K_{e3} decay the only interesting case where such a condition applies appears to be that of $\Sigma^-\rightarrow ne\bar{\nu}$, where a condition similar to Eq. (2) for both the vector and axial form factors may be obtained.

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Inelastic Contribution to the p-n Mass Difference in Cottingham's Formula

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We have calculated the inelastic contribution to the p-n mass difference coming from some low-lying nucleon resonances by using Cottingham's expression for the electromagnetic self-energy of the nucleons. The contribution of the Roper resonance has been estimated numerically, and is found to be too small to reverse the wrong sign of the elastic contribution. This result suggests the importance of the high-energy diffraction region in the inelastic contribution rather than the low-energy resonance region.

I. INTRODUCTION

It is an embarrassing fact that the neutron is heavier than the proton, contrary to simple-minded expectations. Many attempts have been made to resolve this puzzle, but none of them has succeeded in giving a satisfactory answer. Among these attempts Cottingham's expression for the p-n mass difference seems to be promising, because in this expression we can relate the p-n mass difference to other experimentally measurable quantities.[1] By assuming an unsubtracted dispersion relation for the forward Compton scattering amplitude, he has rewritten the expression for the p-n mass difference given by Cini, Ferrari, and Gatto.[2] There is, however, an argument by Harari[3] that the difference between the forward Compton scattering amplitude of the proton and that of the neutron needs one subtraction according to the Regge hypothesis. He suggested that the subtraction term may give the correct sign of the p-n mass difference by taking the on-shell limit for the spacelike photon, $q^2\rightarrow+0$. The conclusion, however, depends on how the subtraction point is chosen and how the on-shell limit of the virtual photon is reached, as suggested by Gibb,[4] who has made a subtraction at $q_0^2=-q^2$ and has taken the limit $q^2\rightarrow-0$ to get the opposite sign for the subtraction term. Moreover he showed that if we need a subtraction in the forward Compton scattering amplitude, then Cottingham's expression itself is likely to diverge.[4,5] It seems rather doubtful whether the subtraction term gives a clear-cut answer.

On the other hand, Theis and Zeiler[6] have recently estimated the inelastic part of Cottingham's expression by assuming a suitable form for the absorptive part of the forward Compton scattering amplitude to incorporate a few experimental data. They obtained -1.5 MeV for $\Delta\equiv m_p-m_n$ by assuming the unsubtracted dispersion relations for the forward Compton scattering amplitude. If their evaluation is reliable, the contribu-

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[1] W. N. Cottingham, Ann. Phys. (N. Y.) **25**, 424 (1963).
[2] M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters **2**, 7 (1959).
[3] H. Harari, Phys. Rev. Letters **17**, 1303 (1966).
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[6] W. R. Theis and J. Zeiler, Der Freien Universität Report, Berlin (unpublished).

tion of the low-lying nucleon resonances through the process $\gamma+N\to N^*\to\gamma+N$ to the p-n mass difference Δ may be fairly large and negative, unless an unexpected cancellation among them occurs.

Thus it seems worthwhile to estimate the contribution of the low-lying nucleon resonances to the p-n mass difference. These estimates of the low-energy contribution in the inelastic part may give some idea about the question of whether the subtraction term or the low-energy inelastic part is important.

Our calculation is based on the zero-width approximation for the nucleon resonances, which is shown to be accurate to about 10%. In particular, the contribution coming from the Roper resonances $N'(1400)$ [7] can be estimated numerically by using the $N'-N-\gamma$ transition form factors which have been related to the nucleon electromagnetic form factors by the present author.[8] We obtain for this contribution to the p-n mass difference

$$\Delta_1=0.010\Delta\gamma^2-0.089\text{ MeV},$$

where $\Delta\gamma^2$ is an unknown parameter related to the slope at $q^2=0$ of the $N'-N-\gamma$ electric transition form factor. This result suggests that if the inelastic part is capable of providing the correct sign of the p-n mass difference Δ, there must be needed at least 20 such $I=\frac{1}{2}$ resonances all contributing with the same sign.[9] Hence it is very hard to believe that the low-lying resonances are responsible for the correct sign of Δ, and it seems that the high-energy part of the inelastic contribution may give a considerable contribution to Δ.

We shall briefly recapitulate Cottingham's formula for the p-n mass difference in Sec. II and estimate the contribution of the low-lying $I=\frac{1}{2}$ nucleon resonances in the zero-width approximation in Sec. III. In Sec. IV, concluding remarks will be given.

II. COTTINGHAM'S FORMULA FOR THE *p-n* MASS DIFFERENCE

From the well-known formula for the second-order electromagnetic self-energy of the nucleon, we can easily obtain the following expression for the p-n mass difference[2,10]:

$$\Delta=\frac{1}{8\pi^2}\int\frac{d^4q\,\Delta T(q^2,q_0)}{q^2},\tag{1}$$

where, with an obvious notation, we can write

$$\Delta T(q^2,q_0)=-\pi i\sum_{\text{spin}}\int d^4x\times e^{-iq\cdot x}\Box\langle p-n|T[A_\mu(x)j^\mu(0)]|p+n\rangle.\tag{2}$$

Here we have chosen, for convenience, the sign of the amplitude $\Delta T(q^2,q_0)$ different from Cottingham's. Rotating the integration contour in the q_0 plane to the imaginary axis, we obtain

$$\Delta=\frac{1}{4\pi}\int_0^\infty\frac{dq^2}{q^2}\int_{-q}^{q}dq_0(q^2-q_0^2)^{1/2}\,\Delta T(q^2,iq_0),\tag{3}$$

which we call Cottingham's formula. If we assume an unsubtracted dispersion relation for the amplitude $\Delta T(q^2,q_0)$ with q^2 fixed, we have

$$\Delta T(q^2,q_0)=\frac{1}{\pi}\int_0^\infty\frac{2\nu d\nu\,\mathrm{Im}\Delta T(q^2,\nu)}{\nu^2-q_0^2},\tag{4}$$

where we have used the crossing symmetry of the amplitude $\Delta T(q^2,q_0)$:

$$\Delta T(q^2,-q_0)=\Delta T(q^2,q_0).\tag{5}$$

Inserting Eq. (4) in Eq. (3), we have

$$\Delta=\frac{1}{2\pi}\int_0^\infty dq^2\int_0^\infty d\nu\frac{\mathrm{Im}\Delta T(q^2,\nu)}{(\nu^2+q^2)^{1/2}+\nu}.\tag{6}$$

It should be remarked here that Eq. (6) is slightly different from the original formula derived by Cottingham [Eqs. (1.14) and (1.15) of Ref. 1]. This is because he assumed unsubtracted dispersion relations for the invariant amplitudes $t_1(q^2,q_0)$ and $t_2(q^2,q_0)$ which are not necessarily compatible with Eq. (4). The difference between his original formula and Eq. (6) is

$$\int_0^\infty dq^2\int_0^\infty\nu d\nu\,\mathrm{Im}\Delta t_2(q^2,\nu),$$

and, when the superconvergence relation

$$\int_0^\infty\nu d\nu\,\mathrm{Im}\Delta t_2(q^2,\nu)=0\tag{7}$$

holds, Eq. (6) is precisely equal to the original formula.[11]

If we need a subtraction for $\Delta T(q^2,q_0)$ we have, instead of Eq. (6),

$$\Delta=\frac{1}{8}\int_0^\infty dq^2\Delta T(q^2,a)+\frac{1}{2\pi}\int_0^\infty\frac{dq^2}{q^2}\int_0^\infty d\nu\times\left(\frac{q^2}{(\nu^2+q^2)^{1/2}+\nu}-\frac{1}{2}\frac{\nu}{\nu^2-a^2}\right)\mathrm{Im}\Delta T(q^2,\nu),\tag{8}$$

where a is the subtraction point which may be a function of q^2. We can choose the subtraction point $a(q^2)$ at very high energy. If we cut off the integral over ν at $\nu=a(q^2)$

[7] L. D. Roper, R. M. Wright, and B. T. Feld, Phys. Rev. **138**, B190 (1965). A complete list of references can be found in A. H. Rosenfeld *et al.*, Rev. Mod. Phys. 40, 77 (1968).

[8] T. Muta (to be published). See, also, D. Lyth, Phys. Rev. **165**, 1786 (1968); T. Muta, Nuovo Cimento **51A**, 1154 (1967).

[9] It is easily seen that the $I=\frac{3}{2}$ nucleon resonances do not contribute to the p-n mass difference.

[10] R. P. Feynman and G. Speisman, Phys. Rev. **94**, 500 (1954).

[11] Y. Taguchi and K. Yamamoto, Progr. Theoret. Phys. (Kyoto) **38**, 1152 (1967). Their Eq. (2.14) is incompatible with the positive definiteness of $h_2(q^2,\nu)$. However, if we take the p-n difference as in Eq. (7), the superconvergence relation may hold.

and include the contribution of the integral over $\nu>a$ into the subtraction term, we can write Eq. (8) as

$$\Delta=C+\frac{1}{2\pi}\int_0^\infty dq^2\int_0^a d\nu\,\frac{\mathrm{Im}\Delta T(q^2,\nu)}{(\nu^2+q^2)^{1/2}+\nu},\tag{9}$$

where the constant C stands for the subtraction term and the high-energy part of the integral. When C is not too large, the dominant contribution to Δ comes from the low-energy part of the integral, and we can use Eq. (6) to approximate it by the low-lying $I=\frac{1}{2}$ resonances. We shall discuss the zero-width resonance approximation for Eq. (6) in Sec. III.

III. ZERO-WIDTH RESONANCE APPROXIMATION

The important feature of Cottingham's formula is that it expresses the mass difference in terms of experimentally obtainable quantities. That is, the absorptive part of the amplitude $\Delta T(q^2,\nu)$ is given by

$$\begin{aligned}\mathrm{Im}\Delta T(q^2,\nu)=&\pi\delta(\nu-q^2/2m)\Delta f(q^2)\\&+\theta[\nu-\mu-(\mu^2+q^2)/2m]\Delta h(q^2,\nu),\end{aligned}\tag{10}$$

where

$$f(q^2)=(e/2\pi)^2[G_E^2(q^2)-(q^2/2m^2)G_M^2(q^2)],\tag{11}$$

$$h(q^2,\nu)=(2\pi)^{-2}(\nu-q^2/2m)[\sigma_L(q^2,\nu)-2\sigma_T(q^2,\nu)].\tag{12}$$

Here we denote by Δf (Δh) the difference of $f(q^2)$ ($h(q^2,\nu)$) between the proton and neutron. $G_E(q^2)$ and $G_M(q^2)$ are the usual electric and magnetic form factors of the nucleons and σ_L and σ_T are the absorption cross sections of longitudinal and transverse photons by the nucleon, respectively.[12] At present we have no satisfactory data on σ_L and σ_T to achieve the integration in Eq. (6). Theis and Zeiler[6] have assumed the factorizability of the function $h(q^2,\nu)$ in the two variables q^2 and $(p+q)^2$, where p is the energy-momentum of the nucleon at rest, and have carried out the integration. This assumption, roughly speaking, corresponds to factoring out the transition form factors between the nucleon and the nucleon resonance and summing over all the relevant resonances. We now make more physically transparent the approximation to the function $h(q^2,\nu)$, i.e., the zero-width resonance approximation

$$\Delta h(q^2,\nu)=\sum_{n=1}^{\Lambda}\pi\delta\left(\nu-\frac{M_n^2-m^2+q^2}{2m}\right)f_n(q^2),\tag{13}$$

where Λ represents the highest resonance which comes into play, m is the nucleon mass, M_n is the mass of the nth nucleon resonance N_n, and $f_n(q^2)$ is expressed by some transition form factors between N_n and N. If we write $m=M_0$ and $f(q^2)=f_0(q^2)$, then we can rewrite Eq. (10) in a more compact form:

$$\mathrm{Im}\Delta T(q^2,\nu)=\sum_{n=0}^{\Lambda}\pi\delta\left(\nu-\frac{M_n^2-m^2+q^2}{2m}\right)f_n(q^2).\tag{14}$$

Inserting Eq. (14) in Eq. (6) we obtain

$$\Delta=m\sum_{n=0}^{\Lambda}\int_0^\infty\frac{dq^2f_n(q^2)}{\{[q^2+(M_n+m)^2][q^2+(M_n-m)^2]\}^{1/2}+M_n^2-m^2+q^2}.\tag{15}$$

The correction to the zero-width approximation due to the nonzero width of the actual resonances is easily estimated by replacing the δ function in Eq. (13) by the Breit-Wigner-type function

$$\pi\delta\left(\nu-\frac{M_n^2-m^2+q^2}{2m}\right)\to\Gamma_n\Big/\left[\left(\nu-\frac{M_n^2-m^2+q^2}{2m}\right)^2+\Gamma_n^2\right],$$

where Γ_n is the total width of the resonance N_n. By this replacement we obtain the following fractional corrections in the first order of Γ_n:

$$2m\Gamma_n/\pi(M_n^2-m^2)\le 0.1,$$

which is negligible for our present purpose.

TABLE I. $I=\frac{1}{2}$ nucleon resonances.

Wave	Mass (MeV)	Total width (MeV)	$\Gamma_{\mathrm{el}}/\Gamma_{\mathrm{tot}}$
(Nucleon)	940	0	...
P_{11}	1470	210	0.65
D_{13}	1520	115	0.55
S_{11}	1530	120	0.35
D_{15}	1680	170	0.50
F_{15}	1960	130	0.45
S_{11}	1710	300	0.80
G_{17}	2220	300	0.35

Now we are in a position to evaluate separately the individual contributions to Δ coming from the nucleon and nucleon resonances. It should be noted that the $I=\frac{3}{2}$ resonances do not contribute to the p-n mass difference Δ at all as stated previously. There are many confirmed $I=\frac{1}{2}$ resonances, which we now list in Table I.[13]

A. Nucleon Pole Contribution (Elastic Part)

For the sake of comparison we briefly discuss the elastic part. Since the function $f_0(q^2)$ is readily given in

[12] R. Wilson, in *Particle Interactions at High Energies*, edited by T. W. Preist and L. L. J. Vick (Oliver and Boyd, London, 1967), p. 156.

[13] We have used the data by the CERN group (A. Donnachie *et al.*) presented at the Informal Theoretical Physics Gathering held at the Rutherford Laboratory, 1967 (unpublished). See also C. Lovelace, in *Proceedings of the Heidelberg International Conference on Elementary Particles* (North-Holland Publishing Co., Amsterdam, 1968), p. 79.

Eq. (11), we can perform straightforwardly the integration for the elastic contribution Δ_0 to the p-n mass difference. We use the dipole form factors which have been determined by recent experimental analysis[14]:

$$G_E{}^p(q^2)=\frac{1}{\mu_p}G_M{}^p(q^2)=\frac{1}{\mu_n}G_M{}^n(q^2)=\left(\frac{m_0{}^2}{q^2+m_0{}^2}\right)^2,\quad G_E{}^n(q^2)=0, \tag{16}$$

where $m_0=850$ MeV. The explicit integration can easily be done, and results in

$$\Delta_0=\frac{\alpha x}{24(1-x)}\left[7-10x+\frac{15-36x+24x^2}{2x}\phi(x)-2(\mu_p{}^2-\mu_n{}^2)\times\left(10x-1+\frac{3+12x-24x^2}{2x}\phi(x)\right)\right], \tag{17}$$

where $x=(m_0/2m)^2$, $\alpha=1/137$ is the fine-structure constant, and

$$\phi(x)=\left(\frac{x}{1-x}\right)^{1/2}\left\{1-\frac{2}{\pi}\tan^{-1}\left[\left(\frac{x}{1-x}\right)^{1/2}\right]\right\}. \tag{18}$$

Inserting the numerical values $x=0.204$, $\mu_p=2.79$, and $\mu_n=-1.91$ in Eq. (17), we obtain

$$\Delta_0=0.66-0.29=0.37 \text{ MeV}.$$

Also, by using Cottingham's original formula which is based on an asymptotic assumption on $\Delta T(q^2,q_0)$ different from that in the present paper, one may get an estimate of the elastic part. The numerical value obtained is 0.78 MeV,[15] which is about twice as large as 0.37 MeV. There is, however, a possibility that the difference between them may be cancelled out by the difference in the inelastic part so that the superconvergence relation, Eq. (7), holds.

B. Roper-Resonance Contribution

The lowest $I=\frac{1}{2}$ nucleon resonance confirmed now is the so-called Roper resonance $N'(1400)$ with a width of about 200 MeV, which is found in the P_{11} channel of π-N scattering.[7] We now call this $N_1(n=1)$ and calculate $f_1(q^2)$ in Eq. (13) by taking the absorptive part of Eq. (2) and keeping only the N_1 state in the intermediate states. We use here the transition form factors $G_E{}^{(1)}(q^2)$ and $G_M{}^{(1)}(q^2)$, which are defined such that

$$\langle N_1|j_\mu|N\rangle=(2\pi)^{-3}(M_1m/p_{10}p_0)^{1/2}\bar{u}(p_1)\times\{F_1{}^{(1)}(q^2)[\gamma_\mu+(M_1-m)q_\mu/q^2]-F_2{}^{(1)}(q^2)i\sigma_{\mu\nu}q^\nu\}u(p), \tag{19}$$

$$G_E{}^{(1)}(q^2)=F_1{}^{(1)}(q^2)-[q^2/(M_1+m)]F_2{}^{(1)}(q^2), \tag{20}$$

$$G_M{}^{(1)}(q^2)=F_1{}^{(1)}(q^2)+(M_1+m)F_2{}^{(1)}(q^2), \tag{21}$$

where p_1 and p are the energy-momenta of the Roper resonance N_1 and the nucleon N, respectively. After some elementary but tedious calculation we obtain

$$f_1(q^2)=\left[1+\left(\frac{M_1-m}{q}\right)^2\right]\left(\frac{M_1+m}{2m}\right)^2\left(\frac{e}{2\pi}\right)^2\times\left[G_E{}^{(1)}(q^2)^2-2\left(\frac{q}{M_1+m}\right)^2G_M{}^{(1)}(q^2)^2\right]. \tag{22}$$

The present author has recently derived the relation between the magnetic transition form factor $G_M{}^{(1)}(q^2)$ and the nucleon magnetic form factor $G_M(q^2)$ by assuming that the Roper resonance dominates the sidewise dispersion relation for the electromagnetic vertex function.[8] The relation is[16]

$$G_M{}^{(1)}(q^2)=(g_{RN\pi}/g_{NN\pi})G_M(q^2), \tag{23}$$

where $g_{RN\pi}{}^2/4\pi\simeq2.5$ is the $N_1-N-\pi$ effective coupling constant determined by the width and $g_{NN\pi}{}^2/4\pi=14.6$ is the π-N coupling constant.[17]

On the other hand, we have less information to determine $G_E{}^{(1)}(q^2)$. We can see, however, that the electric transition form factor $G_E{}^{(1)}(q^2)$ should vanish at $q^2=0$, as is well known in the multipole analysis of the $\gamma+p\to\pi+N$ reaction. This can be seen by observing the relations (19) and (20); in Eq. (19), $F_1{}^{(1)}(q^2)$ should vanish at $q^2=0$ to cancel the spurious pole at $q^2=0$ and therefore $G_E{}^{(1)}(q^2)$ by Eq. (20). Thus it can be assumed that $G_E{}^{(1)}(q^2)$ has the following form near $q^2=0$:

$$G_E{}^{(1)}(q^2)\propto q^2G_M{}^{(1)}(q^2). \tag{24}$$

We have, however, no idea how to determine the high-q^2 behavior of $G_E{}^{(1)}(q^2)$. Here we shall simply assume that the relationship (24) holds for any value of q^2, though there is the possibility of multiplying the right-hand side of Eq. (24) by $1/(q^2+C)$ to improve the asymptotic behavior. At present we have no information how to determine the slope of the function $G_E{}^{(1)}(q^2)$ at $q^2=0$. We leave it as an unknown parameter γ, which we define by

$$G_E{}^{(1)}(q^2)=\frac{\gamma q^2}{2m(M_1+m)}\left(\frac{m_0{}^2}{q^2+m_0{}^2}\right)^2. \tag{25}$$

Inserting Eqs. (23) and (25) in Eq. (22) and performing the integration (15) for $n=1$ numerically, we obtain

$$\Delta_1=0.010\Delta\gamma^2-0.089 \text{ MeV}, \tag{26}$$

[14] See, e.g., W. K. H. Panofsky, in *Proceedings of the Heidelberg International Conference on Elementary Particles* (North-Holland Publishing Co., Amsterdam, 1968), p. 371.

[15] G. Barton and J. Brennan (unpublished); see also Ref. 6.

[16] A different assumption (Ref. 8) on the asymptotic behavior of the vertex function continued analytically in the nucleon mass leads to a relation different from Eq. (23), e.g., $F_2{}^{(1)}(q^2)=(g_{RN\pi}/g_{NN\pi})F_2(q^2)$. We, however, prefer Eq. (23) because it best fits the recent experimental analysis on the photopion production from nucleons [Y. C. Chau, N. Dombey, and R. G. Moorhouse, Phys. Rev. 163, 1632 (1967)].

[17] J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. 35, 737 (1963).

where $\Delta\gamma^2=\gamma^2(N_1^+)-\gamma^2(N_1^0)$. Since it is unlikely that $\Delta\gamma^2$ is very large, the contribution of the Roper resonance Δ_1 is too small to overcompensate the elastic contribution Δ_0, though it has the right sign for $\Delta\gamma^2\leq 8.9$. If the low-energy region of the inelastic part is responsible for reversing the sign of the mass difference, we must have at least 20 resonances contributing comparably to the Roper resonance.

It should be remarked here that, if we use Cottingham's original formula[1] to evaluate the Roper-resonance contribution, we have, for the magnetic contribution to Δ_1,

$$\Delta_{1,\mathrm{mag}}=-0.014\ \mathrm{MeV}.$$

This is even smaller than the value we obtained in Eq. (26).

C. $S_{11}(1530)$ Contribution

We shall call this S_{11} resonance $N_2(n=2)$. The vertex function $\langle N_2|j_\mu|N\rangle$ has the following general form:

$$\langle N_2|j_\mu|N\rangle=(2\pi)^{-3}(M_2m/p_{20}p_0)^{1/2}\bar{u}(p_2)\gamma_5 \times\{F_1^{(2)}(q^2)[\gamma_\mu+(M_2+m)q_\mu/q^2] -F_2^{(2)}(q^2)i\sigma_{\mu\nu}q^\nu\}u(p), \quad (27)$$

where p_2 is the energy-momentum of N_2. It is very easy now to express the function $f_2(q^2)$ by the invariant form factors $F_1^{(2)}(q^2)$ and $F_2^{(2)}(q^2)$,

$$f_2(q^2)=\left[1+\left(\frac{M_2+m}{q}\right)^2\right]\left(\frac{M_2-m}{2m}\right)^2\left(\frac{e}{2\pi}\right)^2 \times\{G_E^{(2)}(q^2)^2-2[q/(M_2-m)]^2G_M^{(2)}(q^2)^2\}, \quad (28)$$

where

$$G_E^{(2)}(q^2)=F_1^{(2)}(q^2)+[q^2/(M_2-m)]F_2^{(2)}(q^2), \quad (29)$$

$$G_M^{(2)}(q^2)=F_1^{(2)}(q^2)-(M_2-m)F_2^{(2)}(q^2). \quad (30)$$

For the present we have no idea how to determine the form factors $G_E^{(2)}(q^2)$ and $G_M^{(2)}(q^2)$ and so we cannot form any quantitative estimate of the S_{11} contribution, Δ_2, to the p-n mass difference. However, we can make a qualitative statement that the S_{11} resonance is likely to contribute less than the Roper resonance does, because $f_2(q^2)$ has a factor $[(M_2-m)/2m]^2$ which is very small compared with the factor $[(M_1+m)/2m]^2$ in $f_1(q^2)$. Thus we may conclude that the contributions of the S_{11} resonance and the Roper resonance are both too small to change the wrong sign of the elastic part Δ_0.

IV. CONCLUDING REMARKS

We have found that the inelastic contribution coming from the low-lying nucleon resonances is too small to overcompensate the elastic contribution. From this point of view it is very difficult to see how Theis and Zeiler[6] have obtained the value -2.6 MeV for the inelastic part. They have evaluated the low-energy region of the inelastic part and therefore the value they obtained should be consistent with our value $\sum_{n=1}^{\Lambda}\Delta_n$. To obtain the value -2.6 MeV by our method we have to add up the contributions of at least 20 or 30 resonances. This seems rather unlikely, because we have chosen the cutoff parameter $a(q^2)$ in Eq. (9) as a point higher than the resonance region and lower than the diffraction region, and we have in the region $\nu<a(q^2)$ less than 20 $I=\frac{1}{2}$ nucleon resonances. In order to explain this discrepancy we may consider two possibilities:

(1) Their assumed form for the function $h(q^2,q_0)$ overestimates the inelastic part;

(2) some of the higher $I=\frac{1}{2}$ resonances have an unexpectedly large contribution to Δ_n.

It seems to us that the second possibility is quite unlikely.

We conclude that the low-energy part of the inelastic contribution is unimportant and therefore, if Cottingham's formula is correct and convergent, there must be some important contributions coming from the high-energy diffraction region in the inelastic part, e.g., the Regge-pole terms.[18]

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[18] S. Okubo, Phys. Rev. Letters **18**, 256 (1967); Y. Liu and S. Okubo, Nuovo Cimento **57A**, 1186 (1967); Y. Srivastava, Phys. Rev. Letters **20**, 232 (1968).