Renormalization of Weak Form Factors, and the Cabibbo Angle*

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We argue that renormalization corrections in K_{e3} decay reduce the magnitude of the decay amplitude $f_+(0)$ from that given by exact $SU(3)$.

IN order to test the Cabibbo[1] postulate of universality of the weak interactions, one compares the Cabibbo angle as measured in K_{e3} decay with that measured in O^{14} β decay. Since accurate measurements of both these processes exist, the chief problem in such a comparison is the evaluation of small corrections to the theoretical rates. With this in mind, we consider the K_{e3} decay rate. The purpose of this paper is to argue that $SU(3)$ breaking corrections to the K_{e3} form factor $f_+(q^2)$ are such at to *decrease* its magnitude, for spacelike q^2, from that expected in the limit of exact $SU(3)$. The basis for this argument is an Ademollo-Gatto[2] relation obtained by considering the appropriate current commutator between *kaon* states, and assuming that the contribution from states of zero strangeness is dominant over states of $|S|=2$. The derivation of this relation and generalizations of it are discussed below; for the present we return to the question of the Cabibbo angle.

If we assume that a reasonable representation of f_+ over a range of q^2 is

$$f_+(q^2)=f_+(0)/(1-\lambda q^2)\,, \qquad (1)$$

where the form factor f_+ is defined by

$$\langle\pi^+(P+q)|V_\mu(0)|\bar{K}_0(P)\rangle = [(2P+q)_\mu f_+(q^2)+q_\mu f_-(q^2)][4P_0(P_0+q_0)]^{-1/2}$$

and $q^2>0$ is timelike in our metric, then our relation is

$$|f_+(q^2)|\leq 1\,, \quad q^2\leq 0 \qquad (2)$$

which implies $|f_+(0)|\leq 1$ and $\lambda\geq 0$ in Eq. (1). Previous estimates of $|f_+(0)|$ have given values both greater and less than 1.[3] Unfortunately the inequality [Eq. (2)] does not provide a simple parameter-free estimate of $f_+(0)$ but it may help to choose between previous estimates. If we believe that the form of Eq. (1) holds also for small timelike q^2 without any significant change in λ, the condition $\lambda\geq 0$ is in agreement with the experimental evidence[4] which gives

$$\lambda m_\pi^2=0.016\pm 0.016\,.$$

As remarked by Sirlin,[5] if $|f_+(0)|\leq 1$ and $\lambda\geq 0$, the effects of the $q^2=0$ renormalization and the q^2 dependence of f_+ tend to cancel one another in calculating θ from the K_{e3} decay rate. The value of λ quoted above gives $-(6\pm 6)\%$ correction to $\sin^2\theta$.[6] Estimates of $|f_+(0)|$ vary, with a deviation from 1 of 5–15%. Thus one would expect a similar correction to the value of $\sin\theta$ obtained from K_{e3} decay using $f_+(q^2)=1$, which means less than 1% correction to $\cos\theta$. This, however, is an interesting quantity, as it is of the same order of magnitude as the possible discrepancy between θ as measured in O^{14} β decay or in K_{e3} decay, and also comparable to the uncertainties in the O^{14} β decay value of θ due to the model dependence of the radiative corrections.[7] In particular, as the estimate of $f_+(0)$ decreases, the cutoff in the radiative correction needed to maintain agreement with Cabibbo theory increases.

We now discuss the derivation of Eq. (2). For all spacelike q we can write a sum rule by the $P\to\infty$ technique, using the commutator

$$[V_0(\mathbf{x},0),V_0^\dagger(\mathbf{x}',0)]=2\delta^3(\mathbf{x}-\mathbf{x}')V_0^3(\mathbf{x},0)\,.$$

For $\mathbf{q}=0$ we need only the charge commutator, so the relation is more general in that case. We find

$$|f_+(q^2)|^2=1-\lim_{\mathbf{P}\to\infty}\sum_{n\neq\pi^+}\delta^3(\mathbf{P}+\mathbf{q}-\mathbf{P}_n) \times\{|\langle\bar{K}_0|V_0^\dagger(0)|n\rangle|^2-|\langle\bar{K}_0|V_0(0)|n\rangle|^2\}\,, \qquad (3)$$

where $\mathbf{q}\cdot\mathbf{P}=0$ and $q^2=-\mathbf{q}^2$. In the exact $SU(3)$ limit the sum $n\neq\pi^+$ vanishes and $|f_+(q^2)|^2=1$. Without having to make the usual arguments about interchanging the limit $P\to\infty$ and the infinite sum $\sum_{n\neq\pi^+}$ in Eq. (3), we can argue plausibly that the correction term in Eq. (3) reduces $|f_+(q^2)|$ from its exact symmetry value. The states contributing positively to the sum in Eq. (3) have the same quantum numbers as the

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[1] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[2] M. Ademollo and R. Gatto, Phys. Rev. Letters 13, 264 (1964); S. Fubini and G. Furlan, Physics 1, 229 (1965); G. Furlan, F. Lannoy, C. Rossetti, and G. Segrè, Nuovo Cimento 38, 1747 (1965).

[3] For example, J. J. Sakurai, Phys. Rev. Letters 12, 79 (1964); G. Furlan, *et al.* (Ref. 2); V. S. Mathur, L. K. Pandit, and R. E. Marshak, Phys. Rev. Letters 16, 947 (1966); A. Bartl and C. P. Korthals Altes, Nuovo Cimento 47, 721 (1967); S. L. Glashow and Steven Weinberg, Phys. Rev. Letters 20, 224 (1968).

[4] R. L. Imlay, P. T. Eschstruth, A. D. Franklin, E. B. Hughes, D. H. Reading, D. R. Bowen, A. K. Mann, and W. K. McFarlane, Phys. Rev. 160, 1203 (1967).

[5] A. Sirlin, Phys. Rev. Letters 16, 872 (1966).

[6] S. Oneda and J. Sucher, Phys. Rev. Letters 15, 927 (1965).

[7] E. S. Abers, D. A. Dicus, R. E. Norton, and H. R. Quinn, Phys. Rev. 167, 1461 (1968).

Born term, while the negative terms differ by two units of charge and, since these are strangeness-changing currents, two units of strangeness. (We assume the validity of the $\Delta Q=\Delta S$ rule for such processes.) For K_{e3} decay the $S=0$ contribution includes many channels and has a contribution from the A_1 and any other abnormal-parity multipion resonances or Regge recurrences, while the $S=-2$ bosonic states include no known resonances. Thus we argue that the former terms can be expected to dominate over the latter, giving a net negative contribution to Eq. (3) from the sum, $\sum_{n\neq\pi^+}$.

To summarize, we obtain Eq. (2) from an Ademollo-Gatto theorem for which we can argue with some plausibility that the dispersion integral contributes with a particular sign. This argument is based on the fact that we can relate the correction term to a difference of squared terms, where the terms of strangeness zero, which include many channels and any abnormal-parity, $S=0$, multipion resonances such as the A_1, contribute with one sign while the terms of the opposite sign have the quantum numbers $B=0$, $Y=-2$ and include no known resonances. In such an integral it is very likely that the former terms dominate over the latter, and thus we infer the sign of the dispersion contribution. Apart from K_{e3} decay the only interesting case where such a condition applies appears to be that of $\Sigma^-\rightarrow ne\bar{\nu}$, where a condition similar to Eq. (2) for both the vector and axial form factors may be obtained.

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Inelastic Contribution to the *p-n* Mass Difference in Cottingham's Formula

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We have calculated the inelastic contribution to the *p-n* mass difference coming from some low-lying nucleon resonances by using Cottingham's expression for the electromagnetic self-energy of the nucleons. The contribution of the Roper resonance has been estimated numerically, and is found to be too small to reverse the wrong sign of the elastic contribution. This result suggests the importance of the high-energy diffraction region in the inelastic contribution rather than the low-energy resonance region.

I. INTRODUCTION

IT is an embarrassing fact that the neutron is heavier than the proton, contrary to simple-minded expectations. Many attempts have been made to resolve this puzzle, but none of them has succeeded in giving a satisfactory answer. Among these attempts Cottingham's expression for the *p-n* mass difference seems to be promising, because in this expression we can relate the *p-n* mass difference to other experimentally measurable quantities.[1] By assuming an unsubtracted dispersion relation for the forward Compton scattering amplitude, he has rewritten the expression for the *p-n* mass difference given by Cini, Ferrari, and Gatto.[2] There is, however, an argument by Harari[3] that the difference between the forward Compton scattering amplitude of the proton and that of the neutron needs one subtraction according to the Regge hypothesis. He suggested that the subtraction term may give the correct sign of the *p-n* mass difference by taking the on-shell limit for the spacelike photon, $q^2\rightarrow+0$. The conclusion, however, depends on how the subtraction point is chosen and how the on-shell limit of the virtual photon is reached, as suggested by Gibb,[4] who has made a subtraction at $q_0^2=-q^2$ and has taken the limit $q^2\rightarrow-0$ to get the opposite sign for the subtraction term. Moreover he showed that if we need a subtraction in the forward Compton scattering amplitude, then Cottingham's expression itself is likely to diverge.[4,5] It seems rather doubtful whether the subtraction term gives a clear-cut answer.

On the other hand, Theis and Zeiler[6] have recently estimated the inelastic part of Cottingham's expression by assuming a suitable form for the absorptive part of the forward Compton scattering amplitude to incorporate a few experimental data. They obtained -1.5 MeV for $\Delta\equiv m_p-m_n$ by assuming the unsubtracted dispersion relations for the forward Compton scattering amplitude. If their evaluation is reliable, the contribu-

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[1] W. N. Cottingham, Ann. Phys. (N. Y.) **25**, 424 (1963).

[2] M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters **2**, 7 (1959).

[3] H. Harari, Phys. Rev. Letters **17**, 1303 (1966).

[4] J. Gibb, Ph.D. thesis, University of Birmingham, 1967 (unpublished).

[5] W. N. Cottingham and J. Gibb, Phys. Rev. Letters **18**, 883 (1967).

[6] W. R. Theis and J. Zeiler, Der Freien Universität Report, Berlin (unpublished).