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Pion-Nucleon Spin-Flip Current-Algebra Sum Rule

EUGEN E. RADESCU
Institute of Atomic Physics, Bucharest, Romania
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Current algebra, unsubtracted dispersion relations, and the hypothesis of partially conserved axial-vector current imply a model-dependent sum rule (derived by Gerstein) for the spin-flip pion-nucleon scattering amplitude, analogous to the one obtained by Weisberger and Adler for the non-spin-flip amplitude. The Gerstein sum rule requires information about the weak amplitude for axial-vector-nucleon scattering, and the purpose of this paper is to evaluate its contribution on the basis of two different models. The first model consists in saturating the weak amplitude with the $N^*(\frac{3}{2},\frac{3}{2})$ (1238 MeV) and $N^{**}(\frac{1}{2},\frac{3}{2})$ (1518 MeV) resonances, using a number of existing theoretical determinations of the N^* and N^{**} axial coupling constants. The second evaluation is done with a very simple assumption of additivity in the quark model, and leads to a poor, but not discouraging, verification of the Gerstein sum rule.

I. INTRODUCTION

RECENTLY Gerstein[1] derived, on the basis of current algebra, unsubtracted dispersion relations, and the hypothesis of partially conserved axial-vector current (PCAC), a sum rule for the spin-flip pion-nucleon scattering amplitude analogous to the one obtained by Weisberger[2] and Adler[3] (W-A) for the non-spin-flip amplitude. This reads

$$F_2(0)=[G_A(0)]^2-F_1(0)-2f_\pi^2\times\frac{1}{\pi}\times\int_{m_\pi(1+m_\pi/2m)}^{\infty}\frac{d\nu}{\nu}\,\mathrm{Im}B^{(-)}(\nu,0,0,0)+\frac{4}{\pi}\int_{m_\pi(1+m_\pi/2m)}^{\infty}\frac{d\nu}{\nu}\,2md_1^{(-)}(\nu,0,0,0).\quad(1)$$

$F_1(0)$, $F_2(0)$, and $G_A(0)$ are, respectively, the electromagnetic and weak axial-vector coupling constants of the nucleon (mass $=m$),[4] and f_π is the pion decay constant.[5] $B^{(-)}$ is defined through the usual decomposition of the πN scattering amplitude[6] [for notation see Eqs. (8) below]:

$$\begin{aligned}T_{\pi N}{}^{ab}(\nu,t,q_1^2,q_2^2)&\equiv-i\int d^4x\,e^{iq_2x}\theta(x_0)\langle p_2|[j_\pi{}^a(x),j_\pi{}^b(0)]|p_1\rangle\\&=\{\delta_{ab}A^{(+)}(\nu,t,q_1^2,q_2^2)+\tfrac{1}{2}[\tau^a,\tau^b]A^{(-)}(\nu,t,q_1^2,q_2^2)\}\\&\quad+\{\delta_{ab}B^{(+)}(\nu,t,q_1^2,q_2^2)+\tfrac{1}{2}[\tau^a,\tau^b]B^{(-)}(\nu,t,q_1^2,q_2^2)\}\times\tfrac{1}{2}[\gamma\cdot(q_1+q_2)].\end{aligned}\quad(2)$$

The quantity

$$D_1^{(-)}(\nu,0,q^2,q^2)\equiv\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{d\nu'}{\nu'-\nu}d_1^{(-)}(\nu',0,q^2,q^2)\quad(3)$$

is the coefficient (taken for the specified values of the kinematical variables) of $\bar{u}i\sigma_{\mu\nu}\frac{1}{2}[\tau^a\tau^b]u$ in the expansion of the axial-vector–nucleon amplitude

$$T_{\mu\nu}{}^{ab}\equiv-i\int d^4x\,e^{iq_2x}\theta(x_0)\langle p_2|[A_\mu{}^a(x),A_\nu{}^b(0)]|p_1\rangle\quad(4)$$

in a complete set of independent covariants. Obviously, the last term in the right-hand side of Eq. (1),

$$\tilde{D}_1^{(-)}(0,0,0,0)\equiv\frac{2}{\pi}\int_{m_\pi(1+m_\pi/2m)}^{\infty}\frac{d\nu}{\nu}d_1^{(-)}(\nu,0,0,0),\quad(5)$$

represents $D_1^{(-)}(0,0,0,0)$ with the nucleon pole extracted.

Gerstein emphasized that, unlike the W-A relation, the spin-flip sum rule [Eq. (1)] is essentially model-dependent (because of the appearance of $\tilde{D}_1^{(-)}$ in the right-hand side). The point is that the covariant $\bar{u}i\sigma_{\mu\nu}u$, for whose coefficient the Fubini procedure[7] cannot yield a formula which eliminates $d_1^{(-)}$ from Eq. (1) (to obtain a model-independent sum rule for the πN spin-flip amplitude), must be included in the expansion of $T_{\mu\nu}{}^{ab}$ in covariants, if kinematical singularities are to be avoided. These circumstances clarify a rather obscure situation in the literature[8,9] where discrepancies are found between the results of previous derivations of the spin-flip current-algebra sum rule.[10]

The purpose of this work is to investigate the validity of Eq. (1) on the basis of two different models. The value of the first integral on the right-hand side of Eq. (1) is

[1] I. S. Gerstein, Phys. Rev. **161**, 1631 (1967).
[2] W. I. Weisberger, Phys. Rev. Letters **14**, 1047 (1965); Phys. Rev. **143**, 1302 (1966).
[3] S. L. Adler, Phys. Rev. Letters **14**, 1051 (1965).
[4] The normalizations are such that $F_1(0)=1$, $F_1(0)+F_2(0)=\mu_p-\mu_n=4.7$.
[5] In terms of f_π the PCAC condition is $\partial_\mu A_\mu{}^a(x)=[-(m_\pi{}^2f_\pi)/\sqrt{2}]\phi_\pi{}^a(x)$.
[6] The Dirac matrices are anti-Hermitian (the metric is $---+$), $\gamma_5=\gamma_0\gamma_1\gamma_2\gamma_3$, $\gamma_5{}^2=-1$, $\sigma_{\mu\nu}\equiv\frac{1}{2}i(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu)$.
[7] S. Fubini, Nuovo Cimento **43A**, 475 (1966).
[8] N. H. Fuchs, Phys. Rev. **150**, 1241 (1966).
[9] C. Bouchiat, G. Flamand, and Ph. Meyer, Orsay Report No. Th/187, 1967 (unpublished).
[10] In the derivation of Ref. 8, both $[G_A(0)]^2$ and the weak amplitude in Eq. (1) are omitted and then a satisfactory agreement with experiment is found, while in Ref. 9 $[G_A(0)]^2$ appears but the weak amplitude does not, the agreement being poorer in the latter case. For a full discussion of all these facts see Ref. 1.

taken from Ref. 8, where it is computed by means of a phase analysis. The first model refers to a determination of $\tilde{D}_1^{(-)}$ by retaining only the $N^*(\frac{3}{2},\frac{3}{2})$(1238 MeV) and $N^{**}(\frac{1}{2},\frac{3}{2})$(1518 MeV) in the continuum contribution to the weak amplitude $T_{\mu\nu}{}^{ab}$. The usual perturbation-theoretic Feynman rules are employed in the evaluation of the Born diagrams. This procedure is similar to the one successfully employed by Schnitzer[11] in calculating the low-energy πN parameters. The method and some of the conclusions presented in Sec. II of this paper differ from the evaluation based on dispersion models given in Ref. 12. Using for the weak NN^* and NN^{**} axial-vector form factors some theoretical numbers given recently by different authors in the literature, the agreement is good only in one case and it is rather poor in the others.

The second model for the axial-vector–nucleon amplitude is considered in Sec. III. It contains an evaluation of the quantity $4m\tilde{D}_1^{(-)}(0,0,0,0)$, using an hypothesis of additivity in the quark model analogous to that employed by Bogolyubov, Matveev, and Tavkhelidze[13] for an invariant function appearing in the amplitude

$$\mathcal{T}^{ab}\equiv\int d^4x\, e^{iq_2x}\theta(x_0)\langle p_2|[\partial_\mu A_\mu{}^a(x),\partial_\nu A_\nu{}^b(0)]|p_1\rangle. \quad (6)$$

These authors succeeded in this manner in deriving the W-A relation, using only this additivity assumption, unsubtracted dispersion relations, PCAC, and the requirement of nonrenormalizability of the axial-vector coupling constant in the weak interactions of the bound quarks (they took $G_A{}^{\text{quark}}=1$), without any current algebra. In our case, using the main features of this simple model in connection with the axial-vector–nucleon amplitude $T_{\mu\nu}{}^{ab}$ [Eq. (4)], we give an estimation of $\tilde{D}_1^{(-)}(0,0,0,0)$. With this value, the sum rule (1) turns out to be in rather poor, but not extremely bad, agreement with experiment.

The conclusions are drawn in Sec. IV.

II. SATURATION WITH N^* AND N^{**}

We use the expansion in covariants of $T_{\mu\nu}{}^{ab}$ [defined by Eq. (4)] given by Gerstein[1]:

$$\begin{aligned}T_{\mu\nu}=\bar{u}[&P_\mu P_\nu(A_1+\bar{A}_1\gamma\cdot Q)+P_\mu q_{2\nu}(A_2+\bar{A}_2\gamma\cdot Q)+P_\mu q_{1\nu}(A_3+\bar{A}_3\gamma\cdot Q)+q_{2\mu}P_\nu(A_4+\bar{A}_4\gamma\cdot Q)+q_{2\mu}q_{2\nu}(A_5+\bar{A}_5\gamma\cdot Q)\\&+q_{2\mu}q_{1\nu}(A_6+\bar{A}_6\gamma\cdot Q)+q_{1\mu}P_\nu(A_7+\bar{A}_7\gamma\cdot Q)+q_{1\mu}q_{2\nu}(A_8+\bar{A}_8\gamma\cdot Q)+q_{1\mu}q_{1\nu}(A_9+\bar{A}_9\gamma\cdot Q)+g_{\mu\nu}(A_{10}+\bar{A}_{10}\gamma\cdot Q)\\&+i\sigma_{\mu\nu}D_1+\tfrac{1}{2}(\gamma_\mu\gamma\cdot Q\gamma_\nu-\gamma_\nu\gamma\cdot Q\gamma_\mu)D_2+P_\mu\gamma_\nu B_1+P_\nu\gamma_\mu B_2+q_{2\mu}\gamma_\nu B_3+q_{2\nu}\gamma_\mu B_4\\&+q_{1\mu}\gamma_\nu B_5+q_{1\nu}\gamma_\mu B_6+(P_\mu i\sigma_{\nu\lambda}Q_\lambda+P_\nu i\sigma_{\mu\lambda}Q_\lambda)C_1+(Q_\mu i\sigma_{\nu\lambda}Q_\lambda+Q_\nu i\sigma_{\mu\lambda}Q_\lambda)C_2\\&+(\Delta_\mu i\sigma_{\nu\lambda}Q_\lambda+\Delta_\nu i\sigma_{\mu\lambda}Q_\lambda)C_3+(\Delta_\mu i\sigma_{\nu\lambda}Q_\lambda-\Delta_\nu i\sigma_{\mu\lambda}Q_\lambda)C_4]u.\end{aligned} \quad (7)$$

The following notation has been introduced:

$$\begin{aligned}&q_2=p_1+q_1-p_2, && \Delta=p_2-p_1,\\&P=\tfrac{1}{2}(p_1+p_2), && \nu=(P\cdot Q)/m,\\&Q=\tfrac{1}{2}(q_1+q_2), && t=\Delta^2, \qquad m=\text{nucleon mass}.\end{aligned} \quad (8)$$

We suppose that $\tilde{D}_1^{(-)}(0,0,0,0)$ is well approximated by the contributions coming from N^* (1238 MeV) and N^{**} (1518 MeV). Then we have to pick up the coefficients of $\bar{u}i\sigma_{\mu\nu}\frac{1}{2}[\tau^a\tau^b]u$ in the corresponding Feynman diagrams.

The axial vector NN^* and NN^{**} form factors are defined by

$$\langle N^*,p+q|A_\mu{}^a|N,p\rangle=\bar{u}_\nu(p+q)\left[g_{\nu\mu}H_1+q_\mu\gamma_\nu\frac{H_2}{m+m^*}-q_\nu(2p+q)_\mu\frac{H_3}{(m+m^*)^2}-q_\nu q_\mu\frac{H_4}{(m+m^*)^2}\right]T_a u(p),\quad (m^*=N^*\text{ mass}),\quad (9)$$

and

$$\langle N^{**},p+q|A_\mu{}^a|N,p\rangle=\bar{u}_\nu(p+q)\gamma_5\left[g_{\nu\mu}h_1+q_\mu\gamma_\nu\frac{h_2}{(m+m^{**})}-q_\nu(2p+q)_\mu\frac{h_3}{(m+m^{**})^2}-q_\nu q_\mu\frac{h_4}{(m+m^{**})^2}\right]\tfrac{1}{2}\tau^a u(p),\quad (m^{**}=N^{**}\text{ mass}).\quad (10)$$

In the above relations, τ^a are the Pauli isospin matrices and T_a are the 2×4 matrices connecting N^* and N isospin

[11] H. Schnitzer, Phys. Rev. **158**, 1471 (1967).
[12] H. Goldberg and F. Gross, Phys. Rev. **162**, 1350 (1967).
[13] N. N. Bogolyubov, V. A. Matveev, and A. N. Tavkhelidze, Report JINR, Dubna, E2876, 1966 (unpublished); Nuovo Cimento **48A**, 132 (1967).

states:

$$T_1=\begin{bmatrix}-1/\sqrt{2} & 0\\ 0 & -1/\sqrt{6}\\ 1/\sqrt{6} & 0\\ 0 & 1/\sqrt{2}\end{bmatrix},\quad T_2=i\begin{bmatrix}1/\sqrt{2} & 0\\ 0 & 1/\sqrt{6}\\ 1/\sqrt{6} & 0\\ 0 & 1/\sqrt{2}\end{bmatrix},\quad T_3=\begin{bmatrix}0 & 0\\ \sqrt{\tfrac{2}{3}} & 0\\ 0 & \sqrt{\tfrac{2}{3}}\\ 0 & 0\end{bmatrix}. \quad (11)$$

The T_a satisfy the relation

$$T_a^\dagger T_b=\tfrac{2}{3}\delta_{ab}-\tfrac{1}{6}[\tau^a\tau^b]. \quad (12)$$

The N^* contribution is obtained from the following expression:

$$T_{\mu\nu}{}^{ab}(\text{Born}, N^*)=\bar{u}(p_2)\left[g_{\alpha\mu}H_1-q_{2\alpha}\gamma_\mu\frac{H_2}{m+m^*}+q_{2\alpha}(2p_2+q_2)_\mu\frac{H_3}{(m+m^*)^2}-q_{2\alpha}q_{2\mu}\frac{H_4}{(m+m^*)^2}\right]\frac{(P+Q)\cdot\gamma+m^*}{(P+Q)^2-m^{*2}}$$
$$\times\left\{-g_{\alpha\beta}+\tfrac{1}{3}\gamma_\alpha\gamma_\beta+\frac{1}{3m^*}[\gamma_\alpha(P+Q)_\beta-\gamma_\beta(P+Q)_\alpha]+\frac{2}{3m^*}(P+Q)_\alpha(P+Q)_\beta\right\}$$
$$\times\left[g_{\beta\nu}H_1+q_{1\beta}\gamma_\nu\frac{H_2}{m+m^*}-q_{1\nu}(2p_1+q_1)_\beta\frac{H_3}{(m+m^*)^2}-q_{1\nu}q_{1\beta}\frac{H_4}{(m+m^*)^2}\right]$$
$$\times T_a^\dagger T_b u(p_1)+\text{crossed term}. \quad (13)$$

One finds

$$\tilde{D}_{1(N^*)}{}^{(-)}(\nu,0,q^2,q^2)=\tfrac{1}{9}(m+m^*)H_1{}^2(q^2)\left(\frac{1}{q^2+2m\nu+m^2-m^{*2}}+\frac{1}{q^2-2m\nu+m^2-m^{*2}}\right)+\cdots, \quad (14)$$

where the dots stand for terms which become zero at $q^2=0$, $\nu=0$. Consequently, we have

$$4m\tilde{D}_{1(N^*)}{}^{(-)}(0,0,0,0)=-\frac{8}{9}\frac{m}{m^*-m}H_1{}^2(0). \quad (15)$$

In deriving Eq. (14), much care must be taken with respect to possible contributions to $\tilde{D}_1{}^{(-)}$ from expressions containing, e.g., the factors

$$\bar{u}(P_\mu i\sigma_{\nu\lambda}Q_\lambda-P_\nu i\sigma_{\mu\lambda}Q_\lambda)u \quad (16)$$

or

$$\bar{u}(Q_\mu i\sigma_{\nu\lambda}Q_\lambda-Q_\nu i\sigma_{\mu\lambda}Q_\lambda)u, \quad (17)$$

which do not appear among the covariants used in the decomposition given in Eq. (7). As a matter of fact, these eventual contributions to $\bar{u}i\sigma_{\mu\nu}u$ vanish in the limit ($q_1=q_2$; $\nu^2=0$, $q^2=0$). This can be seen from the identities Eqs. (5) and (6) of Ref. 1, which we rewrite below for completeness:

$$\begin{aligned}0=\bar{u}[&P^2\tfrac{1}{2}(\gamma_\mu\gamma\cdot Q\gamma_\nu-\gamma_\nu\gamma\cdot Q\gamma_\mu)-m^2\nu i\sigma_{\mu\nu}\\&-\tfrac{1}{2}(P_\mu\Delta_\nu-P_\nu\Delta_\mu)\gamma\cdot Q+\tfrac{1}{2}Q\cdot\Delta(P_\mu\gamma_\nu-P_\nu\gamma_\mu)\\&-\tfrac{1}{2}m\nu(\Delta_\mu\gamma_\nu-\Delta_\nu\gamma_\mu)-m(P_\mu i\sigma_{\nu\lambda}Q_\lambda-P_\nu i\sigma_{\mu\lambda}Q_\lambda)]u, \quad (18)\end{aligned}$$

$$\begin{aligned}0=\bar{u}[&\Delta^2K^2i\sigma_{\mu\nu}+2K^2(P_\mu\Delta_\nu-P_\nu\Delta_\mu)\\&-2Q\cdot\Delta(P_\mu K_\nu-P_\nu K_\mu)+2m\nu(\Delta_\mu K_\nu-\Delta_\nu K_\mu)\\&-2m(\Delta_\mu K_\nu-\Delta_\nu K_\mu)\gamma\cdot Q+2mK^2(\Delta_\mu\gamma_\nu-\Delta_\nu\gamma_\mu)\\&-2mQ\cdot\Delta(K_\mu\gamma_\nu-K_\nu\gamma_\mu)\\&+\Delta^2(K_\mu i\sigma_{\nu\lambda}Q_\lambda-K_\nu i\sigma_{\mu\lambda}Q_\lambda)]u. \quad (19)\end{aligned}$$

Here K_μ is defined by

$$K_\mu\equiv Q_\mu-\frac{m\nu}{P^2}P_\mu-\frac{Q\cdot\Delta}{\Delta^2}\Delta_\mu \quad (20)$$

and one finds

$$K^2=Q^2-\frac{m^2\nu^2}{P^2}-\frac{(Q\cdot\Delta)^2}{\Delta^2}. \quad (21)$$

In a similar manner we obtain the N^{**} contribution in the form

$$4m\tilde{D}_{1(N^{**})}{}^{(-)}(0,0,0,0)=-\frac{2m}{3(m^{**}+m)}h_1{}^2(0). \quad (22)$$

With these results, taking

$$-2f_\pi{}^2\frac{1}{\pi}\int_{m_\pi(1+m_\pi/2m)}^{\infty}\frac{d\nu}{\nu}\,\text{Im}B^{(-)}(\nu,0,0,0)\simeq4.9 \quad (23)$$

as given in Ref. 8 and recalling that $[G_A(0)]^2\simeq1.4$, we have, on the left-hand side of Eq. (1), 3.7, while on the right-hand side we find

$$5.3-2.8[H_1(0)]^2, \quad (24)$$

if $\tilde{D}_1{}^{(-)}(0,0,0,0)$ is approximated only with the N^* in the intermediate states and

$$5.3-2.8[H_1(0)]^2-0.25[h_1(0)]^2 \quad (25)$$

if $\tilde{D}_1{}^{(-)}$ is calculated with both N^* and N^{**}.

TABLE I. Various theoretical values of $[H_1(0)]^2$ and the corresponding values of the right-hand side of Eq. (1).

References	a	a	a	b	c	c	c
$[H_1(0)]^2$	$\frac{3}{2}(0.85)^2$	$\frac{3}{2}(0.65)^2$	$\frac{3}{2}(1.15)^2$	$\frac{3}{2}(0.52)^2$	$1.4\times1.4/2$	$1.2\times1.4/2$	$1.7\times1.4/2$
Right-hand side of Eq. (1) (with N^* only)	2.3	3.5	−0.3	4.1	2.6	3.0	2.0
Right-hand side of Eq. (1) (with N^* and N^{**})	1.5	2.7	−1.1	3.3	1.8	2.2	1.2

[a] Reference 14.
[b] Reference 15.
[c] Reference 11.

For $H_1(0)$, theoretical determinations furnish rather different values. The numbers given by Albright,[14] and Furlan, Jengo, and Remiddi,[15] using a current-algebra approach, and by Schnitzer[11] are listed (in our normalizations) in Table I. The value of $[H_1(0)]^2$ appearing in the fifth column was obtained in Ref. 11 by fitting a pole to the W-A relation. The value of $[H_1(0)]^2$ from the third column must not be taken too seriously, because it corresponds to the symmetry limit and, consequently, it seems to us more doubtful. As concerns $h_1(0)$, no sound theoretical determination has been done to date. It is only possible to correlate $h_1(0)$ through the known πNN^{**} coupling constant; although a rough evaluation of this kind is quite questionable, we shall adopt this procedure here because of the lack of other information. We take therefore for the $[h_1(0)]^2$ the value given in Ref. 12 (calculated as stated above), which in our normalization is

$$[h_1(0)]^2=3.2. \quad (26)$$

The last two lines of Table I contain the corresponding values of the right-hand side of Eq. (1) computed with N^* and with both N^* and N^{**} contributions in the $\tilde{D}_1^{(-)}$ term.

As is seen from Table I, the agreement can be considered as satisfactory only when one uses for $H_1(0)$ the value given in Ref. 15. [Recall that on the left-hand side of Eq. (1) we have the number $F_2(0)=3.7$.]

III. QUARK-MODEL EVALUATION

For convenience, we prefer to rewrite the sum rule (1) using the definition of $\tilde{D}_1^{(-)}$ in the form

$$F_2(0)=[G_A(0)]^2-F_1(0)-2f_\pi^2 \times\frac{1}{\pi}\int_{m_\pi(1+m_\pi/2m)}^{\infty}\frac{d\nu}{\nu}\,\mathrm{Im}B^{(-)}(\nu,0,0,0) +4mD_1^{(-)}(0,0,0,0) -4mD_{1\,(\text{nucleon pole})}^{(-)}(0,0,0,0). \quad (27)$$

With the usual expression of the axial-vector–nucleon vertex

$$\langle N_1p+q|A_\mu{}^a|N_1p\rangle =\bar{u}(p+q)\tfrac{1}{2}\tau^a[i\gamma_5\gamma_\nu G_A(q^2)+iq_\nu\gamma_5F_p(q^2)]u(p), \quad (28)$$

the nucleon contribution to the weak amplitude $D_1^{(-)}$ comes from

$$T_{\mu\nu}{}^{ab}\text{ (Born, nucleon)} =\bar{u}(p_2)[i\gamma_5\gamma_\mu G_A(q_2^2)-iq_{2\mu}\gamma_5F_p(q_2^2)]\tfrac{1}{4}\tau_a\tau_b \times\frac{(p+Q)\cdot\gamma+m}{(P+Q)^2-m^2}[i\gamma_5\gamma_\nu G_A(q_1^2)+iq_{1\nu}\gamma_5F_p(q_1^2)]u(p_1) +\text{crossed term}. \quad (29)$$

After antisymmetrizing in the isospin indices, we pick up the coefficient of $\bar{u}i\sigma_{\mu\nu}u$. As we need $D_{1\,(\text{nucleon pole})}^{(-)}$ for all its arguments equal to zero, we set $q_1=q_2\equiv q$, $p_1=p_2\equiv p$ and obtain

$$D_{1\,(\text{nucleon pole})}^{(-)}(\nu,0,q^2,q^2)=\frac{[G_A(q^2)]^2mq^2}{q^4-4\nu^2m^2}+\cdots, \quad (30)$$

where the dots represent terms which vanish for $q^2=0$, $\nu^2=0$ [$F_p(q^2)$ is assumed as customarily nonsingular at $q^2=0$]. In order to obtain a well-defined value for $D_{1\,(\text{nucleon pole})}^{(-)}(0,0,0,0)$ from Eq. (30) we shall perform the limit $q^2\to0$ and $\nu^2\to0$ by keeping $q^2=\nu^2$. Then we will have from Eq. (30)

$$(-4m)D_{1\,(\text{nucleon pole})}^{(-)}(0,0,0,0) =\lim_{q^2=\nu^2\to0}(-4m)D_{1\,(\text{nucleon pole})}^{(-)}(\nu,o,q^2,q^2) =[G_A(0)]^2. \quad (31)$$

We compute now $4mD_1^{(-)}(0,0,0,0)$ under an additivity assumption in the quark model, which we formulate as follows: The quantity $\frac{1}{2}[\tau^a\tau^b][4mD_1^{(-)}(0,0,0,0)]$ is additively composed of the contributions from the quarks forming the nucleon. Therefore, we have

$$\tfrac{1}{2}[\tau^a\tau^b][4mD_1^{(-)}(0,0,0,0)] =\sum_{i=1}^{3}\tfrac{1}{2}[\tau^a\tau^b]_i[4M_i{}^{\text{quark}}D_{1i}^{(-)}(0,0,0,0)]. \quad (32)$$

$D_{1i}^{(-)}$ is the appropriate coefficient from the axial-

[14] C. H. Albright, Phys. Letters 24, B100 (1967).
[15] G. Furlan, R. Jengo, and E. Remiddi, Phys. Letters 20, 679 (1966); 21, 720(E) (1966).

vector-current-(i)-quark scattering amplitude:

$$T_{\mu\nu}^{ab}(i)=-i\int d^4x\, e^{iq_2x}\theta(x_0)$$
$$\times\langle\text{quark } i,p_2|[A_\mu^a(x),A_\nu^b(0)]|\text{quark } i,p_2\rangle,$$
$$(M_i^{\text{quark}}=(i)-\text{quark mass}). \quad (33)$$

The following hypotheses are used: (1) The amplitude $D_{1i}^{(-)}(0,0,0,0)$ is well approximated by the Born part (one quark in the intermediate states). (2) The axial-vector coupling constant of the bound quarks is unrenormalized, that is, $G_A^{\text{quark } i}(0)=1$. (The $V-A$ form of weak interactions has been ascribed to quarks.) The above hypotheses are compatible with the elementarity of the quark.

Thus, having also in mind that in our case we are dealing with n and p quarks which can be supposed to have equal masses, we obtain immediately

$$\tfrac{1}{2}[\tau^a\tau^b][4mD_1^{(-)}(0,0,0,0)]$$
$$=\sum_{i=1}^{3}\tfrac{1}{2}[\tau^a\tau^b]_i\left[4M^{\text{quark}}\frac{[G_A^{\text{quark}}(0)]^2}{-4M^{\text{quark}}}\right]$$
$$=-\tfrac{1}{2}[\tau^a\tau^b][G_A^{\text{quark}}(0)]^2=-\tfrac{1}{2}[\tau^a\tau^b],$$

and hence in this model

$$4mD_1^{(-)}(0,0,0,0)=-1. \quad (34)$$

The same convention $q^2=\nu^2\rightarrow 0$ as in the above calculation of the nucleon Born part has been employed.

Summing up the results, we find in the framework of this model that the right-hand side of the spin-flip sum rule [Eq. (1)] [or equivalently Eq. (27)] is 5.7 (the left-hand side being 3.7). The agreement is rather poor, but not discouraging.

IV. CONCLUSIONS

As concerns the saturation of the model-dependent term in the spin-flip sum rule (1) with N^* and N^{**}, we wish to emphasize that the situation is not yet clear. In fact, the propagators for the spin-$\frac{3}{2}$ particles used in our evaluations (Sec. II) are not uniquely defined off the mass shell. Also to be noted is the great uncertainty existing in the literature on the values of the weak-coupling constants of resonances. A definite statement on the validity of the spin-flip current-algebra sum rule can not be made until some experimental information becomes available for the weak axial-vector–nucleon amplitude, at least for the axial NN^* and NN^{**} form factors.

The simple calculation with the additivity assumption presented in Sec. III gives a poor, but not extremely bad, value for $F_2(0)$. We point out that in any case the quark-model evaluation of the Fourier transform of the retarded product of two axial divergences (as in the paper of Bogolyubov, Matveev, and Tavkhelidze[13]) works better than it does here for the axial-vector–nucleon amplitude defined in Eq. (4). In the future, the real significance of this kind of estimate for these weak axial quantities must be clarified.

Note added in proof. I wish to thank Dr. H. Goldberg (who derived independently in a paper written in collaboration with Dr. F. Gross[12] the same sum rule as that one found by Gerstein) for a useful communication about his work. Actually, it became apparent that the spin-flip πN sum rule can also be obtained from asymptotic $SU_3\times SU_3$ arguments and PCAC without any use of current commutators [see Fayyazuddin and Faheem Hussain, Phys. Rev. **164**, 1864 (1967)].

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