Vector and Axial-Vector Meson Dominance in the $K_{3\pi}$ Decays*

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A model for $K\to 3\pi$ decays is proposed in which the amplitudes are assumed to be dominated by vector and axial-vector meson poles. S-wave and P-wave amplitudes are treated on the same footing in this scheme. The strong-coupling constants appearing in the model can be determined from strong decays. The $K\to 3\pi$ slopes and average amplitude ratios are then determined in terms of one parameter characterizing the $\Delta I=\frac{1}{2}$ part of the weak Hamiltonian. Estimates are made of the effects of a $\Delta I=\frac{3}{2}$ contribution to the weak interaction.

I. INTRODUCTION

THE assumption of pole dominance of interaction amplitudes has enjoyed a great deal of success in the past. In the following we will present a pole-model description of the three-pion decays of the K meson ($K_{3\pi}$). The model contains one parameter and predicts values for the slopes and average amplitude ratios which are in reasonable agreement with experiment.

Not long after its application to hyperon nonleptonic decays,[1] pole dominance was suggested[2] as an explanation of the energy-dependent (P-wave) part of the $K_{3\pi}$ amplitude. Subsequent treatments[3–9] of the $K_{3\pi}$ amplitudes have made use pole models for both the S-wave (energy-independent) and P-wave amplitudes.

It has been usual in these approaches to deal with the S- and P-wave amplitudes in completely different ways. The S-wave amplitudes are generally taken to involve π- or K-meson poles, while the P-wave amplitudes are dominated by a combination of pseudoscalar and vector meson poles. While the weak vertex is the same (a $K\to\pi$ transition) for both S- and P-wave amplitudes, the strong vertices are quite different. The S-wave amplitude is usually taken to be proportional to an extrapolated value of the π-π scattering amplitude. The P-wave amplitudes, on the other hand, are proportional to vector meson-pseudoscalar meson coupling constants. It has also been suggested[7,9] that scalar meson poles play an important role in the $K_{3\pi}$ decays. However the existence of the appropriate scalar mesons is uncertain at the present time. The model proposed here treats the S-wave and P-wave amplitudes on an equal footing.

It was recently suggested by Sudarshan[10] that all particle interactions, strong, electromagnetic, and weak, are mediated by both vector and axial-vector mesons. In the spirit of this theory we present here a model for the $K_{3\pi}$ decays in which the decay amplitudes are dominated by vector and axial-vector meson poles. The vector mesons are the well-known ρ and $K^*(890)$. The axial-vector mesons we employ are the A_1 at 1080 MeV and the K_A at 1320 MeV. The S- and P-wave amplitudes are treated symmetrically in this scheme, each being proportional to two factors, and one arising from a vector meson pole and the other resulting from an axial-vector meson pole. The S-wave amplitudes, then, do not need contributions from pesudoscalar or scalar mesons.

Three types of coupling constants occur in our model; one characterizes the weak interaction, while the other two parametrize the strong trilinear meson couplings. The magnitude of the strong-coupling constants may be obtained from the experimental decay widths of the vector and axial-vector mesons. The relative signs are assumed to be given by $SU(3)$. The weak interaction involves a transition between a pseudoscalar and an axial-vector meson. Its form is analogous to that of the interaction proposed by Sakurai[11] in his pole-model treatment of the hyperon and $K_{2\pi}$ nonleptonic decays.

We will begin by assigning the weak interaction to an $SU(3)$ octet ($\Delta I=\frac{1}{2}$). The effects of introducing a small $\Delta I=\frac{3}{2}$ contribution, assigned to the 27-plet of $SU(3)$, will subsequently be discussed. It will be shown that our model is capable of yielding a reasonably good description of the $K_{3\pi}$ decays for values of the strong coupling constants falling within the experimentally determined ranges. Estimates of the $\Delta I=\frac{3}{2}$ contribution improve the description provided by the model.

In Sec. II we describe the model in detail. We derive the form of the $K_{3\pi}$ amplitudes resulting from the model in Sec. III. Section IV deals with the determination of the strong coupling constants parametrizing the model and the presentation and discussion of our results. Section V is a summary and conclusion.

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II. MODEL

The purpose of any model description of a given process is, of course, to provide an efficient determination of the relevant parameters of the process. The model proposed here accomplishes this for the $K_{3\pi}$ decays

$$K^+ \to \pi^+ + \pi^- + \pi^+ , \quad K_2{}^0 \to \pi^+ + \pi^- + \pi^0 ,$$
$$K^+ \to \pi^+ + \pi^0 + \pi^0 , \quad K_2{}^0 \to \pi^0 + \pi^0 + \pi^0 ,$$

denoted by $K_{+-+}{}^+$, $K_{+00}{}^+$, $K_{+-0}{}^2$, and $K_{000}{}^2$, respectively. The $K_{3\pi}$ amplitude A may be represented as

$$A = A_{\text{av}}[1 + S((3T - Q)/2Q)] , \tag{1}$$

where A_{av} is the average value of the amplitude, S is the slope (ratio of P-wave to S-wave contributions), T is the kinetic energy of the odd pion, and $Q = K - 3\mu$.[12] The "relevant parameters" to be determined are the three slopes $S_{+-+}{}^+$, $S_{+00}{}^+$, $S_{+-0}{}^2$, and the three ratios

$$\tfrac{1}{2}|A_{\text{av}}(K_{+-+}{}^+)| / |A_{\text{av}}(K_{+00}{}^+)| ,$$
$$\tfrac{1}{2}|A_{\text{av}}(K_{+-+}{}^+)| / |A_{\text{av}}(K_{+-0}{}^2)| ,$$

and

$$\tfrac{1}{2}|A_{\text{av}}(K_{+-+}{}^+)| / \tfrac{1}{3}|A_{\text{av}}(K_{000}{}^2)| .$$

Our first, and the most important, assumption is that the $K_{3\pi}$ amplitudes are dominated by vector and axial-vector meson poles. The second main assumption is that the weak interaction manifests itself in the transition (axial-vector meson) $\to$ (pseudoscalar meson). The Feynman diagrams that contribute to the $K_{3\pi}$ decay amplitudes are shown in Fig. 1.

The weak interaction is given by[13]

$$H_w = f \, \mathrm{Tr}(\{\mathcal{A}_\mu, \partial_\mu \mathcal{P}\} \lambda_6) , \tag{2}$$

where $\mathcal{A}$ and $\mathcal{P}$ are the $SU(3)$-matrix representations of the octets of axial-vector and pseudoscalar mesons, respectively. It is seen that H_w transforms like an $SU(3)$ octet (D type). Hence the $\Delta I = \frac{1}{2}$ relations

$$\begin{gathered} |A(K_{+-+}{}^+)| = 2|A(K_{+00}{}^+)| , \\ |A(K_{+00}{}^+)| = |A(K_{+-0}{}^2)| = \tfrac{1}{3}|A(K_{000}{}^2)| , \\ S_{+-+}{}^+ = -\tfrac{1}{2} S_{+00}{}^+ , \\ S_{+00}{}^+ = S_{+-0}{}^2 \end{gathered} \tag{3}$$

would hold identically if we neglect the mass splittings within each meson isomultiplet. We will use the physical masses[14] of all particles in our calculations (i.e., $\mu^+ \neq \mu^0$) and this will introduce deviations from the $\Delta I = \frac{1}{2}$ relations. We will also discuss the effect on the $K_{3\pi}$ amplitudes of adding to H_w an explicit $\Delta I = \frac{3}{2}$ term.

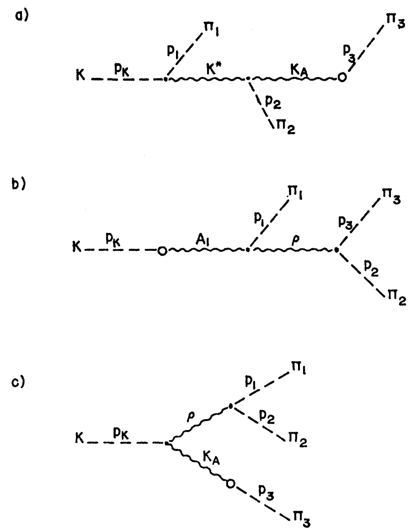

FIG. 1. Diagrams responsible for $K \to 3\pi$ decays.

As can be seen from Fig. 1, two types of strong vertices contribute to the amplitudes. The effective Hamiltonians describing the required couplings are[12]

$$H_{VPP} = \tfrac{1}{2} g_{\rho\pi\pi} \boldsymbol{\rho}_\mu \cdot \boldsymbol{\pi} \times \overleftrightarrow{\partial}_\mu \boldsymbol{\pi} + i g_{K^*K\pi} K_\mu{}^{*\dagger} \boldsymbol{\tau} K \cdot \overleftrightarrow{\partial}_\mu \boldsymbol{\pi} , \tag{4a}$$

$$H_{AVP} = g_{A_1\rho\pi} \mathbf{A}_{1\mu} \cdot \boldsymbol{\rho}_\mu \times \boldsymbol{\pi} + i g_{K_AK^*\pi} K_{A\mu}{}^\dagger \boldsymbol{\tau} K_\mu{}^* \cdot \boldsymbol{\pi} + i g_{K_A\rho K} K_{A\mu}{}^\dagger \boldsymbol{\tau} K \cdot \boldsymbol{\rho}_\mu . \tag{4b}$$

We do not assume the $SU(3)$ symmetric relations between the above coupling constants, with the exception that we take $g_{K_AK^*\pi} = g_{K_A\rho K}$. We do, however, assume that the relative signs of the couplings are given by $SU(3)$.[15] We have also made use of the theoretical prediction[16] that the axial-vector couplings to the vector and pseudoscalar mesons are pure S-wave.

III. DETERMINATION OF $K_{3\pi}$ AMPLITUDES

We are now in a position to determine the $K_{3\pi}$ amplitudes from the three types of diagram shown in Fig. 1. It is a straightforward but lengthy procedure. We will provide some of the intermediate steps in the calculation of the $K_{+-+}{}^+$ amplitude which involves the fewest number of Feynman diagrams.

The diagrams contributing to the $K_{+-+}{}^+$ amplitude are given in Fig. 2. In terms of the coupling constants

[12] In what follows the mass of a particular particle and the field corresponding to it will be represented by its symbol. We will take $m(\pi) = \mu$, however.

[13] It is important to emphasize here that we are postulating a specific form for the weak interaction. An approach based on a current-current weak interaction would include transitions of the form (vector meson) $\to$ (vector meson) and (pseudoscalar meson) $\to$ (pseudoscalar meson). The omission of the latter transition is certainly in keeping with our assumption of vector and axial-vector meson dominance. The neglect of vector meson transitions is perhaps harder to justify. This type of transition can not occur in any other weak process and so there is no way of estimating its importance here.

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[16] S. G. Brown and G. B. West, Phys. Rev. Letters **19**, 812 (1967).

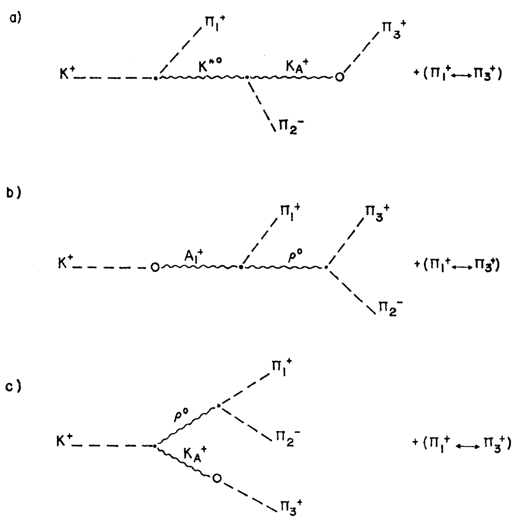

FIG. 2. Diagrams for $K_{+-+}{}^+$ decay.

discussed in Sec. II, the amplitude is found to be

$$A^{\Delta I=1/2}(K_{+-+}{}^+)_{\text{Fig. 2(a)}} = -2\frac{(g_{K^*K\pi}g_{K_AK^*\pi}f^{\Delta I=1/2})}{K_A{}^2[K^{*2}-(K^+-\mu^+)^2]}\left[(p_K+p_1)\cdot p_3 - \frac{(K^{+2}-\mu^{+2})}{K^{*2}}(p_K-p_1)\cdot p_3\right]+(p_1\leftrightarrow p_3), \quad \text{(5a)}$$

$$A^{\Delta I=1/2}(K_{+-+}{}^+)_{\text{Fig. 2(b)}} = 4\alpha\beta\frac{(g_{K^*K\pi}g_{K_AK^*\pi}f^{\Delta I=1/2})}{A_1{}^2[\rho^{02}-(K^+-\mu^+)^2]}\times p_K\cdot(p_3-p_2)+(p_1\leftrightarrow p_3), \quad \text{(5b)}$$

$$A^{\Delta I=1/2}(K_{+-+}{}^+)_{\text{Fig. 2(c)}} = -2\alpha\gamma\frac{(g_{K^*K\pi}g_{K_AK^*\pi}f^{\Delta I=1/2})}{K_A{}^2[\rho^{02}-(K^+-\mu^+)^2]}\times p_3\cdot(p_1-p_2)+(p_1\leftrightarrow p_3), \quad \text{(5c)}$$

where α, β, and γ are the factors due to the broken $SU(3)$ symmetry, and defined as

$$\alpha\equiv\left|\frac{g_{\rho\pi\pi}}{2g_{K^*K\pi}}\right|, \quad \text{(6a)}$$

$$\beta\equiv\left|\frac{g_{A_1\rho\pi}}{2g_{K_AK^*\pi}}\right|, \quad \text{(6b)}$$

$$\gamma\equiv\left|\frac{g_{K_A\rho K}}{g_{K_AK^*\pi}}\right|. \quad \text{(6c)}$$

In Eqs. (5) p_K (p_i) is the momentum of the K meson (the ith pion). We will always work in the rest frame of the decaying K meson, and will take π_2 to be the odd pion. Noting that

$$(3T_2-Q)=(3E_2-K^+),$$

where E_2 is the energy of the odd pion, we find

$$\begin{aligned}
(p_K+p_1)\cdot p_3+(p_1\leftrightarrow p_3)&=(K^{+2}-\mu^{+2})-K^+(3T_--Q_{+-+}),\\
(p_K-p_1)\cdot p_3+(p_1\leftrightarrow p_3)&=(\tfrac{1}{3}K^{+2}+\mu^{+2})+\tfrac{1}{3}K^+(3T_--Q_{+-+}),\\
p_K\cdot(p_3-p_2)+(p_1\leftrightarrow p_3)&=-K^+(3T_--Q_{+-+}),\\
p_3\cdot(p_1-p_2)+(p_1\leftrightarrow p_3)&=-K^+(3T_--Q_{+-+}).
\end{aligned} \quad \text{(7)}$$

When Eqs. (7) are used in Eqs. (5) we arrive at the following expression for the amplitude:

$$A^{\Delta I=1/2}(K_{+-+}{}^+)=-2\frac{(g_{K^*K\pi}g_{K_AK^*\pi}f^{\Delta I=1/2})}{K_A{}^2[K^{*2}-(K^+-\mu^+)^2]}\left\{(K^{+2}-\mu^{+2})\left(1-\frac{\frac{1}{3}K^{+2}+\mu^{+2}}{K^{*2}}\right)+\left[-\left(1+\frac{K^{+2}-\mu^{+2}}{3K^{*2}}\right)+\alpha\left(-\gamma+2\beta\frac{K_A{}^2}{A_1{}^2}\right)\frac{[K^{*2}-(K^+-\mu^+)^2]}{[\rho^{02}-(K^+-\mu^+)^2]}\right]K^+(3T_--Q_{+-+})\right\}. \quad \text{(8)}$$

It should be noticed at this point that diagrams of the type shown in Fig. 1(a) give rise to S-wave as well as P-wave contributions. In this way S- and P-wave amplitudes are accommodated in our model without having to introduce pseudoscalar-meson or scalar-meson poles.

Similarly, with the aid of Figs. 3, 4, and 5 we get the amplitudes for $K_{+00}{}^+$, $K_{+-0}{}^2$, and $K_{000}{}^2$ as

$$A^{\Delta I=1/2}(K_{+00}{}^+)=-2\frac{(g_{K^*K\pi}g_{K_AK^*\pi}f^{\Delta I=1/2})}{K_A{}^2[K^{*2}-(K^+-\mu)^2]}\left\{\tfrac{1}{2}(K^{+2}-\mu^2)\left(1-\frac{\frac{1}{3}K^{+2}+\mu^{+2}}{K^{*2}}\right)-\left[-\left(1+\frac{K^{+2}-\mu^{+2}}{3K^{*2}}\right)+\alpha\left(-\gamma+2\beta\frac{K_A{}^2}{A_1{}^2}\right)\frac{[K^{*2}-(K^+-\mu)^2]}{[\rho^{+2}-(K^+-\mu^0)^2]}\right]K^+(3T_+-Q_{+00})\right\}, \quad \text{(9)}$$

$$A^{\Delta I=1/2}(K_{+-0}{}^2)=2\frac{(g_{K^*K\pi}g_{K_AK^*\pi}f^{\Delta I=1/2})}{K_A{}^2[K^{*2}-(K^0-\mu)^2]}\left\{\tfrac{1}{2}(K^{02}-\mu^2)\left(1-\frac{\frac{1}{3}K^{02}+\mu^{02}}{K^{*2}}\right)\right.$$
$$\left.-\left[-\left(1+\frac{K^{02}-\mu^{02}}{3K^{*2}}\right)+\alpha\left(-\gamma+2\beta\frac{K_A{}^2}{A_1{}^2}\right)\frac{[K^{*2}-(K^0-\mu)^2]}{[\rho^{+2}-(K^0-\mu^+)^2]}\right]K^0(3T_0-Q_{+-0})\right\}, \quad (10)$$

$$A^{\Delta I=1/2}(K_{000}{}^2)=\frac{(g_{K^*K\pi}g_{K_AK^*\pi}f^{\Delta I=1/2})}{K_A{}^2[K^{*2}-(K^0-\mu^0)^2]}\left\{3(K^{02}-\mu^{02})\left[1-\frac{\frac{1}{3}K^{02}+\mu^{02}}{K^{*2}}\right]\right\}. \quad (11)$$

In Eqs. (9) and (10) μ is the mean pion mass.

IV. EXPERIMENTAL INPUTS AND PREDICTIONS

While it is possible, in principle, to determine all of the strong coupling constants of our model directly from decay data, the experimental situation is uncertain. Especially in the case of the axial-vector meson decays, the widths and branching ratios are not known accurately. We will proceed by first taking the mean values of the widths[14] as imputs in our calculation of the $K_{3\pi}$ parameters. We will discuss later the effects of taking other values within the range of uncertainty.

The vector meson decays are apparently the least uncertain. We have

$$\Gamma(\rho\to\pi\pi)=128 \text{ MeV}, \quad \Gamma(K^*\to K\pi)=49.6\pm1.4 \text{ MeV}. \quad (12)$$

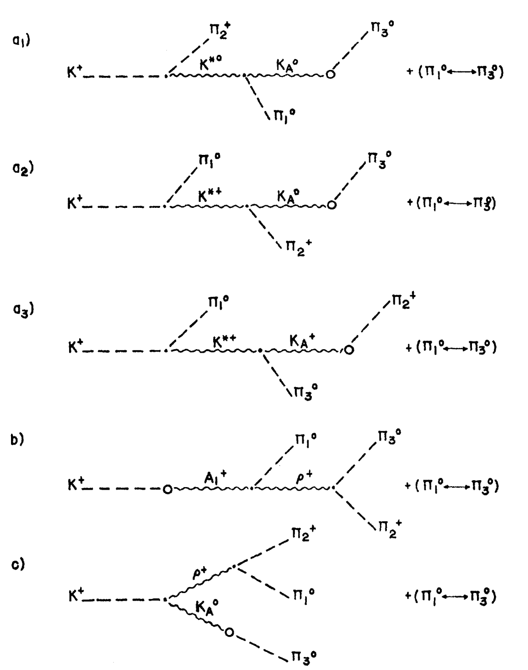

FIG. 3. Diagrams for $K_{+00}{}^+$ decay.

Using Eq. (4a) we get for the mean values

$$|g_{\rho\pi\pi}|=5.78, \quad |g_{K^*K\pi}|=3.22, \quad \alpha=0.897. \quad (13)$$

The widths of the relevant axial-vector meson decays are

$$\Gamma(A_1\to\rho\pi)=80\pm50 \text{ MeV}, \quad \Gamma(K_A\to K^*\pi \text{ and } \rho K)=70\pm45 \text{ MeV}. \quad (14)$$

From these, Eq. (4b), and the assumption that $\gamma=1.0$,

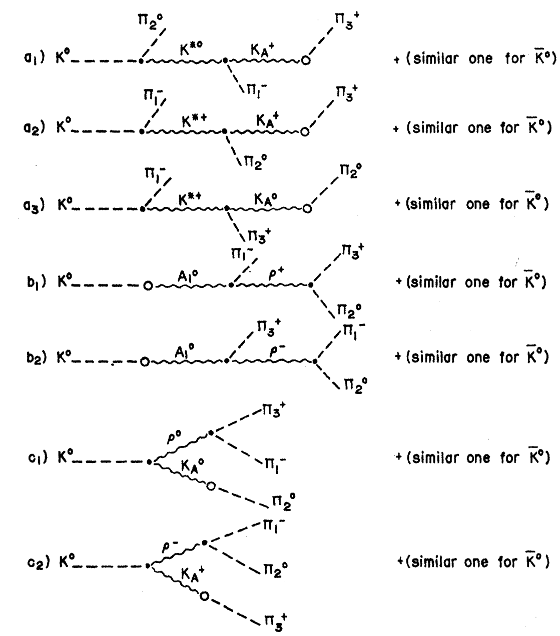

FIG. 4. Diagrams for $K_{+-0}{}^2$ decay.

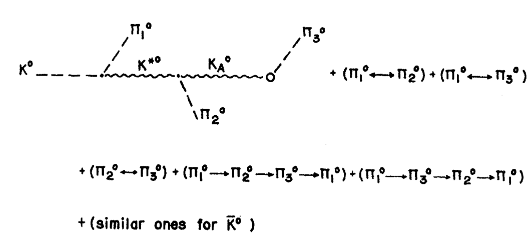

FIG. 5. Diagram for $K_{000}{}^2$ decay.

we find for the mean values

$$|g_{A_1\rho\pi}| = 15.4\mu^+ ,\quad |g_{K_AK^*\pi}| = 9.68\mu^+ ,\quad \beta = 0.793 . \tag{15}$$

These mean values, when substituted into the expressions for the $K_{3\pi}$ amplitudes, Eqs. (8)–(10), yield the results shown in Table I in the third column, labeled $\Delta I=\frac{1}{2}$. These are to be compared with the experimental values[17] given in the second column. It is seen that the amplitude ratios come close to satisfying the $\Delta I=\frac{1}{2}$ relations Eqs. (3). This is to be expected since the weak interaction contains only the $\Delta I=\frac{1}{2}$ term and the only violation can arise from electromagnetic mass differences. The slope $S_{+-+}{}^{+}$ falls within the experimental range. The other slopes, however, are smaller in magnitude by about 20% but have the correct sign.

It is now of interest to estimate the effects of a $\Delta I=\frac{3}{2}$ contribution to H_w in order to see if the agreement of the model with experiment may be improved. To this end we add a term to H_w which transforms like an $SU(3)$ 27-plet and is pure $\Delta I=\frac{3}{2}$.[18] The corrections to the amplitudes are shown in the Appendix. The magnitude (coupling constant) of the $\Delta I=\frac{3}{2}$ term is taken to be 3% of that of the $\Delta I=\frac{1}{2}$ part.[19]

The results of including both the $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ parts of H_w in the calculation of the $K_{3\pi}$ amplitudes are shown in the appropriate column of Table I. The agreement of the amplitude ratios $\frac{1}{2}|A_{\rm av}(K_{+-+}{}^{+})|/|A_{\rm av}(K_{+-0}{}^{2})|$ and $\frac{1}{2}|A_{\rm av}(K_{+-+}{}^{+})|/\frac{1}{3}|A_{\rm av}(K_{000}{}^{2})|$ with experiment is improved, the latter moving closer to the midpoint of the experimental range. The ratio $\frac{1}{2}|A_{\rm av}(K_{+-+}{}^{+})|/|A_{\rm av}(K_{+00}{}^{+})|$ remains unchanged. It is interesting that, by including $\Delta I=\frac{3}{2}$ effects, we improve agreement with the $\Delta I=\frac{1}{2}$ relations. This implies a compensation by the weak $\Delta I=\frac{3}{2}$ term to the corrections due to the electromagnetic mass differences entering the model. $S_{+-+}{}^{+}$ is seen to remain the same (within the experimental range) and $S_{+00}{}^{+}$ enters the experimental range. However, $S_{+-0}{}^{2}$ changes in the wrong direction.[20] Our relative lack of success with $S_{+-0}{}^{2}$ is possibly due to our neglect of CP-violating effects, which are known to occur in $K_2{}^0$ decays. The magnitude of such CP-violating effects could be comparable to that of the $\Delta I=\frac{3}{2}$ CP-nonviolating contribution.[21] Such effects, if properly taken into account, might improve the agreement of $S_{+-0}{}^{2}$ with experiment while maintaining the other results.

TABLE I. Average amplitude ratios and slopes.

	Experiment[a]	Theory $\Delta I=\frac{1}{2}$	Theory $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$
$\frac{1}{2}\lvert A_{\rm av}(K_{+-+}{}^{+})\rvert/\lvert A_{\rm av}(K_{+00}{}^{+})\rvert$	1.02±0.03	0.99	0.99
$\frac{1}{2}\lvert A_{\rm av}(K_{+-+}{}^{+})\rvert/\lvert A_{\rm av}(K_{+-0}{}^{2})\rvert$	1.08±0.06	0.92	1.01
$\frac{1}{2}\lvert A_{\rm av}(K_{+-+}{}^{+})\rvert/\frac{1}{3}\lvert A_{\rm av}(K_{000}{}^{2})\rvert$	1.04±0.08	0.96	1.05
$S_{+-+}{}^{+}$	0.23±0.03	0.26	0.26
$S_{+00}{}^{+}$	−0.70±0.06	−0.56	−0.71
$S_{+-0}{}^{2}$	−0.67±0.06	−0.55	−0.39

[a] Reference 17.

We investigated the fits to the $K_{3\pi}$ amplitudes given by other choices of β and γ [within the experimental ranges shown in Eq. (14)]. It turns out that to obtain a reasonable fit we must always take $\gamma=1$ which corresponds to $g_{K_AK^*\pi}\approx g_{K_A\rho K}$. By varying β within its allowed range, it is possible to fit all the slopes to within about 15% (the average amplitude ratios remain unchanged) by using the $\Delta I=\frac{1}{2}$ part of the weak interaction only. Then if the $\Delta I=\frac{3}{2}$ contribution to the weak interaction has less influence than was estimated above, we would have a good fit to the data for all slopes.

It is amusing to consider the extension of our model to a description of all nonleptonic decays. This can be accomplished by taking the following universal weak coupling:

$$H_w = (f'/\mu)m_V{}^2\mathcal{V}_\mu\partial_\mu\mathcal{P} + (f'/\mu)m_A{}^2\mathcal{A}_\mu\partial_\mu\mathcal{P} , \tag{16}$$

where μ, m_V, and m_A represent the mean masses of the pseudoscalar, vector, and axial-vector meson octets, respectively. We have divided by μ to compensate for the derivative and have inserted the (mass)^2 factors[10] to make f' dimensionless. The first term in H_w is responsible for parity-violating nonleptonic decays, while the second produces the parity-conserving decays. Note that $f=f'm_A{}^2/\mu$.

With H_w as given by Eq. (16) it is possible to fit the $K\to 2\pi$ parity-violating hyperon and $K\to 3\pi$ nonleptonic decay amplitudes to within 30%, provided we take the highest allowed value of the axial-vector coupling constants. This model obviously cannot account for the parity-conserving hyperon decays, without modifications, since it forbids $\Sigma^+\to n+\pi^+$.

V. SUMMARY AND CONCLUSION

We have presented a model for the $K_{3\pi}$ decays in which it is assumed that the $K_{3\pi}$ amplitudes are dominated by vector and axial-vector meson poles. A natural

[17] Y. Hara and Y. Nambu, Phys. Rev. Letters **16**, 875 (1966).

[18] If we were to assume that our effective weak interaction is induced by the Cabibbo Hamiltonian, the 27-plet contribution would be mainly $\Delta I=\frac{3}{2}$.

[19] We take the relative order of magnitude from our experience with $K\to 2\pi$ decays.

[20] The $\Delta I=\frac{3}{2}$ correction to the slopes is seen to be larger than one would expect on the basis of the assumed relative strengths of the $\Delta I=\frac{1}{2}$ and $\frac{3}{2}$ parts of H_w. The reason for this can be seen from Eqs. (9), (10), (A2), and (A3). The last terms in the P-wave amplitudes of both $A^{\Delta I=1/2}$ and $A^{\Delta I=3/2}$ are dominant. In the $\Delta I=\frac{3}{2}$ case for $K_{+00}{}^{+}$ and $K_{+-0}{}^{2}$ the parameters β and γ enter with the same sign and in the $\Delta I=\frac{1}{2}$ case with the opposite sign, leading to a larger effect in the former amplitude. Also $A^{\Delta I=3/2}(K_{+-0}{}^{2})$ has an additional factor of 2 multiplying β.

[21] The CP-violating interaction is believed to be weaker than the CP-conserving part by several orders of magnitude. We have seen in the $\Delta I=\frac{3}{2}$ case, however, that a small correction in the interaction can manifest itself in a considerable change in the slope.

framework, in which both S-wave and P-wave amplitudes receive contributions of the same type, is given by our model. Previously it was found necessary, in pole-model approaches to the $K_{3\pi}$ decays, to treat the S-wave amplitudes by one type of Feynman diagram and the P-wave amplitudes by other types.

The weak interaction in our model is assumed to have definite $SU(3)$ transformation properties (octet +27-plet). The strong interactions are assumed to be $SU(2)$ symmetric only; all the strong coupling constants appearing in the model may, in principle, be determined from outside the $K_{3\pi}$ decays.

The model provides an economical description of the $K_{3\pi}$ decays involving, as it does, one free parameter (to first order) with which we can predict the three slopes and three average amplitude ratios. By including only the octet ($\Delta I=\frac{1}{2}$) part of the weak Hamiltonian, it has been shown that the model is capable of yielding values of these six quantities in fair agreement with experiment.

It was found that the introduction of a $\Delta I=\frac{3}{2}$ part in the weak interactions could improve five of the six predictions. The $\Delta I=\frac{3}{2}$ correction to the slope $S_{+-0}{}^2$ was seen to be in the wrong direction. It was argued that the neglect of CP-violating effects, known to occur in $K_2{}^0$ decays, could be responsible for the discrepancy between our predicted value of $S_{+-0}{}^2$ and experiment.

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APPENDIX

$$A^{\Delta I=3/2}(K_{+-+}{}^{+})=-2\frac{(g_{K^*K\pi}g_{K_AK^*\pi}f^{\Delta I=3/2})}{K_A{}^2[K^{*2}-(K^+-\mu^+)^2]}\Bigg\{(K^{+2}-\mu^{+2})\left(1-\frac{\frac{1}{3}K^{+2}+\mu^{+2}}{K^{*2}}\right)$$
$$+\left[-\left(1+\frac{K^{+2}-\mu^{+2}}{3K^{*2}}\right)+\alpha\left(-\gamma+2\beta\frac{K_A{}^2}{A_1{}^2}\right)\frac{[K^{*2}-(K^+-\mu^+)^2]}{[\rho^{02}-(K^+-\mu^+)^2]}\right]K^+(3T_--Q_{+-+})\Bigg\}, \quad \text{(A1)}$$

$$A^{\Delta I=3/2}(K_{+00}{}^{+})=-2\frac{(g_{K^*K\pi}g_{K_AK^*\pi}f^{\Delta I=3/2})}{K_A{}^2[K^{*2}-(K^+-\mu)^2]}\Bigg\{\frac{1}{2}(K^{+2}-\mu^2)\left(1-\frac{\frac{1}{3}K^{+2}+\mu^{+2}}{K^{*2}}\right)$$
$$-\left[\frac{1}{2}\left(1+7\frac{K^{+2}-\mu^2}{3K^{*2}}\right)+2\alpha\left(\gamma+\beta\frac{K_A{}^2}{A_1{}^2}\right)\frac{[K^{*2}-(K^+-\mu)^2]}{[\rho^{+2}-(K^+-\mu^0)^2]}\right]K^+(3T_+-Q_{+00})\Bigg\}, \quad \text{(A2)}$$

$$A^{\Delta I=3/2}(K_{+-0}{}^{2})=2\frac{(g_{K^*K\pi}g_{K_AK^*\pi}f^{\Delta I=3/2})}{K_A{}^2[K^{*2}-(K^0-\mu)^2]}\Bigg\{-(K^{02}-\mu^2)\left(1-\frac{\frac{1}{3}K^{02}+\mu^{02}}{K^{*2}}\right)$$
$$-\left[\frac{1}{2}\left(1-5\frac{K^{02}-\mu^2}{3K^{*2}}\right)-\alpha\left(\gamma+4\beta\frac{K_A{}^2}{A_1{}^2}\right)\frac{[K^{*2}-(K^0-\mu)^2]}{[\rho^{+2}-(K^0-\mu^+)^2]}\right]K^0(3T_0-Q_{+-0})\Bigg\}, \quad \text{(A3)}$$

$$A^{\Delta I=3/2}(K_{000}{}^{2})=-2\frac{(g_{K^*K\pi}g_{K_AK^*\pi}f^{\Delta I=3/2})}{K_A{}^2[K^{*2}-(K^0-\mu^0)^2]}\left[3(K^{02}-\mu^{02})\left(1-\frac{\frac{1}{3}K^{02}+\mu^{02}}{K^{*2}}\right)\right]. \quad \text{(A4)}$$