cating that the width is much more sensitive to the detailed structure of the background force. Although improved values of the mass and width can be obtained in this manner, for example, with $b\approx -60$, $M^*\approx 1220$ MeV, and $\Gamma\approx 200$ MeV, the phase shift is still found to flatten out a little above the resonance position. In addition, quantitative accuracy for both M^* and Γ is not possible for any value of b. On the basis of these considerations, we conclude that the decrease in the background forces is only a minor factor in the correct description of $\delta(w)$ above the resonance energy. Other factors, for example inelasticity, are probably of greater importance in this respect.

In conclusion, we feel that the two most important factors in the low-energy calculation of the P_{33} amplitude are nucleon exchange and the background forces which represent the effects of the centrifugal barrier. These are plotted in Fig. 2 and are seen to be of comparable importance. It is clear that the simplified reciprocal-bootstrap statement that N and N^* exchange alone are sufficient to determine the N and N^* parameters must be abandoned. Instead, as has been emphasized by Chew, the bootstrap hypothesis requires the proper treatment of all relevant forces, and the present model suggests that the background forces will play an important role in self-consistent calculations.

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Nonleptonic Decays of Hyperons and Current Algebra*

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Nonleptonic decays of baryons are studied within the framework of current algebra, current-current interaction, and hard pions. A formalism which avoids the usual ambiguity of off-mass-shell extrapolation for the P-wave decays is developed, from which formulas for the decay amplitudes can be derived. Two models are discussed. The first model contains the equal-time commutator and the baryon pole terms, but allows for the $SU(3)$ symmetry breaking through the presence of the parity-violating spurion matrix elements between baryons. The second model adds the $\frac{3}{2}+$ decuplet and $Y_0^*(1405)$ contributions to the first model, but the $SU(3)$ symmetry-breaking matrix elements are not considered. Reasonable agreement with experiment is obtained in both models.

I. INTRODUCTION

FOLLOWING the papers of Sugawara and Suzuki[1] there have been a number of articles on the theory of nonleptonic decays of baryons using the methods of current algebra.[2] Attempts have been made to extend the formalism of current algebra to include the P-wave amplitudes. These involve the study of various forms of phenomenological Lagrangian for the decays. Within the framework of current-current interaction and current algebra the P-wave amplitudes have been found to be about a factor of 2 smaller than the experimental values.[3] Further, the decay formula usually derived suffers from ambiguity in the masses to be used, and the problem of extrapolation of the four-momentum of the pion to zero for the P-wave amplitudes is not correct. We would like to present here a method of deriving a decay formula which is devoid of such difficulties. The formalism is in analogy to the treatment of the P-wave pion-nucleon scattering lengths by Schnitzer.[4] The details will be discussed in Sec. II. Following that two models are proposed separately in Secs. III and IV. In model 1 the baryon pole is studied in detail. $SU(3)$ symmetry breaking is introduced through the use of physical masses of the baryons and the existence of parity-violating (pv) spurion matrix elements between baryons. These pv spurion matrix elements vanish in the limit of exact $SU(3)$.[5] In our consideration it is included as an unknown parameter. This approach is similar in spirit to that of Kumar and Pati.[6] The problem is formulated in the language of $SU(3)$ so as to treat the various decays on equal footing as far as the strong interaction is concerned. The decay amplitudes can then be expressed in terms of six unknown reduced

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matrix elements allowing for small violation of the $\Delta I=\frac{1}{2}$ rule. In model 2, additional intermediate states are investigated. We consider in particular the $\frac{3}{2}^+$ decuplet and $Y_0^*(1405)$ contributions to the invariant amplitudes and assume that the pv spurion matrix elements between $\frac{1}{2}^+$ octet baryons studied in model 1 are negligible. In this model octet dominance is assumed for simplicity. The results are good to within 10%. It is interesting to point out that the $\frac{1}{2}^+$ baryon pole alone can give a consistent fit for P-wave amplitudes. In the last section a brief summary is made.

II. GENERAL FORMALISM

Consider the fictitious scattering process denoted by the conservation equation $p_\alpha+k\rightarrow p_\beta+q$, where p_α, p_β, q, and k are the four-momenta of the initial baryon, final baryons, pion, and spurion, respectively. The decay problem can be looked upon as the limiting case of such a scattering problem with possible parity violation, the weak Hamiltonian (H_w) being a spurion with two possible parities. We will work in the center-of-mass coordinate system. The Mandelstam variables are $s=(p_\alpha+k)^2$, $u=(p_\beta-k)^2$, $t=(p_\beta-p_\alpha)^2$, with $s+u+t=m_\alpha^2+m_\beta^2+q^2+k^2$. Other useful kinematic relations are $s=W^2$,

$$|\mathbf{q}|=\{[(W+m_\beta)^2-q^2][(W-m_\beta)^2-q^2]\}^{1/2}/2W,$$

and

$$|\mathbf{k}|=\{[(W+m_\alpha)^2-k^2][(W-m_\alpha)^2-k^2]\}^{1/2}/2W.$$

We define the off-mass-shell scattering amplitude for the process as

$$T=(2\pi)^3\left(\frac{E_\alpha}{m_\alpha}\right)^{1/2}\left(\frac{E_\beta}{m_\beta}\right)^{1/2}\frac{(\mu^2-q^2)}{C_\pi}\int d^4x\, e^{iq\cdot x}\times\langle p_\beta|T\{\partial_\mu A_\mu{}^i(x),H_w(0)\}|p_\alpha\rangle, \quad (1)$$

where we have used the standard reduction technique and defined the pion field of isospin i by means of the hypothesis of partially conserved axial-vector current (PCAC),

$$\partial_\mu A_\mu{}^i(x)=C_\pi\varphi_\pi{}^i(x). \quad (2)$$

Equation (2) relates the pion-field operator $(\varphi_\pi{}^i)$ off the mass shell to the divergence of the axial-vector current with the constant C_π. From the basic identity

$$T\{\partial_\mu A_\mu{}^i(x),H_w(0)\}=\frac{\partial}{\partial x_\mu}T\{A_\mu{}^i(x),H_w(0)\}-\delta(x_0)[A_0{}^i(x),H_w(0)], \quad (3)$$

we obtain

$$T=(2\pi)^3\left(\frac{E_\alpha}{m_\alpha}\right)^{1/2}\left(\frac{E_\beta}{m_\beta}\right)^{1/2}\frac{(\mu^2-q^2)}{C_\pi}\left\{-iq_\mu R_\mu-\int d^4x\times e^{iq\cdot x}\delta(x_0)\langle p_\beta|[A_0{}^i(x),H_w(0)]|p_\alpha\rangle\right\}, \quad (4)$$

where

$$R_\mu=\int d^4x\, e^{iq\cdot x}\langle p_\beta|T\{A_\mu{}^i(x),H_w(0)\}|p_\alpha\rangle. \quad (5)$$

The scattering amplitude T is now decomposed into invariant amplitudes,

$$T=\bar{u}(p_\beta)\{A-\tfrac{1}{2}\gamma\cdot(q+k)B+i\gamma_5C-\tfrac{1}{2}i\gamma\cdot(q+k)\gamma_5D\}u(p_\alpha). \quad (6)$$

In terms of Pauli spinor (χ) this can be expressed in the center of mass as

$$F=\chi^\dagger\{f_1+(\sigma\cdot\hat{q})(\sigma\cdot\hat{k})f_2+(\sigma\cdot\hat{q})f_3+(\sigma\cdot\hat{k})f_4\}\chi, \quad (7)$$

where

$$F=[(m_\alpha m_\beta)^{1/2}/4\pi W]T. \quad (8)$$

The amplitudes f_1, f_2, f_3, and f_4 are given in terms of the invariant amplitudes by the relations

$$f_1=(1/16\pi W^2)[(W+m_\beta)^2-q^2]^{1/2}[(W+m_\alpha)^2-k^2]^{1/2}\times[A-\tfrac{1}{2}(2W-m_\alpha-m_\beta)B], \quad (9a)$$

$$f_2=(1/16\pi W^2)[(W-m_\beta)^2-q^2]^{1/2}[(W-m_\alpha)^2-k^2]^{1/2}\times[-A-\tfrac{1}{2}(2W+m_\alpha+m_\beta)B], \quad (9b)$$

$$f_3=(1/16\pi W^2)[(W-m_\beta)^2-q^2]^{1/2}[(W+m_\alpha)^2-k^2]^{1/2}\times[C+\tfrac{1}{2}(2W+m_\beta-m_\alpha)D], \quad (9c)$$

$$f_4=(1/16\pi W^2)[(W+m_\beta)^2-q^2]^{1/2}[(W-m_\alpha)^2-k^2]^{1/2}\times[-C+\tfrac{1}{2}(2W-m_\beta+m_\alpha)D]. \quad (9d)$$

Using the method of partial-wave decomposition for the parity-conserving and parity-violating amplitudes separately, we have obtained for the total partial-wave amplitude

$$f_{l\pm}=\frac{1}{2}\int_{-1}^{1}dz[(f_1+f_4)P_l(z)+(f_2+f_3)P_{l\pm1}(z)], \quad (10)$$

where $l\pm$ stands for the final states with orbital angular momentum l and total angular momentum $j=l\pm1$. For the parity-conserving amplitude

$$f=\sum_l(2l+1)(\boldsymbol{\sigma}\cdot\hat{q})[f_{l+}\Lambda_{l+}+f_{l-}\Lambda_{l-}]P_l(\cos\theta)=(\boldsymbol{\sigma}\cdot\hat{q})f_3+(\boldsymbol{\sigma}\cdot\hat{k})f_4, \quad (11)$$

while for the parity-violating amplitude

$$f=\sum_l(2l+1)[f_{l+}\Lambda_{l+}+f_{l-}\Lambda_{l-}]P_l(\cos\theta)=f_1+(\boldsymbol{\sigma}\cdot\hat{q})(\boldsymbol{\sigma}\cdot\hat{k})f_2, \quad (12)$$

where Λ_{l+} and Λ_{l-} are the projection operators,

$$[(l+1)+\boldsymbol{\sigma}\cdot\mathbf{L}]/(2l+1)$$

and

$$(l-\boldsymbol{\sigma}\cdot\mathbf{L})/(2l+1).$$

The above expressions can be inverted to give Eq. (10).

From Eqs. (9) and (10) the S-wave and P-wave amplitudes can be expressed in terms of the invariant amplitudes. The decay amplitude is supposed to be the analytic continuation of the scattering amplitude in its variables. It is obtained from the scattering amplitude by setting $k_\mu=0$, $s=m_\alpha^2$, $u=m_\beta^2$, and $q^2=\mu^2$, corresponding to the point of decay. The assumption of PCAC says that the invariant amplitudes will extrapolate smoothly to $q^2=0$, as is justified by the Goldberger-Treiman relation. Thus, the decay amplitude can be approximated by the fictitious scattering amplitude at $k=0$, $s=m_\alpha^2$, $u=m_\beta^2$, and $q^2=0$, and hence $t=0$:

$$\begin{aligned} f_{s\frac{1}{2}}\cong(1/8\pi m_\alpha)[(m_\alpha+m_\beta)^2-\mu^2]^{1/2} \\ \times[A(s=m_\alpha^2,\, q^2=0,\, t=0)-\tfrac{1}{2}(m_\alpha-m_\beta) \\ \times B(s=m_\alpha^2,\, q^2=0,\, t=0)], \end{aligned} \tag{13}$$

$$\begin{aligned} f_{p\frac{1}{2}}\cong(1/8\pi m_\alpha)[(m_\alpha-m_\beta)^2-\mu^2]^{1/2} \\ \times[C(s=m_\alpha^2,\, q^2=0,\, t=0)+\tfrac{1}{2}(m_\alpha+m_\beta) \\ \times D(s=m_\alpha^2,\, q^2=0,\, t=0)]. \end{aligned} \tag{14}$$

The conventional S-wave and P-wave nonleptonic decay amplitudes denoted by S and P as defined by the decay-width formula,

$$\Gamma=[|\mathbf{q}|(E_\beta+m_\beta)/4\pi m_\alpha](|S|^2+|P|^2), \tag{15}$$

are related to $f_{s\frac{1}{2}}$ and $f_{p\frac{1}{2}}$ by a common factor

$$[(m_\alpha+m_\beta)^2-\mu^2]^{1/2}/8\pi m_\alpha\,.$$

They can be worked out easily from Eqs. (8) and (15). With this correction we have

$$\begin{aligned} S=A(s=m_\alpha^2,\, q^2=0,\, t=0)-\tfrac{1}{2}(m_\alpha-m_\beta) \\ \times B(s=m_\alpha^2,\, q^2=0,\, t=0), \end{aligned} \tag{16}$$

$$\begin{aligned} P=K[C(s=m_\alpha^2,\, q^2=0,\, t=0)+\tfrac{1}{2}(m_\alpha+m_\beta) \\ \times D(s=m_\alpha^2,\, q^2=0,\, t=0)], \end{aligned} \tag{17}$$

where

$$K=\left[\frac{(m_\alpha-m_\beta)^2-\mu^2}{(m_\alpha+m_\beta)^2-\mu^2}\right]^{1/2}. \tag{18}$$

In the following section it is assumed that the term R_μ in Eq. (4) obeys an unsubtracted dispersion relation, and various contributions to the invariant amplitudes will be discussed.

III. MODEL 1

In essence this model contains contributions to the invariant amplitudes from the baryon pole and equal-time commutator term. The spurion matrix element between two octet baryons can be defined as follows:

$$\begin{aligned} &\langle\delta,p_\delta|H_w^{(\nu_4,\nu_5)}(0)|\alpha,p_\alpha\rangle \\ &=\langle\delta,p_\delta|H_w^{c(\nu_4,\nu_5)}(0)+H_w^{v(\nu_4,\nu_5)}(0)|\alpha,p_\alpha\rangle \\ &=\frac{1}{(2\pi)^3}\left(\frac{m_\delta}{E_\delta}\right)^{1/2}\left(\frac{m_\alpha}{E_\alpha}\right)^{1/2}\bar{u}(p_\delta) \\ &\qquad\times(S_{\delta\alpha}^{c(\nu_4,\nu_5)}+i\gamma_5 S_{\delta\alpha}^{v(\nu_4,\nu_5)})u(p_\alpha), \end{aligned} \tag{19}$$

where the weak Hamiltonian has been decomposed into a parity-conserving (pc) part H_w^c and a parity-violating (pv) part H_w^v. The spurions S^c (S^v) in the unitary-spin space are given, respectively, by

$$\begin{aligned} S_{\delta\alpha}^{c(\nu_4,\nu_5)}\,(S_{\delta\alpha}^{v})=\sum_\nu\begin{pmatrix}8 & 8 & 27\\ \nu_4 & \nu_5 & \nu\end{pmatrix}\begin{pmatrix}8 & 27 & 8\\ \alpha & \nu & \delta\end{pmatrix}a_{27}^c\,(a_{27}^v) \\ +\sum_\nu\begin{pmatrix}8 & 8 & 8s\\ \nu_4 & \nu_5 & \nu\end{pmatrix}\left[\begin{pmatrix}8 & 8 & 8s\\ \alpha & \nu & \delta\end{pmatrix}a_{8s}^c\,(a_{8s}^v)\right. \\ \left.+\begin{pmatrix}8 & 8 & 8a\\ \alpha & \nu & \delta\end{pmatrix}a_{8a}^c\,(a_{8a}^v)\right]. \end{aligned} \tag{20}$$

The subscripts α, β, etc., serve both as particle and $SU(3)$ indices. Our $SU(3)$ notation follows that of Sugawara, with $\nu_4=(0,1,-1)$ and $\nu_5=(1,\frac{1}{2},\frac{1}{2})$ representing the quantum number (Y,I,I_z). It is implicitly assumed that the pv and pc spurions have the same unitary-spin property given by Eq. (20), with their respective reduced matrix elements a_{8s}^c (a_{8s}^v) and a_{8s}^c (a_{8a}^v).[7] The weak Hamiltonian which is a product of two currents belonging to the same octet must have transformation properties corresponding to the symmetric product of two identical octets **8**$\times$**8**, namely, **1**, **8**, and **27**. For nonleptonic decay, which is strangeness-changing, only the **8** and **27** representations are relevant. This is expressed by Eq. (20), with the appropriate Clebsch-Gordan coefficients. For our purpose, only the transformation property of the Hamiltonian is needed; hence the constant factors of the Hamiltonian have been omitted and may be considered as constants entering into the reduced matrix elements.

The axial-vector current matrix element between two baryons is given by

$$\begin{aligned} \langle\beta,p_\beta|A_\mu^i(0)|\delta,p_\delta\rangle=\frac{1}{(2\pi)^3}\left(\frac{m_\beta}{E_\beta}\right)^{1/2}\left(\frac{m_\delta}{E_\delta}\right)^{1/2}\bar{u}(p_\beta) \\ \times[(g_A(q^2))_{\beta\delta}i\gamma_\mu\gamma_5+(h_A(q^2))_{\beta\delta}iq_\mu\gamma_5]u(p_\delta), \end{aligned} \tag{21}$$

where the axial-vector coupling constant $(g_A)_{\beta\delta}$ is related to the Yukawa coupling constant $g_{\beta\delta}$ by the Goldberger-Treiman relation

$$\mu^2/C_\pi\cong g_{\beta\delta}/(m_\beta+m_\delta)(g_A)_{\beta\delta}. \tag{22}$$

[7] The decomposition (20) is appropriate to the Cabibbo Hamiltonian for both parity-conserving and parity-violating decays. For parity-violating decays, *in the $SU(3)$ limit*, the reduced matrix elements a_{27}^v, a_{8s}^v, and a_{8a}^v are all zero. When $SU(3)$ violations are considered, the reduced matrix elements a^v are proportional to the baryon mass differences, which differ by one unit of strangeness, multiplied by a quantity which depends on the states in question. That is to say, $(a^v)_{\alpha\delta}\simeq(m_\alpha-m_\delta)(b^v)_{\alpha\delta}$, where $(b^v)_{\alpha\delta}$ can also be expanded in terms of $SU(3)$ invariants. We have made the simplifying assumption that only the singlet term is kept in this expansion for $(b^v)_{\alpha\delta}$, which means $(b^v)_{\alpha\delta}\approx b^v$. We further approximate the baryon mass differences as pure F type (which neglects the $\Sigma-\Lambda$ mass difference). Together this means that we approximate the reduced matrix elements $(a_{27}^v)_{\alpha\delta}$, $(a_{8a}^v)_{\alpha\delta}$, and $(a_{8s}^v)_{\alpha\delta}$ by the constants a_{27}^v, a_{8a}^v, and a_{8s}^v.

The induced pseudoscalar form factor $(h_A)_{\beta\delta}$ does not come into the discussion when $q^2=0$. In the spherical basis $g_{\beta\delta}$ is related to the pion-nucleon coupling constant g as follows:

$$g_{\beta\delta}=(-1)^{Q_{\nu_i}}\left[\sqrt{3}\begin{pmatrix}8 & 8 & 8a\\ \beta & \nu_i & \delta\end{pmatrix}f+(5/3)^{1/2}\begin{pmatrix}8 & 8 & 8a\\ \beta & \nu_i & \delta\end{pmatrix}d\right]g. \quad (23)$$

The symmetric and antisymmetric coupling constants have the normalization $f+d=1$.

With the help of Eqs. (19), (21), and (22) the octet baryon pole contribution to the invariant amplitudes at $q^2=0$ can be obtained:

$$A=-\left[\frac{1}{m_\beta+m_\delta}+\frac{m_\beta+m_\alpha-2m_\delta}{2(m_\delta^2-s)}\right]g_{\beta\delta}S_{\delta\alpha}{}^{v(\nu_4,\nu_5)}-\left[\frac{1}{m_\alpha+m_\gamma}+\frac{m_\alpha+m_\beta-2m_\gamma}{2(m_\gamma^2-u)}\right]S_{\beta\gamma}{}^{v(\nu_4,\nu_5)}g_{\gamma\alpha}, \quad (24)$$

$$B=\frac{1}{m_\delta^2-s}g_{\beta\delta}S_{\delta\alpha}{}^{v(\nu_4,\nu_5)}-\frac{1}{m_\gamma^2-u}S_{\beta\gamma}{}^{v(\nu_4,\nu_5)}g_{\gamma\alpha}, \quad (25)$$

$$C=-\left[\frac{1}{m_\beta+m_\delta}+\frac{m_\beta-m_\alpha-2m_\delta}{2(m_\delta^2-s)}\right]g_{\beta\delta}S_{\delta\alpha}{}^{c(\nu_4,\nu_5)}-\left[\frac{1}{m_\alpha+m_\gamma}+\frac{m_\alpha-m_\beta-2m_\gamma}{2(m_\gamma^2-u)}\right]S_{\beta\gamma}{}^{c(\nu_4,\nu_5)}g_{\gamma\alpha}, \quad (26)$$

$$D=\frac{1}{m_\delta^2-s}g_{\beta\delta}S_{\delta\alpha}{}^{c(\nu_4,\nu_5)}+\frac{1}{m_\gamma^2-u}S_{\beta\gamma}{}^{c(\nu_4,\nu_5)}g_{\gamma\alpha}. \quad (27)$$

The amplitudes A and B give to the S wave a combined contribution of the order of baryon mass difference. The relation of the amplitudes C and D to P-wave decay amplitudes is essentially the usual Born approximation, as expected. The previous calculations attempt to extrapolate the P-wave pion four-momentum to zero, as explained in Ref. 3. Here extrapolation is made from $q^2=\mu^2$ to $q^2=0$, which is allowed by PCAC. Our P-wave amplitude contains a small correction term of the order of $1/2m$ omitted in the earlier references.

For the equal-time commutator term (ETC) in Eq. (4) we take the point of view that one can always find a frame of reference such that the pion momentum is timelike, so that this term can be written as the commutator of axial charge with H_w; by covariance this can be generalized to arbitrary frame so that possible noncovariant contributions are paired off with the possible Schwinger term and are irrelevant for our considerations.[8] With this viewpoint the ETC contribution to the invariant amplitudes is

$$A=-\frac{g}{2m_Ng_A(0)}\left\{\sqrt{3}\begin{pmatrix}8 & 8 & 8a\\ \nu_i & \nu_4 & \nu_6\end{pmatrix}S_{\beta\alpha}{}^{c(\nu_5,\nu_6)}+(\nu_4\leftrightarrow\nu_5)\right\}, \quad (28)$$

$$C=-\frac{g}{2m_Ng_A(0)}\left\{\sqrt{3}\begin{pmatrix}8 & 8 & 8a\\ \nu_i & \nu_4 & \nu_6\end{pmatrix}S_{\beta\alpha}{}^{v(\nu_5,\nu_6)}+(\nu_4\leftrightarrow\nu_5)\right\}, \quad (29)$$

where summation over ν_6 is implied. The same assumption is made for the pv and pc spurions S^v and S^c as before, except that the quantum numbers here are different. Equation (28) gives essentially the results derived in Ref. 1 in which the $\Delta I=\frac{1}{2}$ rule is obtained for the S-wave Λ and Ξ decays and the pseudo-($\Delta I=\frac{1}{2}$) rule for the Σ decays.

When Eqs. (24)–(29) are substituted in Eqs. (16) and (17), we have

$$S(\alpha\to\beta\pi^i)=\left(-\frac{1}{m_\delta+m_\beta}+\frac{1}{m_\delta+m_\alpha}\right)g_{\beta\delta}S_{\delta\alpha}{}^{v(\nu_4,\nu_5)}+\left(-\frac{1}{m_\gamma+m_\alpha}+\frac{1}{m_\gamma+m_\beta}\right)S_{\beta\gamma}{}^{v(\nu_4,\nu_5)}g_{\gamma\alpha}-\frac{g}{2m_Ng_A(0)}\left[\sqrt{3}\begin{pmatrix}8 & 8 & 8a\\ \nu_i & \nu_4 & \nu_6\end{pmatrix}S_{\beta\alpha}{}^{c(\nu_5,\nu_6)}+(\nu_4\leftrightarrow\nu_5)\right], \quad (30)$$

$$P(\alpha\to\beta\pi^i)=K(\alpha\to\beta\pi^i)\left\{\left[-\frac{1}{m_\delta+m_\beta}-\frac{1}{m_\alpha-m_\delta}\right]g_{\beta\delta}S_{\delta\alpha}{}^{c(\nu_4,\nu_5)}+\left[-\frac{1}{m_\gamma+m_\alpha}+\frac{1}{m_\gamma-m_\beta}\right]S_{\beta\gamma}{}^{c(\nu_4,\nu_5)}g_{\gamma\alpha}-\frac{g}{2m_Ng_A(0)}\left[\sqrt{3}\begin{pmatrix}8 & 8 & 8a\\ \nu_i & \nu_4 & \nu_6\end{pmatrix}S_{\beta\alpha}{}^{v(\nu_5,\nu_6)}+(\nu_4\leftrightarrow\nu_5)\right]\right\}. \quad (31)$$

What has gone into the formulas for the decay amplitudes in this model, as given in Eqs. (30) and (31), is essentially the baryon pole approximation to R_μ. The pv spurion matrix element between two baryon octets which is usually neglected because of the extended "charge-conjugation invariance" to a $SU(3)$

[8] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966); Lowell S. Brown, *ibid.* **150**, 1338 (1966).

TABLE I. The contributions of various terms to the S-wave amplitudes and comparison with experiment[a] for model 1 with Yukawa coupling $d/f=2$, and spurion reduced matrix elements as in text.

Amplitude	ETC	Baryon	Total	Experiment
$S(\Lambda_-^0)\times 10^6$	−0.27	−0.06	−0.33	−0.33
$S(\Lambda_0^0)\times 10^6$	−0.19	−0.06	−0.25	−0.23
$S(\Sigma_+^+)\times 10^6$	0	−0.01	−0.01	0
$S(\Sigma_0^+)\times 10^6$	0.42	−0.02	0.40	0.33
$S(\Sigma_-^-)\times 10^6$	−0.60	0.06	−0.54	−0.40
$S(\Xi_-^-)\times 10^6$	−0.50	−0.01	−0.51	−0.44
$S(\Xi_0^0)\times 10^6$	0.35	0.03	0.38	0.34

[a] See Ref. 9.

TABLE II. The contributions of various terms to the P-wave amplitudes and comparison with experiment[a] for model 1 with Yukawa coupling $d/f=2$, and spurion reduced matrix elements as in text.

Amplitude	ETC	Baryon	Total	Experiment
$P(\Lambda_-^0)\times 10^7$	0.37	0.71	1.08	1.25
$P(\Lambda_0^0)\times 10^7$	0.26	0.50	0.76	0.87
$P(\Sigma_+^+)\times 10^7$	0.51	2.91	3.42	4.02
$P(\Sigma_0^+)\times 10^7$	−0.26	2.19	1.93	2.54
$P(\Sigma_-^-)\times 10^7$	−0.12	−0.18	−0.30	(−0.21)−0.03
$P(\Xi_-^-)\times 10^7$	−0.16	0.90	−0.74	−0.90
$P(\Xi_0^0)\times 10^7$	−0.11	0.64	0.53	0.57

[a] See Ref. 9.

multiplet has been included here as an unknown parameter. This occurs in other examples such as K-meson decays into two pions, which also violates such charge-conjugation invariance. Writing out the expressions for the 14 decay amplitudes with the help of $SU(3)$ Clebsch-Gordan coefficients, there are six unknown reduced matrix elements. The experimental decay data[9] can be fitted with these six unknowns for the baryon-meson coupling d/f ratio equal to two in agreement with the analysis of the semileptonic decays.[10] The results of the fit are given in Tables I and II. They correspond to $a_{8s}^c=2.382\times10^{-4}$, $a_{8a}^c=-3.579\times10^{-4}$, $a_{27}^c\cong 0$, $a_{8s}^v=1.842\times10^{-4}$, $a_{8a}^v=4.458\times10^{-4}$, and $a_{27}^v=4.728\times10^{-4}$. On the whole the results are good to 10–20% which is within the accuracy expected of PCAC. One thing to be emphasized is that our formulas for the P-wave amplitudes have been derived quite straightforwardly and without the previously mentioned difficulty.

IV. MODEL 2

In this model we include the additional contributions spin-$\frac{3}{2}^+$ decuplet and the spin-$\frac{1}{2}^-$ unitary singlet $Y_0^*(1405)$ to R_μ, but omit the pv spurion matrix elements between octet baryons considered in model 1.

The $Y_0^*(1405)$ contributes to the Σ_+^+ and Σ_-^- decays only and in equal magnitude through the u channel. For our purpose it is appropriate to introduce two parameters y_s and y_p to represent its contribution to the S- and P-wave amplitudes.

The spin-$\frac{3}{2}^+$ decuplet is assumed to be described by the Rarita-Schwinger field. The spin-$\frac{3}{2}$ wave function (u_μ) obeys $(\gamma\cdot p-m)u_\mu(p)=0$ and $\gamma_\mu u_\mu(p)=0$. It can contribute to R_μ when it is off the mass shell, since it then has a spin-$\frac{1}{2}^-$ component. There are four independent form factors for the decuplet-octet axial-vector vertex[11] which can be chosen such that three of them are transverse to the pion four-momentum and hence do not enter into the calculation. The relevant term can be written as

$$\langle\gamma,p_\gamma^*|A_\mu{}^i(0)|\alpha,p_\alpha\rangle=\frac{1}{(2\pi)^3}\left(\frac{m_\gamma^*}{E_\gamma^*}\right)^{1/2}\left(\frac{m_\alpha}{E_\alpha}\right)^{1/2}\times\bar{u}_\mu(p_\gamma^*)(g_A^*(q^2))_{\gamma^*\alpha}{}^i u(p_\alpha)\,,\quad(32)$$

where the form factor $g_A^*(q^2)$ at $q^2=0$ is given by

$$(g_A^*)_{\beta\delta^*}{}^i=(-1)^{\bar{\nu}_i}\begin{pmatrix}8 & 8 & 10\\ \beta & \nu_i & \delta^*\end{pmatrix}g^*.\quad(33)$$

The spurion matrix element between octet baryon and decuplet can be defined accordingly as

$$\langle\delta,p_\delta^*|H_w{}^{(\nu_4,\nu_5)}(0)|\alpha,p_\alpha\rangle=\frac{1}{(2\pi)^3}\left(\frac{m_\delta^*}{E_\delta^*}\right)^{1/2}\left(\frac{m_\alpha}{E_\alpha}\right)^{1/2}\times\bar{u}_\nu(p_\delta^*)p_\alpha{}^\nu(S_{\delta^*\alpha}{}^v+i\gamma_5S_{\delta^*\alpha}{}^c)u(p_\alpha)\,,\quad(34)$$

where the pv spurion $S_{\delta^*\alpha}{}^v$ $(S_{\delta^*\alpha}{}^c)$ in the unitary-spin space is given by

$$S_{\delta^*\alpha}{}^c\;(S_{\delta^*\alpha}{}^v)=\sum_\nu\begin{pmatrix}8 & 8 & 27\\ \nu_4 & \nu_5 & \nu\end{pmatrix}\begin{pmatrix}8 & 27 & 10\\ \alpha & \nu & \delta^*\end{pmatrix}b_{27}{}^c\;(b_{27}{}^v)+\sum_\nu\begin{pmatrix}8 & 8 & 8s\\ \nu_4 & \nu_5 & \nu\end{pmatrix}\begin{pmatrix}8 & 8 & 10\\ \alpha & \nu & \delta^*\end{pmatrix}b_8{}^c\;(b_8{}^v)\,,\quad(35)$$

with their reduced matrix elements $b_{27}{}^c$ $(b_{27}{}^v)$ and $b_8{}^c$ $(b_8{}^v)$.

The appropriate spin-$\frac{3}{2}^+$ positive-energy projection operator is

$$\sum_{\text{spin}}u_\mu(p_\delta^*)\bar{u}_\nu(p_\delta^*)=\left[g_{\mu\nu}-\frac{2}{3m_\delta^{*2}}(p_\delta^*)_\mu(p_\delta^*)_\nu-\tfrac{1}{3}\gamma_\mu\gamma_\nu-\frac{1}{3m_\delta^*}(p_\delta^*)_\mu\gamma_\nu+\frac{1}{3m_\delta^*}\gamma_\mu(p_\delta^*)_\nu\right]\left[\frac{\gamma\cdot p_\delta^*+m_\delta^*}{2m_\delta^*}\right].\quad(36)$$

[9] S. A. Bludman, Cargèse Lectures, 1966 (unpublished). For the Σ_-^- decay, the data in parenthesis in Table II are from D. Berley *et al.*, Phys. Rev. Letters **19**, 979 (1967).

[10] W. Willis *et al.*, Phys. Rev. Letters **13**, 291 (1964); N. Brene, L. Veje, M. Roos, and C. Cronstrom, Phys. Rev. **149**, 1288 (1966).

[11] J. D. Bjorken and J. D. Walecka, Ann. Phys. (N. Y.) **38**, 35 (1966).

With Eqs. (32), (34), and (36) the computation of the contribution to R_μ from the decuplet intermediate state is rather lengthy but straightforward. In terms of the invariant amplitudes in Eqs. (16) and (17) the results are

$$A=-[g/2m_Ng_A(0)]\{[C_1+\tfrac{1}{2}(m_\alpha-m_\beta)C_2](g_A{}^*)_{\beta\delta^*}S_{\delta^*\alpha}{}^v+[C_5+\tfrac{1}{2}(m_\alpha-m_\beta)C_6]S_{\beta\gamma^*}{}^v(g_A{}^*)_{\gamma^*\alpha}\}\,, \quad (37)$$

$$B=[g/2m_Ng_A(0)]\times\{C_2(g_A{}^*)_{\beta\delta^*}S_{\delta^*\alpha}{}^v+C_6S_{\beta\gamma^*}{}^v(g_A{}^*)_{\gamma^*\alpha}\}\,, \quad (38)$$

$$C=-[g/2m_Ng_A(0)]\{[C_3-\tfrac{1}{2}(m_\alpha+m_\beta)C_4](g_A{}^*)_{\beta\delta^*}S_{\delta^*\alpha}{}^c-[C_7+\tfrac{1}{2}(m_\alpha+m_\beta)C_8]S_{\beta\gamma^*}{}^c(g_A{}^*)_{\gamma^*\alpha}\}\,, \quad (39)$$

$$D=[g/2m_Ng_A(0)]\times\{C_4(g_A{}^*)_{\beta\delta^*}S_{\delta^*\alpha}{}^c+C_8S_{\beta\gamma^*}{}^c(g_A{}^*)_{\gamma^*\alpha}\}\,, \quad (40)$$

where the constant C_i's are functions of masses; for example,

$$C_1=[6m_\delta{}^{*2}(m_\delta{}^*-m_\alpha)]^{-1}(m_\alpha{}^2-m_\beta{}^2)\times(m_\delta{}^*+m_\beta)(3m_\delta{}^*-2m_\alpha)\,, \quad (41a)$$

$$C_2=[6m_\delta{}^{*2}(m_\delta{}^*-m_\alpha)]^{-1}[(m_\delta{}^*-m_\alpha)\times(2m_\alpha{}^2-3m_\beta{}^2-m_\alpha m_\delta{}^*)-m_\alpha(m_\delta{}^*+m_\beta)^2]\,. \quad (41b)$$

Combining with the pc baryon pole and equal-time commutator terms, we have

$$S(\alpha\to\beta\pi^i)=-\frac{g}{2m_Ng_A(0)}\Bigg\{\sqrt{3}\left[\begin{pmatrix}8 & 8 & 8a\\ \nu_i & \nu_4 & \nu_6\end{pmatrix}S_{\beta\alpha}{}^{(\nu_5,\nu_6)}+(\nu_4\leftrightarrow\nu_5)\right]$$
$$+\frac{m_\alpha-m_\beta}{6m_\delta{}^{*2}}[(m_\alpha+m_\beta)(m_\delta{}^*+2m_\alpha)+2m_\beta m_\delta{}^*](g_A{}^*)_{\beta\delta^*}S_{\delta^*\alpha}{}^{v(\nu_4,\nu_5)}$$
$$+\frac{m_\alpha-m_\beta}{6m_\gamma{}^{*2}}[(m_\alpha+m_\beta)(m_\gamma{}^*+2m_\beta)+2m_\alpha m_\gamma{}^*]S_{\beta\gamma^*}{}^{v(\nu_4,\nu_5)}(g_A{}^*)_{\gamma^*\alpha}\Bigg\}+y_s\delta_{\beta N}\delta_{\alpha\Sigma}\,, \quad (42)$$

$$P(\alpha\to\beta\pi^i)=K(\alpha\to\beta\pi^i)\Bigg\{-\frac{m_\alpha+m_\beta}{(m_\beta+m_\delta)(m_\alpha-m_\delta)}g_{\beta\delta}S_{\delta\alpha}{}^{(\nu_4,\nu_5)}+\frac{m_\alpha+m_\beta}{(m_\gamma+m_\alpha)(m_\gamma-m_\beta)}S_{\beta\gamma}{}^{(\nu_4,\nu_5)}g_{\gamma\alpha}$$
$$+\frac{g}{2m_Ng_A(0)}\Bigg[\frac{m_\alpha+m_\beta}{6m_\delta{}^{*2}}[(m_\alpha-m_\beta)(2m_\alpha-m_\delta{}^*)+2m_\beta m_\delta{}^*]((g_A{}^*)_{\beta\delta^*})S_{\delta^*\alpha}{}^{c(\nu_4,\nu_5)}$$
$$+\frac{m_\alpha+m_\beta}{6m_\gamma{}^{*2}}[-(m_\alpha-m_\beta)(2m_\beta-m_\gamma{}^*)+2m_\alpha m_\gamma{}^*]S_{\beta\gamma^*}{}^{c(\nu_4,\nu_5)}(g_A{}^*)_{\gamma^*\alpha}\Bigg]\Bigg\}+y_p\delta_{\beta N}\delta_{\alpha\Sigma}\,, \quad (43)$$

where the Kronecker $\delta_{\beta N}\delta_{\alpha\delta}$ is introduced to mean that the Y^* resonance contributes only to the $\Sigma_+{}^+$ and $\Sigma_-{}^-$ decays.

In this model it is perhaps more appropriate to assume octet dominance since the **27** reduced matrix element of the baryon a_{27} violates the $\Delta I=\frac{1}{2}$ rule for the P wave and the **27** reduced matrix element of the decuplet b_{27} violates the $\Delta I=\frac{1}{2}$ rule for the S wave. With this assumption there are six unknown parameters in the decay formula. If we define

$$A_{8s}=[2m_Ng_A(0)]^{-1}(1/20)a_{8s}g\,, \quad (44a)$$

$$A_{8a}=[2m_Ng_A(0)]^{-1}(1/20)a_{8a}g\,, \quad (44b)$$

$$B_s=-[2m_Ng_A(0)]^{-1}(1/4\sqrt{30})\mu b_8(s)g^*g\,, \quad (44c)$$

$$B_p=-[2m_Ng_A(0)]^{-1}(1/4\sqrt{30})\mu b_8(p)g^*g\,, \quad (44d)$$

the S- and P-wave amplitudes can be written as

$$S(\Lambda_-{}^0)=1.732A_{8s}+3.873A_{8a}+2.619B_s\,, \quad (45)$$

$$S(\Sigma_+{}^+)=0+0+2.357B_s+y_s\,, \quad (46)$$

$$S(\Sigma_0{}^+)=3.000A_{8s}-2.236A_{8a}-7.907B_s\,, \quad (47)$$

$$S(\Sigma_-{}^-)=-4.243A_{8s}+3.162A_{8a}+13.54B_s+y_s\,, \quad (48)$$

$$S(\Xi_-{}^-)=-1.732A_{8s}+3.873A_{8a}+7.305B_s\,, \quad (49)$$

and

$$P(\Lambda_-{}^0)=-2.387A_{8s}-2.363A_{8a}-0.4957B_p\,, \quad (50)$$

$$P(\Sigma_+{}^+)=4.320A_{8s}+0.1119A_{8a}-0.4337B_p+y_p\,, \quad (51)$$

$$P(\Sigma_0{}^+)=1.395A_{8s}-1.040A_{8a}+1.817B_p\,, \quad (52)$$

$$P(\Sigma_-{}^-)=2.505A_{8s}+1.608A_{8a}-3.058B_p+y_p\,, \quad (53)$$

$$P(\Xi_-{}^-)=-3.146A_{8s}-1.373A_{8a}-1.501B_p\,, \quad (54)$$

where we have taken $g_A(0)=1.3$ and the Yukawa coupling $d/f=1.75$. A consistent fit of the experimental data[9] is presented in Tables III and IV. The results correspond to $A_{8s}=9.323\times10^{-8}$, $A_{8a}=-1.462\times10^{-7}$, $B_s=3.6\times10^{-8}$, $B_p=0.5\times10^{-8}$, $y_s=-8.485\times10^{-8}$, and $y_p=1.400\times10^{-8}$. The essential features of the $\Delta I=\frac{1}{2}$

TABLE III. The contributions of various terms to the S-wave amplitudes and comparison with experiment[a] for model 2 with Yukawa coupling $d/f=1.75$, and spurion reduced matrix elements as in text.

Amplitude	ETC	Decuplet	Y_0^*	Total	Experiment
$S(\Lambda_-^0)\times10^6$	-0.40	0.09	0	-0.31	-0.33
$S(\Sigma_+^+)\times10^6$	0	0.08	-0.08	0.00	0
$S(\Sigma_0^+)\times10^6$	0.61	-0.29	0	0.32	0.33
$S(\Sigma_-^-)\times10^6$	-0.86	0.49	-0.08	-0.45	-0.40
$S(\Xi_-^-)\times10^6$	-0.73	0.26	0	-0.47	-0.44

[a] See Ref. 9.

TABLE IV. The contributions of various terms to the P-wave amplitudes and comparison with experiment[a] for model 2 with Yukawa coupling $d/f=1.75$, and spurion reduced matrix elements as in text.

Amplitude	Baryon	Decuplet	Y_0^*	Total	Experiment
$P(\Lambda_-^0)\times10^7$	1.23	-0.02	0	1.21	1.25
$P(\Sigma_+^+)\times10^7$	3.86	-0.02	0.14	3.98	4.02
$P(\Sigma_0^+)\times10^7$	2.82	0.09	0	2.91	2.54
$P(\Sigma_-^-)\times10^7$	-0.02	-0.15	0.14	-0.03	-0.03
$P(\Xi_-^-)\times10^7$	-0.93	-0.07	0	-1.00	-0.90

[a] See Ref. 9.

rule and the Lee-Sugawara relations have been expressed in the fit. The agreement with experiment is better than in model 1 and the prediction lies within experimental uncertainty.

A few comments about this model may be of interest. First, this is a good $SU(3)$ model in the sense that the usual argument of CP invariance and $SU(3)$ symmetry does not apply to the pv matrix elements between the decuplet and the octet baryon. Secondly, since the four-momentum of the pion need not vanish in our approach, it is natural to examine other possible intermediate states that may contribute to R_μ in Eq. (4). The t-channel poles, K and K^* mesons, have been considered and found to be insignificant.[12] Here the decuplet contributes since it is off the mass shell. From Eqs. (45)–(54) it is clear that the kinematic favors the S-wave amplitudes for the decuplet. In fact, the P-wave fit is already very good without the decuplet and the Y^* so that their presence helps the S-wave amplitudes primarly. Thirdly, the **27** amplitude is expected to be small because of the $\Delta I=\frac{1}{2}$ rule, which is quite good for the Λ and Ξ decays. However, the **27** amplitude between octet baryons gives comparatively the largest contribution to the Σ_+^+ and Σ_-^- decays so that the possibility of replacing the Y^* terms appears to exist. This can be done except that the $\Delta I=\frac{1}{2}$ rule for the P-wave Ξ decyas would be violated rather badly.

[12] Lowell S. Brown and H. Crater (private communication to H. J. Schnitzer).

V. CONCLUSION

To sum up briefly: We have developed a formalism based on which decay amplitudes can be expressed in terms of the invariant amplitudes for the fictitious scattering, spurion+baryon $\rightarrow$ pion+baryon. With the momentum of the spurion going to zero, the S and P partial-wave amplitudes are indeed connected with the pv and pc decay amplitudes. The decay amplitudes are obtained by assuming a model for the term R_μ. We have considered the baryon pole in detail and have arrived at a formula for the decay amplitudes avoiding the usual ambiguity of the extrapolation for the P-wave decays. Two models are studied which give good agreement with experiment. Although there is no direct way by which these models can be tested, we tend to believe that the current-algebra approach is compatible with the current-current picture when such additional terms are taken into consideration.

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