Centrifugal-Barrier Effects in N/D Calculations*

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It is shown in a simple, zero-parameter model that inclusion of the background forces representing the effects of the centrifugal barrier substantially modifies the results of partial-wave calculations.

THE simplified version of the reciprocal-bootstrap hypothesis[1] for the nucleon (N) and the 3,3 resonance (N^*) roughly states that given the N, the N^* is determined,[2] and vice versa. Forces due to other particles or resonances enter, but it is generally assumed that their contributions give only small corrections to the basic calculation. We will show, with a very simple zero-parameter model, that this is not likely to be the case and that the "background" forces remaining after the one-particle-exchange (OPE) forces are isolated are equally important in determining the output-particle parameters.

Let us restrict our attention to the first half of the bootstrap, namely, the N^* calculation. In the usual approach, N exchange is taken as the primary driving force, seconded by N^* exchange. The resulting partial-wave dispersion relation is solved by the N/D technique.[3] When this is done, two points are noticed: (i) The p-wave amplitude does not have the correct threshold behavior. (ii) N exchange is too strong; it is so attractive that the zero of $\mathrm{Re}D(w)$ is pushed below threshold.[4] The 3,3 resonance comes out as a bound state.

Previously, these two problems have been treated separately. To avoid the threshold difficulty, it has been the practice to write a dispersion relation for a modified amplitude that does not vanish at threshold. To reduce the N-exchange force, a cutoff function is usually employed. Both of these manipulations are undesirable: The latter introduces an arbitrary cutoff parameter and the former is mathematically inconsistent unless it is accompanied by the introduction of additional, arbitrary threshold parameters. It is obvious that the solution is sensitive to the value of the cutoff, and Simmons[5] has shown that this applies to the threshold parameters as well. Furthermore, Simmons was unable to find a resonant N^* solution for reasonable values of the threshold parameters when using an undamped nucleon-exchange force.

It is the purpose of this note to show that these two problems are related. When the threshold difficulty is treated in the proper manner, the problem with the strength of the nucleon-exchange force is no longer present. To demonstrate this point, we present the results of a calculation of the 3,3 resonance that contains no arbitrary parameters and in which the 3,3 amplitude has the correct threshold behavior at $w=M+\mu$. The phase shift plotted in Fig. 1 is seen to be in qualitative agreement with experiment.

The correct threshold behavior is obtained by the inclusion of forces other than those associated with

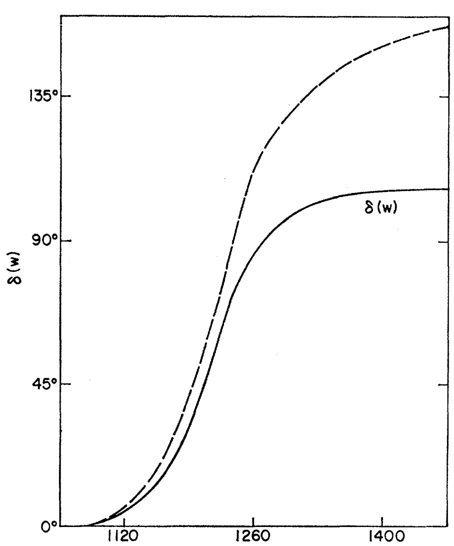

FIG. 1. 3,3 phase shift for the zero-parameter model. The dashed curve gives the values of $\delta(w)$ obtained in the phase-shift analysis of Roper, Wright, and Feld (Ref. 14).

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[1] G. F. Chew, Phys. Rev. Letters **9**, 233 (1962).

[2] Of course, N^* enters as an exchanged particle along with N, as well as emerging from the calculation as a pole on the second sheet in the energy variable.

[3] G. F. Chew and S. Mandelstam, Phys. Rev. **119**, 467 (1960).

[4] A. W. Martin and J. L. Uretsky, Phys. Rev. **135**, B803 (1964). In this paper, the N/D calculation was done with a modified amplitude and the N-exchange force was not damped. The N^* emerged as a bound state that became a resonance only when the pion-nucleon coupling constant was reduced to $g_{N\pi}^2/4\pi=6.2$. See this article for further references to the literature.

[5] L. M. Simmons, Jr., Phys. Rev. **144**, 1157 (1966).

one-particle exchange.[6] We stress the fact that these forces must be present because OPE contributions vanish as $q \to 0$ (for $l \neq 0$), yet it is well known[4,7] that the complete partial-wave driving force must be nonvanishing at threshold. Furthermore, since these background forces are found to be comparable in magnitude with the sum of N- and N^*-exchange forces (see Fig. 2), they cannot be considered as "small corrections" to the bootstrap.

Let $B_b(w)$ be the contribution of the background forces to the Cauchy integral around the unphysical branch cuts, and let $\sum_p B_{\mathrm{OPE}}(w)$ be the sum of the OPE forces. The unsubtracted dispersion relation for the 3,3 partial-wave amplitude, $f(w)$, is

$$f(w) = B_b(w) + \sum_p B_{\mathrm{OPE}}(w) + U(w) = B(w) + U(w),$$

where $U(w)$ is the Cauchy integral over the physical (unitarity) cuts. Since $U(w)$ cannot, in general, vanish at the physical thresholds,[4,7] the correct threshold behavior of $f(w)$ requires cancellation of $B_b(w)$ and $U(w)$ as $q \to 0$. If $B_b(w)$ is omitted from the dispersion relation, the solution of the N/D equations cannot be expected to have the right threshold behavior, and it is not surprising that previous attempts to obtain such behavior, while neglecting $B_b(w)$, met with difficulties.

For simplicity in our calculation, we shall not impose unitarity on the d-wave physical cut from $-\infty$ to $-w_0$ where $w_0 = M + \mu$. Instead, we shall include the contribution of this cut, which has been found to be small in the physical p-wave region,[4] in $B_b(w)$. Accordingly, we are concerned only with the threshold behavior at w_0. We expect the dominant contributions to the background forces to arise from regions of the unphysical cuts relatively distant from the physical region; the nearby singularities should be well approximated by the OPE terms. As a consequence, we expect $B_b(w)$ to be a slowly varying function in the low-energy region, and we shall approximate it by its value at threshold[8] in our method of solution.

Of the OPE terms we keep only N and N^* exchange,[9] and the dispersion relation reads

$$f(w) = B_N(w) + B_{N^*}(w) + B_b(w_0) + \frac{1}{\pi}\int_{w_0}^{\infty} \frac{dw' q(w') r(w') |f(w')|^2}{w' - w}, \quad (1)$$

where $r(w)$ is the ratio of the total to the elastic partial-wave cross section, and $B_b(w_0)$ is determined by the threshold requirement

$$B_b(w_0) = -\frac{1}{\pi}\int_{w_0}^{\infty} \frac{dw' q(w') r(w') |f(w')|^2}{w' - w_0} < 0. \quad (2)$$

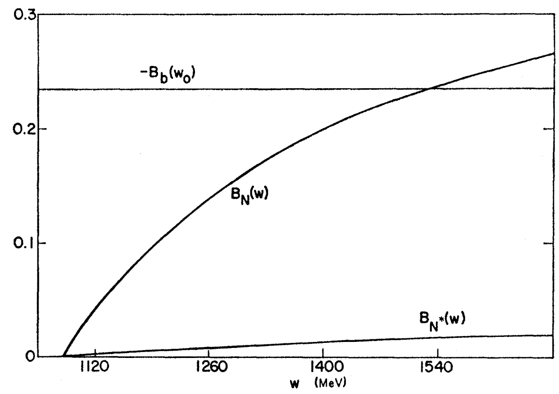

FIG. 2. Driving forces for the zero-parameter model.

The relationship between the threshold problem and the strength of the OPE forces is now evident. The background forces that must exist to provide the equivalent of the centrifugal barrier in the Schrödinger equation correspond in the relativistic case to a short-range repulsive force upon which the OPE forces are superimposed. In a first approximation, the background forces strongly affect the magnitude but not the structure of the driving force.

We solve the N/D equations for the dispersion relation (1) in Pagels's approximation.[10] As Dilley has pointed out,[11] the advantage of using the Pagels approximation is that (2) reduces to an algebraic equation that is trivially solved.[12] It is of interest to compare the results of this approximation method with numerical solutions of the N/D integral equations.[4] Following the usual procedure, we neglected $B_b(w)$ and defined a new 3,3 amplitude as $h(w) = \rho(w) f(w)$, where $\rho(w) = w/(w - w_0)$. In the Pagels approximation, the solution of this modified problem is characterized by a bound 3,3 state, and the bound state becomes a resonance only when the pion-nucleon coupling constant is reduced to $g_{N\pi}^2/4\pi = 9.5$. The resonance has the correct mass for $g_{N\pi}^2/4\pi = 7.7$.

On the basis of this qualitative agreement with previous numerical solutions, we expect the Pagels form to be a reasonable approximation for the problem at hand. In this approximation, the amplitude is given by $f(w) = N(w)/D(w)$, with[10]

$$N(w) = B(w) - RB(a)(w - a + R)^{-1},$$
$$D(w) = 1 - \Delta(w) N(w),$$

[6] By "one-particle exchange" forces we mean the partial-wave projections of that part of the scattering amplitude associated with the Feynman diagram for single-particle exchange.

[7] G. Frye and R. L. Warnock, Phys. Rev. **130**, 478 (1963).

[8] For angular momentum states with $l > 1$ the threshold conditions determine $B_b(w_0)$ and $(l-1)$ derivatives of $B_b(w)$ at $w = w_0$. See, for example, Ref. 11.

[9] For the N- and N^*-exchange forces we use the physical masses and coupling constants of these particles, namely, $g_{N\pi}^2/4\pi = 14.5$ and $g_{NN^*\pi}^2/4\pi = 0.36$. In numerical solutions of the N/D integral equations the N^*-exchange force will require a cutoff. This fact has essentially no effect on our low-energy approximate solution.

[10] H. Pagels, Phys. Rev. **140**, B1599 (1965).

[11] J. Dilley, Nuovo Cimento **50**, 837 (1967).

[12] In numerical solutions of the N/D integral equations the threshold condition will require an iterative procedure in order to determine $B_b(w_0)$.

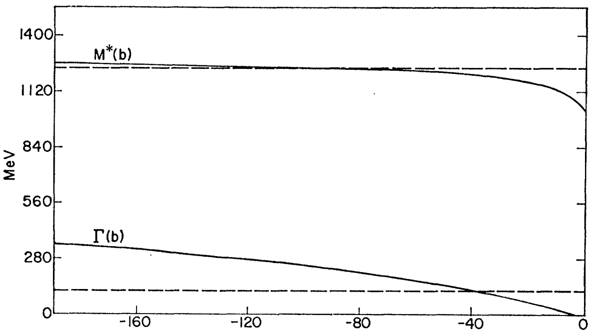

FIG. 3. Output mass and width of the N^* in the one-parameter model. The pole position b is measured in units of the pion mass, and the dashed lines are the experimental values.

where

$$\Delta(w)=F(w)-cw^2(a-w)^{-1},$$
$$F(w)=\frac{w^2}{\pi}\int_{w_0}^{\infty}\frac{dw'q(w')r(w')}{w'^2(w'-w)},$$
$$R=ca^2B(a)[1-ca^2B'(a)]^{-1}.$$

The two parameters a and c of the one-pole kinematic approximation are determined by minimizing $\Delta(w)$ on the short nucleon cut ($w\approx M$) and at the beginning of the u-channel cut ($w=M-\mu$). In this determination, we have set the inelasticity factor $r(w)=1$, corresponding to the elastic approximation. Although the scattering in the P_{33} state is quite elastic at low energies, the inelasticity for larger W, entering through the dispersion integrals, can have a strong effect[13] on the quantitative aspects of the problem. However, we would not expect the inclusion of inelasticity to produce a marked qualitative change in the low-energy solution.

The best fit was obtained with $a=12.42$ and $c=0.157$ in units $\mu=1$. With these values $\Delta(w)$ is also relatively small on the cut running along the imaginary axis. Equation (2) for $B_b(w_0)$ now becomes

$$N(w_0)=B_b(w_0)-RB(a)(w_0-a+R)^{-1}=0,$$

which has the unique solution

$$B_b(w_0)=R_0B_0(a)(w_0-a-R_0)^{-1}=-0.235, \quad (3)$$

where $B_0(w)=B_N(w)+B_{N^*}(w)$ and $R_0=R$, with $B(a)$ and $B'(a)$ replaced with $B_0(a)$ and $B_0'(a)$, respectively. The 3,3 amplitude is now completely determined with no arbitrary parameters. The phase shift is defined by

$$f(w)=\sin\delta(w)e^{i\delta(w)}/q(w)$$

and is plotted in Fig. 1. As usual, the mass and width of the N^* are obtained from the equations

$$\mathrm{Re}D(M^*)=0,$$
$$\Gamma=-2q(M^*)N(M^*)/\mathrm{Re}D'(M^*), \quad (4)$$

which yield the values $M^*=1270$ MeV and $\Gamma=374$ MeV. The experimental values are $M^*=1236$ MeV and $\Gamma=120$ MeV.

Since our phase shift does not continue to rise for $w\gtrsim 1320$ MeV (this is reflected in the overly large width obtained), we might suspect that the magnitude of the repulsive background has been overestimated in our model.

First, we show that this is not the case for $w=w_0$. Equation (2) is an exact relation, and it provides a means of calculating a lower limit for $|B_b(w_0)|$ in terms of the experimental values of $\mathrm{Im}f(w)$. Taking these from the phase-shift analyses of Roper *et al.*[14] and Bareyre *et al.*,[15] we find that

$$-\frac{1}{\pi}\int_{w_0}^{15.5\mu}dw'\frac{\mathrm{Im}f(w')}{w'-w_0}\approx-0.2.$$

Since the integrand is positive throughout the entire integration range,

$$-B_b(w_0)=\frac{1}{\pi}\int_{w_0}^{\infty}dw'\frac{\mathrm{Im}f(w')}{w'-w_0}>\frac{1}{\pi}\int_{w_0}^{15.5\mu}dw'\frac{\mathrm{Im}f(w')}{w'-w_0}$$

and

$$|B_b(w_0)|>0.2,$$

which is consistent with the value obtained in our model, namely, $B_b(w_0)=-0.235$.

Second, we consider the energy dependence of the background force. Its magnitude presumably decreases from its value at w_0 as w is increased. Perhaps our neglect of this variation in $B_b(w)$ is responsible for the large width and the poor behavior of $\delta(w)$ for larger w. To examine this possibility, we have repeated the calculation with the background forces represented by a simple pole,

$$B(w)=B_N(w)+B_{N^*}(w)+\beta(b)/(w-b).$$

The pole position b represents the adjustable parameter and the residue $\beta(b)$ is determined by the threshold condition.

In the Pagels approximation, it is easy to show that

$$\beta(b)=B_b(w_0)(a-b)(w_0-b)/(\lambda-b),$$

where

$$\lambda=a+2(a-w_0)B_b(w_0)/B_0(a)$$

and $B_b(w_0)$ is given by (3). Note that as $b\to-\infty$,

$$\beta(b)/(w-b)\to B_b(w_0).$$

The mass and width of the N^* as determined from (4) are plotted as functions of b in Fig. 3. The output mass shows very little variation with b until the pole position nears the origin. The width, on the other hand, varies considerably over a wide range of pole positions, indi-

[13] The results of a study by G. C. Oades on the effects of inelasticity is given in *High Energy Physics*, edited by E. H. S. Burhop (Academic Press Inc., New York, 1967), Vol. I, p. 245.

[14] L. Roper, R. Wright, and B. Feld, Phys. Rev. **138**, B190 (1965).

[15] P. Bareyre, C. Bricman, and G. Villet, Phys. Rev. **165**, 1730 (1968).

cating that the width is much more sensitive to the detailed structure of the background force. Although improved values of the mass and width can be obtained in this manner, for example, with $b\approx -60$, $M^*\approx 1220$ MeV, and $\Gamma\approx 200$ MeV, the phase shift is still found to flatten out a little above the resonance position. In addition, quantitative accuracy for both M^* and Γ is not possible for any value of b. On the basis of these considerations, we conclude that the decrease in the background forces is only a minor factor in the correct description of $\delta(w)$ above the resonance energy. Other factors, for example inelasticity, are probably of greater importance in this respect.

In conclusion, we feel that the two most important factors in the low-energy calculation of the P_{33} amplitude are nucleon exchange and the background forces which represent the effects of the centrifugal barrier. These are plotted in Fig. 2 and are seen to be of comparable importance. It is clear that the simplified reciprocal-bootstrap statement that N and N^* exchange alone are sufficient to determine the N and N^* parameters must be abandoned. Instead, as has been emphasized by Chew, the bootstrap hypothesis requires the proper treatment of all relevant forces, and the present model suggests that the background forces will play an important role in self-consistent calculations.

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Nonleptonic Decays of Hyperons and Current Algebra*

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Nonleptonic decays of baryons are studied within the framework of current algebra, current-current interaction, and hard pions. A formalism which avoids the usual ambiguity of off-mass-shell extrapolation for the P-wave decays is developed, from which formulas for the decay amplitudes can be derived. Two models are discussed. The first model contains the equal-time commutator and the baryon pole terms, but allows for the $SU(3)$ symmetry breaking through the presence of the parity-violating spurion matrix elements between baryons. The second model adds the $\frac{3}{2}+$ decuplet and $Y_0^*(1405)$ contributions to the first model, but the $SU(3)$ symmetry-breaking matrix elements are not considered. Reasonable agreement with experiment is obtained in both models.

I. INTRODUCTION

FOLLOWING the papers of Sugawara and Suzuki[1] there have been a number of articles on the theory of nonleptonic decays of baryons using the methods of current algebra.[2] Attempts have been made to extend the formalism of current algebra to include the P-wave amplitudes. These involve the study of various forms of phenomenological Lagrangian for the decays. Within the framework of current-current interaction and current algebra the P-wave amplitudes have been found to be about a factor of 2 smaller than the experimental values.[3] Further, the decay formula usually derived suffers from ambiguity in the masses to be used, and the problem of extrapolation of the four-momentum of the pion to zero for the P-wave amplitudes is not correct. We would like to present here a method of deriving a decay formula which is devoid of such difficulties. The formalism is in analogy to the treatment of the P-wave pion-nucleon scattering lengths by Schnitzer.[4] The details will be discussed in Sec. II. Following that two models are proposed separately in Secs. III and IV. In model 1 the baryon pole is studied in detail. $SU(3)$ symmetry breaking is introduced through the use of physical masses of the baryons and the existence of parity-violating (pv) spurion matrix elements between baryons. These pv spurion matrix elements vanish in the limit of exact $SU(3)$.[5] In our consideration it is included as an unknown parameter. This approach is similar in spirit to that of Kumar and Pati.[6] The problem is formulated in the language of $SU(3)$ so as to treat the various decays on equal footing as far as the strong interaction is concerned. The decay amplitudes can then be expressed in terms of six unknown reduced

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[2] M. Gell-Mann, Physics **1**, 63 (1964).

[3] L. S. Brown and C. M. Sommerfield, Phys. Rev. Letters **16**, 751 (1966); Y. Hara, Y. Nambu, and J. Schechter, *ibid.* **16**, 380 (1966); S. Badier and C. Bouchiat, Phys. Letters **20**, 529 (1966); Y. T. Chiu and J. Schechter, Phys. Rev. Letters **16**, 1022 (1966); S. N. Biswas, Aditya Kumar, and R. P. Saxena, *ibid.* **17**, 268 (1966); Y. T. Chiu, J. Schechter, and Y. Ueda, Phys. Rev. **150**, 1201 (1967).

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[6] Arvind Kumar and J. G. Pati, Phys. Rev. Letters **18**, 1230 (1967); G. S. Guralnik, V. S. Mathur, and L. K. Pandit, Phys. Rev. **168**, 1866 (1968).