taken as unity. As in all the elastic scattering calculations, the unitarity condition is a useful check on the computations. Table V shows the convergence of the results and the unitarity condition for $\lambda_1=1$, $\lambda_2=1$, $\lambda_3=-3.206\cdots$, $E_b=-0.05$, $p=0.272$. From the convergence of the approximations and the accuracy of our input values we estimate the accuracy of the results to be between 1 and 0.1%. All the results reported here concerning short-range potentials were obtained in about 2 min of IBM 360 computer time, where 12 values of $(\tan\delta)/p$ were calculated for each bound-state energy E_b.

VI. CONCLUSIONS

With this method we have been able to obtain precise values for some model three-body scattering phase shifts and amplitudes in a simple and efficient way. We have not, however, been able to calculate breakup amplitudes, probably because of the neglect of the three-body logarithmic threshold. Despite this failing, we believe that the method employed here can be a useful tool in the solution of a wide variety of scattering problems.

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Embedding of $SU(3)$ in $SU(8)$*

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The fact that $SU(8)$ symmetry has recently been applied to the nonleptonic decays of baryons, both in a pole model and in a current-algebra model, suggests a closer look at this symmetry. The $SU(8)$ algebra is constructed so that the $SU(8)$ structure is preserved. The possible application of other physical processes is then considered. It is shown that with certain restrictive assumptions, approximate octet dominance follows from a current-current interaction.

I. INTRODUCTION

THE use of $SU(8)$ symmetry in the parity-conserving baryon-pole model by Lee[1] and by Graham, Pakvasa, and Rosen[2] has supplied the motivation for a more careful look at $SU(8)$. More recently, in fact, $SU(8)$ has been applied to parity-violating baryon decays in the pole model[3] and in a current-algebra model.[4] If one should believe that $SU(3)$ might not be the smallest possible internal symmetry that has relevance to particle physics, then it seems to be important to consider the possibility of a more general application of $SU(8)$.

The $SU(8)$ algebra of Ref. 3 was constructed in terms of the Gell-Mann or Hermitian basis. Here, the algebra will be constructed in terms of the 8×8 traceless matrices A_j^i, $i,j=1,\cdots,8$, which satisfy the commutation relations

$$[A_j^i,A_l^k]=\delta_j^kA_l^i-\delta_l^iA_j^k.$$

As will be shown in Secs. II and III, the actual construction will be a generalization of the Elliott model of $SU(3)$.[5]

The basic requirement for the construction of such a higher symmetry is that the $SU(3)$ structure must be preserved. One example of such a symmetry would be the $SU(4)$ model,[6] which is described by

$$SU(4)=SU(3)\times U(1).$$

That is, a new quantum number, "supercharge," is added to the $SU(3)$ algebra, enlarging it to $SU(4)$. In the construction of $SU(8)$, however, it will not be necessary to assume the existence of any new quantum numbers since, as mentioned, the structure is simply a generalization of the Elliott model. For this reason, it is useful to describe this model briefly.

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[6] P. Tarjanne and V. L. Teplitz, Phys. Rev. Letters **11**, 447 (1963).

II. ELLIOTT MODEL

It is not necessary to construct the Elliott model of $SU(3)$ in terms of creation and annihilation operators in a nuclear potential well. It is necessary only to assume the existence of some object having spin one. The physical interpretation of this spin is not important as far as the construction of the algebra is concerned. It matters only that the object transforms as a 3-dimensional representation of the rotation group in 3 dimensions. It is desired that this object be embedded in the 3-dimensional representation of $SU(3)$ by a simple one-to-one correspondence. Explicitly, let

$$|1\ 1\rangle \sim \phi_2,\quad |1\ 0\rangle \sim \phi_3,\quad |1\ -1\rangle \sim \phi_1. \tag{1}$$

Here $|jm\rangle$ represents the state of spin j and third spin component m, and ϕ_μ transforms as the 3-dimensional representation of $SU(3)$.

Let this embedding be denoted by

$$3=[3],$$

where the term in the bracket represents the dimensionality of the $R(3)$ representation. Since

$$3\times 3^*=1+8, \tag{2}$$

the adjoint representation of $SU(3)$ decomposes with respect to $R(3)$ as

$$8=[3]+[5].$$

From the 8, therefore, a set of operators can be selected which are to be identified as the generators of $R(3)$. The explicit form of these generators is obtained in terms of the 3×3 traceless matrices $A_j{}^i$, by demanding that Eq. (1) is properly transformed. Therefore, in terms of $SU(3)$ indices, the generators transform as

$$J_0=(A_2{}^2-A_1{}^1),$$
$$J_+=-(A_1{}^3+A_3{}^2),$$
$$J_-=(A_3{}^1+A_2{}^3).$$

Here

$$J_\pm=(1/\sqrt{2})[\mp J_x-iJ_y],$$

and the commutation relations among the J_μ are given by

$$[J_\mu,J_\nu]=\sqrt{2}\langle 1\nu 1\mu|1\lambda\rangle J_\lambda.$$

From Eq. (2), the remaining $SU(3)$ generators transform as a second-rank tensor Q_μ with respect to the $R(3)$ subalgebra. The components of Q transform as

$$Q_{+2}=-\sqrt{2}A_1{}^2,$$
$$Q_{+1}=A_3{}^2-A_1{}^3,$$
$$Q_0=(2A_3{}^3-A_1{}^1-A_2{}^2),$$
$$Q_{-1}=A_3{}^1-A_2{}^3,$$
$$Q_{-2}=-\sqrt{2}A_1{}^2.$$

From the symmetry properties of the Clebsch-Gordan coefficients, the following definition of the reduced matrix elements, and, since

$$(T_\mu{}^{(\lambda)})^\dagger=(-1)^\mu(T_{-\mu}{}^{(\lambda)})$$

for a spherical tensor of rank λ and component μ, the reduced matrix elements of ν must satisfy

$$\frac{\langle j'\|Q\|j\rangle}{\langle j\|Q\|j'\rangle}=(-1)^{j'-j}\left[\frac{(2j+1)}{(2j'+1)}\right]^{1/2}.$$

This places a condition of the normalization of the Q_μ, the result being that Q and J have the same normalization. The relative phase of Q_μ, however, is arbitrary. The definition of the reduced matrix element used in the above is

$$\langle j'm'|T_\mu{}^{(\lambda)}|jm\rangle=\sum_{j'}\langle j'm'\lambda\mu|jm\rangle\langle j'\|Q\|j\rangle.$$

The remaining commutation relations are

$$[Q_\mu,J_\nu]=(\sqrt{6})\langle 1\nu 2\mu|2\lambda\rangle Q_\lambda,$$
$$[Q_\mu,Q_\nu]=-(\sqrt{10})\langle 2\nu 2\mu|1\lambda\rangle J_\lambda.$$

Thus, $SU(3)$ even though it is a second-rank algebra can be described in terms of the first-rank algebra $R(3)$. The operator Q_0, which commutes with J_0, instead of defining a quantum number simply is described in terms of its transformation properties with respect to the $R(3)$ subalgebra. It is this structure which will be generalized for the construction of the $SU(8)$ algebra.

III. $SU(8)$ ALGEBRA

By applying the arguments of the last section, $SU(3)$ may be embedded in $SU(8)$ without the introduction of any new quantum numbers. Therefore, let the 8-dimensional representation of $SU(3)$ be embedded in the 8-dimensional representation of $SU(8)$. In terms of explicit $SU(8)$ indices, let the embedding be

$$\Sigma^+\sim B_1,\quad \Sigma^0\sim B_3,\quad \Sigma^-\sim B_2,\quad \Lambda\sim B_6,\quad p\sim B_4,\quad n\sim B_5,\quad \Xi^0\sim B_7,\quad \Xi^-\sim B_8. \tag{3}$$

The particle symbols represent those states with the appropriate (T,T_3,Y) quantum numbers, and B_i is the 8-dimensional representation of $SU(8)$.

Let this embedding be denoted

$$8=[8],$$

where the bracket refers to the dimensionality of the $SU(3)$ representation. Since

$$8\times 8^* = \mathbf{1}+\mathbf{63},$$

the adjoint or 63-dimensional representation of $SU(8)$ decomposes with respect to $SU(3)$ as

$$\mathbf{63}=[8]+[8]+[10]+[10^*]+[27].$$

Therefore, from the **63** a set of operators can be collected which transforms the **8**, Eq. (3), as in $SU(3)$. These operators are identified as the generators of $SU(3)$, and are called the F-type octet. In terms of $SU(8)$ indices these generators transform as

$$\begin{aligned}
T_3 &= A_2{}^2 - A_1{}^1 + \tfrac{1}{2}(A_5{}^5 - A_4{}^4 + A_8{}^8 - A_7{}^7),\\
T_+ &= (T_-)^\dagger = -\sqrt{2}(A_3{}^2 + A_1{}^3) - (A_4{}^5 + A_7{}^8),\\
Y &= A_7{}^7 + A_8{}^8 - A_4{}^4 - A_5{}^5,\\
K_+ &= (K_-)^\dagger = -[(A_1{}^7 + A_5{}^2) + \tfrac{1}{2}\sqrt{3}(A_6{}^8 + A_4{}^6)\\
&\qquad + (1/\sqrt{2})(A_3{}^8 + A_4{}^3)],\\
L_+ &= (L_-)^\dagger = -[(A_2{}^8 - A_4{}^1) + \tfrac{1}{2}\sqrt{3}(A_5{}^6 - A_6{}^7)\\
&\qquad + (1/\sqrt{2})(A_3{}^7 - A_5{}^3)].
\end{aligned}$$

The notation for these octet operators agrees basically with that of de Swart.[7] The phase convention is determined by choosing the nonvanishing matrix elements of K_+ to be positive.

It is necessary to express these operators in terms of the spherical generators of $SU(3)$, which are denoted $F^{(\mu)}$, with $\mu=(T,T_3,Y)$:

$$\begin{aligned}
\sqrt{2}F^{(110)} &= -T_+,\\
\sqrt{2}F^{(1-10)} &= T_-,\\
F^{(100)} &= T_3,\\
(2/\sqrt{3})F^{(000)} &= Y,\\
-\sqrt{2}F^{(\frac{1}{2}\,\frac{1}{2}\,1)} &= K_+,\\
\sqrt{2}F^{(\frac{1}{2}-\frac{1}{2}-1)} &= K_-,\\
-\sqrt{2}F^{(\frac{1}{2}-\frac{1}{2}\,1)} &= L_+,\\
-\sqrt{2}F^{(\frac{1}{2}\,\frac{1}{2}-1)} &= L_-.
\end{aligned}$$

The commutation relations among the $F^{(\mu)}$ can then be written

$$[F^{(\mu)},F^{(\nu)}]=\sqrt{3}\begin{pmatrix}8 & 8 & 8_2\\ \nu & \mu & \lambda\end{pmatrix}F^{(\lambda)},$$

where

$$\begin{pmatrix}8 & 8 & 8_2\\ \nu & \mu & \lambda\end{pmatrix}$$

is the Clebsch-Gordan coefficient of $SU(3)$ defined in Ref. 7. These also satisfy

$$(F^{(\mu)})^\dagger = (-1)^{Q_\mu}F^{(-\mu)}, \qquad (4)$$

[7] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).

TABLE I. Explicit form for some of the $SU(8)$ generators.

$$\begin{aligned}
D^{(000)} &= (\sqrt{\tfrac{3}{5}})[-\tfrac{3}{2}(A_4{}^4+A_5{}^5+A_7{}^7+A_8{}^8)-2A_6{}^6],\\
P^{(100)} &= (1/\sqrt{12})(A_2{}^2+A_4{}^4+A_7{}^7-A_1{}^1-A_5{}^5-A_8{}^8)+\tfrac{1}{2}(A_3{}^6-A_6{}^3),\\
R^{(000)} &= (2/\sqrt{15})(2A_6{}^6-A_4{}^4-A_5{}^5-A_7{}^7-A_8{}^8),\\
R^{(200)} &= -(2/\sqrt{6})(A_1{}^1+A_2{}^2-2A_3{}^3).
\end{aligned}$$

with

$$Q_\mu = (T_3+\tfrac{1}{2}Y)_\mu \quad\text{and}\quad -\mu=(T,-T_3,-Y).$$

With the identification of the F-type octet, the $[8_D]$, $[10]$, $[10^*]$, and $[27]$ components of the **63** can be constructed. In spherical tensor form, these components are denoted, respectively, $D^{(\mu)}$, $P^{(\mu)}$, $\bar{P}^{(\mu)}$, and $R^{(\mu)}$. These satisfy

$$(N^{(\mu)})^\dagger = (-1)^{Q_\mu}\bar{N}^{(-\mu)}, \qquad (5)$$

with $N=D$, P, $\bar{P}$, or R. The transformation properties of a few of these operators are given in Table I.

The commutation relations of the $SU(8)$ algebra are determined by the $SU(3)$ reduced matrix elements. These may, in principle, be determined by means of the Casimir operators of $SU(8)$. The construction of these operators is too formidable an algebraic task, and so the necessary reduced matrix elements can be determined by means of explicit construction of all $SU(8)$ elements. For the $SU(8)$ algebra, the most general form of the commutation relation may be written

$$\sum_{[N^{(\mu_1)},N^{(\mu_2)}]=N_3,\mu_3} \langle N_3\|N_1\|N_2\rangle \begin{pmatrix}N_2 & N_1 & N_3\\ \mu_2 & \mu_1 & \mu_3\end{pmatrix} N_3{}^{(\mu_3)}. \qquad (6)$$

Here $N_i=F$, D, P, $\bar{P}$, or R, and $\langle N_3\|N_1\|N_2\rangle$ is the $SU(3)$ reduced matrix element.

By means of Eqs. (4)–(6) and the symmetry properties of the $SU(3)$ Clebsch-Gordan coefficients,[7] the following relation will hold between reduced matrix elements:

$$\frac{\langle N_3\|N_2\|N_1\rangle}{\langle N_1\|N_2\|N_3\rangle} = \xi_1(3\,2^*:1)\xi_2(2^*3:1)\xi_3(2^*1^*:3)\xi_1(21:3)\left(\frac{N_1}{N_3}\right)^{1/2}.$$

The $\xi_i(jk:l)$ are ± 1 depending only on N_j, N_k, or N_l. Just as in the Elliott model, the consequence is that all $SU(3)$ components must be normalized to the same number. The phase is arbitrary, except that the relative phase of $\bar{P}$ with respect to P is determined by Eq. (5). The same general comments will hold for other representations of $SU(8)$.

Therefore, the $SU(8)$ algebra has been constructed as a generalization of the Elliott model. All of the operators are characterized by their transformation properties with respect to the $SU(3)$ subalgebra. This algebra could be applied to the $SU(8)$ pole-model calculations if it were desirable to do so. The possibility of other possible applications are considered next.

IV. POSSIBLE PARTICLE ASSIGNMENTS IN $SU(8)$

The baryon-pole model of Ref. 3 has assumed specific $SU(8)$ classifications for the baryon octet and for the pseudoscalar-meson octet. In particular, the baryons were assigned to the **8**, and the pseudoscalar mesons were assumed to belong to an arbitrary linear combination of F- and D-type octets in the **63**. These are certainly convenient assignments in this case, but it seems appropriate to consider other possibilities. Also, assignments for other classes of particles might be considered.

In this section, therefore, some possible particle assignments will be considered. In order to do so, it is necessary to set up certain assumptions on the $SU(8)$ properties of the operators so that the $SU(3)$ results may be duplicated. In addition, these assumptions should be so chosen that those predictions which go beyond $SU(3)$ seem as reasonable as possible. The easiest assumption to make is that all effects of $SU(3)$ will be embodied in the **63** of $SU(8)$. This would correspond to replacing octet dominance by the somewhat more general principle of **63** dominance.

As an example of what this means, consider the electromagnetic-mass-splitting interaction. In $SU(3)$ the interaction is assumed to transform as $Q\times Q$, which decomposes into a 27-plet and F- and D-type octets. Since in $SU(8)$, Q will belong to the **63**, $Q\times Q$ would contain a **1232** contribution. However, in order to reproduce the results of $SU(3)$, the simplest possible assumption to make is that the interaction belongs to the **63** and transforms as an arbitrary linear combination of the U-spin invariant member of the F- and D-type octets and the 27-plet.

Of course, the most important interaction to consider is the $SU(3)$ mass-breaking interaction. With the previous assumption, the interaction would be expected to transform as

$$F^{(000)}+\lambda_D{}^{(100)},$$

where λ is arbitrary. This would not introduce any mass splitting between common mass terms of $SU(3)$ multiplets belonging to the same representation of $SU(8)$. Since there is little evidence for many $SU(3)$ multiplets of the same $(J)^P$ having nearly equal masses, there must be some provision for introducing mass splitting between different $SU(3)$ multiplets. This would seem to rule out $SU(8)$ as an invariance group. Presumably $SU(8)$ should be regarded as a noninvariance group.[8]

The simplest possible way to introduce such a mass splitting is to assume that the common mass term of a given $SU(3)$ multiplet is completely independent of that of another $SU(3)$ multiplet belonging to the same $SU(8)$ representation. This is somewhat similar to the common mass term that arises in the $SU(3)$ electromagnetic mass formula for, say, the baryon octet. The difference is, of course, that one can find in $SU(3)$ a relation between these common mass terms, whereas in $SU(8)$ this does not seem to be possible.

Therefore, if we believe that $SU(8)$ may possibly have some relevance to particle physics, then the first things to test are the mass relations. Some of the representations of lower dimensionality are the **28**, the **36**, and the **56**. Their transformation properties, in terms of the characteristic indices which represent the Young tableaux, $(p_1p_2p_3p_4p_5p_6p_7)$, and their $SU(3)$ content are given in Table II.

The 28-dimensional representation, which transforms as $G_{ij}=-G_{ji}$ contains [8], [10], and [10*] $SU(3)$ components. With the assumption for the mass operator as given above, the difference relations for the masses are obtained:

$$(N_{3/2}{}^*-Y_1{}^*)=\tfrac{1}{2}(N-\Xi)+\tfrac{1}{4}(\Sigma-\Lambda),$$
$$(Z_0{}^*-N_{1/2}{}^*)=\tfrac{1}{2}(N-\Xi)+\tfrac{1}{4}(\Lambda-\Sigma).$$

Here, N, Σ, Λ, and Ξ refer to the masses of numbers of the octet, and $N_{3/2}{}^*$, $Y_1{}^*$ and $Z_0{}^*$, $N_{1/2}{}^*$ refer, respectively, to masses of members of the [10] and [10*]. Of course, the $SU(3)$ sum rules hold within each multiplet.

The 36-dimensional representation transforms as $D_{ij}=D_{ji}$ and contains [1]+[8]+[27] components. Examples of mass difference relations are

$$Z_1{}^{**}-\Omega_1{}^{**}=2(N-\Xi),$$
$$Y_2{}^{**}-Y_1{}^{**}=2(\Lambda-\Sigma),$$

where unstarred symbols again represent the **8** masses, and doubly starred symbols represent the masses of the [27].

Also, the 56-dimensional representation is considered. The **56**, which contains [1], [8], [10], [10*], and [27] components, transforms as the completely antisymmetric N_{ijk}. With the same assumption for the mass-breaking interaction, some representative difference relations are

$$N_{3/2}{}^*-Y_1{}^*=\tfrac{1}{2}(N-\Xi)+\tfrac{1}{2}(\Lambda-\Sigma),$$
$$Z_0{}^*-N_{1/2}{}^*=\tfrac{1}{2}(N-\Xi)+\tfrac{1}{2}(\Sigma-\Lambda),$$
$$Z_1{}^{**}-\Omega_1{}^{**}=2(N-\Xi).$$

The notation is the same as that employed above.

TABLE II. $SU(3)$ content of some $SU(8)$ representations.

$(p_1p_2p_3p_4p_5p_6p_7)$	Dimensionality	$SU(3)$ content
(1000000)	8	[8]
(0100000)	28	[8]+[10]+[10*]
(2000000)	36	[1]+[8]+[27]
(1000001)	63	[8]+[8]+[10]+[10*]+[27]
(0010000)	56	[1]+[8]+[10]+[10*]+[27]
(0001000)	70	[8]+[8]+[27]+[27]

[8] N. Mukunda, L. O'Raifeartaigh, and E. C. G. Sudarshan, Syracuse University Report No. NYO-3399-30 (unpublished).

These mass relations have assumed no mixing between $SU(3)$ multiplets. Presumably, if all of these states were to exist at nearly the same mass, there would be a great deal of $SU(3)$ mixing. For simple cases of mixing, $SU(8)$ could be used to obtain some rules of the Schwinger type.[9] For example, if we assume only $[1]+[8]$ mixing in either the **36** or the **56**, then we find

$$\Lambda\Lambda' \cong \Lambda_8(\Lambda+\Lambda'-\Lambda_8). \quad (7)$$

Here Λ and Λ' are taken to be the physical $T=Y=0$ masses, with Λ mostly octet and Λ' mostly unitary singlet, and $3\Lambda_8=2(N+\Xi)-\Sigma$.

This relation, Eq. (7), follows if the physical mass matrix is related to the mathematic matrix by a unitary transformation, so that

$$\Lambda+\Lambda'=\Lambda_1+\Lambda_8\,,$$
$$\Lambda\Lambda'=\Lambda_1\Lambda_8-k^2\langle\Lambda_8|M|\Lambda_1\rangle^2.$$

Here, k is a numerical factor depending on whether the **36** or the **56** is considered. Since only the D-type term of the mass operator contributes to the matrix element, this term is proportional to $[2\Sigma-(N+\Xi)]^2$, which is expected to be small compared with the other terms.

It has been suggested[10] that mixing of this type is observed in the $(\frac{3}{2})^-$ resonances. It is observed that Eq. (7) is well satisfied by these resonances. Of course, this equation will hold for any model in which it is possible to evaluate the off-diagonal matrix elements in terms of the masses of the other states.

Before looking at the pseudoscalar-meson assignment, it is convenient to consider how to describe the Yukawa couplings in $SU(8)$. The simpliest possible assumpton that the interaction is a scalar in $SU(8)$ space. Whether it might prove necessary, later, to introduce some way of distinguishing between $SU(3)$ components of the same $SU(8)$ representation, is not considered. The pseudoscalar mesons are assumed to belong to the **63**, since this is the self-adjoint representation of lowest dimensionality. Therefore, if the baryon octet is assigned to the **8**, or to any representation **R** such that $\mathbf{R}^*\times\mathbf{R}$ contains **63** only once, the pseudoscalar mesons must be assigned to an arbitrary linear combination of the F- and D-type octets in the **63**. This is necessary, of course, in order to have both D- and F-type couplings. With these assumptions there are some simple selection rules that would hold for strong decays. For example,

$$\mathbf{8}\not\leftrightarrow\mathbf{R}+\mathbf{63}\,,$$
$$\mathbf{28}\not\leftrightarrow\mathbf{56}+\mathbf{63}\,,$$
$$\mathbf{36}\not\leftrightarrow\mathbf{56}+\mathbf{63}\,,$$

where

$$R=\mathbf{28},\ \mathbf{36},\ \text{or}\ \mathbf{56}.$$

Since the pseudoscalar mesons are, thus, to be assigned to an arbitrary linear combination of F- and D-type octet, there are no mass sum rules relating the physical particles with the other states of the **63**. These other states in the **63** form a $\mathbf{10}+\mathbf{10}^*$, or icosuplet[11] and a 27-plet. However, in terms of the mass for the F- and D-type octets (which do not represent physical observables), the following difference relations are satisfied:

$$(K_F-\pi_F)=(5/3)(\pi_D-K_D)$$
$$=3(\pi_1{}^*-K_{3/2}{}^*)=(K_{3/2}{}^{**}-\pi_2{}^{**})\,.$$

Here the symbols represent the masses (squared), respectively, of members of the F-type octet, the D-type octet, the icosuplet, and the 27-plet.

Since there is no conclusive evidence at this time for any of the higher multiplets of $SU(3)$, it is not clear whether any meaning can be given to $SU(8)$. Perhaps some of the extra multiplets will be discovered at higher energies. Perhaps it will be necessary, in order to make use of $SU(8)$, to assume that some of the common mass terms are arbitrarily high, and thus would not be expected to be seen. If this latter interpretation were used, the $SU(8)$ algebra could still be potentially useful. A possible application of this will be given in Sec. V.

V. CURRENT-CURRENT MODEL FOR NONLEPTONIC DECAYS

The application of $SU(8)$ to nonleptonic baryon decays in the current-algebra model and in the pole model has been mentioned. If we maintain the assumption suggested above, namely that the $SU(8)$ model can be interpreted so that all $SU(3)$ properties can be included in the **63**, then $SU(8)$ becomes very useful in the current-current model as well. In fact, approximate octet dominance is the result.

The approach is very similar to that of abovementioned calculations.[3,4] The assumption is that the Cabibbo current[12,13] transforms as an arbitrary linear combination of F- and D-type octets in the **63**:

$$j\sim\cos\theta\,(F^{(110)}+\mu D^{(110)})+\sin\theta\,(F^{(\frac{1}{2}\frac{1}{2}1)}+\mu D^{(\frac{1}{2}\frac{1}{2}1)})\,.$$

The V and A indices are suppressed here. F and D represent currents which transform as the F- and D-type octets.

The nonleptonic Hamiltonian, therefore, is taken to transform as

$$H_{NL}=\{F^{(1-10)},F^{(\frac{1}{2}\frac{1}{2}1)}\}+\mu^2\{D^{(1-10)},D^{(\frac{1}{2}\frac{1}{2}1)}\}$$
$$+\mu\{F^{(1-10)},D^{(\frac{1}{2}\frac{1}{2}1)}\}+\mu\{F^{(\frac{1}{2}\frac{1}{2}1)},D^{(1-10)}\}\,. \quad (8)$$

Since we assume no contributions other than the **63**, these anticommutation relations are the same as those

[9] J. Schwinger, Phys. Rev. **135**, B816 (1964).

[10] G. B. Yodh, Phys. Rev. Letters **18**, 510 (1967); N. Masuda and S. Mikamo, Phys. Rev. **162**, 1517 (1967).

[11] B. W. Lee, S. Okubo, and J. Schechter, Phys. Rev. **135**, B219 (1964).

[12] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[13] S. P. Rosen, S. Pakvasa, and E. C. G. Sudarshan, Phys. Rev. **146**, 1118 (1966).

satisfied by the generators:

$$\{F^{(1-10)},F^{(\frac{1}{2}\,\frac{1}{2}\,1)}\}+5/3\{D^{(1-10)},D^{(\frac{1}{2}\,\frac{1}{2}\,1)}\}=0\,,$$
$$\{F^{(1-10)},D^{(\frac{1}{2}\,\frac{1}{2}\,1)}\}+\{F^{(\frac{1}{2}\,\frac{1}{2}\,1)},D^{(1-10)}\}=-(3\sqrt{\tfrac{2}{5}})F^{(\frac{1}{2}-\frac{1}{2}\,1)}\,,$$
$$\{D^{(1-10)},D^{(\frac{1}{2}\,\frac{1}{2}\,1)}\}=-\tfrac{1}{2}(18/5)^{1/2}[-\tfrac{3}{5}D^{(\frac{1}{2}\,\frac{1}{2}\,1)} +(2/3\sqrt{5})(R^{(\frac{3}{2}-\frac{1}{2}\,1)}+(1/\sqrt{5})R^{(\frac{1}{2}-\frac{1}{2}\,1)})]\,.$$

Therefore the resulting Hamiltonian will transform as

$$H_{NL}\sim F^{(\frac{1}{2}-\frac{1}{2}\,1)}+\epsilon[D^{(\frac{1}{2}-\frac{1}{2}\,1)}-(2/9)(\sqrt{5})(R^{(\frac{3}{2}-\frac{1}{2}\,1)} +(1/\sqrt{5})R^{(\frac{1}{2}-\frac{1}{2}\,1)})]\,,$$

where ϵ is proportional to $(\mu^2-5/3)$.

The parameter μ, of course, represents the D/F ratio for the Cabibbo current. It is important to observe that the normalization for the D-type operator is different from that in Ref. 3. In fact, with this normalization the D/F ratio obtained from the semileptonic decay processes would not be about $\sqrt{3}$, but it would be about $(5/3)^{1/2}$.[3,14] Therefore, the parameter ϵ is small, and the 27-plet and D-type are suppressed with respect to the F-type term; the Hamiltonian is nearly pure F type.

In terms of explicit $SU(8)$ indices, the Hamiltonian, Eq. (8), will transform as

$$(1/\sqrt{2})(T_3{}^7-T_5{}^3)+(\sqrt{\tfrac{3}{2}})(T_5{}^6-T_6{}^7)+(T_2{}^8-T_4{}^1) +\epsilon[(1/\sqrt{2})(T_3{}^7+T_5{}^3)+(1/\sqrt{6})(T_6{}^7+T_5{}^6) +k(T_2{}^8+T_4{}^1)]\,.$$

The parameter k is introduced for convenience, because $k=1$ corresponds to a pure octet interaction, and $k=\frac{1}{3}$ is the value that arises from the anticommutator.

Assuming that the pseudoscalar mesons belong to an arbitrary linear combination of F- and D-type octets in the **63**, it is not difficult to obtain sum rules for the decay amplitudes. If an octet assignment is assumed, one finds three sum rules for both S waves and P waves. In addition to the current-current relations of $SU(3)$,[13] we have

$$\Delta_S'(\Sigma)=\sqrt{3}\Delta(\Lambda)\,,$$

where

$$\Delta_S'(\Sigma)=\sqrt{2}A(\Sigma_0{}^+)-A(\Sigma_+{}^+)-A(\Sigma_-{}^-)\,.$$

That is, $\Delta_S'(\Sigma)$ is the "pseudo-$\Delta T=\frac{1}{2}$" rule for Σ decays.

[14] W. J. Willis, in Proceedings of the Argonne International Conference on Weak Interactions, 1965 [Argonne National Laboratory Report No. ANL-7130, (unpublished)].

For P waves, a Suzuki-type relation,[15]

$$\Delta_P(\Lambda)=-\Delta_P(\Xi)\,,$$

holds, plus two other relations which are too complicated to be interesting.

The **28** is a particularly interesting assignment for the baryons as far as the nonleptonic predictions are concerned. In this case there are no P-wave predictions, but for the S wave we have

$$\Delta_S(\Lambda)=\Delta_S(\Xi)=\Delta_S'(\Sigma)=0\,,$$
$$\Delta_S(L-S)=(\sqrt{\tfrac{3}{2}})A(\Sigma_+{}^+)\,,$$

with

$$\Delta_S(\Lambda)=A(\Lambda_-{}^0)-\sqrt{2}A(\Lambda_0{}^0)\,,$$
$$\Delta_S(\Xi)=A(\Xi_-{}^-)+\sqrt{2}A(\Xi_0{}^0)\,,$$
$$\Delta_S(L-S)=\sqrt{3}A(\Sigma_0{}^+)+A(\Lambda_-{}^0)-2A(\Xi_-{}^-)\,.$$

Thus, even in the presence of a $\Delta T=\frac{3}{2}$ interaction the $\Delta T=\frac{1}{2}$ relations hold for Λ and Ξ decays. A careful analysis will show that this is true because the 27-plet does not couple to the D part of the meson assignment.

VI. DISCUSSION

The success of the eightfold way together with the apparent lack of evidence for any $SU(3)$ representation of nonzero triality has suggested that the most natural candidate for a higher symmetry is the rotation group in eight dimensions.[16] However, the basic $SU(3)$ structure seems somewhat obscured in this scheme. Also there are operators for which it seems impossible to find a physical interpretation. $SU(8)$ would seem to have certain conceptual advantages over such a scheme.

The basic question is what interpretation can be given to the meaning of $SU(8)$. As mentioned, the existing particle spectrum seems to rule out $SU(8)$ as an invariance group. It has been suggested[3] that $SU(8)$ may be regarded as a noninvariance group for the baryon-pole model. Perhaps $SU(8)$ will prove to have a more general application.

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[15] M. Suzuki, Phys. Rev. **137**, B1062 (1965).

[16] Y. Ne'eman and I. Osvath, Phys. Rev. **138**, B1474 (1965); W. M. Fairbairn, Nucl. Phys. **B1**, 127 (1967).