Second-Order Weak Processes and Weak-Interaction Cutoff*

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The problem of the weak-interaction cutoff Λ has been studied with the universal current-current theory (UFI) and the intermediate-vector-boson (IVB) model, using the Bjorken technique to treat the high-energy behavior and the "hard meson" technique of Weinberg and Schnitzer. With these methods, the most divergent contributions to the second-order weak decay $K_L^0 \to \mu\bar{\mu}$ and K_L^0-K_S^0 mass difference Δm are essentially independent of strong interaction. From $K_L^0 \to \mu\bar{\mu}$ decay, $\Lambda \lesssim 35$–100 BeV, and from Δm, $\Lambda \simeq 3$–4 BeV. With this latter value of Λ, the predicted rate of $K_L^0 \to \mu\bar{\mu}$ is reduced to a value obtained by Bég treating this process as first-order in weak and second-order in electromagnetic interaction. The Bég calculation is redone with the algebra of currents, and the soft-kaon contribution to $K_L^0 \to \mu\bar{\mu}$ is calculated with both the UFI and IVB models. A possible origin of the low value of Λ deduced from Δm is discussed.

1. INTRODUCTION

A MAJOR problem in weak-interaction physics is to decide whether the universal (V,A) current-current interaction represents effectively a phenomenological description of lowest-order weak processes or can be used to calculate higher-order processes in a self-consistent way. As is well known, the weak current-current interaction is badly nonrenormalizable when it is viewed as a field-theoretic four-fermion Lagrangian and leads to lowest-order cross sections which exceed the unitary limit at energies in excess of $\Lambda \simeq 350$ BeV (in the c.m. system). The quantity Λ has been called the "weak-interaction cutoff" and, clearly, if it attained its "unitarity" value, higher-order weak processes would be expected to be comparable to lowest-order weak processes. This last observation is contrary to experience since the lowest-order calculations of weak processes, where applicable, seem to provide a very good description of the physical world. In some sense, therefore, the weak-interaction cutoff Λ must be smaller than its unitarity limit, and it is perhaps not premature to attempt estimates of Λ from the available experimental data.

Some years ago, Ioffe undertook a program[1] to derive upper limits on Λ from a consideration of higher-order purely-leptonic weak processes (e.g., $\mu \to e+\gamma$, $\mu \to 3e$) which are forbidden in lowest order of the usual current-current interaction and are free from strong-interaction complications. However, the need to assign different lepton numbers to the (ν_e,e) and (ν_μ,μ) lepton pairs (because of the discovery that $\nu_e \neq \nu_\mu$) forbids processes like $\mu \to e+\gamma$ and $\mu \to 3e$ to *all* orders and eliminates the study of higher-order purely leptonic processes as a practical source of information concerning Λ. If we wish to gain any insight into the nature of the weak-interaction cutoff Λ, we must turn to semileptonic and nonleptonic weak processes and attempt to isolate effects which are essentially independent of the strong interaction. Processes suitable for investigation in this respect are "second-order" neutral decays like $K_L^0 \to \mu\bar{\mu}$ and the "second-order" $K_L^0 - K_S^0$ mass difference. The main object of this paper is to derive estimates of the weak cutoff Λ from a study of the decay $K_L^0 \to \mu\bar{\mu}$ and the $K_L^0 - K_S^0$ mass difference.

In Sec. 2, we study the process $K_L^0 \to \mu\bar{\mu}$ in detail using techniques of current algebra and the Bjorken[2] limit. For completeness we review the work of Ioffe,[3] who uses the intermediate-vector-boson (IVB) hypothesis. We then calculate the most divergent contribution to the decay rate for $K_L^0 \to \mu\bar{\mu}$ with the universal (V,A) current-current interaction (UFI) and employ the hard-meson technique of Schnitzer and Weinberg[4] to study a 3-point function that emerges in this study. From these results we derive rather high-upper limits on Λ. It is also possible to apply the "soft-kaon" technique and Weinberg sum rules to derive a low-energy theorem for the $K_L^0 \to \mu\bar{\mu}$ amplitude, which is then free of divergences; estimates for the "soft-kaon" decay rate can then be obtained with both the UFI and IVB models. Finally, we recall that the decay $K_L^0 \to \mu\bar{\mu}$ can proceed through a combination of "first-order" weak and second-order electromagnetic interaction and a current-algebra calculation substantiates the earlier work of Bég.[5]

In Sec. 3, we investigate the $K_L^0 - K_S^0$ mass difference and, using the Bjorken technique to determine the high-energy behavior and certain additional approximations to permit an evaluation of the most divergent contribution, we derive an actual (and appreciably lower) value for the weak cutoff Λ. This value of Λ depends rather insensitively on whether the UFI or IVB model is used. We conclude Sec. 3 with some brief comments on the recent calculations by Biswas and Smith[6] and Cosslett[6] of the $K_L^0 - K_S^0$ mass difference, using the "soft-kaon" technique. Finally, in Sec. 4, we review our results and

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[1] B. L. Ioffe, Zh. Eksperim. i Teor. Fiz. **38**, 1608 (1960) [English transl.: Soviet Phys.—JETP **11**, 1158 (1960)]; J. Nilsson and R. E. Marshak, in Proceedings of CERN Conference on Very High Energy Processes, 1961, p. 29 (unpublished); A. Pais, *Trieste Lectures* (International Atomic Energy Agency, Vienna, 1962), p. 591.

[2] J. D. Bjorken, Phys. Rev. **148**, 1467 (1966).

[3] B. L. Ioffe, in *Proceedings of the International Conference on Particles and Fields, Rochester, 1967* (John Wiley & Sons, Inc., New York, 1968), p. 447.

[4] H. Schnitzer and S. Weinberg, Phys. Rev. **164**, 1828 (1966).

[5] M. A. B. Bég, Phys. Rev. **132**, 426 (1963).

[6] S. N. Biswas and J. Smith, Phys. Rev. Letters **19**, 729 (1967); S. R. Cosslett, *ibid.* **20**, 634 (1968).

remark on the possible meaning of the weak-interaction cutoff derived from the $K_L^0-K_S^0$ mass difference. A preliminary version of our work has been reported.[7]

2. DECAY $K_L^0 \to \mu\bar{\mu}$

The S matrix for the decay process $K_L^0 \to \mu\bar{\mu}$ receives contributions from the diagrams given in Figs. 1 and 2. Figure 1 represents the $K_L^0 \to \mu\bar{\mu}$ decay as first order in the weak and second order in the electromagnetic interaction. This mechanism was considered by Bég[5] and is reconsidered here in Sec. 2 D using the algebra of currents. Figures 2(a) and 2(b) represent the decay $K_L^0 \to \mu\bar{\mu}$ as second order in the weak interaction, Fig. 2(a) on the basis of the UFI model and Fig. 2(b) on the basis of the IVB model. These two diagrams yield information concerning Λ and in Secs. 2 A–2 C, we shall restrict our attention to them.

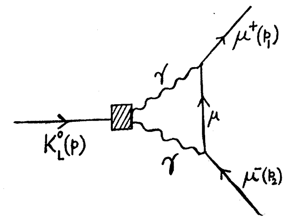

FIG. 1. First-order weak and second-order electromagnetic contribution to the decay $K_L^0 \to \mu\bar{\mu}$.

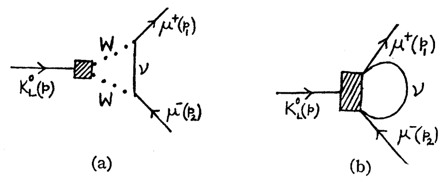

FIG. 2. Second-order weak contributions to the decay $K_L^0 \to \mu\bar{\mu}$.

A. Ioffe's Treatment

Ioffe[3] has used Fig. 2(b), and the matrix element for the process is

$$\langle\mu\bar{\mu}|S|K_L^0\rangle=(2\pi)^4 i\delta^4(p-p_1-p_2)M\,,$$

$$M=2\sin\theta\cos\theta\,(4\pi g^2)^2\left(\frac{m_l^2}{2p_0p_{10}p_{20}V^3}\right)^{1/2}$$
$$\times\int\frac{d^4q}{(2\pi)^4}\frac{(\delta_{\lambda\mu}+q_\lambda q_\mu/m_W^2)}{q^2+m_W^2}$$
$$\times\frac{[\delta_{\sigma\nu}+(q-p)_\sigma(q-p)_\nu/m_W^2]}{(q-p)^2+m_W^2}M_{\mu\nu}(q,p)\bar{u}(p_1)$$
$$\times\gamma_\sigma(\not{q}+\not{p}_1)^{-1}\gamma_\lambda(1+\gamma_5)v(p_2)\,,$$

where

$$M_{\mu\nu}(q,p)=i\int d^4x$$
$$\times e^{iq\cdot x}\langle 0|T\{J_{\mu1}^2(x)J_{\nu3}^1(0)\}|K^0(p)\rangle(2p_0V)^{1/2},\quad(1)$$

where m_W is the IVB mass, m_l the muon mass, and θ the Cabibbo angle, and the rest of the notation is standard. It is easy to see that the matrix element M is divergent and therefore we keep the term with the highest divergence, which is

$$M\simeq 2\sin\theta\cos\theta\frac{(4\pi g^2)^2}{m_W^4}\left(\frac{m_l^2}{2p_0p_{10}p_{20}V^3}\right)^{1/2}\int\frac{d^4q}{(2\pi)^4}$$
$$\times\frac{q_\mu q_\lambda q_\sigma q_\nu}{q^4}\bar{u}(p_1)\gamma_\sigma\not{q}^{-1}\gamma_\lambda(1+\gamma_5)v(p_2)M_{\mu\nu}(q,p)\,.\quad(2)$$

But

$$\lim_{q^2\to\infty}q_\nu M_{\mu\nu}=(2p_0V)^{1/2}i\langle 0|A_{\mu3}^2|K^0\rangle=f_Kp_\mu\,,\quad(3)$$

and using

$$4\pi g^2/m_W^2=G_W/\sqrt{2}\,,$$

$$M\simeq\tfrac{1}{4}G_W^2\sin\theta\cos\theta\,f_K\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\bar{u}(p_1)\not{p}$$
$$\times(1+\gamma_5)v(p_2)\left(\frac{m_l^2}{2p_0p_{10}p_{20}V^3}\right)^{1/2}$$
$$\simeq\frac{G_W^2}{32\pi^2}\sin\theta\cos\theta\,f_K\Lambda^2\bar{u}(p_1)\not{p}$$
$$\times(1+\gamma_5)v(p_2)\left(\frac{m_l^2}{2p_0p_{10}p_{20}V^3}\right)^{1/2}.\quad(4)$$

Comparing this with the observed upper limit[8] for the partial decay rate of $K_L^0 \to \mu\bar{\mu}$ ($\lesssim 1.6\times 10^{-6}$), one finds

$$\Lambda\lesssim 100\text{ BeV}.$$

This is to be compared with the "unitarity" cutoff of 350 BeV. It is interesting that the value of Λ is independent of the IVB mass [cf. Eq. (4)].

B. Use of Bjorken Technique

Here, we use the matrix element for the diagram of Fig. 2(a) which employs the (V,A) current-current pictures; the matrix element is

$$M=\tfrac{1}{2}G_W^2\sin\theta\cos\theta\left(\frac{m_l^2}{2p_0p_{10}p_{20}V^3}\right)^{1/2}\int\frac{d^4q}{(2\pi)^4}$$
$$\times\bar{u}(p_1)\gamma_\mu(\not{q}+\not{p}_1)^{-1}\gamma_\nu(1+\gamma_5)v(p_2)M_{\mu\nu}(q,p)\,,\quad(5)$$

where $M_{\mu\nu}(q,p)$ has been defined earlier in Eq. (1). If we now examine the structure of $M_{\mu\nu}(p,q)$, one is tempted to use the Bjorken technique to study the

[7] R. N. Mohapatra, J. Subba Rao, and R. E. Marshak, Phys. Rev. Letters 20, 1081 (1968).

[8] A. H. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).

highest divergence in the matrix element[9]; neglecting terms of order $(1/q_0^2)$, we find

$$\lim_{q=0,\,q_0\to\infty} M_{\mu\nu}\simeq\frac{(2p_0V)^{1/2}}{q_0}\int d^3x\langle 0|[V_{\mu 1}{}^2(x)+A_{\mu 1}{}^2(x),\; V_{\nu 3}{}^1(0)+A_{\nu 3}{}^1(0)]_{x_0=0}|K^0(p)\rangle \quad (6a)$$

$$=\frac{(2p_0V)^{1/2}}{q_0}\{\langle 0|[V_{i1}{}^2(t),V_{j3}{}^1(0)] -i\delta_{\mu 4}A_{\nu 3}{}^2-i\delta_{\nu 4}A_{\mu 3}{}^2+[V_{i1}{}^2(t),A_{i3}{}^1(0)] -i\delta_{\mu 4}A_{\nu 3}{}^2-i\delta_{\nu 4}A_{\mu 3}{}^2-[V_{i3}{}^1(t),A_{j1}{}^2(0)] +[A_{i1}{}^2(t),A_{j3}{}^1(0)]_{t=0}|K^0\rangle\}. \quad (6b)$$

If we next take account of the commutation relations, Eq. (6b) becomes

$$\lim_{q=0,\,q_0\to\infty} M_{\mu\nu}=\frac{(2p_0V)^{1/2}}{q_0}\langle 0|[ia\epsilon_{ijk}A_{k3}{}^2 +2(i\delta_{\mu 4}A_{\nu 3}{}^2+i\delta_{\nu 4}A_{\mu 3}{}^2)+id\delta_{ij}A_{43}{}^2+id'\delta_{ij}A_{43}{}^2 +ia'\epsilon_{ijk}A_k+\text{vector currents}]|K^0(p)\rangle. \quad (6c)$$

If we finally use Eq. (3) and go to a relativistic frame

$$\left[\text{so that, }\frac{i\delta_{\mu 4}}{q_0}\to-\frac{q_\mu}{q^2},\quad \delta_{ij}\to\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2},\quad \text{etc.}\right],$$

Eq. (6c) reduces to

$$\lim_{q^2\to\infty} M_{\mu\nu}(q,p)=\frac{f_K}{q^2}\left[-2\left(p_\mu q_\nu+p_\nu q_\mu-p\cdot q\frac{q_\mu q_\nu}{q^2}\right) -(d+d')p\cdot q\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)-(a+a')\epsilon_{\mu\nu\sigma\lambda}q_\lambda p_\sigma\right]. \quad (7)$$

In Eqs. (6c) and (7), the coefficients a, a', d, d' result from the commutation relations among the space-space components of the currents as follows:

$$\begin{aligned}
\delta(x_0)[V_{i1}{}^2(x),V_{j3}{}^1(0)]&=\delta^4(x)\{ia\epsilon_{ijk}A_{k3}{}^2(0)+ib\delta_{ij}V_{43}{}^2(0)+\cdots\},\\
\delta(x_0)[V_{i1}{}^2(x),A_{j3}{}^1(0)]&=\delta^4(x)\{ie\epsilon_{ijk}V_{k3}{}^2(0)+id\delta_{ij}A_{43}{}^2(0)+\cdots\}, \quad (8)\\
\delta(x_0)[A_{i1}{}^2(x),A_{j3}{}^1(0)]&=\delta^4(x)\{ia'\epsilon_{ijk}A_{k3}{}^2(0)+ib'\delta_{ij}V_{43}{}^2(0)+\cdots\}.
\end{aligned}$$

The coefficients a, a', d, d' all vanish in the "field-algebra" model,[10] whereas in the current-algebra (quark-type) model[11]

$$a=a'=1\,,\qquad d=d'=-1\,.$$

If we accept the "field-algebra" model, then the most divergent term in M becomes

$$M\simeq 2G_W{}^2\cos\theta\sin\theta\left(\frac{m_l{}^2}{2p_0p_{10}p_{20}V^3}\right)^{1/2}f_K \times\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2}\bar{u}(p_1)p(1+\gamma_5)v(p_2)$$

$$\simeq G_W{}^2\cos\theta\sin\theta\, f_K\frac{\Lambda^2}{4\pi^2}\bar{u}(p_1) \times p(1+\gamma_5)v(p_2)\left(\frac{m_l{}^2}{2p_0p_{10}p_{20}V^3}\right)^{1/2}, \quad (9)$$

and hence $\Lambda\lesssim 36$ BeV. On the other hand, the current-algebra model yields

$$M\simeq\tfrac{5}{8}G_W{}^2\cos\theta\sin\theta\, f_K\frac{\Lambda^2}{4\pi^2}\bar{u}(p_1) \times p(1+\gamma_5)v(p_2)\left(\frac{m_l{}^2}{2p_0p_{10}p_{20}V^3}\right)^{1/2}, \quad (10)$$

and hence $\Lambda\lesssim 45$ BeV. Thus the weak-interaction cutoff within the framework of the UFI approach turns out to be model-dependent, but rather weakly so. We note that the value of Λ found with the UFI model is not the value obtained with the IVB model in the limit $m_W\to\infty$. This is due to the fact that the limiting procedure $q\to\infty$ [which leads to Eq. (2)] and the limit $m_W\to\infty$ are, so to speak, not interchangeable.

C. Hard-Meson Technique

The same matrix element as in Eq. (5) is used, but the hard-meson treatment of Weinberg and Schnitzer[4] is applied to the matrix element $M_{\mu\nu}$. We have four three-point functions in $M_{\mu\nu}$. For convenience, we make several approximations:

(1) We drop the following 3-point functions:

$$\int e^{iq\cdot x}d^4x\langle 0|T\{V_{\mu 1}{}^2(x)V_{\nu 3}{}^1(0)\}|K^0(p)\rangle, \quad (11)$$

$$\int e^{iq\cdot x}d^4x\langle 0|T\{A_{\mu 1}{}^2(x)A_{\nu 3}{}^1(0)\}|K^0(p)\rangle. \quad (12)$$

[9] B. L. Ioffe (Ref. 3) has expanded the spinors in Eq. (5) in the following form: $\bar{u}\gamma_\mu q\gamma_\nu(1+\gamma_5)v=\bar{u}[\gamma_\mu q_\nu+\gamma_\nu q_\mu-q\delta_{\mu\nu}-\epsilon_{\mu\nu\lambda\sigma}q_\lambda\gamma_\sigma]v$, and he throws out the last two terms. On the other hand, we treat all the terms and find that the last two terms of Ioffe are essentially equivalent to our model-dependent terms; in certain models, we can give an exact evaluation of these terms (see text). *Note added in proof*. After this work was submitted for publication, a paper by B. L. Ioffe and E. P. Shabalin {Zh. Eksperim. i Teor. Fiz. Pis'ma v Redaktsiyu **6**, 978 (1967) [English transl.: Soviet Phys.—JETP Letters **6**, 390 (1967)]} arrived, in which this model dependence is recognized and evaluated with the same result.

[10] T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters **18**, 1029 (1967).

[11] M. Gell-Mann, Phys. Letters **8**, 214 (1964); G. Zweig, CERN Report, 1964 (unpublished); Z. Maki, Progr. Theoret. Phys. (Kyoto) **31**, 331 (1964); H. Bacry, J. Nuyts, L. Van Hove, Phys. Letters **9**, 279 (1964). It should be remarked that the independence of the coefficients a, a', d, d' of the particular version of quark model is due to the fact that the unitary indices are different in Eq. (8) [cf. R. E. Marshak, in Proceedings of International Spring School on Elementary Particles, Yalta, 1965, p. 139 (unpublished)]; M. Y. Han and Y. Nambu, Phys. Rev. **139**, B1006 (1965).

The justification is that, as we saw in Sec. 2 B, these terms are model-dependent; since our result was insensitive to the current versus field-algebra model, we work with the latter model, where the contribution of these terms vanishes.

(2) We assume exact $SU(3)$ since it is then possible to take $\partial_\mu V_{\mu 1}{}^3(0)=0$. In principle, we can relate this divergence to a κ-meson field and repeat the whole calculation; but we believe that the results will not be significantly altered.

Thus, using the prescriptions of Ref. 4, we proceed as follows: We have

$$M_{\mu\nu}=\frac{\sqrt{2}}{c_K}(p^2+m_K{}^2)\int d^4x d^4y \times e^{iq\cdot x-ip\cdot y}\langle 0|T\{\partial_\alpha A_{\alpha 2}{}^3(y)J_{\mu 1}{}^2(x)J_{\nu 3}{}^1(0)\}|0\rangle, \quad (13)$$

where we have used kaon PCAC (hypothesis of partially conserved axial-vector current). We now examine a typical matrix element:

$$\int d^4x d^4y\, e^{iq\cdot x-ip\cdot y}\langle 0|T\{\partial_\alpha A_{\alpha 2}{}^3(y)A_{\mu 1}{}^2(x)V_{\nu 3}{}^1(0)\}|0\rangle$$
$$=\frac{c_K G_\rho G_{KA}{}^*}{\sqrt{2}(p^2+m_K{}^2)}\Delta_{\mu\sigma}{}^{KA^*}(q)\Delta_{\nu\lambda}{}^\rho(k)\Gamma_{\sigma\lambda}{}^1(q,p) +\frac{c_K{}^2G_\rho q_\mu\Delta_{\nu\sigma}{}^\rho(k)}{2m_K{}^2(p^2+m_K{}^2)(q^2+m_K{}^2)}\Gamma_\sigma{}^1(q,p), \quad (14a)$$

where the Γ's are vertex functions and

$$\Delta_{\mu\nu}(q)=\left(\delta_{\mu\nu}+\frac{q_\mu q_\nu}{m^2}\right)(q^2+m^2)^{-1}.$$

Using Ward identities and the conservation of currents, one gets the following approximations for the vertex functions:

$$\Gamma_{\sigma\lambda}{}^1(q,p)\simeq f_\pi{}^{-1}[-2m_\rho{}^2\delta_{\sigma\lambda}-(\delta_{\sigma\lambda}q^2-q_\sigma q_\lambda) +(\delta_{\sigma\lambda}k^2-k_\sigma k_\lambda)+\delta(\delta_{\sigma\lambda}p\cdot k-p_\lambda k_\sigma)],$$
$$\Gamma_\sigma{}^1(q,p)\simeq\sqrt{2}f_\pi{}^{-1}m_\rho{}^{-1}[m_\rho{}^2(p+q)_\sigma +\tfrac{1}{4}(1+\delta)\{k^2(p+q)_\sigma-k_\sigma(q^2-p^2)\}]. \quad (14b)$$

In Eqs. (14b), we assume smooth variation of the vertex functions with momenta and use $\delta=-\frac{1}{2}$ as in Ref. 4. Similar computations are repeated for the other 3-point functions and then one finds for the matrix element

$$M_{K_L{}^0\to\mu\bar{\mu}}=\frac{27}{8}\left(\frac{G_W{}^2}{4}\right)\cos\theta\,\sin\theta\,\frac{\Lambda^2}{8\pi^2}f_K \times\left(\frac{m_l{}^2}{2p_0p_{10}p_{20}V^3}\right)^{1/2}\bar{u}(p_1)\not{p}(1+\gamma_5)v(p_2), \quad (14c)$$

whence $\Lambda\lesssim 50$ BeV. It is interesting that the weak cutoff derived by means of the "hard-kaon" technique is so close to the value obtained with the Bjorken technique.

D. Other Calculations of $K_L{}^0\to\mu\bar{\mu}$

In order to interpret the meaning of the weak-cutoff Λ derived from a consideration of the high-energy behavior of the matrix element for $K_L{}^0\to\mu\bar{\mu}$, it is important to evaluate the low-energy contribution. This is done by applying the soft-kaon technique, which involves the assumption that the extrapolation to zero kaon four-momentum of the matrix element is smooth, i.e., $M_{\mu\nu}(q,p)\simeq M_{\mu\nu}(q,0)$. Let us then contract $K^0(p)$ in the definition of $M_{\mu\nu}$ [Eq. (1)] and go to the soft-kaon limit; we obtain

$$M_{\mu\nu}=\frac{\sqrt{2}}{f_K}[\Delta_{\mu\nu}{}^{A\pi}(q)+\Delta_{\mu\nu}{}^{V\pi}(q)-\Delta_{\mu\nu}{}^{AK}(q)-\Delta_{\mu\nu}{}^{VK}(q)],$$

where

$$\Delta_{\mu\nu}{}^{A\pi}(q)=\int e^{iq\cdot x}d^4x\langle 0|T\{A_{\mu 2}{}^1(x)A_{\nu 1}(0)^2\}|0\rangle,$$
$$\Delta_{\mu\nu}{}^{AK}(q)=\int e^{iq\cdot x}d^4x\langle 0|T\{A_{\mu 3}{}^1(x)A_{\nu 1}{}^3(0)\}|0\rangle, \quad (15)$$

and similar expressions for $\Delta_{\mu\nu}{}^{V\pi}$ and $\Delta_{\mu\nu}{}^{VK}$. We must now calculate with Eq. (15), using the UFI and IVB models. To obtain a convergent answer on the UFI picture, we need both Weinberg sum rules[12] (see below) whereas to get a convergent answer on the IVB picture, we need only the first Weinberg sum rule. The sum rules satisfied by the spectral functions are derived on the assumption of asymptotic $SU(3)\otimes SU(3)$, namely,

$$\int\frac{dm^2}{m^2}[\rho_V{}^\pi(m^2)-\rho_A{}^K(m^2)]=f_K{}^2,$$
$$\int\frac{dm^2}{m^2}[\rho_V{}^K(m^2)-\rho_A{}^\pi(m^2)]=f_\pi{}^2; \quad (16)$$

$$\int dm^2[\rho_V{}^K(m^2)-\rho_A{}^\pi(m^2)]=0,$$
$$\int dm^2[\rho_V{}^\pi(m^2)-\rho_A{}^K(m^2)]=0. \quad (17)$$

If Eqs. (16) and (17) are substituted into (15), we find

$$M_{\mu\nu}=\frac{\sqrt{2}}{f_K}(\delta_{\mu\nu}-q_\mu q_\nu/q^2) \times\int\frac{\rho_A{}^\pi(m^2)+\rho_V{}^\pi(m^2)-\rho_A{}^K(m^2)-\rho_V{}^K(m^2)dm^2}{q^2+m^2}, \quad (18)$$

which does not contain any term involving kaon momenta, since we have gone to the soft-kaon limit. This is

[12] S. Weinberg, Phys. Rev. Letters **18**, 507 (1967); T. Das, V. S. Mathur, and S. Okubo, *ibid.* **18**, 761 (1967).

to be compared with Eqs. (3) and (7), which contain the "hard-kaon" contribution and hence terms multiplied by the kaon momenta. We next saturate the spectral functions in (18) by the low-lying poles $A_1(1020)$, $\rho(760)$, $K_A{}^*(1320)$, and $K^*(890)$, respectively, and derive the following estimates for the branching ratio $\alpha=\Gamma(K_L{}^0\to\mu\bar{\mu})/\Gamma(K^+\to\mu^+\nu_\mu)$:

UFI with Weinberg second sum rule ($G_\rho{}^2=G_{K^*}{}^2$):

$$\alpha\simeq 2\times 10^{-10};$$

IVB model without Weinberg second sum rule ($G_{K^*}{}^2=\frac{1}{2}G_\rho{}^2$):

$$\alpha\simeq 10^{-11}\ (m_W=10\ \text{BeV}).$$

Before we draw any further conclusions concerning the value of Λ derived from Secs. 2 B and 2 C, we must check the (convergent) value of α predicted by Fig. 1, which is first order in the weak and second order in the electromagnetic interaction. We consider the S-matrix element from Fig. 1 as follows:

$$\langle\mu\bar{\mu}|S|K_L{}^0\rangle=(2\pi)^4\delta^4(p-p_1-p_2)M(Q^2),\quad Q=p_1+p_2. \tag{19}$$

We next write an unsubtracted dispersion relation for M in the Q^2 variable and evaluate it using the pion pole. Thus following Bég's procedure closely, we write

$$\Gamma(K_L{}^0\to\mu\bar{\mu})=\frac{1}{8\pi}(m_K{}^2-4m_\mu{}^2)^{1/2}\times\left|\frac{f_{K_L{}^0\pi^0}F_{\pi\mu\bar{\mu}}(m_\pi{}^2)}{m_K{}^2-m_\pi{}^2}\right|^2, \tag{20a}$$

where

$$f_{K_L{}^0\pi^0}=(4p_0q_0V^2)^{1/2}\langle K_L{}^0(p)|H_W|\pi^0(q)\rangle, \tag{20b}$$

$$\langle\pi^0|\mu\bar{\mu}\rangle=-\left(\frac{m_l{}^2}{2p_0p_{10}p_{20}V^3}\right)^{1/2}F_{\pi\mu\bar{\mu}}(Q^2)\times\bar{u}(p_1)\gamma_5 v(p_2). \tag{20c}$$

To evaluate $f_{K_L{}^0\pi^0}$, we use current algebra and Weinberg sum rules for asymptotic $SU(3)\otimes SU(3)$ [Eqs. (16) and (17)] within the framework of the IVB picture, viz.,

$$H_W=\int d^4x\ \Delta_{\mu\nu}(x)T(J_{\mu1}{}^3(x)J_{\nu2}{}^1(0)). \tag{21}$$

Inserting Eq. (21) into (20b), we obtain

$$\langle K_L{}^0|H_W|\pi^0\rangle=\int d^4x\ e^{iq\cdot x}\int d^4q\ \Delta_{\mu\nu}(q)\times\langle K_L{}^0(p)|T(J_{\mu1}{}^3(x)J_{\nu2}{}^1(0))|\pi^0(k)\rangle. \tag{22}$$

One now contracts both kaon and pion and discards σ terms by the usual argument of Weinberg; going to the soft-kaon and soft-pion limit, one finds

$$(4p_0q_0V^2)^{1/2}\langle K_L{}^0(p)|H_W|\pi^0(q)\rangle=\frac{2}{f_Kf_\pi}\int d^4q\ \Delta_{\mu\nu}(q)(\delta_{\mu\nu}-q_\mu q_\nu/q^2)\times\int\frac{\rho_A{}^\pi(m^2)+\rho_V{}^\pi(m^2)-\rho_A{}^K(m^2)-\rho_V{}^K(m^2)}{q^2+m^2}dm^2. \tag{23}$$

If one then uses the Weinberg sum rules [Eqs. (16) and (17)], and saturates the spectral functions with low-lying poles, one arrives at

$$|f_{K_L{}^0\pi^0}|\simeq 8\times 10^{-8}M_n{}^2, \tag{24}$$

and, therefore,

$$\alpha\simeq 5\times 10^{-8}. \tag{25}$$

This value is to be compared with Bég's value: $\alpha\simeq 2\times 10^{-8}$. A calculation of $f_{K_L{}^0\pi^0}$ on the basis of the UFI model would lead to a value of α even closer to that of Bég.

3. $K_L{}^0-K_S{}^0$ MASS DIFFERENCE

Since the second-order decay $K_L{}^0\to\mu\bar{\mu}$ only yields an upper limit for the weak-interaction cutoff, it is of great interest to attempt an evaluation of Λ from the second-order $K_L{}^0-K_S{}^0$ mass difference Δm. The complicating feature of the Δm calculation is the occurrence of four hadron currents rather than the two currents required for $K_L{}^0\to\mu\bar{\mu}$ decay. Nevertheless, it is possible, making plausible approximations, to derive a value for Λ with the same combination of techniques.

In the current-current picture[13]

$$\Delta E(K_L{}^0)-\Delta E(K_S{}^0)=-\text{Re}(2\pi)^3i\int d^4x\times\langle K^0|T\{H_W(x)H_W(0)\}|\bar{K}^0\rangle, \tag{26}$$

where

$$H_W(x)=\frac{G_W}{\sqrt{2}}\cos\theta\sin\theta\,[V_{\mu2}{}^1(x)V_{\mu1}{}^3(x)+A_{\mu2}{}^1(x)A_{\mu1}{}^3(x)+V_{\mu2}{}^1(x)A_{\mu1}{}^3(x)+A_{\mu2}{}^1(x)V_{\mu1}{}^3(x)+\text{H.c.}]. \tag{27}$$

We now propose to evaluate (26) in the standard fashion by approximating the four-point functions by means of products of two-point functions and by insertion of the vacuum intermediate state alone.[14] When this is

[13] V. Barger and E. Kazes, Nuovo Cimento **28**, 394 (1963).

[14] Using a generalized Bjorken technique, Dr. P. Olesen has shown that the "Tamm-Dancoff" separation of 4-point functions into 2-point functions and keeping the vacuum intermediate state alone can be justified. He also has shown that the one-particle intermediate states will not make any contribution. We thank Dr. Olesen for communicating his results to us prior to publication.

done, we obtain three types of terms:

$$\text{I:}\quad \langle K^0|H_W(x)|0\rangle\langle 0|H_W(0)|\bar{K}^0\rangle,$$

$$\text{II:}\quad \langle K^0|T\{V_{\mu 2}{}^1(x)V_{\nu 1}{}^3(0)\}|0\rangle,\quad \langle K^0|T\{A_{\mu 2}{}^1(x)A_{\nu 1}{}^3(0)\}|0\rangle,\quad \text{etc.,} \tag{28a}$$

$$\text{III:}\quad \langle K^0|T\{V_{\mu 2}{}^1(x)A_{\nu 1}{}^3(0)\}|0\rangle,\cdots.$$

With regard to type-I terms, one can give an argument on the basis of perturbation theory to show that these terms do not contribute to the $K_L{}^0-K_S{}^0$ mass difference. The argument[15] is as follows: Suppose we write

$$\Delta E \equiv \Delta E(K_L{}^0)-\Delta E(K_S{}^0) = (2\pi)^6\sum_n \delta^3(\mathbf{p}-\mathbf{p}_n)\frac{|\langle n|H_W|\bar{K}^0\rangle|^2}{m_K-E_n}. \tag{28b}$$

If $|n\rangle=|0\rangle$, then

$$\Delta E^{(0)}=(2\pi)^3\frac{|\langle K^0|H_W|0\rangle|^2}{m_K};$$

on the other hand, if we select the intermediate states

$$|n\rangle=|K^0\bar{K}^0\rangle,$$

their contribution is

$$\Delta E^{(K^0\bar{K}^0)}=(2\pi)^3\sum_n \delta^3(\mathbf{p}n)\frac{|\langle K^0|H_W|K^0\bar{K}^0\rangle|^2}{-m_K}. \tag{28c}$$

But

$$\langle K^0|H_W|K^0\bar{K}^0\rangle=\langle 0|H_W|\bar{K}^0\rangle+\text{connected parts},$$

and so

$$\Delta E^{(K^0\bar{K}^0)}=-(2\pi)^3\frac{|\langle K^0|H_W|0\rangle|^2}{m_K}+\text{connected parts}.$$

On adding up these two contributions, one sees that the vacuum contribution cancels. For the reasons given at the beginning of Sec. 2 C, we also drop the terms of type II. We are left with type-III terms, which we recognize as products of terms evaluated in Sec. 2 B by means of the Bjorken technique. Then we find

$$\Delta m \simeq \frac{2.5}{32\pi^2}G_W{}^2\sin^2\theta\cos^2\theta\, f_K{}^2 m_K\Lambda^2, \tag{29}$$

from which we derive

$$\Lambda\simeq 3\text{ BeV}.$$

It is also of interest to calculate Λ from the $K_L{}^0-K_S{}^0$ mass difference on the basis of the IVB model. The calculation proceeds as follows:

$$\Delta E(K_L{}^0)-\Delta E(K_S{}^0)=-\mathrm{Re}(2\pi)^3 i\int d^4x\int d^4y \times\int d^4z(4\pi g^2)^2\sin^2\theta\cos^2\theta\,\Delta_{\mu\nu}{}^B(x-y)\Delta_{\lambda\sigma}{}^B(z) \times\langle K^0|T\{J_{\mu 1}{}^3(x)J_{\nu 2}{}^1(y)J_{\lambda 1}{}^3(z)J_{\sigma 2}(0)^1\}|\bar{K}^0\rangle, \tag{30}$$

where $\Delta_{\mu\nu}{}^B$ and $\Delta_{\lambda\sigma}{}^B$ are IVB propagators. The most divergent term in Eq. (30) is

$$\Delta E(K_L{}^0)-\Delta E(K_S{}^0)=-\frac{(4\pi g^2)^2\sin^2\theta\cos^2\theta}{m_W{}^4(2\pi)^8}\mathrm{Re}(2\pi)^3 i \times\int d^4x d^4y d^4z d^4q d^4q' e^{iq\cdot(x-y)+iq'\cdot z}\frac{q_\mu q_\nu q_\lambda' q_\sigma'}{q^2q'^2} \times\langle K^0(k)|T\{J_{\mu 1}{}^3(x)J_{\nu 2}{}^1(y)J_{\lambda 1}{}^3(z)J_{\sigma 2}{}^1(0)\}|\bar{K}^0(k)\rangle. \tag{31}$$

We next use q_μ and q_λ' to contract the currents and by means of current commutation relations, we get

$$\Delta E(K_L{}^0)-\Delta E(K_S{}^0)=\frac{4(4\pi g^2)^2\sin^2\theta\cos^2\theta}{(2\pi)^8 m_W{}^4}\mathrm{Re}(2\pi)^3 i \times\int\frac{d^4x d^4q d^4q'}{q^2q'^2}q_\nu q_\sigma' e^{i(q-q')\cdot x}\langle K^0(k)|T \times\{J_{\sigma 2}{}^3(x)J_{\nu 2}{}^3(0)\}|\bar{K}^0(k)\rangle. \tag{32}$$

At this stage, we keep only vacuum as the intermediate state and find

$$\Delta m\simeq\frac{m_K f_K{}^2 G_W{}^2\sin^2\theta\cos^2\theta\,\Lambda^2}{32\pi^2}, \tag{33}$$

so that $\Lambda\simeq 4$ BeV.

It is gratifying that the weak cutoff derived from the $K_L{}^0-K_S{}^0$ mass difference on the basis of the IVB model almost coincides with the value of Λ obtained with UFI. While it is true that the IVB calculation has neglected the divergence of both vector and axial-vector currents, this may be justified on the grounds that we are interested only in the high-q^2 limit and in this limit we have asymptotic $SU(3)\otimes SU(3)$ symmetry. Once we neglect the divergence of currents (in the asymptotic limit), we are able to reduce the four-point function Eq. (31) to a two-point function Eq. (32) in a more natural way than in the UFI case.

The low-energy contribution to the $K_L{}^0-K_S{}^0$ mass difference has been calculated by means of the "soft-kaon" technique by several authors.[6] Biswas and Smith[6] find the soft-kaon contribution to Δm in the UFI picture to be appreciable (of the order of $\frac{1}{2}$), but Cosslett[6] has questioned this result. The latter author has repeated the soft-kaon calculation with the IVB model and is unable to recover the UFI result in the limit $m_W\to\infty$. [This difficulty cannot be explained in the same fashion as the difference between the UFI and IVB "hard-kaon" calculations of the decay $K_L{}^0\to\mu\bar{\mu}$ mentioned in Sec. 2 B.] Cosslett[6] does show how to match Δm in the IVB model by a special choice of coupling constants $G_\rho{}^2\simeq\frac{1}{2}G_{K^*}{}^2$; however, this particular choice is arbitrary. In any case, an appreciable "soft-kaon" contribution to Δm would only further reduce the weak cutoff Λ which governs the "hard-kaon" contribution.

[15] The argument is due to Professor S. Okubo to whom we express our thanks.

4. DISCUSSION

We have derived estimates of the weak-interaction cutoff Λ from a consideration of two second-order processes: the decay $K_L^0 \to \mu\bar{\mu}$ and the $K_L^0 - K_S^0$ mass difference. The $K_L^0 \to \mu\bar{\mu}$ decay provides the best upper limit for a semileptonic weak process and we have attempted to calculate both the low- and high-energy contributions to this process: the low-energy contributions by means of a combination of "soft-kaon" technique and current algebra, the high-energy contribution by combining the Bjorken or "hard-kaon" technique with current or field algebra. Both the current-current interaction model and the intermediate-vector-boson model were employed in the calculations. Our results may be summarized in a convenient fashion by writing α (the branching ratio for $K_L^0 \to \mu\bar{\mu}$) in the form

$$\alpha = C + D(\Lambda) m_K^2. \tag{34}$$

In Eq. (34), C is the convergent low-energy contribution to α, and $D(\Lambda)$ is the divergent high-energy contribution depending on Λ. We have found $C \sim 10^{-11}$–10^{-10}, which is extremely small compared to the observed upper limit $\alpha \lesssim 1.6 \times 10^{-6}$, and Λ is in the range $\lesssim 35$–100 BeV when $D(\Lambda)$ is matched to this upper limit. Both C and D are fairly model-independent.

Since the $K_L^0 - K_S^0$ mass difference is the most sensitive measure known of the higher-order weak-interaction effects, we have applied the Bjorken technique to the calculation of Δm in both the UFI and IVB models, despite the fact that the Δm calculation is on shakier ground than the α calculation. The presence of four hadron currents in the expression for Δm greatly complicates the calculation compared to that for α (which only involves two hadron currents), but we believe that the additional assumptions required to make the Δm calculation tractable are reasonable and the insensitivity of the result to the UFI or IVB model is encouraging. The $K_L^0 - K_S^0$ mass difference requires an exceptionally low value of Λ, which is not unexpected in view of the extremely small value of Δm.[16] If this value is inserted into Eq. (9) or (14c) for $K_L^0 \to \mu\bar{\mu}$ decay, the value of the branching ratio is reduced roughly to the Bég value, i.e., $\alpha \simeq 10^{-8}$. This result makes it worthwhile to push down the experimental upper limit on α by at least a factor of 10, since detection of the $K_L^0 \to \mu\bar{\mu}$ decay, say, at the 10^{-6}–10^{-7} level, could only be due to the "second-order" weak interaction (and not to the combined first-order weak second-order electromagnetic interaction) and would imply a larger value of Λ than is derived from Δm.

If we accept the value $\Lambda \sim 3$–4 BeV as a plausible estimate of the weak-interaction cutoff, we would be tempted to measure the effective strength of the weak interaction by the dimensionless coupling constant $(G\Lambda^2) \sim 10^{-4}$—which is indeed small and justifies the procedure of calculating "higher-order" weak processes despite the unrenormalizability of the theory (on either the UFI or IVB model). However, we are now faced with the problem of explaining a weak cutoff as low as 4 BeV. A possible reason was given by Ioffe[3] in the IVB picture, wherein he assumes that the weak cutoff has an electromagnetic origin, and therefore the weak interaction will be cut off by the electromagnetic form factor of the W boson at energies where the electromagnetic interaction becomes strong, i.e., $\Lambda \sim m_W/e$. Ioffe used his estimate of $\Lambda \sim 100$ BeV from $K_L^0 \to \mu\bar{\mu}$ decay and deduced $m_W \sim 10$ BeV. Apart from the failure to explain why "electromagnetic interactions become strong" for W, the above formula yields $m_W \lesssim 0.4$ BeV, when $\Lambda \sim 3$–4 BeV which is manifestly absurd.

In our opinion, a weak cutoff as low as 3–4 BeV could have its origin in a truly "strong" form factor of the W boson, which would then provide a natural cutoff at $\Lambda \sim m_W$. For example, an IVB model of weak interactions has been suggested[17] within the framework of $SU(3)$ symmetry which at the same time allows for a strong quadratic interaction of W's with hadrons. Such a strong quadratic interaction—suitably modified—could account for the large mass of W (i.e., $m_W \sim 3$–4 BeV), provide the automatic cutoff for higher-order weak processes at this energy, which seems to be required to understand the $K_L^0 - K_S^0$ mass difference, and perhaps be responsible for the anomalous production of high-energy muons by short-lived particles observed recently in the cosmic radiation.[18–20]

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[16] For example, a perturbation-theoretic estimate of Λ from the $(K_L^0 - K_S^0)$ mass difference yields $\Lambda \sim 1$–4 BeV [cf. J. Nilsson, Nuovo Cimento **22**, 414 (1961)].

[17] S. V. Pepper, C. Ryan, S. Okubo, and R. E. Marshak, Phys. Rev. **137**, B1259 (1965); C. Ryan, S. Okubo, and R. E. Marshak, Nuovo Cimento **34**, 753 (1964); cf. also, T. Ericson and S. L. Glashow, Phys. Rev. **133**, B130 (1964).

[18] H. E. Bergeson *et al.*, Phys. Rev. Letters **19**, 1487 (1967).

[19] S. Glashow, H. Schnitzer, and S. Weinberg [Phys. Rev. Letters **19**, 205 (1967)] have done a soft-kaon and soft-pion calculation of $K_s^0 \to 2\pi$ decay in the IVB model, in which they obtain a value for the IVB mass, $m_W \sim 8$ BeV, which is close to the value of cutoff obtained in the present paper.

[20] *Note added in proof.* At the Massachusetts Institute of Technology Dedication on 21 March, M. Gell-Mann reported on (unpublished) calculations by M. Gell-Mann, M. Goldberger, N. Kroll, and F. Low on the $K_L^0 - K_S^0$ mass difference similar to those presented in this paper. These workers were apparently led—as were we—to consider as the source of low cutoff a possible quadratic strong interaction of W bosons with hadrons along the lines spelled out in Ref. 17 [and, again, in a recent paper by C. G. Callan, Jr., Phys. Rev. Letters **20**, 809 (1968)]. See also F. Low, Comments Nucl. Part. Phys. **2**, 33 (1968).