The result III is therefore applicable, and we conclude that in this model (i) the parity-conserving amplitudes satisfy the Lee triangle (3.3) and (ii) the two relations (3.6) will hold if and only if the parameters are restricted such that (3.7) is satisfied.

Writing $\xi=\alpha/(1-\alpha)$, $\eta=y/x$, the condition (3.7) gives

$$\xi\eta=-1\,,\quad \xi+\eta=-\tfrac{4}{3}\,, \tag{4.4}$$

for which the solution with positive ξ is

$$\xi=0.54\,,\quad \eta=-1.87 \tag{4.5}$$

to two decimal places.[18] The value for ξ is in good agreement with other determinations[13] of the F/D ratio at the $B\bar{B}M$ vertex. If one neglects the t-channel meson exchange, η becomes the actual F/D ratio at the spurion vertex. The fact that the above value for η is not drastically different from estimates by other authors[14–17] for this ratio indicates that the meson exchange is not a major contribution.

Finally, we remark that since the question of whether or not conditions such as (3.7) are satisfied is completely determined by the actual values of parameters like F/D ratios, one may conclude that within $SU(3)$, relations of the type (3.6) must be of purely dynamical origin. The Lee relation (3.3), on the other hand, can be a symmetry effect in that it is a direct consequence of abnormal charge conjugation in models with this property. Since every dynamical model for the parity-conserving decays must include the pole model in some approximation, it seems that a large abnormal component is in any case inevitable.

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Backward ($\theta=180°$) πN Dispersion Relations: Applications to the Interference Model, P and P' Trajectories, and to the Mechanical Form Factors of the Nucleon

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On the basis of Mandelstam analyticity, crossing, and the observed drop of the backward (180°) $\pi^{\pm}p$ differential cross sections with energy, a set of unsubtracted dispersion relations is written for the πN amplitudes $A^{\pm}$, $B^{\pm}$ at fixed $\theta=\pi$. A further application of crossing allows the derivation of separate sum rules on A^- and B^+, which are *not* of the superconvergent variety, and which provide us with information about $\bar{N}N\to\pi\pi$ scattering. In particular, we are able to deduce a value of the spin-flip $f^0(1250)$-N-N coupling constant, which is shown to permit the following observations: (1) The residue function of the P (or P') trajectory in πN scattering changes sign between $t=0$ and $t=m_f^2=1.56$ GeV^2, and (2) universal coupling of the $f^0(1250)$ meson to the gravitational stress-energy density and the knowledge of the aforementioned coupling constant fixes the zero-momentum-transfer values of two of the three mechanical form factors. The values are given in the text. Lastly, we present an extended discussion of the Barger–Cline model within the context of backward dispersion relations.

I. INTRODUCTION

In this paper, we make use of the Mandelstam analyticity, crossing, and the *observed* high-energy behavior of the backward $\pi^{\pm}p$ differential cross sections to derive unsubtracted backward ($\theta=180°$) πN dispersion relations. From these we obtain sum rules on the two invariant amplitudes A^- and B^+. These are *not* superconvergent relations at $u=0$; unlike such relations,[1,2] the sum rules in the present work (1) do not make use of the Regge postulates $\alpha_N(0)$ and/or $\alpha_\Delta(0)<-\frac{1}{2}$, or, equivalently, $\lim_{s\to\infty}sA^{\pm}$, $sB^{\pm}=0$ (the validity of any of these assumptions is at best in doubt[3]) and (2) clearly separate the $I=0$ and $I=1$ contributions in the $N\bar{N}\to\pi\pi$ channel. Thus we are able to estimate (with a fair degree of confidence) the spin-flip coupling of the $f(1250)$ to the nucleon and thence to proceed to the results mentioned in the abstract. An outline of the paper is as follows: Sec. II: derivation of the dispersion relations [Eqs. (35)–(38)] and the sum rules [Eqs. (41) and (42)]; Sec. III: saturation of the sum rules by known resonances; Sec. IV: numerical estimate of the coupling of the $f(1250)$ to $\pi\pi$ and $N\bar{N}$;

[1] D. S. Beder and J. Finkelstein, Phys. Rev. **160**, 1363 (1967).

[2] D. Griffiths and W. Palmer, Phys. Rev. **161**, 1606 (1967).

[3] A. Ashmore, C. J. S. Damerell, W. R. Frisken, R. Rubinstein, J. Orear, D. P. Owen, F. C. Peterson, A. L. Read, D. G. Ryan, and D. H. White, Phys. Rev. Letters **19**, 460 (1967).

Sec. V: implications for the P and P' trajectories; Sec. VI: implications for the mechanical form factors of the nucleon; and Sec. VII: relevance of the present work to the Barger-Cline model.[4] The reader interested in the applications could omit the derivation presented in Sec. II.

II. DERIVATION OF BACKWARD DISPERSION RELATIONS AND SUM RULES

Consider the invariant amplitudes $A^\pm$ and $B^\pm$ of the πN elastic-scattering matrix.[5] These satisfy the crossing relations

$$A^\pm(s,t,u)=\pm A^\pm(u,t,s)\,, \tag{1}$$

$$B^\pm(s,t,u)=\mp B^\pm(u,t,s)\,, \tag{2}$$

or, equivalently,

$$A^\pm(s,\cos\theta_s)=\pm A^\pm(u,\cos\theta_u)\,, \tag{3}$$

$$B^\pm(s,\cos\theta_s)=\mp B(u,\cos\theta_u)\,, \tag{4}$$

where

$$\cos\theta_s=1+t/2k_s^2\,, \tag{5}$$

$$\cos\theta_u=1+t/2k_u^2\,, \tag{6}$$

and

$$\begin{aligned}\Sigma&=2M^2+2\mu^2\,,\\ 4k_s^2&=s-\Sigma+r^2/s\,,\\ 4k_u^2&=u-\Sigma+r^2/u\,,\\ r&=M^2-\mu^2\,.\end{aligned} \tag{7}$$

M is the mass of the nucleon, and μ is the mass of the pion.

At 180° in the c.m. system of the s channel, we have

$$\cos\theta_s=-1\,. \tag{8}$$

As a consequence of (8), the usual kinematic relations then provide us with the following:

$$u=r^2/s\,, \tag{9}$$

$$t=\Sigma-s-r^2/s\,, \tag{10}$$

$$\cos\theta_u=-1\,. \tag{11}$$

In the kinematical configuration specified by (8)–(11), the crossing relations (3) and (4) become

$$A^\pm(s,-1)=\pm A^\pm(r^2/s,-1)\,, \tag{12}$$

$$B^\pm(s,-1)=\mp B^\pm(r^2/s,-1)\,. \tag{13}$$

It is especially the kinematical curiosity

$$\cos\theta_s=-1\leftrightarrow\cos\theta_u=-1$$

(untrue in the equal-mass case) which makes (12) and (13) and the subsequent analysis possible.

We now make the following two assumptions: (I) $\lim_{s\to\infty}s^\eta A^\pm(s,-1)=0$ for some $\eta>0$ with a similar equation for $B^\pm(s,-1)$; and (II) $A^\pm(s,-1)$, $B^\pm(s,-1)$ contain only those s-plane singularities imposed by the Mandelstam representation.

Assumption I is plausible either from (a) the observation that $\pi^\pm p$ differential cross sections in the backward direction $[\propto|A^+\pm A^-+M(B^+\pm B^-)|^2$ as $s\to\infty]$ appear to fall according to some power law[3] or from (b) the Freedman-Wang conclusion[6] that A, $B\propto_{s\to\infty}s^{\alpha(0)-1/2}$ at $\theta=\pi$ [$\alpha(\sqrt{u})$ is the leading u-channel trajectory], coupled with the indication that $\alpha(0)<\frac{1}{2}$ for every $Y=1$, $B=1$ trajectory.[3,4,7] Note that we do *not* require $sA^\pm(s,-1)$, $sB^\pm(s,-1)\to0$ [or, equivalently, $\alpha(0)<-\frac{1}{2}$ for all trajectories], as would be the case in order to obtain superconvergence relations.[1,2] The indications, in fact, are that $\alpha_\Delta(0)$, $\alpha_N(0)>-\frac{1}{2}$.[3]

Using assumption II, we may catalog the complex s-plane singularities of A, B at fixed $\cos\theta_s=-1$ as follows:

(i) unitary cut, $s\geq(M+\mu)^2$;
(ii) u-channel cut, $0\leq s\leq(M-\mu)^2$;
(iii) t-channel cuts, $4\mu^2\leq t\leq4M^2\to$ circle $|s|=r$
$t>4M^2\to-\infty<s\leq0$;
(iv) poles in the B amplitudes at $s=M^2$, $s=(r/M)^2$.

The Cauchy theorem, assumption I, and the Schwartz reality condition then allow us to write the following *unsubtracted* dispersion relations:

$$\begin{aligned}A^\pm(s)=&\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}\frac{\mathrm{Im}A^\pm(s'+i\epsilon)ds'}{s'-s}\\&+\frac{1}{\pi}\int_0^{(M-\mu)^2}\frac{\mathrm{Im}A^\pm(s'+i\epsilon)ds'}{s'-s}\\&+\frac{1}{\pi}\int_{-\infty}^{0}\frac{\mathrm{Im}A^\pm(s'+i\epsilon)ds'}{s'-s}\\&+\frac{1}{2\pi i}\oint_{|s|=r}\frac{\Delta A^\pm(s')ds'}{s'-s}=I_1+I_2+I_3+I_4\,,\end{aligned} \tag{14}$$

$$\begin{aligned}B^\pm(s)=&\frac{G^2}{M^2-s}\pm\frac{G^2(r^2/M^4)}{(r/M)^2-s}+\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}\frac{\mathrm{Im}B^\pm(s'+i\epsilon)ds'}{s'-s}\\&+\frac{1}{\pi}\int_0^{(M-\mu)^2}\frac{\mathrm{Im}B^\pm(s'+i\epsilon)ds'}{s'-s}\\&+\frac{1}{\pi}\int_{-\infty}^{0}\frac{\mathrm{Im}B^\pm(s'+i\epsilon)}{s'-s}\\&+\frac{1}{2\pi i}\oint_{|s|=r}\frac{\Delta B^\pm(s')ds'}{s'-s}\,.\end{aligned} \tag{15}$$

[4] V. Barger and D. Cline, Phys. Rev. **155**, 1792 (1967).
[5] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).
[6] D. Z. Freedman and J. M. Wang, Phys. Rev. **153**, 1596 (1967).
[7] C. B. Chiu and J. D. Stack, Phys. Rev. **153**, 1575 (1967).

We note briefly the following:

(a) The circle integral is taken counterclockwise, with

$$\Delta A^{\pm}(s')=A^{\pm}((r-\epsilon)e^{i\varphi})-A^{\pm}((r+\epsilon)e^{i\varphi}) = A_{\mathrm{in}}{}^{\pm}-A_{\mathrm{out}}{}^{\pm} \quad (16)$$

and a similar equation for $\Delta B^{\pm}$. Here s' on the circle has been parametrized as

$$s'=re^{i\varphi}, \quad -\pi<\varphi<\pi. \quad (17)$$

(b) The argument $\cos\theta_s=-1$ has been suppressed.

(c) The residue of $B^{\pm}$ at $s=(r/M)^2$ is obtained by letting $s\to(r/M)^2$ in the usual crossed-nucleon pole term $\mp G^2/(M^2-u)$, with $u=r^2/s$. ($G^2/4\pi=14.4$.)

We now proceed to recast these dispersion relations into a useful form. This will be done term by term in Eqs. (14). An exactly similar analysis can be done for Eq. (15), but we shall just state the result in that case.

Integral I_1 of Eq. (14). This integral is over the physical $A^{\pm}$ in the backward direction and remains as is.

Integral I_2 of Eq. (14). Perform a change in the integration variable $s'\to r^2/s'$. The result is

$$I_2=\frac{r^2}{\pi}\int_{(M+\mu)^2}^{\infty}\frac{\mathrm{Im}A^{\pm}((r^2/s')+i\epsilon)ds'}{s'^2[(r^2/s')-s]} = \frac{r^2}{\pi}\int_{(M+\mu)^2}^{\infty}\frac{\mathrm{Im}A^{\pm}(r^2/(s'-i\epsilon'))ds'}{s'(r^2-ss')}, \quad (18)$$

with $\epsilon'=(s'/r)^2\epsilon>0$. The use of the crossing relations (12) then gives

$$\mathrm{Im}A^{\pm}(r^2/(s'-i\epsilon'))=\pm\,\mathrm{Im}A^{\pm}(s'-i\epsilon') = \mp\mathrm{Im}A^{\pm}(s'+i\epsilon'), \quad (19)$$

which, inserted in (18), yields

$$I_2=\mp\frac{r^2}{\pi}\int_{(M+\mu)^2}^{\infty}\frac{\mathrm{Im}A^{\pm}(s'+i\epsilon)ds'}{s'(r^2-ss')}. \quad (20)$$

Integral I_3 of Eq. (14). I_3 may be converted into a dispersion integral over physical scattering ampiltudes for the $N\bar{N}\to\pi\pi$ process.

First, Eq. (10) tells us that for $-\infty<s'<0$, $\cos\theta_s=-1$ (the domain of I_3), t' is a single-valued function of s, varying from $t'_{\min}=4M^2$ when $s=-r$ to $t'_{\max}=\infty$ when $s=-\infty$ or 0. At the same time, the scattering angle in the c.m. system of $\bar{N}(p_1)+N(p_2)\to\pi_\alpha(q_1)+\pi_\beta(q_2)$ is given by[8]

$$\cos\theta_{t'}=\hat{p}_1\cdot\hat{q}_1=\frac{s'-u'}{(t'-4M^2)^{1/2}(t'-4\mu^2)^{1/2}} = \frac{s'-r^2/s'}{[(s'-r^2/s)^2]^{1/2}} \quad (21)$$
$$=-1 \quad \text{for} \quad -\infty<s'<-r$$
$$=+1 \quad \text{for} \quad -r<s'<0.$$

[8] W. Frazer and J. Fulco, Phys. Rev. **117**, 1603 (1960).

[Use has been made of Eq. (10).] Hence the kinematical region corresponds to that of physical $N\bar{N}\to\pi\pi$ scattering in the forward (or backward) direction.

We effect the change of variable $s'\to t'$, which involves solving Eq. (10) for $s'(t')$ in the two regions of s':

Region I $(-\infty<s'<-r)$:
$$s'=-\tfrac{1}{2}\{(t'-\Sigma)+[(t'-\Sigma)^2-4r^2]^{1/2}\};$$
Region II $(-r<s'<0)$:
$$s'=-\tfrac{1}{2}\{(t'-\Sigma)-[(t'-\Sigma)^2-4r^2]^{1/2}\}. \quad (22)$$

Finally, we note from Eq. (10) that in

$$\text{Region I:} \quad s'+i\epsilon\to t'-i\epsilon',$$
$$\text{Region II:} \quad s'+i\epsilon\to t'+i\epsilon', \quad \epsilon,\epsilon'>0. \quad (23)$$

After some algebra, Eqs. (21)–(23) and the substitution rule allow us to express I_3 in Eq. (14) as

$$I_3=-\frac{1}{\pi}\int_{4M^2}^{\infty}\frac{1}{\sigma}\left(\frac{\tau+\sigma}{\tau+\sigma-2s}\right)\times\mathrm{Im}A^{\pm}{}_{N\bar{N}\to\pi\pi}(t'-i\epsilon,\cos\theta_{t'}=-1)$$
$$-\frac{1}{\pi}\int_{4M^2}^{\infty}\frac{1}{\sigma}\left(\frac{\tau-\sigma}{\tau-\sigma-2s}\right)\times\mathrm{Im}A^{\pm}{}_{N\bar{N}\to\pi\pi}(t'+i\epsilon,\cos\theta_{t'}=+1), \quad (24)$$

where

$$\tau=t'-\Sigma,$$
$$\sigma=(\tau^2-4r^2)^{1/2}=(t'-4m^2)^{1/2}(t'-4\mu^2)^{1/2}. \quad (25)$$

Recall also that for the $N\bar{N}\to\pi\pi$ channel[8]

$$A^{+}=6^{-1/2}A^0,$$
$$A^{-}=\tfrac{1}{2}A^1. \quad (26)$$

Since $\cos\theta_t$ is odd under $s\leftrightarrow u$, it follows from Eq. (1) that

$$A^{\pm}(t,\cos\theta_t)=\pm A^{\pm}(t,-\cos\theta_t). \quad (27)$$

Equations (24) and (27) then give rise to the final form of I_3,

$$I_3=-\frac{1}{\pi}\int_{4M^2}^{\infty}\frac{1}{\sigma}\left[\frac{\tau-\sigma}{\tau-\sigma-2s}\mp\frac{\tau+\sigma}{\tau+\sigma-2s}\right]\times\mathrm{Im}A^{\pm}(t'+i\epsilon,1), \quad (28)$$

with σ and τ defined in Eq. (25). The channel description $N\bar{N}\to\pi\pi$ will henceforth be understood if the energy argument is t'.

Integral I_4 of Eq. (14). The identification of $\Delta A^{\pm}(s')$ proceeds as follows: Write a fixed s' dispersion relation for $A^{\pm}(s',t')$,

$$A^{\pm}(s',t')=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{A_t{}^{\pm}(t'',s')dt''}{t''-t'} +\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}\frac{A_u{}^{\pm}(u'',s')du''}{u''-u'}, \quad (29)$$

where $s'+t'+u'=\Sigma$. We wish to calculate the discontinuity of $A^{\pm}(s',t')$ across the circle $|s'|=r$ for $\cos\theta_s'=-1$.

A_t and A_u have singularities only for s' real and negative.[9] It is then easy to see that the discontinuities in $A^{\pm}(s',t')$ for s' complex will come entirely from the vanishing of the denominator $t'-t''$ in Eq. (29).

From Eq. (10) and the parametrization (17) we find the following:

$$\begin{aligned}&0<\varphi<\pi:\\ &\quad s'=(r\pm\epsilon)e^{i\varphi}\rightarrow t'=\Sigma-2r\cos\varphi\mp i\epsilon',\quad \epsilon'>0;\\ &-\pi<\varphi<0:\\ &\quad s'=(r\pm\epsilon)e^{i\varphi}\rightarrow t'=\Sigma-2r\cos\varphi\pm i\epsilon',\quad \epsilon'>0.\end{aligned}\tag{30}$$

Equations (29) and (30) then yield the following expressions for $\Delta A^{\pm}(s')$:

$$\begin{aligned}\Delta A^{\pm}(re^{i\varphi})&=2iA_t(\Sigma-2r\cos\varphi,re^{i\varphi}),\quad 0<\varphi<\pi\\ &=-2iA_t(\Sigma-2r\cos\varphi,re^{i\varphi}),\quad -\pi<\varphi<0.\end{aligned}\tag{31}$$

The substitution of Eq. (31) into (14) [with the parametrization (17) of s'] gives the following expression for I_4:

$$\begin{aligned}I_4=&\frac{ir}{\pi}\int_0^{\pi}\frac{d\varphi\, A_t^{\pm}(\Sigma-2r\cos\varphi,re^{i\varphi})}{r-se^{-i\varphi}}\\ &-\frac{ir}{\pi}\int_{-\pi}^{0}\frac{d\varphi\, A_t^{\pm}(\Sigma-2r\cos\varphi,re^{i\varphi})}{r-se^{-i\varphi}}\\ =&-\frac{2r}{\pi}\int_0^{\pi}d\varphi\,\mathrm{Im}\left[\frac{A_t^{\pm}(\Sigma-2r\cos\varphi,re^{i\varphi})}{r-se^{-i\varphi}}\right].\end{aligned}\tag{32}$$

In the last step, we have made use of the reality of s and of the Schwartz property $A_t(t,s^*)=[A_t(t,s)]^*$ for t real.

From the Mandelstam representation and crossing, we obtain[10]

$$A_t^{\pm}(t,s)=\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}ds'\frac{\rho_{st}(s',t)}{s'-s}\pm\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}ds'\frac{\rho_{st}(s',t)}{s'-u}.\tag{33}$$

The substitution of

$$t=\Sigma-2r\cos\varphi,\quad s=re^{i\varphi},\quad u=r^2/s=re^{-i\varphi}$$

into Eq. (33) shows that

$$\begin{aligned}&\mathrm{Im}A_t^{+}(\Sigma-2r\cos\varphi,re^{i\varphi})=0,\\ &\mathrm{Re}A_t^{-}(\Sigma-2r\cos\varphi,re^{i\varphi})=0.\end{aligned}\tag{34a}$$

For the B amplitudes these conditions read

$$\begin{aligned}&\mathrm{Re}B_t^{+}(\Sigma-2r\cos\varphi,re^{i\varphi})=0,\\ &\mathrm{Im}B_t^{-}(\Sigma-2r\cos\varphi,re^{i\varphi})=0.\end{aligned}\tag{34b}$$

Finally, we note that we may use the Legendre expansion for the continuation of $A_t^{\pm}(t,s)$, $B_t^{\pm}(t,s)$ to complex s, since it is easy to verify that $\cos\theta_t=+1$ for $0<\varphi<\pi$.

Collecting all our information, we can write the following four unsubtracted dispersion relations (suppressing $\cos\theta_s=-1$):

$$\begin{aligned}A^{+}(s)=&\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}ds'\,\mathrm{Im}A^{+}(s')\left[\frac{1}{s'-s}-\frac{r^2}{s'(r^2-ss')}\right]-\frac{s}{\pi}\int_{4M^2}^{\infty}\frac{\mathrm{Im}A^{+}(t',1)dt'}{r^2-s(t'-\Sigma)+s^2}\\ &+\frac{2rs}{\pi}\int_0^{\pi}\sin\varphi d\varphi\frac{A_t^{+}(\Sigma-2r\cos\varphi,re^{i\varphi})}{r^2+s^2-2rs\cos\varphi},\end{aligned}\tag{35}$$

$$\begin{aligned}B^{-}(s)=&\frac{G^2}{M^2-s}-\frac{G^2(r^2/M^4)}{(r/M)^2-s}+\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}ds'\,\mathrm{Im}B^{-}(s')\left[\frac{1}{s'-s}-\frac{r^2}{s'(r^2-ss')}\right]-\frac{s}{\pi}\int_{4M^2}^{\infty}\frac{\mathrm{Im}B^{-}(t',1)dt'}{r^2-s(t'-\Sigma)+s^2}\\ &+\frac{2rs}{\pi}\int_0^{\pi}\sin\varphi d\varphi\frac{B_t^{-}(\Sigma-2r\cos\varphi,re^{i\varphi})}{r^2+s^2-2rs\cos\varphi},\end{aligned}\tag{36}$$

$$\begin{aligned}A^{-}(s)=&\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}ds'\,\mathrm{Im}A^{-}(s')\left[\frac{1}{s'-s}+\frac{r^2}{s'(r^2-ss')}\right]-\frac{1}{\pi}\int_{4M^2}^{\infty}\frac{2r^2-s(t'-\Sigma)}{r^2-s(t'-\Sigma)+s^2}\frac{\mathrm{Im}A^{-}(t',1)}{[(t'-4M^2)(t'-4\mu^2)]^{1/2}}dt'\\ &+\frac{2ir}{\pi}\int_0^{\pi}d\varphi\frac{(r-s\cos\varphi)}{r^2+s^2-2rs\cos\varphi}A_t^{-}(\Sigma-2r\cos\varphi,re^{i\varphi}),\end{aligned}\tag{37}$$

[9] See, e.g., S. C. Frautschi and J. D. Walecka, Phys. Rev. **120**, 1486 (1960).

[10] The lower limits should extend only to the boundary of the double spectral function.

$$B^+(s)=\frac{G^2}{M^2-s}+\frac{G^2(r^2/M^4)}{(r/M)^2-s}+\int_{(M+\mu)^2}^{\infty} ds'\,\mathrm{Im}B^+(s')\left[\frac{1}{s'-s}+\frac{r^2}{s'(r^2-ss')}\right]$$

$$-\frac{1}{\pi}\int_{4M^2}^{\infty}\frac{2r^2-s(t'-\Sigma)}{r^2-s(t'-\Sigma)+s^2}\frac{\mathrm{Im}B^+(t',1)}{[(t'-4M^2)(t'-4\mu^2)]^{1/2}}dt'+\frac{2ir}{\pi}\int_0^{\pi} d\varphi\frac{r-s\cos\varphi}{r^2+s^2-2rs\cos\varphi}B_t^+(\Sigma-2r\cos\varphi,re^{i\varphi}). \quad (38)$$

We have made explicit use of Eqs. (34a) and (34b) in writing these relations.

The sum rules of interest emerge when we note that the crossing relations (12) and (13) imply

$$A^-(\pm r,-1)=0, \quad (39)$$

$$B^+(\pm r,-1)=0. \quad (40)$$

Substituting $s=\pm r$ into (37) and (38), we obtain

$$0=\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}\frac{\mathrm{Im}A^-(s',-1)ds'}{s'}-\frac{1}{2\pi}\int_{4M^2}^{\infty}\frac{\mathrm{Im}A^1(t',1)dt'}{[(t'-4M^2)(t'-4\mu^2)]^{1/2}}+\frac{i}{\pi}\int_0^{\pi}d\varphi\,A_t^-(\Sigma-2r\cos\varphi,re^{i\varphi}), \quad (41)$$

$$0=\frac{G^2}{M^2}+\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}\frac{\mathrm{Im}B^+(s',-1)ds'}{s'}-\frac{1}{(\sqrt{6})\pi}\int_{4M^2}^{\infty}\frac{\mathrm{Im}B^0(t',1)dt'}{(t'-4M^2)^{1/2}(t'-4\mu^2)^{1/2}}+\frac{i}{\pi}\int_0^{\pi}d\varphi\,B_t^+(\Sigma-2r\cos\varphi,re^{i\varphi}). \quad (42)$$

The amplitudes $A^1(t',1)$ and $B^0(t',1)$ denote $I=1$ and $I=0$ forward scattering in the $N\bar{N}\rightarrow\pi\pi$ channel with c.m. energy $=\sqrt{t'}$.

The same sum rules may also be obtained by recognizing that as a result of our assumption I and the crossing relations (12) and (13),

$$A^-(0,-1)=0, \quad (43)$$

$$B^+(0,-1)=0, \quad (44)$$

and then substituting $s=0$ in Eqs. (37) and (38).

Finally, we remark that the integrals over φ can be trivially transformed into t' integrals, viz.,

$$\frac{i}{\pi}\int_0^{\pi}d\varphi\,A_t^-(\Sigma-2r\cos\varphi,re^{i\varphi})$$

$$=\frac{i}{\pi}\int_{4\mu^2}^{4M^2}\frac{dt'A_t^-(t',1)}{[(4M^2-t')(t'-4\mu^2)]^{1/2}},$$

with a similar equation for B_t^+. We prefer to use the φ parametrization.

III. SATURATION OF SUM RULES BY KNOWN RESONANCES

A. A^- Sum Rule

Contribution from πN scattering. The contributions from the various resonances to the first integral in Eq. (41) were calculated from the data of Rosenfeld *et al.*,[11] using Breit-Wigner forms. The 33 resonance does not dominate this sum rule, but does make a sizable contribution. The separate contributions of the resonances are given in Table I[12] and the sum total is

$$\frac{1}{\pi}\int_{(M+\mu)^2}^{\infty}\frac{\mathrm{Im}A^-(s',-1)ds'}{s'}=4\pi(-1.77)\text{ GeV}^{-1}. \quad (45)$$

Contributions from $\rho(760)$. The ρ pole contributes to the integral I_4 of Eq. (14). We deduce this contribution from the ρ-pole expression for A^-:

$$A^-(s,t,u)=-\frac{F_2^V(0)}{2M}\frac{\gamma_{\rho\pi\pi}\gamma_{\rho NN}(s-u)}{m_\rho^2-t}, \quad (46)$$

where

$$F_2^V(0)=1.85,$$
$$\gamma_{\rho\pi\pi}=(4\pi\times 2.4)^{1/2}, \quad (47)$$

TABLE I. Contributions of the πN resonances to the A^- and B^+ sum rules [Eqs. (41) and (42)].

Resonance [MeV (I,J^P)]	Contribution to A^- sum rule (GeV^{-1})	Contribution to B^+ sum rule (GeV^{-2})
Nucleon pole	...	+16.36
$1236(\frac{3}{2},\frac{3}{2}^+)$	−1.01	−10.47
$1920(\frac{3}{2},\frac{7}{2}^+)$	−0.25	− 1.40
$2420(\frac{3}{2},\frac{11}{2}^+)$	−0.07	− 0.31
$1525(\frac{1}{2},\frac{3}{2}^-)$	+0.46	− 0.93
$2190(\frac{1}{2},\frac{7}{2}^-)$	+0.28	− 0.32
$1400(\frac{1}{2},\frac{1}{2}^+)$ ("Roper")	−0.60	+ 1.31
$1688(\frac{1}{2},\frac{5}{2}^+)$	−0.60	+ 0.96
High-energy continuum ($\omega_L>5$ GeV)	−0.01	0.00

[11] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **40**, 77 (1968).

[12] The high-energy contribution was estimated using the Regge parametrization of Ref. 7.

and $\gamma_{\rho NN}=\gamma_{\rho\pi\pi}=\gamma_\rho$ if the ρ is universally coupled to the isospin current.[13,14] For our purposes, this is a sufficient approximation; we then obtain from (45) and (29)

$$A_t^-(t,s)=-\pi\frac{F_2^V(0)}{2M}\gamma_\rho^2(2s+t-\Sigma)\delta(t-m_\rho^2),\quad (48)$$

giving the quantity desired in Eq. (41)

$$A_t^-(\Sigma-2r\cos\varphi,re^{i\varphi})=-i\pi\frac{F_2^V(0)}{2M}\gamma_\rho^2 \times\sin\varphi\,\delta\left(\cos\varphi-\frac{\Sigma-m_\rho^2}{2r}\right).\quad (49)$$

Equations (49) and (47) then yield the ρ contribution

$$\frac{i}{\pi}\int A_t^-(\Sigma-2r\cos\varphi,re^{i\varphi})d\varphi =\frac{F_2^V(0)}{2M}\gamma_\rho^2=4\pi(2.36)\text{ GeV}^{-1}.\quad (50)$$

Note, however, that this estimate could be changed to $4\pi(1.74)$ GeV^{-1} by keeping universality but obtaining $\gamma_{\rho\pi\pi}$ from the e^+e^- annihilation data of Ausslender *et al.*[15] (corresponding to $\Gamma_\rho\simeq 95$ MeV), or to $4\pi(2.75)$ GeV^{-1} by deducing $\gamma_{\rho\pi\pi}\gamma_{\rho NN}$ from the assumption that the entire isospin-flip πN amplitude at threshold is given by ρ exchange.[13,16] Other possibilities are also available. We average among the three given here and change the right-hand side of Eq. (50) to read $4\pi(2.28\pm 0.40)$ MeV^{-1}.

The sum rule now takes the form

$$4\pi(-0.51\pm0.40)=\frac{i}{\pi}\int_0^\pi d\varphi\,A_t^{-\prime}(\Sigma-2r\cos\varphi,re^{i\varphi}) +\frac{1}{2\pi}\int_{4M^2}^\infty\frac{\text{Im}A^1(t',1)dt'}{[(t'-4M^2)(t'-4\mu^2)]^{1/2}},\quad (51)$$

where $A^{-\prime}$ denotes all contributions except for the ρ contribution. The error reflects the uncertainty about the ρ.

The continuum part of the second integral is assumed to be small, since $p\bar{p}$-annihilation data in the region 0–3.3 GeV/c indicate only small branching (<0.5%) into $\pi\pi$.[17] The contributions of the higher $I=1$ resonances[11,18,19] to A_t^- (and to the second integral) are, of course, unknown. [We shall discover later some evidence that the contributions of one (or more) of these resonances to B_t^- are not small.] As a rough approximation, we may retain the only state which seems to have sizable coupling to $\pi\pi$, the $g^0(1650)$ [11,19] (with suspected $J^P=3^-$). Equation (51) will then provide a cancelling relation between the two constants characterizing the spin-flip and non-spin-flip coupling of the g^0 to the nucleon, since these both enter into A_t^-.

Keeping this comment in mind for future reference, we turn now to the B^+ sum rule.

B. B^+ Sum Rule

Contributions from N pole and πN scattering. The first term on the right-hand side of (42) is equal to $4\pi(16.36)$ GeV^{-2}. The second term is again saturated with the known resonances (here the 33 makes the dominant contribution; see Table I), giving $4\pi(-11.16)$ GeV^{-2}. Thus the sum rule becomes

$$-\frac{i}{\pi}\int_0^\pi d\varphi\,B_t^+(\Sigma-2r\cos\varphi,re^{i\varphi}) +\frac{1}{\sqrt{6}}\frac{1}{\pi}\int_{4M^2}^\infty\frac{\text{Im}B^0(t',1)dt'}{[(t'-4M^2)(t'-4\mu^2)]^{1/2}} =4\pi(16.36-11.16)=4\pi(5.20)\text{ GeV}^{-2}.\quad (52)$$

Annihilation contribution. The *total* $p\bar{p}\to\pi\pi$ cross section is known to be very small between 0 and 3.3 GeV/c.[17] The only $I=0$ resonant structure suspected[20] (at $M=2380$ MeV) has a small decay rate at least into $p\bar{p}$.[21] This leads us to safely neglect the second integral in (52).

Contribution of resonances with mass <2M. The present data[11] indicate no $I=0$, $G=+$ mesonic activity with sizable coupling to $\pi\pi$ in the region below 1876 MeV except for the $J^P=2^+$ $f^0(1250)$. The 2^+ $f'(1500)$ seems to have only small coupling to $\pi\pi$, while any possible 0^+ σ meson[22] does not contribute to B^+. With a slope of 1 GeV^{-2}, the 4^+ recurrence of the $f^0(1250)$ would have a mass very close to $N\bar{N}$ threshold. There would then be a severe suppression due to the angular-momentum-threshold factor $p_{N\bar{N}}^3$, since the relevant $N\bar{N}$ state is $^3F_4$ (in spectroscopic notation).

f^0 contribution. The coupling of a spin 2^+ meson to the hadrons π and N may be described by the covariant

[13] J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960); Phys. Rev. Letters **17**, 1021 (1966).

[14] M. Gell-Mann and F. Zachariasen, Phys. Rev. **124**, 953 (1961).

[15] V. L. Ausslender *et al.*, Phys. Letters **25B**, 433 (1967).

[16] An equivalent result is obtained from current algebra [S. Weinberg, Phys. Rev. Letters **17**, 616 (1966)] combined with the Kawarabayashi-Suzuki relation $m_\rho=f_\pi f_\rho$.

[17] T. Ferbel *et al.*, Phys. Rev. **143**, 1096 (1966); W. M. Katz, B. Forman, and T. Ferbel, Phys. Rev. Letters **19**, 265 (1967); G. R. Lynch *et al.*, Phys. Rev. **131**, 1287 (1963).

[18] M. N. Focacci *et al.*, Phys. Rev. Letters **17**, 890 (1966).

[19] D. J. Crennell *et al.*, Phys. Rev. Letters **18**, 323 (1967).

[20] R. J. Abrams *et al.*, Phys. Rev. Letters **18**, 1209 (1967).

[21] The results of Ref. 20 show that $(2J+1)x=0.8$, where J is the spin of resonance, $x=\Gamma_{\text{el}}/\Gamma_{\text{tot}}$. If we construct a linear Chew-Frautschi plot for the f^0 trajectory with a slope$=1$ GeV^{-2}, we discover that at $t=(2360\text{ MeV})^2$, Re $\alpha=6$. From the data above we find $x=0.8/13=0.06$.

[22] W. D. Walker, J. Carroll, A. Garfinkel, and B. Y. Oh, Phys. Rev. Letters **18**, 630 (1967); E. Malamud and P. Schlein, *ibid.* **19**, 1056 (1967).

vertex functions[23]

$$\mathcal{L}(f\pi\pi):\quad \delta_{ab}\gamma_{f\pi\pi}2Q_\mu Q_\nu \epsilon^{\mu\nu}(\Delta)\,, \tag{53}$$

$$\mathcal{L}(fNN):\quad \mathbf{1}[\gamma_{fNN}{}^{(1)}\tfrac{1}{2}(P_\mu\gamma_\nu+P_\nu\gamma_\mu) + (\gamma_{fNN}{}^{(2)}/M)P_\mu P_\nu]\epsilon^{\mu\nu}(\Delta)\,, \tag{54}$$

with the momenta and isotopic indices as shown in Fig. 1, and the definitions

$$Q=\tfrac{1}{2}(q_1+q_2)\,,$$
$$P=\tfrac{1}{2}(p_1+p_2)\,,$$
$$\Delta=q_2-q_1\,.$$

1 is the unit matrix in nucleon isotopic space, and $\epsilon^{\mu\nu}(\Delta)$ is the spin-2 wave function.

Using these couplings and the spin-2 projection operator[24]

$$\sum_{\lambda=1}^{5}\epsilon_{\mu\nu}{}^{(\lambda)}(\Delta)\epsilon_{\rho\sigma}{}^{(\lambda)*}(\Delta)=\tfrac{1}{2}(g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}-\tfrac{2}{3}g_{\mu\nu}g_{\rho\sigma})$$
$$-(1/2m^2)(g_{\sigma\nu}\Delta_\mu\Delta_\rho+g_{\nu\rho}\Delta_\mu\Delta_\sigma+g_{\sigma\mu}\Delta_\rho\Delta_\nu+g_{\mu\rho}\Delta_\nu\Delta_\sigma)$$
$$+(1/3m^2)(g_{\mu\nu}\Delta_\rho\Delta_\sigma+g_{\rho\sigma}\Delta_\mu\Delta_\nu)$$
$$+(2/3m^4)\Delta_\mu\Delta_\nu\Delta_\rho\Delta_\sigma\,, \tag{55}$$

we obtain

$$B^+(s,t,u)=\tfrac{1}{2}\gamma_{f\pi\pi}\gamma_{fNN}{}^{(1)}(s-u)/(m_f{}^2-t)\,, \tag{56}$$

whence

$$-\frac{i}{\pi}B_t{}^+(\Sigma-2r\cos\varphi,re^{i\varphi}) =\tfrac{1}{2}\gamma_{f\pi\pi}\gamma_{fNN}{}^{(1)}\sin\varphi\,\delta[\cos\varphi-(\Sigma-m_f{}^2)/2r]\,. \tag{57}$$

For the reasons given above, we saturate the sum rule (52) with (57), obtaining[25]

$$\gamma_{f\pi\pi}\gamma_{fNN}{}^{(1)}/4\pi=+10.4\ \text{GeV}^{-2}. \tag{58}$$

From Eqs. (53) and (55) we may calculate the decay rate $f^0\to\pi\pi$. This turns out to be

$$\Gamma_{f^0\to\pi\pi}=\tfrac{2}{5}(\gamma_{f\pi\pi}{}^2/4\pi)(k^5/m_f{}^2)\,, \tag{59}$$

where $2k=(m_f{}^2-4\mu^2)^{1/2}$. The assignments $\Gamma_{f^0}=120$ MeV, $m_{f^0}=1250$ MeV [11] yield

$$\gamma_{f\pi\pi}{}^2/4\pi=5.32\ \text{GeV}^{-2}. \tag{60}$$

Equation (58) and (60) then combine to give

$$\gamma_{f\pi\pi}=\pm 2.30(4\pi)^{1/2}\ \text{GeV}^{-1}, \tag{61}$$

$$\gamma_{fNN}{}^{(1)}=\pm 4.51(4\pi)^{1/2}\ \text{GeV}^{-1}. \tag{62}$$

We now turn to the applications of this result.

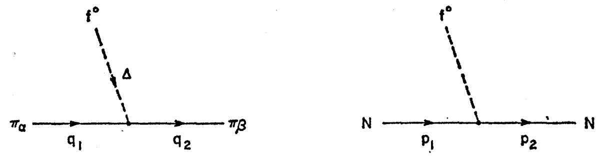

FIG. 1. $f\pi\pi$ and fNN vertices.

IV. APPLICATION TO P AND P' TRAJECTORIES

Equation (56) approximates $B^+(s,t)$ for any s as long as $t\simeq m_f{}^2$. In particular,

$$B^+(s,t)\simeq 2M\gamma_{f\pi\pi}\gamma_{fNN}{}^{(1)}\omega_L/(m_f{}^2-t) \tag{63}$$

(where ω_L is the total pion lab energy) is a good approximation as long as $\omega_L\gtrsim{\sim}4$ GeV and $t\simeq m_f{}^2$.

The Regge form for B^+ corresponding to P (or P') exchange is[26]

$$B^+(\omega_L,t)\underset{\omega_L\gg t}{=}-\pi\alpha(t)\frac{(1+e^{-i\pi\alpha(t)})}{\sin\pi\alpha(t)}\beta(t)\left(\frac{\omega_L}{\omega_0}\right)^{\alpha(t)-1}, \tag{64}$$

with ω_0 as an arbitrary scale factor.

If the $f^0(1250)$ lies on the P (or P') trajectory, then near $t=m_f{}^2$ Eq. (64) becomes

$$B^+(s,t)\underset{\substack{m_f{}^2\simeq t,\\ \alpha\simeq 2}}{\simeq}\frac{4\beta(m_f{}^2)(\omega_L/\omega_0)}{\alpha'(m_f{}^2)(m_f{}^2-t)}, \tag{65}$$

leading [in conjunction with Eq. (63)] to the relationship

$$\beta(m_f{}^2)=\tfrac{1}{2}M\omega_0\gamma_{f\pi\pi}\gamma_{fNN}{}^{(1)}\alpha'(m_f{}^2)=61.4\alpha'(m_f{}^2)\,. \tag{66}$$

The parameter ω_0 has been set equal to 1 GeV and we have, in the last step, made use of Eq. (58).

We compare (66) with some fairly recent experimental fits,[27] which provide us with $\beta(t)$ in a range $-0.6\ \text{GeV}^2<t\leq 0$. In terms of the parametrization of Ref. 27,

$$B^+(\omega_L,t)=-\frac{D_0(1+e^{-i\pi\alpha})}{\sin\pi\alpha}\alpha^2(\alpha+1) \times\exp(D_1t)(\omega_L/\omega_0)^{\alpha-1}, \tag{67}$$

[23] We have omitted terms proportional to $g_{\mu\nu}$, $\Delta_\mu\Delta_\nu$, since these contract to zero with $\epsilon^{\mu\nu}(\Delta)$. Also omitted are the usual wave-function normalization factors $(2E)^{-1/2}$ for each boson, $(M/E)^{1/2}$ for each nucleon.

[24] The spin-2 wave function $\epsilon_{\mu\nu}{}^{(\lambda)}(\Delta)$ is obtained from the rest-frame $J=2$, $J_\rho=\lambda$ wave function via a Lorentz transformation along **Δ**. Also $\epsilon_{\mu\nu}{}^{(\lambda}=\sum_{\lambda'}C(112;\ \lambda-\lambda',\ \lambda')\epsilon_\mu{}^{(\lambda-\lambda')}\epsilon_\nu{}^{(\lambda')}$. For a field-theoretic derivation of the spin-2 propagator, see S. Weinberg, Phys. Rev. **133**, B1318 (1964); S.-J. Chang, Phys. Rev. **148**, 1259 (1966).

[25] A value of +4.6 had previously been obtained via the superconvergence postulate in Ref. 2. However, the authors there had made use of the questionable properties $\lim_{s\to\infty}sA^\pm$, $sB^\pm=0$ (see Ref. 3). In addition, (i) their value varies widely with the value of u in the fixed-u sum rules and (ii) their assumption of 33 saturation is not supported in our A^- sum rule (see Table I). However, we do agree in sign and order of magnitude, and this will be significant in the applications.

[26] V. Singh, Phys. Rev. **129**, 1889 (1963).

[27] C. B. Chiu, R. J. N. Phillips, and W. Rarita, Phys. Rev. **153**, 1485 (1967).

TABLE II. Comparison of $\beta(0)$ (Ref. 27) and $\beta(m_f^2)$ (present work).

Solution (Ref. 27)	If f^0 lies on	$\alpha'(0)$ (GeV^{-2})	$\alpha'(m_f^2)$ (GeV^{-2})	$\alpha(0)$	D_0 (GeV^{-2})	$\beta(0)$ (GeV^{-2})	$\beta(m_f^2)$ (GeV^{-2})
(a)	P	0.34	0.94	1.00	− 70.0	−44.5	+57.7
(a)	P'	0.34	1.20	0.72	−212	−83.4	+73.2
(b)	P	0.23	1.05	1.00	− 9.1	− 5.8	+64.5
(b)	P'	0.93	0.80	0.65	− 23.0	− 7.9	+49.1

one obtains from (64)

$$\beta(0)=(1/\pi)D_0\alpha(0)[\alpha(0)+1]. \qquad (68)$$

In Ref. 27, there are two solutions (*a* and *b*), each of which demands that D_0 be *negative* for both the P and P' trajectories. *Hence Eqs. (66) and (68) indicate that the residue of the P (or P') trajectory changes sign for $0<t<1.56$ GeV^2.* The value of $\beta(0)$ with which to compare $\beta(m_f^2)$ of Eq. (66) now depends on (1) whether the f^0 lies on P or P' and (2) which of the solutions (a) and (b) of Ref. 27 we choose. Also, none of the slopes $\alpha'(0)$ given in Ref. 27 sends its respective trajectories (P, P') through $\alpha=2$ at $t=1.56$ GeV^2; this dictates curvature for $\alpha(t)$ and hence the inequality of $\alpha'(m_f^2)$ [entering Eq. (66)] and $\alpha'(0)$ (provided by the data). We tackle this question first in a crude approximation.

Expand each trajectory in a power series

$$\alpha(t)=\alpha(0)+\alpha'(0)t+\tfrac{1}{2}\alpha''(0)t^2, \qquad (69)$$

with $\alpha(0)$ and $\alpha'(0)$ assumed known from the data. Fix $\alpha''(0)$ by setting $\alpha(m_f^2)=2$. $\alpha'(m_f^2)$ is then computed from (69) and turns out to be

$$\alpha'(m_f^2)=2(2-\alpha(0))/m_f^2-\alpha'(0)$$
$$=1.28(2-\alpha(0))-\alpha'(0)\ \text{GeV}^{-2}. \qquad (70)$$

Now calculate $\beta(m_f^2)$ from (66).

The comparisons are given in Table II. It is clear that $|\beta(m_f^2)-\beta(0)|$ is considerably smaller for the choice of solution (b). There is no clear choice between the P or P' as the trajectory associated with the f^0.

In summary, our determination of $\gamma_{f\pi\pi}\gamma_{fNN}^{(1)}$, the identification of the f^0 as a particle on the P or P' trajectories, and the available Regge fits to $\pi^\pm p$ data near $t=0$ provide us with two conclusions: (1) that $\beta(t)$ changes sign between $t=0$ and $t=m_f^2$ (and thus has a zero) and (2) that solution (b) of Chiu, Phillips, and Rarita[27] is to be preferred to solution (a) on the criterion of minimal variation of $\beta(t)$. We may remark at this point that the Regge fits to $\pi^-p\to\pi^0n$ near $t=0$ [27,28] and to $\pi^+p\to\pi^+p$ near $u=0$ [7] both impose as a condition on their parametrization that $\beta_\rho(t)$ [or $\beta_N(\sqrt{u})$] extrapolate to its coupling constant value for $t=m_\rho^2$ or $\sqrt{u}=m_N$. The analytic form employed to parametrize $\beta(t)$ in Ref. 27 [Eq. (67)], if used to extrapolate β_P or $\beta_{P'}$ to $t=m_f^2$, immensely increases the variation $|\beta(m_f^2)-\beta(0)|$ already present. (See Table II.) It is then suggested, on the basis of the present analysis, that future parametrization of the P, P' residues take account of the sign of $\beta(m_f^2)$ as given in Eq. (6), or at least attempt to minimize the variation $|\beta(m_f^2)-\beta(0)|$. At any rate, we have one more peculiarity associated with this troublesome aspect of Regge theory.

[28] F. Arbab and C. B. Chiu, Phys. Rev. 147, 1045 (1966).

V. APPLICATION TO MECHANICAL FORM FACTORS

An isoscalar, symmetric, rank-2, tensor density $\theta_{\mu\nu}(x)$, satisfying $\partial^\mu\theta_{\mu\nu}(x)=0$, has the matrix elements[29]

$$\langle\pi_\beta q_2|\theta_{\mu\nu}(0)|\pi_\alpha q_1\rangle=[\delta_{ab}/(2q_{10}2q_{20})^{1/2}]$$
$$\times[2G_1^\pi(q^2)Q_\mu Q_\nu+G_2^\pi(q^2)(q^2g_{\mu\nu}-q_\mu q_\nu)] \qquad (71)$$

and

$$\langle Np_2|\theta_{\mu\nu}(0)|Np_1\rangle=(M^2/p_{10}p_{20})^{1/2}\bar{u}(p_2)$$
$$\times[\tfrac{1}{2}G_1^N(q^2)(P_\mu\gamma_\nu+P_\nu\gamma_\mu)+G_2^N(q^2)P_\mu P_\nu$$
$$+G_3^N(q^2)(q^2g_{\mu\nu}-q_\mu q_\nu)]u(p_1) \qquad (72)$$

between pions and nucleons, respectively, where

$$q=p_2-p_1 \quad\text{or}\quad q_2-q_1,$$
$$P=\tfrac{1}{2}(p_2+p_1),$$
$$Q=\tfrac{1}{2}(q_2+q_1).$$

If, moreover, $\theta_{\mu\nu}(x)$ is the energy-momentum stress tensor, then, in particular, $\theta_{00}(x)$ is the Hamiltonian density, and the conditions[29]

$$G_1^\pi(0)=1, \qquad (73)$$

$$G_1^N(0)+G_2^N(0)=1 \qquad (74)$$

are enforced by the equivalence principle.

We now inquire whether one can separately fix $G_1^N(0)$ and $G_2^N(0)$. The answer, under an assumption relevant to this work, is yes. The assumption is that f^0 is universally coupled to $\theta_{\mu\nu}(x)$. The elucidation of universalitity in the present context will follow presently.

If the f^0 is coupled to $\theta_{\mu\nu}$, then we may relate the coupling constants $\gamma_{f\pi\pi}$ and $\gamma_{fNN}^{(i)}$ ($i=1$, 2) [defined in Eqs. (53) and (54)] to G_1^π and G_i^N ($i=1$, 2) in the usual way:

$$\gamma_{f\pi\pi}=\gamma_\pi G_1^\pi(0)=\gamma_\pi, \qquad (75)$$

$$\gamma_{fNN}^{(1)}=\gamma_N G_1^N(0), \qquad (76)$$

$$\gamma_{fNN}^{(2)}=\gamma_N G_2^N(0). \qquad (77)$$

Universality (as with the coupling of the ρ to the isospin current[13,14]) is taken to mean

$$\gamma_\pi=\gamma_N. \qquad (78)$$

This postulate implies [via Eqs. (74)–(77)] the conditions

$$G_1^N(0)=\gamma_{fNN}^{(1)}/\gamma_{f\pi\pi}=1-G_2^N(0) \qquad (79)$$

[29] H. Pagels, Phys. Rev. 144, 1250 (1966), and references therein.

and

$$\gamma_{fNN}^{(1)}+\gamma_{fNN}^{(2)}=\gamma_{f\pi\pi}. \quad (80)$$

Combining (79), (80), (61), and (62) leads to the numerical results

$$G_1^N(0)=1.96\simeq 2, \quad (81)$$

$$G_2^N(0)=0.92\simeq -1, \quad (82)$$

$$\gamma_{fNN}^{(2)}=\mp 2.50(4\pi)^{1/2}\ \text{GeV}^{-1}. \quad (83)$$

The "symmetrical" case $G_1^N(0)=G_2^N(0)=\frac{1}{2}$ would require $\gamma_{FNN}^{(1)}/\gamma_{f\pi\pi}=\frac{1}{2}$, which combines with (60) to give $\gamma_{fNN}^{(1)}\gamma_{f\pi\pi}/4\pi=2.66$, differing considerably from the value 10.4 needed to saturate the sum rule. An obvious check on some of the physics of this section is to verify the relationship (80). This can only be done with some reference to the A^+ amplitude.

As an example of such a test, the contribution of f^0 exchange to the (+) πN scattering-length combination can be calculated using Eqs. (63) and (79). It turns out to be

$$\mu(a_{1/2}+2a_{3/2})$$

$$=\frac{4M\mu^3}{m_f^2(1+\mu/M)}\frac{\gamma_{f\pi\pi}(\gamma_{fNN}^{(1)}+\gamma_{fNN}^{(2)})}{4\pi}$$

$$=0.0057[\gamma_{f\pi\pi}(\gamma_{fNN}^{(1)}+\gamma_{fNN}^{(2)})/4\pi], \quad (84)$$

with the object in square brackets expressed in GeV^{-2}. The result of universality, Eq. (79), gives

$$\mu(a_{1/2}+2a_{3/2})=0.0057\gamma_{f\pi\pi}^2/4\pi=0.03, \quad (85)$$

negligibly perturbing the value zero obtained from vector dominance[13] or current algebra.[30] The significance of this test may be gauged by allowing $\gamma_{fNN}^{(2)}\sim\gamma_{fNN}^{(1)}\sim\pm 4.5(4\pi)^{1/2}\ \text{GeV}^{-1}$ as in Eq. (62). This gives

$$\mu(a_{1/2}+2a_{3/2})\simeq\pm 0.12, \quad (86)$$

far outside the Hamilton-Woolcock value[31] -0.005 ± 0.009.

VI. APPLICATION TO BARGER-CLINE MODEL

The presence of interesting interference patterns in 1–3-GeV πN backward scattering[32] has prompted Barger and Cline[4] to propose an explanatory model for these data. This consists of assuming that the π^-p scattering amplitude in the backward direction,

$$f=(1/8\pi)[(1+r/s)(A^-+A^+)+M(1-r/s)(B^-+B^+)] \quad (87)$$

(where $r=M^2-\mu^2$), is given by a sum of two terms (the "interference model")

$$f=f_{\text{res}}+\tilde{f}, \quad (88)$$

where f_{res} contains all the direct-channel resonances (including the nucleon pole) in Breit-Wigner approximations, and a background (bg) term $\tilde{f}$ is given in this model by Reggeized $N^*(1236)$ exchange, with one parameter, a constant residue function, left to be determined by fitting. The resulting fit to the data is remarkably good. As an application of the present work, we examine whether the backward dispersion relations (35)–(38) can lend support to the real part of $\tilde{f}$ as given by $\text{Re}f_{\text{Regge}}$ in the Barger-Cline Model. (We can, of course, say nothing about $\text{Im}\tilde{f}$.)

One simple comparison can be made as follows: It can be shown that the parametrization of the trajectory $\alpha(\sqrt{u})$ as a function of u alone (one of the assumptions in Ref. 4) leads to the prediction

$$B^{\pi^-p}{}_{\text{Regge}}(s,\cos\theta=-1)=0. \quad (89)$$

Hence we may calculate (as best as we can) $B^{\pi^-p}=B^-+B^+$, using our dispersion relations (36) and (38), and compare with (89). Otherwise stated, the contribution of $\text{Re}\tilde{B}$ to $\text{Re}\tilde{f}$ [as given by Eq. (84)] in the energy region considered should be small compared with $\text{Re}f_{\text{Regge}}$, as predicted by Barger and Cline.

From Eqs. (36) and (38) we may extract the following expression for $\text{Re}\tilde{B}=\text{Re}B_{\text{bg}}=\text{Re}[B_{\pi^-p}-$(direct-channel resonances+nucleon pole)]:

$$\text{Re}\tilde{B}(s)=\frac{r^2}{\pi}\int\frac{\text{Im}B^{I=3/2}(s')ds'}{s'(r^2-ss')}-\frac{2r}{\pi}\int_0^\pi d\varphi\,\text{Im}\left[\frac{B_t^++B_t^-}{r-se^{-i\varphi}}\right]+(N\bar{N}\to\pi\pi), \quad (90)$$

where the last term denotes a dispersion over unknown physical $N\bar{N}\to\pi\pi$ scattering amplitudes.

One can first note, of course, that $\tilde{B}$ cannot be identically zero, as demanded by $\alpha(\sqrt{u})=\alpha(-\sqrt{u})$. Of more practical interest is its value in an important region of the analysis of Ref. 4, namely, at the "2190" dip position, $s=(2.19)^2=4.80\ \text{GeV}^2$.

The first integral (giving the crossed π^+p contributions) is completely dominated by the 33 resonance. It is evaluated using standard procedures and gives

$$\tilde{B}_{N^*}(4.80)=+0.68\ \text{GeV}^{-2}\times 4\pi. \quad (91)$$

In the second integral, the ρ contribution is[33]

$$\tilde{B}_\rho(4.80)=+1.15\ \text{GeV}^{-2}\times 4\pi. \quad (92)$$

The value of $\gamma_{f\pi\pi}\gamma_{fNN}^{(1)}$ deduced in Sec. IV may be used to evaluate B_t^+ and the f^0 contribution to (89):

$$\tilde{B}_{f^0}(4.80)=-0.03\times 4\pi. \quad (93)$$

[30] S. Weinberg, Phys. Rev. Letters **17**, 616 (1966).

[31] J. Hamilton and W. S. Woolcock, Rev. Mod. Phys. **35**, 737 (1963), Eq. (4.49).

[32] S. W. Kormanyos *et al.*, Phys. Rev. Letters **16**, 709 (1966).

[33] This is obtained from $B_t^-=\pi 2G_M f_\rho^2\delta(t-m_\rho^2)$, with $f_\rho^2=4\pi\times 2.4$, $G_M=2.35$.

Finally, consistent with the postulate $\alpha(\sqrt{u}) \simeq\alpha(-\sqrt{u})$ in the region of $u=0$, there is no asymptotic contribution to ImB via the first and third integrals in (90).

The contributions (91)–(93) sum to

$$\mathrm{Re}\tilde{B}(4.80)=+1.80\ \mathrm{GeV}^{-2}\times 4\pi + \text{higher resonant contributions.} \quad (94)$$

The known part of Eq. (94) is a far cry from zero. In fact, from Eq. (84) we see that Re$\tilde{f}$ is itself augmented by $\frac{1}{2}(1+r/s)M\times\mathrm{Re}\tilde{B}/4\pi$, which is

$$\mathrm{Re}\tilde{f}=0.70\ \mathrm{GeV}^{-1} \quad \text{at} \quad 4.80\ \mathrm{GeV}^2. \quad (95)$$

The value of Ref_{Regge} at this point is [via Eqs. (17) and (21) of Ref. 4]

$$\mathrm{Re}f_{\mathrm{Regge}}=+0.07\ \mathrm{GeV}^{-1}. \quad (96)$$

It is therefore clear that agreement with the postulate $\alpha=\alpha(u)$ near $u=0$ will only come about if one or more of the higher boson resonances[11] have substantial coupling to the $\pi\pi$ states *and* to $N\bar{N}$ states in which the helicities are opposite (only "triplet states" enter B). The only higher resonance which has indicated substantial coupling to $\pi\pi$ is the $g^0(1650)$.[11,19] If this removes the discrepancy just discussed, then the seeming saturation of the A^- sum rule [Eq. (51)] is really the result of large cancellations between $N\bar{N}$ helicity flip and nonflip couplings,[34] as mentioned at the end of the discussion of the A^- sum rule in Sec. III. The B^+ sum rule [Eq. (52)] would then still give a good indication of $\gamma_{f\pi\pi}\gamma_{fNN}{}^{(1)}$, since there are no indications yet of other low-lying $I=0$ resonances.[11]

We may also try to evaluate the $\tilde{A}$ piece of Eq. (84) using Eq. (35). We quote the results:

$$\tilde{A}_{N^*}(4.80)=+0.13\times 4\pi\ \mathrm{GeV}^{-1},$$
$$\tilde{A}_{\rho}(4.80)=-0.57\times 4\pi\ \mathrm{GeV}^{-1},$$
$$A_{f^0}(4.80)=+0.78\times 4\pi\ \mathrm{GeV}^{-1}. \quad (97)$$

In the f^0 evaluation, in lieu of any other information, we have made use of the f^0 coupling constants found in Secs. IV and V. The results above, folded into Eq. (84), lead to

$$\mathrm{Re}\tilde{f}=+0.20\ \mathrm{GeV}^{-1} \text{ due to } \tilde{A}$$
$$=+0.70\ \mathrm{GeV}^{-1} \text{ due to } \tilde{B}. \quad (98)$$

These differ greatly from the Ref_{Regge} given in Eq. (95).

We have omitted from the $\tilde{A}$ calculation a consideration of the σ.[22] An estimate of its contribution, commensurate with the bounds on its proposed mass and width,[22] the dispersion fits to NN scattering given by Ball, Scotti, and Wong,[35] and the results of πN partial-wave dispersion relations,[36] shows that it can at most add an entry

$$|\tilde{A}_{\sigma}(4.80)|<0.5\times 4\pi\ \mathrm{GeV}^{-1} \quad (99)$$

to Eq. (98) which has the possibility of cancelling (or doubling, if the sign is as given in Ref. 36) the contribution of Re$\tilde{A}$ to Re$\tilde{f}$ shown in Eq. (98). However, the σ does nothing to the A^- or B^+ sum rules, Eq. (52).

At this stage these numbers should be merely suggestive of the need for coupling to higher resonances in order to obtain the Re$\tilde{f}$ of Barger and Cline (which, irrespective of its origin, gives a good fit to the data).

Expecially interesting is the gross violation by our calculated $\tilde{B}$ of the "$\sqrt{u}$-evenness" condition embodied in Eq. (89). Several alternatives present themselves concerning this point: (1) The $I=1$ $g(1650)$ meson and possibly other $I=1$ mesons[11,18] show large enough coupling to $\pi\pi$ (and perhaps experimentally to $N\bar{N}$ if their mass $>2M$) to allow a cancellation of the $\tilde{B}$ amplitude thus far calculated. In this way, there would be no need to alter our B^+ sum rule. (2) There exist also higher $I=0$ resonances with substantial coupling to $\pi\pi$ and $N\bar{N}$. These would then force a reconsideration of the B^+ sum rule and of the derivation of $\gamma_{fNN}{}^{(1)}$. (3) None of the higher mesonic resonances has substantial couplings to both $\pi\pi$ and to $N\bar{N}$. This latter possibility would cause us to take more seriously our evaluation of $\tilde{B}$. The Regge *model* used by Barger and Cline to obtain $\tilde{f}$ would then need to be reexamined. The over-all question of the sensitivity of their analysis to Re$\tilde{f}$ could also be studied.[37,38]

[34] See Ref. 9 for the helicity expansions of A and B.

[35] J. S. Ball, A. Scotti, and D. Y. Wong, Phys. Rev. **142**, 1000 (1966).

[36] J. Hamilton, P. Menotti, G. C. Oades, and L. L. J. Vick, Phys. Rev. **128**, 1881 (1962).

[37] Most of the cancellation near the dip is between Imf_{res} and Im$\tilde{f}_{\mathrm{Regge}}$, since Re$f_{\mathrm{res}}\simeq 0$ at the resonance.

[38] C. B. Chiu and A. V. Stirling, Phys. Letters **26B**, 236 (1968).