This can be analyzed according to the previous techniques and exhibits the same singularities and $i\epsilon$ prescriptions in the natural boundary value as (2.1), provided we assume the Feynman $i\epsilon$ prescription as discussed above. Such properties are normally preserved under infinite summation so that Fig. 1 now applies to the whole model theory defined by Fig. 5 and, we suggest, to the complete theory.

Under the normal threshold cut, (5.8) has poles of order up to $(l+1)^{-n+1}$. The normal threshold discontinuity is, using a generalization of (3.2),[4]

$$\mathrm{disc}_N G_n = \rho \sum_{r=1}^{n-1} G_r^{I} G_{n-r}^{II}/(l+1).$$

Thus our behavior near $l=-1$ agrees with unitarity which, in nth order, reads

$$a_I{}^n - a_{II}{}^n = \rho \sum_{r=1}^{n-1} a_I{}^r a_{II}{}^{n-r}.$$

Notice the importance of the two-dimensional nature of the integration in this for it means that the residues at the poles factor into products. This is not the case for analogous calculations for the diagrams of Fig. 4. The residues in that case still contain integrations and this is what gives rise to the GP essential singularity in that case.[12]

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Parity-Conserving Hyperon Nonleptonic Decays in $SU(3)$

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General restrictions on the $SU(3)$ invariants are found such that an experimentally satisfactory one-parameter solution of the parity-conserving nonleptonic hyperon decays is obtained under octet dominance and CP invariance. Both normal and abnormal charge conjugation are considered. The pole model is discussed as an example. In this model the restrictions determine the $B\bar{B}M$ and spurion F/D ratios, for which reasonable values are obtained.

I. INTRODUCTION

It is known that within the framework of strict $SU(3)$ symmetry, the situation with respect to the (p-wave) parity-conserving nonleptonic hyperon decays remains in its pre-$SU(3)$ state of the $\Delta I=\frac{1}{2}$ rule when one applies octet dominance and CP invariance, together with the charge-conjugation properties of the current-current theory of the weak interaction. With these assumptions, one $SU(3)$ restriction is imposed on the parity-violating amplitudes.[1] This is the well-known Lee-Sugawara[2] relation. It is found to be experimentally good for both the parity-violating and the parity-conserving decays, and can be deduced for the parity-conserving decays as well under one or the other of some further symmetry assumptions.[2–4]

In a recent paper,[5] a somewhat general result is obtained on the conditions under which an $SU(3)$ model leads to a one-parameter solution to the parity-violating hyperon nonleptonic decays in agreement with experiment. In particular, it is shown that beyond the above assumptions, it is sufficient to neglect the decuplet contributions in both the $B\pi$ and $B\bar{B}$ channels. It is our purpose in this paper to apply similar considerations to the parity-conserving amplitudes.

We start by finding what can be deduced about the parity-conserving decays under the assumptions of Ref. 5 mentioned above. Two relations among the four independent amplitudes result:

$$2B(\Xi^- \to \Lambda\pi^-) = B(\Lambda \to p\pi^-) + \sqrt{3}B(\Sigma^+ \to p\pi^0), \quad (1.1)$$

$$B(\Sigma^+ \to p\pi^0) = -\sqrt{3}B(\Lambda \to p\pi^-). \quad (1.2)$$

The relation (1.1) is the Lee triangle known to be in agreement with experiment. Relation (1.2) is also well satisfied experimentally.[6]

Using (1.1) and (1.2) together with

$$B(\Sigma^- \to n\pi^-) = 0 \quad (1.3)$$

to characterize the experimental situation, we then state necessary and sufficient conditions on the $SU(3)$ invariants, such that an experimentally satisfactory one-parameter solution is obtained. A special case of

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these conditions, stated in result II.2, is to neglect the **27** in the $B\pi$ channel as well as the previous decuplets.

There is, however, in the case of the parity-conserving decays, good motivation for going beyond this to consider the abnormal charge-conjugation case ($\mathcal{C}=-1$), which, under CP invariance and the current-current picture, becomes possible only as a violation of $SU(3)$. Whether it is considered against exact $SU(3)$ or the current-current interaction (CP cannot be seriously blamed for this), the abnormal component in the parity-conserving decays seems to be important—possibly dominant—both in bootstrap calculations[7] and in the pole model.[8] A further argument[7] for considering the abnormal component is that in its absence $K_1^0 \to 2\pi$ is forbidden,[1,9] in direct conflict with experiment.

The case of abnormal charge conjugation is considered in Sec. III. The Lee relation (1.1) results without further assumptions,[3,4] and we find that (1.2) and (1.3) are satisfied if and only if the invariants A_i of the direct channel satisfy the simple restriction

$$A_{27}=A_{10}=A_{ss}, \qquad (1.4)$$

where A_{ss} is the octet invariant $A(8_s \to 8_s)$. This restriction therefore ensures a one-parameter solution satisfying experiment.

In Sec. IV, we discuss the pole model,[10,11] under a plausible approximation on the mass spectrum,[8,11] using the same formalism as in Secs. II and III. It is straightforward to see that this model is effectively abnormal[12] and the results of Sec. III therefore apply. The Lee triangle holds, and the restriction (1.4) leads to a simple quadratic equation with numerical coefficients (i.e., not involving the decay amplitudes) for the F/D ratio ξ at the $B\bar{B}M$ vertex and to a simultaneous equation ($\xi\eta=-1$) determining the effective F/D ratio η at the spurion vertex. To two decimal places, the solution with positive ξ is

$$\xi=0.54, \quad \eta=-1.87. \qquad (1.5)$$

The value for ξ is in good agreement with the estimate $\xi=0.58$ obtained[13] from data on semileptonic processes. The η value, which includes the effect of the meson exchange in the $B\bar{B}$ channel and becomes the actual F/D ratio at the spurion vertex only if the meson exchange is negligible, is in line with other theoretical estimates.[14–18]

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II. NORMAL CHARGE CONJUGATION

We decompose the isospin eigenamplitudes

$$M_I(B' \to B\pi)$$

for the parity-conserving part of the process $B' \to B\pi$ in terms of the $SU(3)$ invariants T_i of the direct channel, using the assumption that the weak interaction transforms like a unitary octet. Normal charge conjugation[1] means that it transforms like the sixth component of the octet:

$$H_w \sim K_0 + \bar{K}_0. \qquad (2.1)$$

Taking into account all the amplitudes relevant to the nonleptonic hyperon decays, one can write a set of eight equations of the form[19]

$$M_I(B_i\pi) \to B_j = \sum_k C_I{}^{ijk} T_k, \qquad (2.2)$$

where $C_I{}^{ijk}$ are numerical coefficients. This set of equations can be solved for the invariants T_i. It yields a one-parameter solution, together with a relation among the amplitudes M_I. Denoting the unknown parameter by λ, the solution is

$$\begin{aligned}
T_1 &= \lambda-(6/5)\alpha+\tfrac{1}{3}\beta-\tfrac{1}{3}\gamma+\delta-\mu+\nu+\eta,\\
T_{27} &= \lambda+\delta,\\
T_{10} &= \lambda,\\
T_{10^*} &= \lambda-\tfrac{1}{5}\alpha+\tfrac{1}{6}\beta-\tfrac{1}{6}\gamma-\tfrac{1}{2}\mu+\tfrac{3}{2}\nu,\\
T_{ss} &= \lambda-\alpha+\delta,\\
(1/\sqrt{5})T_{sa} &= \tfrac{1}{5}\alpha-\tfrac{1}{6}\beta+\tfrac{1}{6}\gamma,\\
(1/\sqrt{5})T_{as} &= -\tfrac{1}{10}\alpha+\tfrac{1}{12}\beta-\tfrac{1}{12}\gamma-\tfrac{1}{4}\mu+\tfrac{1}{4}\nu,\\
T_{aa} &= \lambda+\tfrac{1}{2}\alpha-(5/12)\beta-\tfrac{1}{12}\gamma-\tfrac{1}{4}\mu+\tfrac{3}{4}\nu,
\end{aligned} \qquad (2.3)$$

where $\alpha, \beta, \cdots, \eta$ are given by

$$\begin{aligned}
\alpha &= (5/\sqrt{6})M_1(\Xi\to\Lambda\pi)+(5/\sqrt{6})M_1(N\to\Lambda\pi),\\
\beta &= 4M_{1/2}(\Lambda\to N\pi)-4M_{1/2}(\Lambda\to\Xi\pi),\\
\gamma &= -6M_1(N\to\Sigma\pi),\\
\delta &= 2M_{3/2}(\Sigma\to N\pi),\\
\mu &= 4M_{1/2}(\Lambda\to N\pi)+4M_{1/2}(\Lambda\to\Xi\pi),\\
\nu &= (\sqrt{6})M_1(\Xi\to\Lambda\pi)-(\sqrt{6})M_1(N\to\Lambda\pi),\\
\eta &= \tfrac{4}{3}(\sqrt{6})M_0(N\to\Sigma\pi).
\end{aligned} \qquad (2.4)$$

The relation, independent of T_i, is

$$\begin{aligned}
(\sqrt{6})M_1(\Xi\to\Lambda\pi)-2M_{1/2}(\Lambda\to\Xi\pi)-\tfrac{1}{2}(\sqrt{6})\\
\times M_1(N\to\Lambda\pi)+M_{1/2}(\Lambda\to N\pi)+\tfrac{3}{2}M_1(N\to\Sigma\pi)\\
-M_{3/2}(\Sigma\to N\pi)-M_{1/2}(\Sigma\to N\pi)=0.
\end{aligned} \qquad (2.5)$$

Now, CP invariance implies that the parity-conserving amplitudes are even under charge conjugation. This

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determines their crossing relations. The $s\leftrightarrow u$ crossing relations (where s and u are, respectively, the direct and crossed $B\pi$ channels) among the isospin eigenamplitudes are

$$\begin{aligned}M_0(N\to\Sigma\pi)&=-\tfrac{1}{3}(\sqrt{6})M_{1/2}(\Sigma\to N\pi)\\&\quad+\tfrac{2}{3}(\sqrt{6})M_{3/2}(\Sigma\to N\pi),\\M_{1/2}(\Lambda\to\Xi\pi)&=\tfrac{1}{2}(\sqrt{6})M_1(\Xi\to\Lambda\pi),\\M_1(N\to\Lambda\pi)&=-\tfrac{1}{3}(\sqrt{6})M_{1/2}(\Lambda\to N\pi),\\M_1(N\to\Sigma\pi)&=\tfrac{2}{3}M_{1/2}(\Sigma\to N\pi)+\tfrac{2}{3}M_{3/2}(\Sigma\to N\pi).\end{aligned}\tag{2.6}$$

One immediately notices that Eqs. (2.6) reduce (2.5) to an identity. It is thus clear that CP invariance gives no restriction on the parity-conserving amplitudes, in contrast to the parity-violating case where (2.5) becomes the Lee relation on using the crossing equations.[5]

Using (2.6), we see from (2.4) that the following relations hold:

$$\beta=-(12/5)\alpha,\quad \nu=\tfrac{1}{2}\mu,\quad \eta=\tfrac{2}{3}\gamma+4\delta.\tag{2.7}$$

These relations reduce the number of independent parameters in (2.3) from eight to five, as is expected from considering the charge-conjugation conditions in the $B\bar{B}$ channel. In fact, on substituting (2.7) into (2.3) and using the $s\leftrightarrow t$ crossing matrix, one directly checks that the $B\bar{B}$ invariants T_i'' satisfy the conditions of unitary-charge conservation:

$$T_{10}''+T_{10^*}''=T_{aa}''=T_{as}''=0.\tag{2.8}$$

(In the octet invariants T_{ij}'', the index j refers to the $B\bar{B}$ state.) The remaining invariants of the t channel are

$$\begin{aligned}T_1''&=8\lambda-\tfrac{1}{2}\alpha+5\delta+\tfrac{3}{8}\mu-\tfrac{1}{4}\gamma,\\T_{27}''&=-(9/10)\alpha+\delta-\tfrac{1}{8}\mu+\tfrac{1}{12}\gamma,\\T_{10}''&=-(9/10)\alpha-\tfrac{1}{8}\mu-\tfrac{1}{4}\gamma,\\T_{ss}''&=(11/10)\alpha+\delta-\tfrac{1}{8}\mu+\tfrac{1}{12}\gamma,\\(1/\sqrt{5})T_{sa}''&=-\tfrac{3}{10}\alpha+\tfrac{1}{8}\mu-\tfrac{1}{12}\gamma.\end{aligned}\tag{2.9}$$

This completes the parametrization of the invariants using octet dominance and CP invariance.

Now, imposing the same conditions that usefully restrict the parity-violating amplitudes[5] to a one-parameter solution in agreement with experiment, we find the following results.

Result II.1

Any $SU(3)$ model with normal charge conjugation, CP invariance, octet dominance, and vanishing decuplet contributions in both the $B\pi$ and $B\bar{B}$ channels gives the following two relations among the parity-conserving amplitudes:

$$2B(\Xi^-\to\Lambda\pi^-)=B(\Lambda\to p\pi^-)+\sqrt{3}B(\Sigma^+\to p\pi^0),\tag{2.10}$$

$$B(\Sigma^+\to p\pi^0)=-\sqrt{3}B(\Lambda\to p\pi^-).\tag{2.11}$$

To see this, note that we have already imposed all but the decuplet conditions. These yield two constraints on the parameters:

$$(9/5)\alpha+\tfrac{1}{2}\gamma+\tfrac{1}{4}\mu=0,\tag{2.12}$$

$$\tfrac{3}{5}\alpha+\tfrac{1}{6}\gamma-\tfrac{1}{4}\mu=0.\tag{2.13}$$

Using (2.4) and (2.6), these can be written

$$\begin{aligned}(\sqrt{6})M_1(\Xi\to\Lambda\pi)-M_{1/2}(\Lambda\to N\pi)-M_{1/2}(\Sigma\to N\pi)&\\-M_{3/2}(\Sigma\to N\pi)&=0,\end{aligned}\tag{2.14}$$

$$\begin{aligned}3M_{1/2}(\Lambda\to N\pi)+M_{1/2}(\Sigma\to N\pi)&\\+M_{3/2}(\Sigma\to N\pi)&=0.\end{aligned}\tag{2.15}$$

Equations (2.14) and (2.15) reduce to (2.10) and (2.11), respectively, on making use of the $\Delta I=\frac{1}{2}$ relations connecting the isospin eigenamplitudes to the observable decay amplitudes $B(Y\to B\pi)$, together with the $\Delta I=\frac{1}{2}$ restriction on the Σ decays. These are

$$\begin{aligned}M_{1/2}(\Lambda\to N\pi)&=-\tfrac{1}{2}(\sqrt{6})B(\Lambda\to p\pi^-),\\M_{1/2}(\Sigma\to N\pi)&=B(\Sigma^+\to n\pi^+)-\tfrac{1}{2}\sqrt{2}B(\Sigma^+\to p\pi^0),\\M_{3/2}(\Sigma\to N\pi)&=-B(\Sigma^-\to n\pi^-),\\M_1(\Xi\to\Lambda\pi)&=-B(\Xi^-\to\Lambda\pi^-),\end{aligned}\tag{2.16}$$

and

$$\begin{aligned}B(\Sigma^+\to n\pi^+)+\sqrt{2}B(\Sigma^+\to p\pi^0)&\\-B(\Sigma^-\to n\pi^-)&=0.\end{aligned}\tag{2.17}$$

We note that both (2.10), which is the Lee relation, and (2.11) are well satisfied experimentally. Thus the conditions of negligible decuplet contributions (with normal charge conjugation) are favorable to both the parity-violating and the parity-conserving amplitudes.

These conditions, however, do not lead to a one-parameter solution for the parity-conserving amplitudes, since we still need one more relation. The following result is a statement on conditions that can ensure a one-parameter solution for the parity-conserving amplitudes under normal charge conjugation.

Result II.2

In an $SU(3)$ model with normal charge conjugation, CP invariance, and octet dominance, the following conditions (i) and (ii) are equivalent:

(i) $T_{10}''=0,\quad T_{27}=T_{10}=T_{10^*},$

(ii) $2B(\Xi^-\to\Lambda\pi^-)=B(\Lambda\to p\pi^-)+\sqrt{3}B(\Sigma^+\to p\pi^0),$
$B(\Sigma^+\to p\pi^0)=-\sqrt{3}B(\Lambda\to p\pi^-),$
$B(\Sigma^-\to n\pi^-)=0.$

The proof of this result, by substitution from the previous equations, is straightforward. We note that the relations (ii) are all well satisfied experimentally, so that the restrictions (i) on the $SU(3)$ invariants for such a model are a good characterization of the experimental situation.

III. ABNORMAL CHARGE CONJUGATION

Abnormal charge conjugation, $\mathcal{C}=-1$, occurs if the weak Hamiltonian transforms like the seventh component of a unitary octet[1]:

$$H_w \sim K - \bar{K}_0. \qquad (3.1)$$

Proceeding in the manner of Sec. II [with the $\Delta Y = -1$ processes acquiring a minus sign in Eqs. (2.2)], we again obtain a parametrization of the group invariants, to be denoted A_i, which simultaneously takes into account the conditions of abnormal charge conjugation, octet dominance, and CP invariance. It turns out that the analysis is similar to the case of the parity-violating amplitudes,[5] and one obtains a three-parameter solution with the Lee relation holding.[3,4] The latter follows from the relation, similar to (2.5),

$$\begin{aligned}(\sqrt{6})M_1(\Xi\to\Lambda\pi)+2M_{1/2}(\Lambda\to\Xi\pi)\\ +\tfrac{1}{2}(\sqrt{6})M_1(N\to\Lambda\pi)+M_{1/2}(\Lambda\to N\pi)\\ -\tfrac{3}{2}M_1(N\to\Sigma\pi)-M_{3/2}(\Sigma\to N\pi)\\ -M_{1/2}(\Sigma\to N\pi)=0,\end{aligned} \qquad (3.2)$$

which, on using the crossing equations and the $\Delta I = \frac{1}{2}$ restrictions, reduces to

$$2B(\Xi^-\to\Lambda\pi^-)=B(\Lambda\to p\pi^-)+\sqrt{3}B(\Sigma^+\to p\pi^0). \qquad (3.3)$$

The three-parameter solution for the direct-channel invariants A_i is

$$\begin{aligned}A_1&=\tfrac{1}{5}\lambda_1-(7/4)\lambda_2-\tfrac{7}{2}\delta,\\ A_{27}&=\tfrac{1}{5}\lambda_1+\tfrac{1}{4}\lambda_2+\tfrac{1}{2}\delta,\\ A_{10}&=\tfrac{1}{5}\lambda_1+\tfrac{1}{4}\lambda_2-\tfrac{1}{2}\delta,\\ A_{10^*}&=\tfrac{1}{5}\lambda_1-\tfrac{3}{4}\lambda_2-\tfrac{1}{2}\delta,\\ A_{ss}&=-\tfrac{4}{5}\lambda_1+\tfrac{1}{4}\lambda_2+\tfrac{1}{2}\delta,\\ A_{sa}&=A_{as}=-\tfrac{1}{4}\sqrt{5}\lambda_2,\\ A_{aa}&=-\tfrac{2}{5}\lambda_1-\tfrac{1}{4}\lambda_2-\tfrac{1}{2}\delta,\end{aligned} \qquad (3.4)$$

where δ is the same parameter as that of Sec. II, and

$$\begin{aligned}\lambda_1&=\tfrac{5}{6}(\sqrt{6})M_1(\Xi\to\Lambda\pi)+(5/3)M_{1/2}(\Lambda\to N\pi),\\ \lambda_2&=4M_{1/2}(\Lambda\to N\pi)-2(\sqrt{6})M_1(\Xi\to\Lambda\pi).\end{aligned} \qquad (3.5)$$

It is now straightforward to use this solution to find the conditions that must be satisfied to reproduce the experimental results in terms of one parameter. We summarize this in the following result, using the experimental conditions (ii) of the result II.2.

Result III

In an $SU(3)$ model with abnormal charge conjugation, CP invariance, and octet dominance, the parity-conserving amplitudes satisfy the Lee triangle. The two relations

$$\begin{aligned}B(\Sigma^+\to p\pi^0)&=-\sqrt{3}B(\Lambda\to p\pi^-),\\ B(\Sigma^-\to n\pi^-)&=0\end{aligned} \qquad (3.6)$$

hold if and only if the invariants A_i satisfy the restriction

$$A_{27}=A_{10}=A_{ss}. \qquad (3.7)$$

Note that (3.7) is a simpler condition than the corresponding one for the case of normal charge conjugation [II.2 (i)] and that it involves only the direct-channel invariants. We find in Sec. IV that (3.7) is applicable to the pole model, leading to two simple algebraic equations that determine the F/D ratios.

IV. POLE MODEL

For the hyperon nonleptonic weak decays, the pole model was first suggested by Feldman, Matthews, and Salam[10] and later employed by several authors either phenomenologically or in connection with current algebras. It consists in approximating the parity-conserving amplitudes by the baryon octet poles in the s and u $B\pi$ channels and the pseudoscalar meson octet pole in the $B\bar{B}$ channel. In order to continue using our formalism, it is convenient to adopt an approximation, used by Lee and Swift,[11] on the baryon mass splitting. For the physical decay process $B_i \to B_j + \pi$ we assume

$$M(B_i)-M(B_j)=\text{const}. \qquad (4.1)$$

This then results in $M(\Lambda)=M(\Sigma)=\frac{1}{2}\{M(\Xi)+M(N)\}$. Besides being a reasonable estimate, considering the other assumptions (such as octet dominance), the approximation (4.1) has two advantages:

(i) It enables us to continue using the $SU(3)$ invariants without complicating the expressions with factors involving the mass ratios and differences.

(ii) It allows us to take implicitly into account the octet meson exchange in the $B\bar{B}$ channel by simply ignoring it. The reason for this[11] is that under (4.1) the contribution of the meson exchange can be absorbed into the antisymmetric component of the spurion vertex.

Representing the effective spurion vertex by $xD+yF$ and the $B\bar{B}M$ vertex by $(1-\alpha)D+\alpha F$ (not to be confused with α of Sec. II), we obtain from the combined contribution of the direct- and crossed-channel baryon poles the following expressions for the invariants:

$$\begin{aligned}A_1&=-(5/3)x(1-\alpha)+2y\alpha,\\ A_{27}&=-\tfrac{1}{3}x(1-\alpha)-y\alpha,\\ A_{10}&=-\tfrac{2}{3}x(1-\alpha)-x\alpha-y(1-\alpha),\\ A_{10^*}&=-\tfrac{2}{3}x(1-\alpha)+x\alpha+y(1-\alpha),\\ A_{ss}&=(13/6)x(1-\alpha)+\tfrac{3}{2}y\alpha,\\ A_{sa}&=A_{as}=\tfrac{1}{2}(\sqrt{5})[x\alpha+y(1-\alpha)],\\ A_{aa}&=\tfrac{5}{6}x(1-\alpha)+\tfrac{3}{2}y\alpha,\end{aligned} \qquad (4.2)$$

where a common multiplicative factor has been dropped.

From (4.2) one can directly check that this is a case of abnormal charge conjugation.[12] In fact, Eqs. (4.2) can be reproduced from Eqs. (2.4) by the following identification[20]:

$$\begin{aligned}\lambda_1&=-\tfrac{5}{2}x(1-\alpha)-\tfrac{5}{2}y\alpha,\\ \lambda_2&=-2x\alpha-2y(1-\alpha),\\ \delta&=\tfrac{1}{3}x(1-\alpha)+x\alpha-y\alpha+y(1-\alpha).\end{aligned} \qquad (4.3)$$

[20] Note that identifying (4.2) with the parametrization of normal charge conjugation leads to the result that all terms must vanish identically. This model therefore exists only in the case of abnormal charge conjugation.

The result III is therefore applicable, and we conclude that in this model (i) the parity-conserving amplitudes satisfy the Lee triangle (3.3) and (ii) the two relations (3.6) will hold if and only if the parameters are restricted such that (3.7) is satisfied.

Writing $\xi=\alpha/(1-\alpha)$, $\eta=y/x$, the condition (3.7) gives

$$\xi\eta=-1\,,\quad \xi+\eta=-\tfrac{4}{3}\,, \tag{4.4}$$

for which the solution with positive ξ is

$$\xi=0.54\,,\quad \eta=-1.87 \tag{4.5}$$

to two decimal places.[18] The value for ξ is in good agreement with other determinations[13] of the F/D ratio at the $B\bar{B}M$ vertex. If one neglects the t-channel meson exchange, η becomes the actual F/D ratio at the spurion vertex. The fact that the above value for η is not drastically different from estimates by other authors[14–17] for this ratio indicates that the meson exchange is not a major contribution.

Finally, we remark that since the question of whether or not conditions such as (3.7) are satisfied is completely determined by the actual values of parameters like F/D ratios, one may conclude that within $SU(3)$, relations of the type (3.6) must be of purely dynamical origin. The Lee relation (3.3), on the other hand, can be a symmetry effect in that it is a direct consequence of abnormal charge conjugation in models with this property. Since every dynamical model for the parity-conserving decays must include the pole model in some approximation, it seems that a large abnormal component is in any case inevitable.

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Backward ($\theta=180°$) πN Dispersion Relations: Applications to the Interference Model, P and P' Trajectories, and to the Mechanical Form Factors of the Nucleon

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On the basis of Mandelstam analyticity, crossing, and the observed drop of the backward (180°) $\pi^{\pm}p$ differential cross sections with energy, a set of unsubtracted dispersion relations is written for the πN amplitudes $A^{\pm}$, $B^{\pm}$ at fixed $\theta=\pi$. A further application of crossing allows the derivation of separate sum rules on A^- and B^+, which are *not* of the superconvergent variety, and which provide us with information about $\bar{N}N\to\pi\pi$ scattering. In particular, we are able to deduce a value of the spin-flip $f^0(1250)$-N-N coupling constant, which is shown to permit the following observations: (1) The residue function of the P (or P') trajectory in πN scattering changes sign between $t=0$ and $t=m_f^2=1.56$ GeV^2, and (2) universal coupling of the $f^0(1250)$ meson to the gravitational stress-energy density and the knowledge of the aforementioned coupling constant fixes the zero-momentum-transfer values of two of the three mechanical form factors. The values are given in the text. Lastly, we present an extended discussion of the Barger–Cline model within the context of backward dispersion relations.

I. INTRODUCTION

In this paper, we make use of the Mandelstam analyticity, crossing, and the *observed* high-energy behavior of the backward $\pi^{\pm}p$ differential cross sections to derive unsubtracted backward ($\theta=180°$) πN dispersion relations. From these we obtain sum rules on the two invariant amplitudes A^- and B^+. These are *not* superconvergent relations at $u=0$; unlike such relations,[1,2] the sum rules in the present work (1) do not make use of the Regge postulates $\alpha_N(0)$ and/or $\alpha_\Delta(0)<-\frac{1}{2}$, or, equivalently, $\lim_{s\to\infty}sA^{\pm}$, $sB^{\pm}=0$ (the validity of any of these assumptions is at best in doubt[3]) and (2) clearly separate the $I=0$ and $I=1$ contributions in the $N\bar{N}\to\pi\pi$ channel. Thus we are able to estimate (with a fair degree of confidence) the spin-flip coupling of the $f(1250)$ to the nucleon and thence to proceed to the results mentioned in the abstract. An outline of the paper is as follows: Sec. II: derivation of the dispersion relations [Eqs. (35)–(38)] and the sum rules [Eqs. (41) and (42)]; Sec. III: saturation of the sum rules by known resonances; Sec. IV: numerical estimate of the coupling of the $f(1250)$ to $\pi\pi$ and $N\bar{N}$;

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