# $K_2^0+p \rightarrow K_1^0+p$ and the Regge Trajectory of the ω Meson*

FREDERICK J. GILMAN
*Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305*
(Received 25 March 1968)

The behavior of the process $K_2^0+p \rightarrow K_1^0+p$ at high energy is shown to be dominated near the forward direction by Reggeized $\omega$ exchange with an intercept $\alpha_\omega(0) \simeq 0.5$. It is shown how a study of the energy dependence and angular distribution in this process will determine the important characteristics of the $\omega$ Regge trajectory and residue functions.

## I. INTRODUCTION

PERHAPS the most impressive success of the Regge-pole theory of high-energy scattering has been the fit of $\rho$ Regge-pole exchange to the $\pi N$ charge-exchange data. In this reaction Reggeized $\rho$ exchange explains three critical features of the data[1]: (1) the energy dependence of the forward amplitude; (2) the shrinkage of the diffraction peak with increasing energy; and (3) the dip at $t \simeq -0.6$ BeV$^2$ due to the vanishing of the spin-flip amplitude. In this and other cases, the Regge-pole model has been most successful in explaining the data when only one, or at most, a few, Regge poles can be exchanged.[2] Working in the other direction, processes where only one or two Regge poles can be exchanged enable us to determine in detail the trajectories and residues of the leading Regge poles.

We propose in this paper that the process $K_2^0+p \rightarrow K_1^0+p$ is dominated by Reggeized $\omega$ exchange and that a study of the energy dependence and angular distribution in this process will allow us to "pick off" the important characteristics of the $\omega$ Regge trajectory in much the same way that study of $\pi^-+p \rightarrow \pi^0+n$ determined the important characteristics of the $\rho$ Regge trajectory. In fact, it appears that the only other important Regge pole contributing to $K_2^0+p \rightarrow K_1^0+p$ is the $\rho$ pole, and we know all its important characteristics precisely from $\pi N$ charge exchange.

A detailed knowledge of the $\omega$ trajectory will also permit us to predict the ratio of the real to imaginary parts of the forward amplitude for $K_2^0+p \rightarrow K_1^0+p$ at high energies. A knowledge of this phase is crucial in experiments to determine the relative phase (of $\eta_{+-}$) of the $CP$-violating amplitude for the decay of neutral $K$ mesons into two pions.[3,4] Conversely, weak-interaction experiments can be used to determine the regeneration phase and thus provide strong restrictions on the possible exchanged Regge trajectories. There is then a rather beautiful overlap of strong and weak interactions in the determination of the energy dependence and phase of the forward amplitude for $K_2^0+p \rightarrow K_1^0+p$. We shall return to this point later after we have investigated the evidence for the dominance of $\omega$ exchange and the energy dependence of the forward amplitude.

## II. REGGEIZED ω EXCHANGE AND $K_2^0+p \rightarrow K_1^0+p$

In the process $K_2^0+p \rightarrow K_1^0+p$ only the exchange of $C=-1$, natural spin-parity, neutral meson trajectories is allowed in the $t$ channel (see Fig. 1). The only known mesons satisfying these criteria are the $\rho$, $\omega$, and $\varphi$ mesons. In the $q\bar{q}$ model of mesons with orbital angular momentum $L=0, 1$, only the neutral $^3S_1$ $(\rho,\omega,\varphi)$ and $^1P_1$ $(B?,H?)$ states have $C=-1$, but the $^1P_1$ states have $J^P=1^+$ and cannot couple to $K_1^0K_2^0$.

Using isospin rotations, the amplitude $A(K_2^0+p \rightarrow K_1^0+p)$ for $K_2^0+p \rightarrow K_1^0+p$ can be related to other $KN$ elastic-scattering amplitudes as follows[5]:

$$\begin{aligned} 2A(K_2^0+p \rightarrow K_1^0+p) &= A(K^0+p \rightarrow K^0+p) - A(\bar{K}^0+p \rightarrow \bar{K}^0+p) \\ &= A(K^++n \rightarrow K^++n) - A(K^-+n \rightarrow K^-+n). \quad (1) \end{aligned}$$

In the forward direction, $t=0$, the imaginary part of the amplitude $A(K_2^0+p \rightarrow K_1^0+p)$ is then related, using the optical theorem, to the total-cross-section difference $\sigma_T(K^+n) - \sigma_T(K^-n)$. In the Regge-pole theory of high-

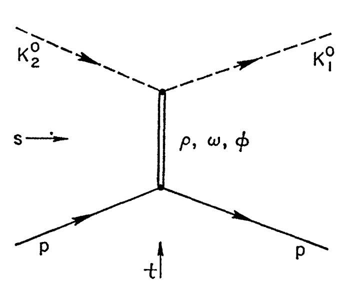


FIG. 1. Diagram representing the Regge-pole contributions to the regeneration amplitude $A(K_2^0+p \rightarrow K_1^0+p)$.

* Work supported by the U. S. Atomic Energy Commission.

[1] See, for example, G. Hohler, J. Baacke, H. Schlaile, and P. Sonderegger, Phys. Letters **20**, 79 (1966); F. Arbab and C. Chiu, Phys. Rev. **147**, 1045 (1966); S. Frautschi, Phys. Rev. Letters **17**, 722 (1966).

[2] A recent review of the Regge-pole model is to be found in V. Barger, in Proceedings of the CERN Topical Conference on High-Energy Collisions of Hadrons, 1968 (unpublished).

[3] V. L. Fitch, R. F. Roth, J. S. Russ, and W. Vernon, Phys. Rev. Letters **15**, 73 (1965); Phys. Rev. **164**, 1711 (1968); M. Bott-Bodenhausen *et al.*, Phys. Letters **20**, 212 (1966); C. Alff-Steinberger *et al.*, *ibid.* **20**, 207 (1966); **21**, 505 (1966).

[4] A. Firestone *et al.*, Phys. Rev. Letters **16**, 556 (1966).

[5] We write $K_1^0=(K^0+\bar{K}^0)/\sqrt{2}$ and $K_2^0=(K^0-\bar{K}^0)/\sqrt{2}$, neglecting for the moment the fact that the actual short- and long-lived neutral $K$ states observed in nature differ slightly from this because of $CP$ noninvariance.

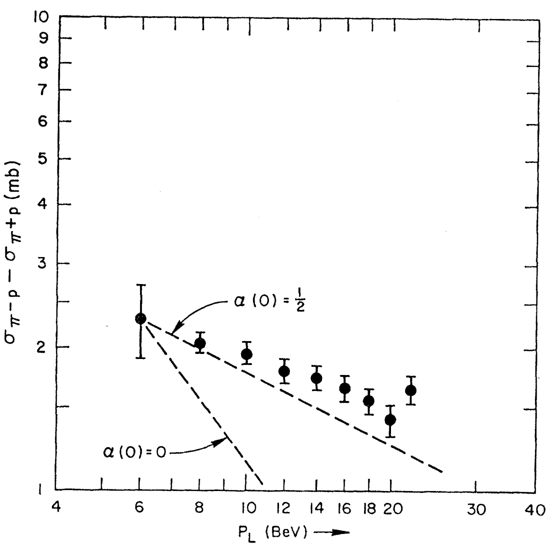


FIG. 2. The total-cross-section difference $\sigma_T(\pi^-p)-\sigma_T(\pi^+p)$. All data are taken from Ref. 8, except for the point at $p_L=6$ BeV, which is from Ref. 7. The dashed lines represent the behavior $p_L{}^{\alpha(0)-1}$ for the cross-section difference. For purposes of normalization, the dashed lines were drawn through the (arbitrarily chosen) point at $p_L=6$ BeV.

energy scattering, such total-cross-section differences are expected to have the behavior[6]

$$\Delta\sigma_T \propto s^{\alpha(0)-1} \propto p_L{}^{\alpha(0)-1}, \qquad (2)$$

where $\alpha(0)$ is the intercept at $t=0$ of the leading Regge trajectory in the crossed channel. For example, in Fig. 2 the total-cross-section difference[7,8] $\sigma_T(\pi^-p)-\sigma_T(\pi^+p)$ is plotted versus $p_L$, and it can be seen that $\alpha(0)\simeq 0.5$ for the leading $t$-channel trajectory, that of the $\rho$ meson.[9]

Turning back to $\sigma_T(K^-n)-\sigma_T(K^+n)$, whose energy dependence is presumed governed by $\rho$, $\omega$, and $\varphi$ exchange, the high-energy data[7] are plotted versus $p_L$ in Fig. 3. Although it is probably possible to at least conclude that $\alpha(0)<1$ for the leading trajectory, it is difficult, given the errors, to conclude much more about the energy dependence of $\sigma_T(K^-n)-\sigma_T(K^+n)$, and, hence, of $A(K_2{}^0+p\to K_1{}^0+p)$.

However, since we know the leading $I=1$, $C=-1$ ($\rho$) trajectory rather well from $\pi N$ charge exchange, and since we can calculate its contribution to $KN$ processes, let us concentrate on the $I=0$, $C=-1$ $\omega$ and $\varphi$ trajectories. Because the deuteron has isospin zero, $\sigma_T(\bar{p}d)-\sigma_T(pd)$ and $\sigma_T(K^-d)-\sigma_T(K^+d)$ only involve $\omega$ and $\varphi$ exchange. Furthermore, these cross sections are free of the screening corrections needed to extract $\sigma_T(K^-n)$ and $\sigma_T(K^+n)$, and the resulting errors are smaller. The data[7] in Fig. 4, expecially those for $\sigma_T(\bar{p}d)-\sigma_T(pd)$, rather clearly indicate that $\alpha(0)\simeq 0.5$.

We thus find that the leading $I=0$, $C=-1$ trajectory has $\alpha(0)\simeq 0.5$, just as the leading $I=1$, $C=-1$ ($\rho$)

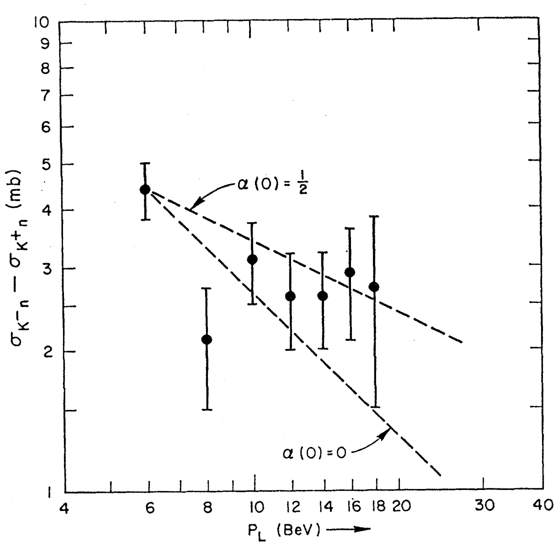


FIG. 3. The total-cross-section difference $\sigma_T(K^-n)-\sigma_T(K^+n)$. All data are from Ref. 7. The dashed lines represent the Regge-pole behavior of $p_L{}^{\alpha(0)-1}$, and are arbitrarily normalized to the point at $p_L=6$ BeV.

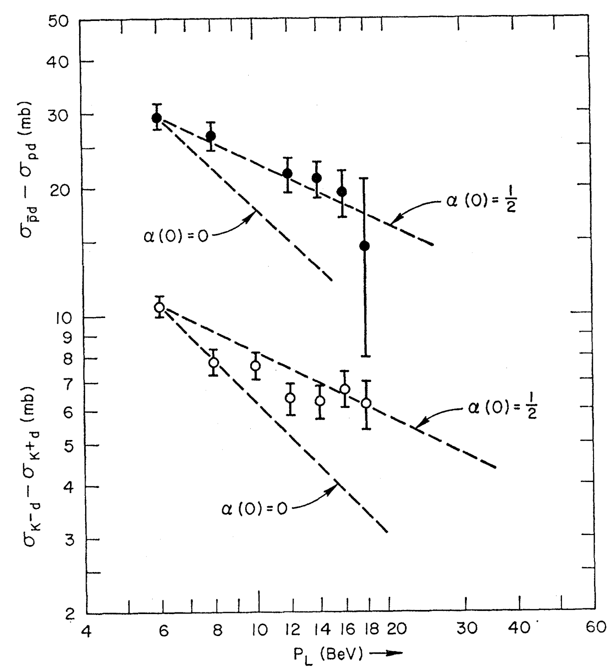


FIG. 4. The total-cross-section differences $\sigma_T(\bar{p}d)-\sigma_T(pd)$ (dark circles) and $\sigma_T(K^-d)-\sigma_T(K^+d)$ (open circles). All data are from Ref. 7. The dashed lines represent the Regge-pole behavior of $p_L{}^{\alpha(0)-1}$ and are arbitrarily normalized to pass through the point at $p_L=6$ BeV.

[6] Actually $p_L$ and $s$ are not exactly proportional, but we neglect terms of order $M_N{}^2/s$ which are only a few percent at the energies we consider here.

[7] W. Galbraith, E. W. Jenkins, T. F. Kycia, B. A. Leontic, R. H. Phillips, A. L. Read, and R. Rubinstein, Phys. Rev. **138**, B913 (1965).

[8] K. J. Foley *et al.*, Phys. Rev. Letters **19**, 330 (1967).

[9] A better fit is $\alpha(0)=0.69$ (Ref. 8) from $\sigma_T(\pi^-p)-\sigma_T(\pi^+p)$ or $\alpha(0)=0.57$ from the $\pi N$ charge-exchange data (Ref. 1).

trajectory does. Furthermore, the leading trajectory with $C=-1, I=0$ should be that of the $\omega$ meson for the following reasons.

(1) With the usual slope of $\sim 1/\text{BeV}^2$,

$$\alpha_\rho(0)\simeq\alpha_\omega(0)\simeq\tfrac{1}{2}, \quad \text{while } \alpha_\varphi(0)\simeq 0.$$

(2) Experimentally, it appears that the $\varphi$ is not coupled to nucleons or nuclei. This is also a consequence of the usual $\varphi$-$\omega$ mixing theory with universal vector-meson couplings (equivalently, the $\varphi$ only couples to particles with strange quarks in the quark model). The $\varphi$ then should not contribute to scattering off nucleons or nuclei.

(3) In the usual $\omega$-$\varphi$ mixing model with universal vector-meson couplings (or quark model), the $\omega$ couplings are proportional to $Y+2B$, where $Y$ is the hypercharge current and $B$ the baryon number current (in the quark model this amounts to a coupling proportional to the number of nonstrange quarks). We immediately have the relation[10]

$$3[\sigma_T(K^-d)-\sigma_T(K^+d)]=\sigma_T(\bar{p}d)-\sigma_T(pd) \qquad (3)$$

which is well satisfied (see Fig. 4). Also, since we have vector exchange and $K^-$ or $\bar{p}$ and $d$ have opposite values of $Y+2B$, $\omega$ exchange is attractive for $K^-d$ or $\bar{p}d$ and predicts $\sigma_T(K^-d)>\sigma_T(K^+d)$ and $\sigma_T(\bar{p}d)>\sigma_T(pd)$, again in agreement with experiment. Thus we find strong experimental and theoretical reasons for believing that the leading $C=-1, I=0$ and 1 Regge trajectories are the $\omega$ (*not* $\varphi$) and $\rho$ and that both $\alpha_\omega(0)$ and $\alpha_\rho(0)$ are approximately 0.5. These are then the leading Regge trajectories to be considered for $K_2^0+p\to K_1^0+p$.

## III. PREDICTION OF $(d\sigma/dt)(K_2^0+p\to K_1^0+p)$ AT $t=0$

From Eq. (1) and the optical theorem it is simple to show that the part of $d\sigma/dt$ at $t=0$ for $K_2^0+p\to K_1^0+p$ which comes from the imaginary part of the forward amplitude is

$$\left(\frac{d\sigma}{dt}\right)_{\text{optical}}=\frac{1}{64\pi}[\sigma_T(K^+n)-\sigma_T(K^-n)]^2. \qquad (4)$$

To determine the real part of the forward amplitude we use the Regge-pole model and the specific results for the $\omega$ and $\rho$ trajectories at $t=0$ determined in the previous section. In the Regge-pole model the phase of the forward amplitude is given by the signature factor,[11] $1-e^{-i\pi\alpha(0)}$. This gives a ratio of real to imaginary part of the forward amplitude of $\tan[\frac{1}{2}\pi\alpha(0)]$. We therefore have for $K_2^0+p\to K_1^0+p$

$$\left.\frac{d\sigma}{dt}\right|_{t=0}=\frac{1}{64\pi\cos^2[\frac{1}{2}\pi\alpha(0)]}[\sigma_T(K^+n)-\sigma_T(K^-n)]^2, \qquad (5)$$

where $\alpha(0)$ is the intercept at $t=0$ of the leading Regge trajectory in the $t$ channel.

Since we found in the previous section that both the leading trajectories ($\rho$ and $\omega$) have $\alpha(0)\simeq\frac{1}{2}$, $\tan\frac{1}{2}\pi\alpha(0)\simeq 1$ and we predict *roughly equal real and imaginary parts* of the forward regeneration amplitude.[12] Equation (5) then predicts that $d\sigma/dt|_{t=0}$ is roughly *twice* the optical theorem value in Eq. (4).

If we choose $p_L=6$ BeV as an arbitrary point to normalize with respect to, we can now summarize the predictions of our Regge-pole model for the forward regeneration amplitude as[13]

$$\left.\frac{d\sigma}{dt}\right|_{t=0}=\frac{([\sigma_T(K^+n)-\sigma_T(K^-n)]_{p_L=6\text{ BeV}})^2}{64\pi}\times\frac{(p_L/6\text{ BeV})^{2\alpha(0)-2}}{\cos^2[\frac{1}{2}\pi\alpha(0)]}. \qquad (6)$$

Taking[14] $[\sigma_T(K^-n)-\sigma_T(K^+n)]_{p_L=6\text{ BeV}}=4.5$ mb, we find

$$\left.\frac{d\sigma}{dt}\right|_{t=0}=\left(260\frac{\mu\text{b}}{\text{BeV}^2}\right)\frac{(p_L/6\text{ BeV})^{2\alpha(0)-2}}{\cos^2[\frac{1}{2}\pi\alpha(0)]}. \qquad (7)$$

The values of $d\sigma/dt|_{t=0}$ are shown in Fig. 5 for the most probable value of $\alpha(0)=\frac{1}{2}$, as well as for $\alpha(0)=\frac{1}{3}$ and $\frac{2}{3}$. It is important to note that the Regge-pole model connects the energy dependence of the forward amplitude with the ratio of its real to imaginary parts, and that *both* of these enter Eqs. (6) and (7). Measurement of both the energy dependence of $d\sigma/dt|_{t=0}$ and of its magnitude in comparison with $(d\sigma/dt)_{\text{optical}}$ is then a critical test of the Regge model of the forward amplitude at high energies and of the value of $\alpha(0)$.

## IV. DETERMINATION OF THE $\omega$ REGGE TRAJECTORY'S PARAMETERS FROM $K_2^0+p\to K_1^0+p$

In Sec. II we saw that the high-energy behavior of the forward $K_2^0+p\to K_1^0+p$ amplitude is governed by the

[10] C. A. Levinson, H. J. Lipkin, and N. S. Wall, Phys. Rev. Letters **17**, 1122 (1966).

[11] Although this signature factor is specific to (odd-signature trajectories like the $\omega$ and $\rho$ in) Regge-pole theory, the connection between the asymptotic energy dependence of the forward amplitude and the ratio of its real to imaginary parts given here is actually quite generally provable using dispersion theory (for amplitudes which are odd under $s\leftrightarrow u$ crossing).

[12] A similar discussion of the phase of the forward regeneration amplitude in Regge theory has been given by N. Cabibbo, Phys. Letters **22**, 212 (1966). It was not shown there, however, that the $\omega$ with $\alpha_\omega(0)\simeq\frac{1}{2}$ is the dominant $I=0$, $C=-1$ trajectory, and not the $\varphi$. With the usual slope of $\sim 1/\text{BeV}^2$, $\alpha_\varphi(0)\simeq 0$, and the regeneration amplitude would be almost purely imaginary.

[13] We actually should write the contribution of the $\rho$ and $\omega$ Regge poles separately, but since they have the same signature and $\alpha_\rho(0)\simeq\alpha_\omega(0)$, we have written Eq. (6) as if only one Regge pole contributes to the forward amplitude. We will in fact show in the next section that the $\omega$ exchange contribution to the regeneration amplitude is much larger than that due to $\rho$ exchange.

[14] W. Galbraith *et al.* (Ref. 7) give $\sigma_T(K^-n)-\sigma_T(K^+n)=4.4\pm0.6$ mb. The weak Johnson-Treiman relation, $\sigma_T(K^-n)-\sigma_T(K^+n)=\sigma_T(K^-p)-\sigma_T(K^+p)-\sigma_T(\pi^-p)+\sigma_T(\pi^+p)$, which is well satisfied (see Ref. 2), gives $\sigma_T(K^-n)-\sigma_T(K^+n)=4.7\pm0.4$ mb from the better measured $K^\pm p$ and $\pi^\pm p$ total cross sections.

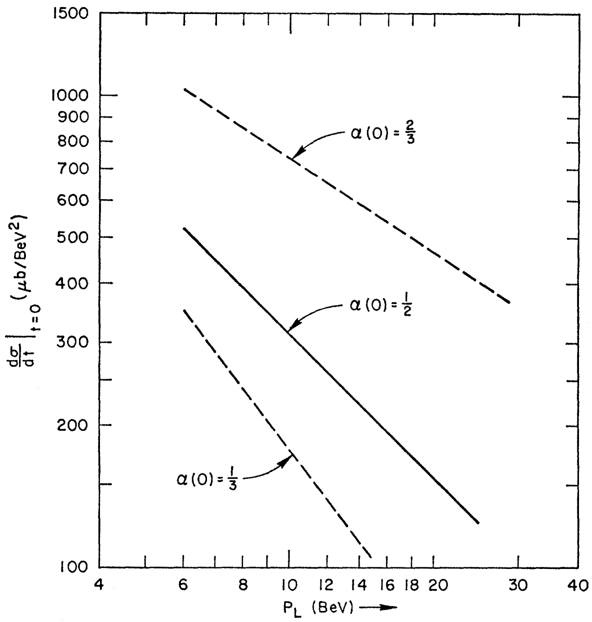


FIG. 5. $d\sigma/dt|_{t=0}$ for the process $K_2^0+p\to K_1^0+p$ for $\alpha(0)=\frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{3}$.

leading $C=-1$, $I=0$ and 1 Regge trajectories, the $\omega$ and $\rho$ trajectories. If we assume that at $t=0$ the Reggeized $\rho$ meson is coupled at the meson-meson-$\rho$ vertex universally through the isospin current[15] [or that the $\rho$-meson-meson Regge residues obey $SU(3)$], then we find that the relative contributions of the $\omega$ and $\rho$ trajectories to $A(K_2^0+p\to K_1^0+p)$ at $t=0$ are proportional to $[\sigma_T(K^-n)-\sigma_T(K^+n)+\frac{1}{2}[\sigma_T(\pi^-p)-\sigma_T(\pi^+p)]$ and $\frac{1}{2}[\sigma_T(\pi^-p)-\sigma_T(\pi^+p)]$. Over the range of $p_L$ from 6 to 20 BeV this gives a ratio of $\omega$ to $\rho$ contributions of 4 or 5 to 1.[16] The forward regeneration amplitude is thus dominately due to $\omega$ exchange and the measurement of $d\sigma/dt|_{t=0}$ as a function of energy (see the previous section) gives $\alpha(0)$ for the $\omega$ meson.

Both the $\rho$ and $\omega$ have spin-nonflip and spin-flip residues at the nucleon vertex and as we move away from $t=0$ there are contributions to $d\sigma/dt$ from both types of residue functions (at $t=0$, only non-flip contributes). In most Regge fits[17] to high-energy $\pi^\pm p$, $K^\pm p$, $pp$, and $\bar{p}p$ scattering the $\omega$ is assumed to contribute predominately through the nonflip residue function, while the $\rho$ gives a large spin-flip contribution.[18] Since we have seen that the $\omega$ is much more important at $t=0$ than the $\rho$, we expect no rise in $d\sigma/dt$ as we move away from $t=0$, as occurs in $d\sigma/dt$ for $\pi^-+p\to\pi^0+n$, where the large $\rho$-meson spin-flip residue dominates.

In fact, the usual Regge-pole models would predict a dip in $(d\sigma/dt)(K_2^0+p\to K_1^0+p)$ at $t\simeq-0.15$ BeV$^2$. This is because the usual Regge-pole models explain the cross-over phenomenon[17] $[(d\sigma/dt)(\bar{p}p)-(d\sigma/dt)(pp)$, $(d\sigma/dt)(\pi^-p)-(d\sigma/dt)(\pi^+p)$, and $(d\sigma/dt)(K^-p)-(d\sigma/dt)(K^+p)$ all change sign at $t\simeq-0.15$ BeV$^2]$ by assuming that there are *dynamical zeros* in the $\rho$ and $\omega$ nonflip residue functions at $t\simeq-0.15$ BeV$^2$. Since the $\omega$ trajectory is expected to have a large nonflip residue and to dominate $A(K_2^0+p\to K_1^0+p)$, we expect a large dip in $(d\sigma/dt)(K_2^0+p\to K_1^0+p)$ at $t\simeq-0.15$ BeV$^2$ *if the usual Regge-pole models are correct.*[19] This is again a critical test of part of the Regge-pole model. We note in passing that the presence of such a dip at $t\simeq-0.15$ BeV$^2$ requires experiments with a resolution in $t$ of $\lesssim0.05$ BeV$^2$ if we are to determine the shape of $d\sigma/dt$ with any accuracy. A dip in $d\sigma/dt$ at $t\simeq-0.15$ BeV$^2$ together with the fact that the bins in $t$ are 0.2 BeV$^2$ wide in the experiment of Firestone *et al.*[4] would help explain why their extrapolated value for $d\sigma/dt|_{t=0}$ is more than a factor of 2 less than $(d\sigma/dt)_{\text{optical}}$.

At values of $t\simeq-0.6$ BeV$^2$ we expect another dip in $d\sigma/dt$ from the vanishing of the $\rho$ and $\omega$ spin-flip residues when $\alpha_\rho$ or $\alpha_\omega$ equals zero. We do not expect the dramatic dip observed in $\pi N$ charge exchange, since here we expect that the $\omega$ nonflip residue is the largest contributor and it has no known zero in the region of $t\simeq-0.6$ BeV$^2$.

Finally, let us return to the overlap of strong and weak interactions in this problem. Given the energy dependence of the forward amplitude, $A(K_2^0+p\to K_1^0+p)$, we have seen that the Regge model gives us its phase, which can then be used in analyzing experiments on the interference between the $CP$-violating amplitude for long-lived neutral $K$ mesons to decay into $\pi^++\pi^-$ and the amplitude for regenerated $K_1^0$'s to decay into $\pi^++\pi^-$. Conversely, knowledge of the phase of $\eta_{+-}$ can be used to get the phase of the forward regeneration amplitude. Even away from $t=0$, the phase of the regeneration amplitude, $A(K_2^0+p\to K_1^0+p)$, can be measured relative to that for elastic scattering, $A(K_2^0+p\to K_2^0+p)$, by measuring the charge asymmetry between the $\pi^-e^+\nu$ and $\pi^-e^-\bar{\nu}$ modes of the scattered neutral $K$ mesons.[20] This is a very much more difficult experiment, however, than those proposed here to simply measure the shape and energy dependence of

[15] This leads to the weak Johnson-Treiman relation which is well satisfied (see Ref. 2).

[16] A ratio of 3 to 1 is predicted assuming universality (pure $F$-type) for all the vector-meson couplings and the usual $\varphi$-$\omega$ mixing. The ratio of 4 or 5 to 1 given here corresponds to the statement in Ref. 10 that the $\omega$ contribution to $KN$ scattering is about 40% larger than the value predicted by $SU(6)$ from the $\rho$ coupling.

[17] See, for example, W. Rarita, R. J. Riddel, C. B. Chiu, and R. J. N. Phillips, Phys. Rev. **165**, 1615 (1968); and the review in Ref. 2.

[18] One expects this in a model for the vector-meson residues based on vector-meson dominance of the electromagnetic form factors where the ratio of spin-flip to spin-nonflip residues are proportional to $G_M^V/G_E^V$ and $G_M^S/G_E^S$ for the $\rho$ and $\omega$ mesons, respectively.

[19] Arguments against the usual assumption of a zero in the $\omega$ residues at $t\simeq-0.15$ BeV$^2$ are found in V. Barger and L. Durand, Phys. Rev. Letters **19**, 1295 (1967). In processes such as $NN$ scattering and $\gamma+p\to\pi^0+p$ the $B$ meson can be exchanged, so $K_2^0+p\to K_1^0+p$ may provide the cleanest test of the vanishing of the $\omega$ residue function alone.

[20] The author thanks Dr. D. Dorfan for a discussion on this subject.

$d\sigma/dt$ (with present beam intensities such a charge asymmetry measurement is a "theorist's experiment" away from $t=0$).

In summary, the following key aspects of the $\omega$ trajectory can be determined from measuring $(d\sigma/dt)$ $(K_2^0+p\rightarrow K_1^0+p)$:

(1) Measurement of the magnitude and energy dependence of $d\sigma/dt|_{t=0}$ gives $\alpha(0)$.

(2) The absence of a rise in $d\sigma/dt$ for very small increasing values of $-t$ would show the dominance of the nonflip $\omega$ residue function (in contrast to the $\rho$).

(3) The energy dependence of $d\sigma/dt$ for fixed $t\neq 0$ determines $\alpha(t)$.

(4) The presence of a dip at $t\simeq -0.15$ BeV$^2$ would demonstrate the presence of a dynamical zero in the nonflip residue functions which is used to explain the cross-over phenomenon in Regge-pole theory.

(5) The size of the dip at $t\simeq -0.6$ BeV$^2$ indicates the magnitude of the $\omega$ and $\rho$ spin-flip residues.

(6) At least in principle, weak interaction experiments can determine the phase of the regeneration amplitude for all $t$ for comparison with the Regge-pole model prediction of the phase, $1-e^{-i\pi\alpha(t)}$.


## ACKNOWLEDGMENTS

The author thanks Professor David Leith for many useful discussions and for putting him on the trail of the $\omega$ in $K_2^0+p\rightarrow K_1^0+p$.


PHYSICAL REVIEW VOLUME 171, NUMBER 5 25 JULY 1968

# Theoretical Study of $K_{l4}$ Decay


F. A. BERENDS* AND A. DONNACHIE
*University of Glasgow, Scotland*

AND

G. C. OADES
*Rutherford High Energy Laboratory, Chilton, Didcot Berkshire, England*


(Received 26 February 1968)

$K_{l4}$ decay is formulated in a way which allows the extraction of form factors and $\pi$-$\pi$ phase-shift differences from data with limited statistics. The only assumptions required are $T$ invariance and a restriction to $S$ and $P$ waves in the final state, no other dynamical assumptions being required. It is shown that the assumption of the $\Delta I=\frac{1}{2}$ rule is not necessary if all $K_{l4}$ decays are taken into account. When the existing available data are analyzed in the simple extension of the Cabibbo-Maksymowicz approach in which all $P$-wave contributions are taken into account, there is a sign ambiguity in the $S$-wave $\pi$-$\pi$ phase shift. This ambiguity is fundamental to this approach and can be solved only by means of another angular distribution. Such a distribution could be obtained from the existing data.

## 1. INTRODUCTION

THERE are several standpoints from which $K_{l4}$ decay is an interesting subject of study. First, it has been realized for some time that it can be used to obtain information on the low-energy $\pi$-$\pi$ phase shifts, and, in particular, Cabibbo and Maksymowicz[1] developed an elegant formalism for angular correlations in which, under certain assumptions, a determination of the low-energy $\pi$-$\pi$ phase shifts can be made. Secondly, it is of considerable interest to determine the various form factors which occur in $K_{l4}$ decays, particularly in view of the predictions of their magnitudes which have been made within the framework of current algebra.[2] Thirdly, is there any way in $K_{l4}$ decay in which violation of the $\Delta I=\frac{1}{2}$ rule could be demonstrated explicitly? Fourthly, is there any way in $K_{l4}$ decay in which $T$ noninvariance could manifest itself unambiguously?

At present, only a few hundred events for

$$K^+\rightarrow \pi^+\pi^-e^+\nu$$

are available,[3] and of the other processes some events of $K^+\rightarrow\pi^+\pi^-\mu^+\nu$ have been observed, sufficient only to give some indication of the rate.[4] In view of the current proposals for experiments which will increase the number of events by one order of magnitude (or thereabouts), it is worthwhile to discuss the possibilities of extracting information on phase shifts and form factors, relaxing some of the assumptions of Ref. 1, in particular the $\Delta I=\frac{1}{2}$ rule and dynamical assumptions on form-factor dependences. It appears that the forth-

* On leave of absence from Instituut-Lorentz, Leiden.

[1] N. Cabibbo and A. Maksymowicz, Phys. Rev. **137**, B438 (1965); **168**, 1926(E) (1968).

[2] S. Weinberg, Phys. Rev. Letters **17**, 336 (1966); and erratum [Phys. Rev. Letters **18**, 1178 (1967)].

[3] Berkeley-UCL-Wisconsin collaboration, paper presented by M. J. Esten at the Physical Society Conference on Elementary Particles, London, 1967 (unpublished).

[4] V. Bisi, R. Cester, A. Marzari Chiesa, and M. Vigone, Phys. Letters **25B**, 572 (1967).