Measurements of C_{NN} in Proton-Proton Scattering from 305 to 415 MeV*

A. Beretvas†
The Enrico Fermi Institute and the Department of Physics, University of Chicago, Chicago, Illinois 60637
(Received 4 March 1968)

The spin-spin correlation parameter C_{NN} for p-p scattering has been measured at four energies between 305 and 415 MeV. Measurements were made over the angular range of 50° to 90° in the c.m. system. A polarized beam, produced by scattering of the internal proton beam of the Chicago cyclotron, and the Argonne polarized proton target were used for the experiment. Results for C_{NN} (90°) are compared with other experiments. The total set of experimental data indicates a strong energy dependence for C_{NN} (90°) with a minimum at approximately 500 MeV. A brief qualitative discussion of the behavior of C_{NN} (90°) is given.

I. INTRODUCTION

PRIOR to 1953, the only aspects of proton-proton scattering that had received experimental attention were the differential and total cross sections. The production of beams of polarized protons[1] made possible measurements of the polarization parameter and the triple-scattering parameters. Still, the measurements were inadequate to determine uniquely the p-p scattering matrix. One further experiment suggested[2] was the measurement of C_{NN}, the correlation between the components of the spins of the final-state protons normal to the scattering plane. The first measurement of C_{NN} was performed by Ashmore *et al.*[3] at 380 MeV, in an attempt to select a unique set of phase shifts from among the five sets found by Stapp *et al.*[4] in an analysis of data at 310 MeV. This attempt was not quite successful because of the unknown energy dependence of C_{NN} between 310 and 380 MeV. The number of ambiguities was reduced theoretically, first by calculating the higher partial waves in the one-pion-exchange approximation,[5] and second by including dispersion relations.[6] Further measurements of C_{NN} were made at 320 MeV[7] and 315 MeV.[8] Other measurements have been made at 52 MeV,[9] 650 MeV,[10] and, with the use of spark chambers, at 400 and 450 MeV.[11] Most of these early measurements had rather large errors and were made at only one angle, 90°, in the c.m. system.

Schumacher and Bethe pointed out that the time-reversed experiment (namely, a measurement of A_{YY}, the dependence of the differential cross section on the correlation between the components of the spins of the initial-state protons normal to the scattering plane) would be equivalent to a measurement of C_{NN}.[12] They also pointed out that the measurement of C_{NN} and the other spin-spin correlation parameters could be simplified by the use of a polarized proton beam and a polarized proton target. Polarized targets are now available, due most notably to the efforts of Abragam and Jeffries.[13]

The first experiment performed with a polarized proton target was the measurement of C_{NN} in proton-proton scattering at 20 MeV.[14] A summary of the more recent measurements is given in Sec. V.

There are several advantages in the use of a polarized proton target, together with a polarized proton beam, to measure spin-spin correlation parameters. We recall that the other method requires scattering the final-state protons off suitable analyzers and measuring their coincidence rates. In the polarized-target method only one scattering need be performed in the experimental area, no counters need be moved, and angles other than 90° can often be measured at the same time. In the present experiment the sign of the beam polarization was reversed by means of a solenoid magnet, and the sign of the target polarization was reversed by means of a slight change in the frequency of the microwaves used to polarize the target. This paper describes mea-

* Work supported by the U. S. Office of Naval Research under contract Nonr-2121(25), and in part by the U. S. Atomic Energy Commission.

† A thesis submitted to the Department of Physics, University of Chicago, in partial fulfillment of the requirements for the Ph.D. degree.

[1] C. L. Oxley, W. F. Cartwright, and J. Rouvina, Phys. Rev. **93**, 806 (1954).

[2] L. Wolfenstein, Phys. Rev. **96**, 1654 (1954).

[3] A. Ashmore, A. N. Diddens, G. B. Huxtable, and K. Skarsvag, Proc. Phys. Soc. (London) **72**, 289 (1958).

[4] H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. **105**, 302 (1957).

[5] P. Cziffra, M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **114**, 881 (1959).

[6] Riazuddin and M. J. Moravcsik, Phys. Letters **4**, 243 (1963).

[7] J. V. Allaby, A. Ashmore, A. N. Diddens, J. Eades, G. B. Huxtable, and K. Skarsvag, Proc. Phys. Soc. (London) **77**, 234 (1961).

[8] I. M. Vasilevsky, V. V. Vishnyzkov, E. I. Ilescu, and A. A. Tyapkin, Zh. Eksperim. i Teor. Fiz. **39**, 889 (1960) [English transl.: Soviet Phys.—JETP **12**, 616 (1961)].

[9] K. Nisimura, J. Sanada, P. Catillon, K. Fukunaga, T. Hasegawa, H. Hasai, N. Ruy, D. C. Worth, and H. Imada, Progr. Theoret. Phys. (Kyoto) **30**, 719 (1963).

[10] B. M. Golovin, V. P. Dzhelepov, and R. Y. Zulkarneev, Zh. Eksperim. i Teor. Fiz. **41**, 83 (1961) [English transl.: Soviet Phys.—JETP **14**, 63 (1962)].

[11] E. Engels, Jr., T. Bowen, J. W. Cronin, R. L. McIlwain, and L. G. Pondrom, Phys. Rev. **129**, 1958 (1963).

[12] C. Schumacher and H. Bethe, Phys. Rev. **121**, 1534 (1961). Since A_{YY} is equivalent by time reversal to C_{NN}, we will use the symbol C_{NN} for both.

[13] A. Abragam, *The Principles of Nuclear Magnetism* (Oxford University Press, London, 1961); C. D. Jeffries, *Dynamic Nuclear Orientation* (Interscience Publishers, Inc., New York, 1963).

[14] A. Abragam, M. Borghini, P. Catillon, J. Coustham, P. Roubeau, and J. Thirion, Phys. Letters **2**, 310 (1962).

surements of C_{NN} in proton-proton scattering at four energies between 305 and 415 MeV.[15]

II. THEORY OF METHOD

The p-p scattering process is most conveniently described in the c.m. system where we define the usual vectors

$$\mathbf{P}=\frac{\mathbf{k}+\mathbf{k}'}{|\mathbf{k}+\mathbf{k}'|},\quad \mathbf{K}=\frac{\mathbf{k}'-\mathbf{k}}{|\mathbf{k}'-\mathbf{k}|},\quad \mathbf{N}=\frac{\mathbf{k}\times\mathbf{k}'}{|\mathbf{k}\times\mathbf{k}'|},\qquad(1)$$

where $\mathbf{k}$ and $\mathbf{k}'$ are, respectively, the incoming and outgoing momenta. The initial state I of the two-proton system may be described by the 4×4 density matrix

$$\rho_I=\tfrac{1}{2}(1+P_{BP}\sigma_P+P_{BK}\sigma_K+P_{BN}\sigma_N)\otimes\tfrac{1}{2}(1+P_{TP}\sigma_P+P_{TK}\sigma_K+P_{TN}\sigma_N),\qquad(2)$$

where P_{Bi} (P_{Tj}) is the polarization of the beam (target) in the $i(j)$th direction.[16] In terms of the M matrix[17] the observables of interest to us, the differential cross section $I(\theta)$, the unpolarized cross section $I_0(\theta)$, the polarization $P(\theta)$, and the spin-spin correlation coefficients $C_{ij}(\theta)$, are given by

$$\begin{aligned}I(\theta)&=\mathrm{Tr}[\rho_F]=\mathrm{Tr}[M^\dagger M\rho_I],\\ I_0(\theta)&=\tfrac{1}{4}\,\mathrm{Tr}[M^\dagger M],\\ I_0(\theta)P(\theta)&=\tfrac{1}{4}\,\mathrm{Tr}[M^\dagger M1\otimes\sigma_N],\\ I_0(\theta)C_{ij}(\theta)&=\tfrac{1}{4}\,\mathrm{Tr}[M^\dagger M\sigma_i\otimes\sigma_j].\end{aligned}\qquad(3)$$

From (3), plus invariance considerations, we obtain

$$\begin{aligned}I(\theta)=I_0(\theta)[1&+(P_{BN}+P_{TN})P(\theta)+P_{BP}P_{TP}C_{PP}(\theta)\\ &+P_{BK}P_{TK}C_{KK}(\theta)+P_{BN}P_{TN}C_{NN}(\theta)\\ &+(P_{BP}P_{TK}+P_{BK}P_{TP})C_{KP}(\theta)].\end{aligned}\qquad(4)$$

Since the directions of $\mathbf{P}$ and $\mathbf{K}$, and consequently the values of P_{BK}, P_{TK}, P_{BP}, and P_{TP}, depend upon the scattering angle, whereas $\mathbf{N}$ does not, Eq. (4) simplifies, when the polarizations are normal to the scattering plane (also true in the laboratory system), to

$$I(\theta)=I_0(\theta)[1+(P_{BN}+P_{TN})P(\theta)+P_{BN}P_{TN}C_{NN}(\theta)].\qquad(5)$$

No relativistic corrections apply because spins normal to the scattering plane are unchanged by Lorentz transformations. In the following, we will always consider the beam and target polarizations to be parallel or antiparallel to the $\mathbf{N}$ direction, so we may drop the subscript and write

$$I(\theta)=I_0(\theta)[1+(\pm P_B)P(\theta)+(\pm P_T)P(\theta)+(\pm P_B)(\pm P_T)C_{NN}(\theta)].\qquad(6)$$

The experiment then consists of measuring $I(\theta)$ for the four possible orientation combinations of P_B and P_T.

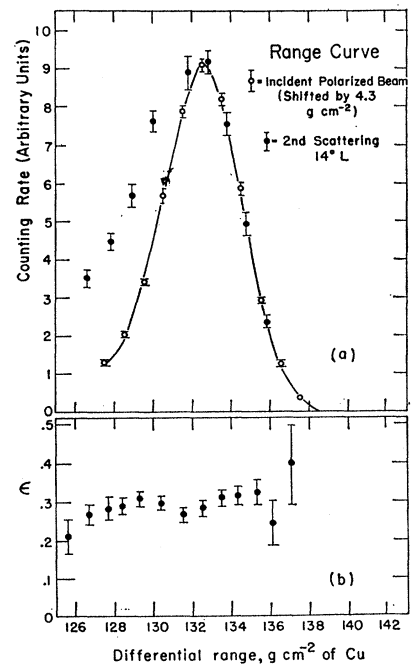

FIG. 1. (a) Typical differential range curves for: (○) polarized beam (1st scattering is 14° Left); (●) twice-scattered beam (1st and 2nd scatterings are 14° Left). (b) Left-right asymmetry in the second scattering as a function of differential range.

III. EXPERIMENTAL PROCEDURE

A. Polarized Beam

The polarized beam was produced by scattering the internal beam of the Chicago cyclotron from an internal Be target, 0.64 cm long $\times$0.16 cm high. The central orbit of the beam selected corresponded to a left scattering of 14.0° in the median plane. This angle was determined by the momentum of the scattered beam, the magnetic field of the cyclotron, and the impact parameter of the scattered beam line. The beam was momentum-analyzed and vertically focused by the fringing field as it emerged from the cyclotron. Two quadrupole triplets were used to transport the beam to the experimental area. The polarization of the beam was measured by the standard double-scattering technique. A Be target, 0.49 cm thick, was used as the second scatterer and the scattered protons were detected in a differential range telescope consisting of a Cu absorber and seven scintillation counters. The telescope was aligned to an accuracy of 1/40° at 14.0°

[15] In a preliminary report on this experiment, data were quoted at five energies. The data at 358 MeV have been deleted because they consisted of only three runs instead of four and hence could not be subject to the same consistency requirements as the rest of the data. The target polarization has been reexamined, resulting in some changes from the earlier values; see A. Beretvas, N. E. Booth, C. Dolnick, R. J. Esterling, R. E. Hill, J. Scheid, D. Sherden, and A. Yokosawa, Rev. Mod. Phys. **39**, 536 (1967).

[16] It is more conventional to write the polarization of the beam and target in terms of the polarization in the laboratory system. See P. Catillon, M. Chapellier, and D. Garretta, Nucl. Phys. **B2**, 93 (1967).

[17] M. J. Moravcsik, *The Two-Nucleon Interaction* (Clarendon Press, Oxford, England, 1963).

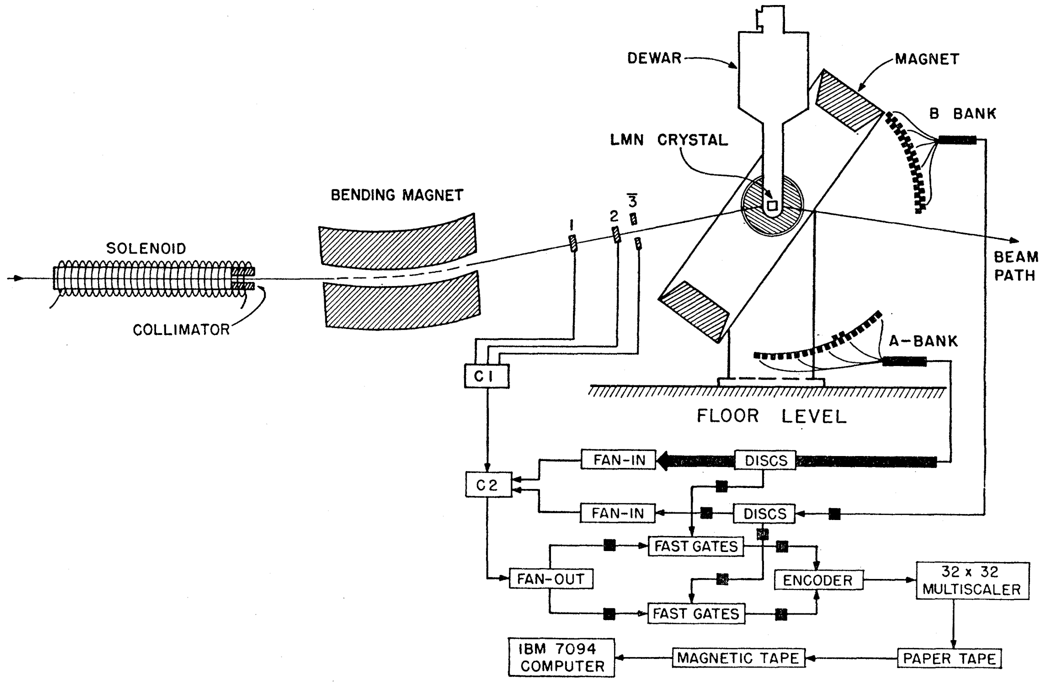

FIG. 2. Schematic view of the apparatus. The heavy lines indicate a cable for each counter of the A and B banks.

left and right of the incident beam direction. A typical differential range curve is shown in Fig. 1(a). We also show a range curve of the incident beam, normalized and shifted to the left by 4.3 g cm^{-2} of Cu. The low-energy tail due to inelastic scattering is apparent. Figure 1(b) shows the measured left-right asymmetry as a function of differential range. The inelastic scattering has little effect on the asymmetry, although its contribution is high at lower values of the range. The final value for the beam polarization is $P_B=0.535 \pm 0.025$. This result is in good agreement with most of the earlier measurements at Chicago.[18,19]

B. Experiment Arrangement

The polarization of the beam as it emerges from the cyclotron is directed vertically upwards. A solenoid magnet in the experimental area permitted the polarization to be rotated to the left or right through an angle of 90° into the horizontal plane. Extreme care was taken in the alignment of the solenoid to prevent any beam shifts which would introduce false asymmetries. Figure 2 shows the arrangement of the apparatus in the experimental area. The bending magnet just downstream of the solenoid was used to elevate the beam to the polarized target magnet. The arrangement was such that the beam was horizontal as it passed through the polarized target. The incident beam was defined and monitored by scintillation counters 1, 2, and 3. Beam intensities were typically 2.5×10^6 per sec.

The polarized target consisted of four rectangular crystals of $(0.99\ \mathrm{La}+0.01\ \mathrm{Nd})_2\mathrm{Mg}_3(\mathrm{NO}_3)_{12}\cdot24\mathrm{H}_2\mathrm{O}$ stacked to give a scatterer 2.9 cm horizontal $\times$1.9 cm

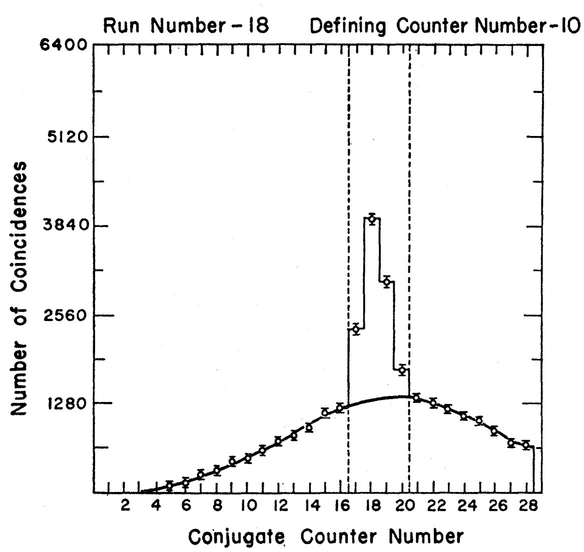

FIG. 3. Typical coincidence distribution curve to illustrate the effectiveness of the angular-correlation technique for selecting scatterings off the free protons of the target crystals.

[18] H. G. de Carvallo, J. Marshall, and L. Marshall, Phys. Rev. **96**, 108 (1954); E. Heiberg, *ibid.* **106**, 1271 (1957); R. March, *ibid.* **120**, 1874 (1960); R. Roth, E. Engles, Jr., S. C. Wright, P. Kloeppel, R. Handler, and L. G. Pondrom, *ibid.* **140**, B1533 (1965).

[19] There is evidence, both experimental and theoretical, that degrading does not change the beam polarization; see E. Heiberg, U. Kruse, J. Marshall, L. Marshall, and F. Solmitz, Phys. Rev. **97**, 250 (1955); L. Wolfenstein, *ibid.* **75**, 1664 (1949).

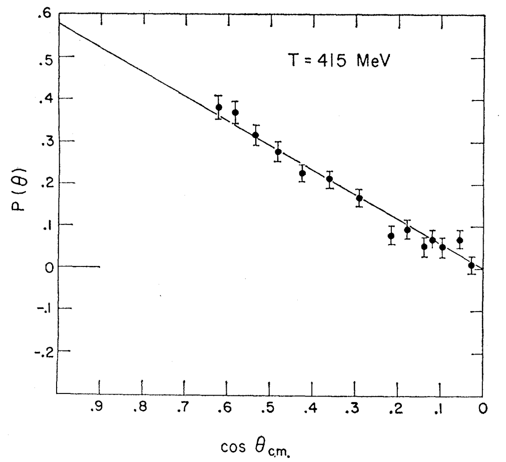

FIG. 4. Polarization in p-p scattering at 415 MeV.

TABLE I. Polarization in proton-proton scattering at 415 MeV.

$\theta_{\mathrm{c.m.}}$ (deg)	$P(\theta)$
96.9	-0.071 ± 0.020
91.9	-0.012 ± 0.020
86.9	0.071 ± 0.020
84.5	0.052 ± 0.021
82.1	0.054 ± 0.021
79.8	0.094 ± 0.021
77.5	0.082 ± 0.021
73.2	0.168 ± 0.020
68.9	0.209 ± 0.021
64.8	0.226 ± 0.020
61.2	0.276 ± 0.022
57.6	0.317 ± 0.023
54.4	0.369 ± 0.024
51.5	0.382 ± 0.027

vertical $\times 2.7$ cm long. The free protons were polarized by the dynamic nuclear-orientation method and the degree of polarization was monitored with an NMR system.[20] The target polarization P_T was perpendicular to the plane of Fig. 2 and could be reversed by means of a small change in the frequency of the microwaves used to polarize the target.

Scatterings in the vertical plane were detected by two arrays of scintillation counters (designated A and B in Fig. 2).[21] The counters were placed alternately at radii of 101.6 and 106.7 cm and had an azimuthal acceptance of $\pm 8°$ and an angular resolution in the c.m. system of $\pm 1.0°$ to $\pm 1.4°$. The requirements for an event in C2 were a beam particle (C1 coincidence) and at least one particle in each of the A and B arrays. If more than one particle was detected in either the A or the B array, the event was rejected in the encoder. The rate of these multiple events was about 10% of the valid events. A typical distribution in the B array of events in a particular A counter is shown in Fig. 3. The background under the elastic peak was computed by fitting the distribution outside the dashed lines. Note that the background has some angular correlation, indicating that at least part of it is due to quasielastic p-p scattering in the complex nuclei of the target.

The energy of the beam at the center of the polarized target was 415 ± 5 MeV with an energy spread of 12 MeV full width at half-maximum (FWHM). Data at reduced energies were obtained by degrading the beam with CH_2 placed between the bending magnet and the defining telescope.[19] Slight adjustments of the current in the bending magnet and the position of the telescope were necessary when the energy was changed to ensure that the beam was centered on the polarized target.

C. Data Reduction

Data were taken with the four orientation combinations of P_B and P_T. The number of elastic events at each angle was determined by the method indicated in Fig. 3. Energy loss of the recoil protons in the target limited us to angles greater than 50° in the c.m. system. For each run the target polarization was computed from the NMR signals.

By using Eq. (6) for the rates one can compute the three asymmetries[22]

$$\begin{aligned}\epsilon_1(\theta)&=P_BP(\theta)\,,\\ \epsilon_2(\theta)&=P_TP(\theta)\,,\\ \epsilon_3(\theta)&=P_BP_TC_{NN}(\theta)\,.\end{aligned}\tag{7}$$

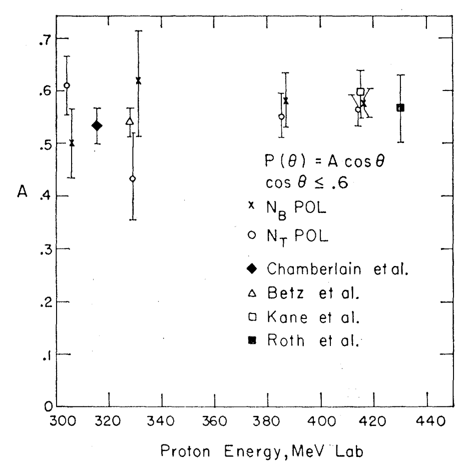

FIG. 5. Comparison of the two methods of computing the polarization $P(\theta)$ in p-p scattering using our data from the beam polarization (×) and from the target polarization (○). Comparison with other experimental data obtained with a polarized beam (◆, □, ■) and with a polarized target (△).

[20] A. Moretti, S. Suwa, and A. Yokosawa, in *Proceedings of the International Conference on Polarization Phenomena of Nucleons, Karlsruhe, 1965*, edited by P. Huber and H. Schopper (W. Rosch and Co., Bern, 1966), p. 128; F. Betz, J. Arens, O. Chamberlain, H. Dost, P. Grannis, M. Hansroul, L. Holloway, C. Schultz, and G. Shapiro, Phys. Rev. **148**, 1297 (1966).

[21] The experimental details are similar to those discussed by S. Suwa, A. Yokosawa, N. E. Booth, R. J. Esterling, and R. E. Hill, Phys. Rev. Letters **15**, 560 (1965).

[22] The actual situation is slightly more complicated because it is not possible to reverse exactly the magnitude of the target polarization. Thus each run is characterized by its own target polarization.

TABLE II. Measurements of the spin-spin correlation parameter $C_{NN}(\theta)$ in proton-proton scattering at 305, 330, 386, and 415 MeV.

$T=415$ MeV		$T=386$ MeV		$T=330$ MeV		$T=305$ MeV	
$\theta_{\text{c.m.}}$ (deg)	$C_{NN}(\theta)$	$\theta_{\text{c.m.}}$ (deg)	$C_{NN}(\theta)$	$\theta_{\text{c.m.}}$ (deg)	$C_{NN}(\theta)$	$\theta_{\text{c.m.}}$ (deg)	$C_{NN}(\theta)$
96.9	0.402±0.045	100.8	0.338±0.059	99.9	0.609±0.104	103.9	0.687±0.093
91.9	0.421±0.045	95.8	0.534±0.058	95.3	0.599±0.111	98.9	0.537±0.079
86.9	0.412±0.044	91.1	0.390±0.057	90.7	0.595±0.107	94.0	0.715±0.079
84.5	0.368±0.048	88.7	0.528±0.063	88.1	0.669±0.117	91.6	0.600±0.093
82.1	0.487±0.048	86.2	0.514±0.058	85.8	0.522±0.109	89.3	0.568±0.087
79.8	0.465±0.048	84.0	0.504±0.064	83.4	0.651±0.123	87.0	0.594±0.088
77.5	0.318±0.047	81.9	0.444±0.063	81.3	0.548±0.120	84.7	0.663±0.089
73.2	0.507±0.045	77.4	0.467±0.058	77.0	0.489±0.118	80.5	0.484±0.081
68.9	0.455±0.047	73.7	0.507±0.062	73.2	0.613±0.106	76.3	0.792±0.086
64.8	0.456±0.047	70.4	0.381±0.059	69.4	0.703±0.132	72.4	0.553±0.085
61.2	0.501±0.050	66.8	0.445±0.062	65.7	0.522±0.156	68.8	0.531±0.086
57.6	0.494±0.052	64.0	0.537±0.067	62.0	0.764±0.170	65.4	0.669±0.092
54.4	0.536±0.057	61.2	0.548±0.071	59.7	0.666±0.144	62.1	0.702±0.100
51.5	0.564±0.061	58.4	0.490±0.075			59.6	0.619±0.113
90°	0.40 ±0.04	90°	0.48 ±0.05	90°	0.60 ±0.08	90°	0.63 ±0.06

Figure 4 and Table I show $P(\theta)$ at 415 MeV computed from $\epsilon_1(\theta)$. We have found that a straight line, $P(\theta)=A\cos\theta$, gives a good fit to $P(\theta)$ versus $\cos\theta$ for $\cos\theta \leqslant 0.6$. Such fits were used to compare values of $P(\theta)$ computed from $\epsilon_1(\theta)$ and $\epsilon_2(\theta)$ and measured in other experiments.[18,20,23] In most cases there is good agreement between the results computed from $\epsilon_1(\theta)$ and $\epsilon_2(\theta)$. Such a comparison is given in Fig. 5. This gives us confidence that we know the target polarization reliably [a weighted average of the data at the four energies yields $A_B/A_T=1.00\pm0.05$, where $A_B(A_T)$ is determined from the straight-line fit to $\epsilon_1(\epsilon_2)$]. We also find reasonable agreement with other measurements of $P(\theta)$.

The results for $C_{NN}(\theta)$ are given in Table II and plotted at 415 MeV in Fig. 6. The indicated errors are statistical; a systematic error of $\pm 8\%$ should be added as a normalization error at each energy to take account of the uncertainties in P_B and P_T. At the bottom of Table II we give $C_{NN}(90°)$ computed by averaging the values for $|\cos\theta| \leqslant 0.1$.

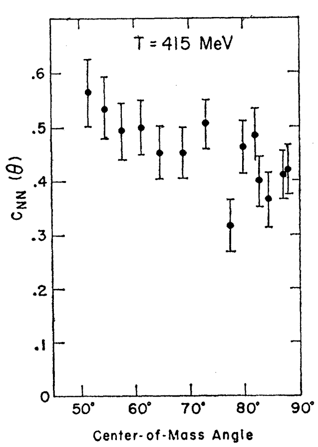

FIG. 6. Angular distribution of C_{NN} at 415 MeV.

[23] A convenient summary of the data may be found in R. Wilson, *The Nucleon-Nucleon Interaction: Experimental and Phenomenological Aspects* (Interscience Publishers, Inc., New York, 1963).

IV. DISCUSSION

Referring to Fig. 6, we notice a trend for C_{NN} to be larger at smaller angles at 415 MeV. At 575 MeV, the angular distributions over the same angular range appear flatter, while at 680 MeV, the trend is the opposite, with $C_{NN}(\theta)$ being smaller at smaller angles.[24] Below 415 MeV our angular distributions become flatter.

To compare our results with those of other experiments, we plot $C_{NN}(90°)$ versus laboratory kinetic energy in Fig. 7.[24] Many of the other experiments measured C_{NN} only at 90°. The majority of the points in Fig. 7 have been obtained in the last few years with polarized targets. Note that our values of $C_{NN}(90°)$ decrease as the energy increases in the range 300 to 400 MeV. Our points plus that of Coignet *et al.* at 575 MeV indicate a definite dip centered at about 500 MeV in the value of $C_{NN}(90°)$ with a subsequent rise and a peak at about 700 MeV. In view of some of the large errors and cases of disagreement, it is difficult to make any more quantitative statement. The rise in $C_{NN}(90°)$ beginning at about 600 MeV has been explained by Hoshizaki and Machida[25] on the basis that single-pion production is dominated by the process $N+N \rightarrow N+N^*$, where N^* is the $I=\frac{3}{2}$, $J=\frac{3}{2}$ resonance. We return to this point later.

[24] Points and errors are those given by the author's results except when not given. In these cases, the weighted average of those points with $|\cos\theta| \leq 0.1$ was used. In addition, a 10% normalization error was added in quadrature for the value given by Dost *et al.*, and one of 8% for the present experiment. N. Jarmie, J. E. Brolley, H. Kruse, H. C. Bryant, and R. Smythe, Phys. Rev. **155**, 1438 (1967); O. N. Jarvis *et al.* (provisional results kindly communicated to us by B. Rose, UKAEA, Harwell); G. Coignet, D. Cronenberger, K. Kuroda, A. Michalowicz, J. C. Oliver, M. Poulet, J. Teillac, M. Borghini, and C. Ryter, Nuovo Cimento **43A**, 708 (1966); Yu. M. Kazarinov, Rev. Mod. Phys. **39**, 509 (1967), quotes the results of B. M. Golvin, R. Ya. Zulkarneyev, V. S. Kiselev, S. V. Medved, V. I. Nikonorov, A. F. Pisarev, and G. L. Semashko; H. E. Dost, J. F. Arens, F. W. Betz, O. Chamberlain, M. J. Hansroul, L. E. Holloway, C. H. Schultz, and G. Shapiro, Phys. Rev. **153**, 1394 (1967).

[25] N. Hoshizaki and S. Machida, Progr. Theoret. Phys. (Kyoto) **29**, 49 (1963); **30**, 575 (1963).

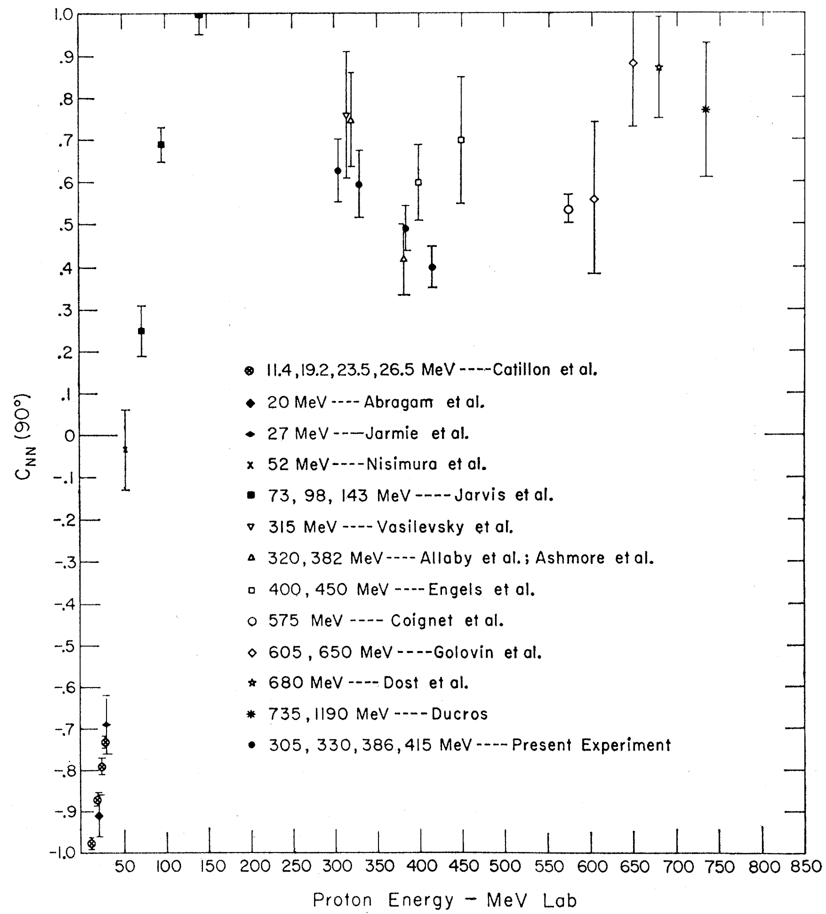

FIG. 7. $C_{NN}(90°)$ as a function of energy.

It has been pointed out by Stapp,[26] among others, that $C_{NN}(90°)$ has the very direct and simple interpretation

$$C_{NN}(90°)=1-2I_s/(I_s+I_t), \tag{8}$$

where I_s and I_t are the singlet and triplet cross sections at 90° and $I_s+I_t=I_0$. Thus $C_{NN}(90°)$ projects out the singlet part of the M matrix which involves only a relatively small number of partial waves.

Neglecting Coulomb terms, we have

$$I_0(90°)[I-C_{NN}(90°)]=\tfrac{1}{2}|M_{SS}(90°)|^2,$$

$$M_{SS}(\theta)=(1/ik)\sum_{l\text{ even}}(2l+1)P_l(\cos\theta)a_l, \tag{9}$$

where $a_l=r_le^{2i\delta_l}-1$; r_l and δ_l (both real) are the absorption coefficient and phase shift, respectively, for the partial wave of orbital angular momentum l. Even at 970 MeV Hama and Hoshizaki[27] consider only three terms in M_{SS}: the $^1S_0$, $^1D_2$, and $^1G_4$.

[26] H. P. Stapp, University of California Radiation Laboratory Report No. UCRL-3098, 1955 (unpublished).

[27] Y. Hama and N. Hoshizaki, Progr. Theoret. Phys. (Kyoto) **31**, 615 (1964).

Thus, if I_0 is known, and the $^1S_0$, $^1D_2$, and $^1G_4$ phase shifts are known, $C_{NN}(90°)$ may be computed; conversely, $C_{NN}(90°)$ may be used to place limits on the $^1S_0$, $^1D_2$, and $^1G_4$ phase shifts.

In Fig. 8 we construct simple vector diagrams to elucidate the relationship between $C_{NN}(90°)$ and the phase shifts at different energies. The vector diagrams are constructed by drawing from a point $(2l+1)P_l(0)$ on the y axis a line segment of length $r_l(2l+1)|P_l(0)|$ in the direction of the origin. This line segment is rotated by an angle $2\delta_l$ clockwise (counterclockwise) away from the y axis if the above-mentioned point falls along the $+(-)$ y axis. The singlet vector is obtained by joining the origin and the end of the rotated line segment. The vector sum of the singlet vectors is the vector M_{SS}. Referring to Fig. 8, we can understand the qualitative aspects of $C_{NN}(90°)$ between 10 and 1000 MeV on the basis of the following five points:

(1) At very low energies (10 MeV) only S-wave (singlet) scattering is possible and $C_{NN}=-1$ as indicated in Fig. 8(a).

(2) As the energy increases, the S-wave phase shift changes from attractive[28] to repulsive at about

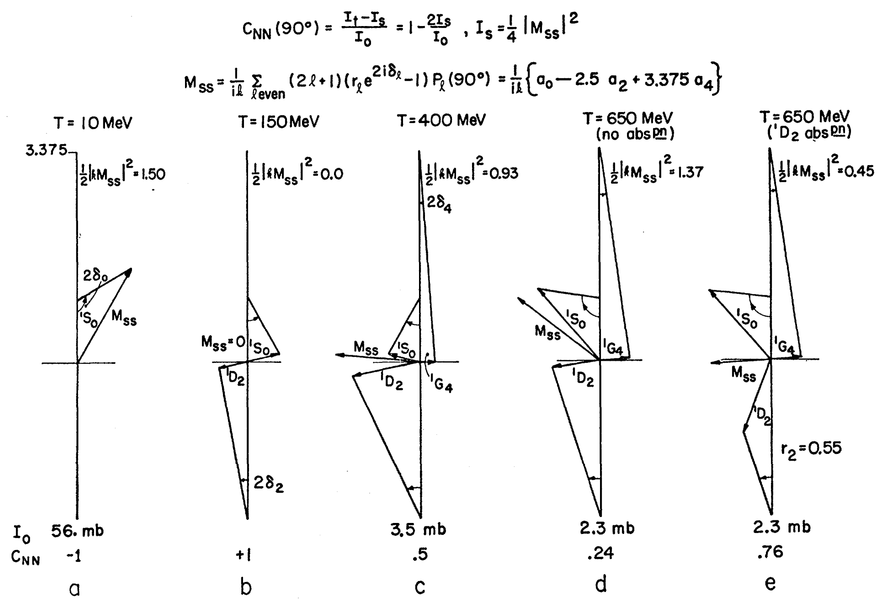

FIG. 8. Vector diagrams for the singlet amplitude $M_{SS}(90°)$ in p-p elastic scattering at several energies.

250 MeV. At 140 MeV the $^1S_0$ amplitude and the $^1D_2$ amplitude cancel. Hence there is no singlet scattering at 90° and $C_{NN}=+1$. [See Fig. 8(b).]

(3) Figure 8(c) shows the vector diagram for the singlet amplitude at 400 MeV. The situation is now more complicated, since the $^1G_4$ amplitude is present. Using the value of the differential cross section, one finds that the scattering is $\frac{3}{4}$ triplet and $\frac{1}{4}$ singlet.

(4) Hoshizaki and Machida have pointed out that, if the Stapp No. 1-type solution is extended to 650 MeV, the expected value for $C_{NN}(90°)$ is 0, which is in bad disagreement with the data. Pion production in S and P states requires that the initial state for the reaction $p+p\rightarrow n+N^*\rightarrow d+\pi^+$ be the $^1D_2$ state. The effect of this absorption in the $^1D_2$ state is illustrated in Figs. 8(d) and 8(e), where the value of the absorption coefficient is taken from an article by Soroko.[29] Furthermore, following the treatment given by Mandelstam,[30] Hoshizaki and Machida conclude that absorption in the triplet states is small. Hence the singlet amplitude is reduced relative to the triplet, yielding a high value for C_{NN}.

(5) The comparison of theory with C_{NN} at higher energies will be relatively straightforward because of the few partial waves involved. Using the peripheral model, Amaldi *et al.*[31] have been able to calculate the absorption coefficients in the energy interval 800 to 1400 MeV. They find absorption in the triplet states increasing in the interval 800 to 1000 MeV, while there is a decrease in the absorption of the $^1D_2$ partial wave. Since the character of the $^1S_0$ and $^1D_2$ are not expected to change, that is, the $^1S_0$ will remain repulsive and the $^1D_2$ attractive, this implies that the value of C_{NN} will decrease in this energy interval. This is in agreement with the recent experiment of Ducros *et al.*[32] which yields a value of $C_{NN}(90°)=0.4$ at 1190 MeV.

ACKNOWLEDGMENTS

My sponsor, Professor N. E. Booth, provided both the inspiration and guidance which made this experiment possible. I also thank my co-workers C. Dolnick, Dr. R. J. Esterling, Dr. R. E. Hill, J. Scheid, D. Sherden, and Dr. A. Yokosawa.

[28] Although the $^1S_0$ phase shift starts out positive, it reaches a maximum at about 8 MeV. It then decreases monotonically at higher energy. See M. A. Preston, *Physics of the Nucleus* (Addison-Wesley Publishing Company, Inc., London, 1962).

[29] L. Soroko, Zh. Eksperim. i Teor. Fiz. **35**, 276 (1958) [English transl.: Soviet Phys.—JETP **8**, 190 (1959)].

[30] S. Mandelstam, Proc. Roy. Soc. (London) **A244**, 491 (1958).

[31] J. Amaldi, Jr., R. Biancastelli, and S. Francaviglio, Nuovo Cimento **47**, 85 (1967).

[32] Y. Ducros, Rev. Mod. Phys. **39**, 531 (1967).