E_{th} is the threshold energy for production of antiprotons. The production rate is given by

$$q=\int_0^{p_{\max}}\frac{dq}{dp}dp. \tag{B2}$$

To obtain an expression for the differential cross section $d\sigma/dp$, we use the results of Wayland and Bowen[24]:

$$\frac{d^2\sigma}{dpd\Omega}=kT^2\mu_2'K_1\left(\frac{\mu_2'}{T_0}\right)\left[\exp\left(-\frac{\mu_1'}{T}\right)\right]\left(1+\frac{\mu_1'}{T}\right)\frac{E'p^2}{E}, \tag{B3}$$

where the primed quantities are in the c.m. system and T is the longitudinal temperature, T_0 is the transverse temperature, $\mu_2^2=p_\perp^2+m^2$, $\mu_1^2=p_{\parallel}^2+m^2$, p is the momentum of produced secondary, m is the mass of produced secondary,

$$k=2V_0/h^3m^2c^4T_0K_2(mc^2/T_0),$$

and V_0 is the interaction volume. As we are working with high energies, $d^2\sigma/dpd\Omega$ decreases very rapidly with increasing θ. Therefore, we can write $\cos\theta\approx 1$ and $\sin\theta\approx\theta$, and extend the range of integration over θ to ∞. Thus we have

$$\frac{d\sigma}{dp}=\int_0^{\infty}\frac{kT^2E'p^2}{E\gamma(1-\beta E/p)}(p'^2\theta^2+\delta^2)^{1/2}K_1\left[\frac{(p'^2\theta^2+\delta^2)^{1/2}}{T_0\gamma(1-\beta E/p)}\right]\times\exp\left(\frac{-\mu_1'}{T}\right)\left(1+\frac{\mu_1'}{T}\right)2\pi\theta d\theta, \tag{B4}$$

$$\frac{d\sigma}{dp}=\Phi\frac{T^2\gamma(E-\beta p)}{E}e^{-\mu_1'/T}\left(1+\frac{\mu_1'}{T}\right), \tag{B5}$$

where

$$\Phi=2\pi T_0m^2K_2(m/T_0)k,$$

$$\mu_1'^2=\gamma^2(p-\beta E)^2+m^2.$$

[24] J. R. Wayland and T. Bowen, Nuovo Cimento 48A, 663 (1967).

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Time Delay of Scattering Processes

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It is shown that Smith's formulation of the collision lifetime matrix can be applied to the relativistic steady-state solutions of Maxwell's equations for the total-reflection case. In this way, a delay time Δt is obtained which, though probably unobservable as a time process ($\Delta t\sim 10^{-15}$ sec), leads to measurable results in a suitable stationary experiment. By multiplying the delay time Δt by the group velocity of the wave in the vacuum, we obtain the displacement Δx of the wave in the vacuum parallel to the surface. If the incident wave is then collimated by a slit, the center of the reflected beam, on reappearing in the dielectric, will show a parallel displacement relative to the beam reflected at the mirror which is given by $\Delta x_{\parallel}=(\lambda/\pi)[(\sin\theta_i)/(\sin^2\theta_i-1/n^2)^{1/2}]$, where λ is the wavelength of the light in the dielectric of refractive index n, and θ_i is the incident angle, greater than or equal to the critical one. This is just the result of the experiment performed as early as 1947 by Goos and Hänchen, who found the parallel displacement to be given by $(\Delta x_{\parallel})_{\exp}=0.52\lambda[1/(\sin^2\theta_i-1/n^2)^{1/2}]$.

THE time delay of scattering processes has not been discussed explicitly until quite recently. Eisenbud,[1] Bohm,[2] and Wigner[3] have pointed out that a simple wave-packet analysis of an elastic collision (in single-channel scattering), which can be described by phase shift δ, implies a delay time which involves the energy derivative of the phase shift:

$$\Delta t=2\hbar\frac{\partial\delta}{\partial E}=-i\hbar\frac{\partial S}{\partial E}S^*, \tag{1}$$

where $S=e^{2i\delta}$ is the scattering matrix. Smith[4] had shown that it is possible to obtain a delay time from the steady-state solution of the time-independent Schrödinger equation for a single energy E. In his paper it was shown that a collision lifetime Q (in single-channel scattering) can be defined as the difference between the time spent

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[1] L. Eisenbud, dissertation, Princeton University, 1948, quoted in Ref. 3 (unpublished).
[2] D. Bohm, *Quantum Theory* (Prentice-Hall, Inc., Englewood Cliffs, N. J., 1951), pp. 257–261.
[3] E. Wigner, Phys. Rev. 98, 145 (1955).
[4] F. T. Smith, Phys. Rev. 118, 349 (1960).

by the particle in the region of the scattering interaction and the time spent in the same region in the absence of the scattering interaction:

$$Q=\lim_{Z\to\infty}\int_{-Z}^{0}[U_E(z)U_E^*(z)-2/v]dz. \qquad (2)$$

In (2), $U_E(z)$ is the steady-state solution normalized to unit incoming flux with a potential function that vanishes at large z and becomes infinite at $z=0$. The term $-2/v$ arises from considering the probability density in the absence of interaction ($V=0$ for all $z<0$ and is infinite for $z=0$). Furthermore, some device is needed to exclude oscillations of the integral. Smith was also able to prove that Q from (2) is identical with Δt as given by (1). Examples of scattering processes in which the form of the incident wave packet has been explicitly taken into account have been considered in detail by Beck[5] and by Nussenzveig.[6] The question of whether the interesting details of onset and straggling effects in the scattered wave packet can be studied experimentally has been posed by Austern.[7] Quite recently Froissart, Goldberger, and Watson[8] have shown that just as the energy derivative of the argument of the S matrix gives a time interval for events, the corresponding derivative with respect to momentum transfer gives a space interval. More generally, they suggested that these two derivatives may provide a basis for introducing space-time intervals into physical theory. Ohmura[9] has shown from a more general wave-packet analysis of a scattering process that the observable delay time is not always given in terms of the scattering matrix only, but depends also on the energy derivative of the phase $\alpha(E)$ of the incident wave packet. Since the latter is independent of the scattering matrix, the suggestion made by Froissart, Goldberger, and Watson would remain valid only under the conditions specified implicitly in their paper.

It is the aim of the present paper to show that Smith's formulation can be applied to the relativistic steady-state solutions of Maxwell's equations for the total-reflection case, and that observable results can be obtained from a suitable stationary experiment. We consider the left half-space filled with a medium of dielectric constant ϵ [$\epsilon(z)=\epsilon$ for all $z\leq 0$ and $=1$ for $z>0$]. We distinguish two cases: (a) the "free" case in which the plane wave incident from the left at an angle θ_i equal to or greater than the critical one [$\theta_{\rm crit}=\sin^{-1}(1/n)$] is totally reflected by a mirror at $z=0$; and (b) the second case in which the wave undergoes total reflection after penetrating some distance in the vacuum.[10]

Taking the electric field of the incident wave polarized normal to the plane of incidence (y axis), the electric fields in each case are, respectively, as follows: case (a)

$$E_y=E_0(e^{ik_zz}-e^{-ik_z})_{z\leq 0}e^{i(k_xx-\omega t)}, \qquad (3)$$

case (b)

$$\mathcal{E}_y=\mathcal{E}_0\left\{(e^{ik_zz}+e^{-2i\delta}e^{-ik_zz})_{z\leq 0}+\left(\frac{2k_z}{k_z^2+K^2}e^{-i\delta}e^{-Kz}\right)_{z\leq 0}\right\}e^{i(k_xx-\omega t)}, \qquad (4)$$

where $K=(1/n)[(n^2-1)k_x^2-k_z^2]^{1/2}$, $n=(\epsilon)^{1/2}$, and $\delta=\tan^{-1}(K/k_z)$. The solution in the absence of a mirror, for $z\leq 0$, satisfies $\partial^2\mathcal{E}_y/\partial z^2+k_z^2\mathcal{E}_y=0$, from which it follows that

$$(z\leq 0),\quad -\frac{1}{2k_z}\frac{\partial}{\partial z}\left[\mathcal{E}_y^*\frac{\partial^2\mathcal{E}_y}{\partial z\partial k_z}-\frac{\partial\mathcal{E}_y}{\partial k_z}\frac{\partial\mathcal{E}_y^*}{\partial z}\right]=\mathcal{E}_y\mathcal{E}_y^*. \qquad (5)$$

Likewise, for $z\geq 0$ it satisfies

$$(z\geq 0),\quad \frac{1}{2K}\frac{\partial}{\partial z}\left[\mathcal{E}_y^*\frac{\partial^2\mathcal{E}_y}{\partial z\partial K}-\frac{\partial\mathcal{E}_y}{\partial K}\frac{\partial\mathcal{E}_y^*}{\partial z}\right]=\mathcal{E}_y\mathcal{E}_y^*. \qquad (6)$$

The integrated excess electromagnetic energy density from $z=-Z$ to $z=+Z$ is given by

$$\Delta U(Z)=\frac{1}{8\pi}\int_{-Z}^{+Z}\epsilon(z)[\mathcal{E}_y\mathcal{E}_y^*-E_yE_y^*]dz, \qquad (7)$$

where Z must be taken always positive. This gives

$$\Delta U(Z)=\epsilon E_0E_0^*\left[-2\frac{\partial\delta}{\partial k_z}+\frac{\sin2(k_zZ-\delta)+\sin2k_zZ}{k_z}-\frac{4k_z^2Z(1+1/2KZ)}{\epsilon(k_z^2+K^2)}e^{-2KZ}\right], \qquad (8)$$

since we take $\mathcal{E}_0=E_0$. For sufficiently large Z the exponential term may be neglected, and the oscillating terms may be avoided by taking the average value of $\Delta U(Z)$ as Z increases. Thus, we have

$$\langle\Delta U\rangle\equiv\lim_{Z\to\infty}\frac{1}{Z}\int_{Z}^{2Z}\Delta U(Z')dZ'=-2\epsilon E_0E_0^*\partial\delta/\partial k_z. \qquad (9)$$

According to Smith's formulation, if we now divide $\langle\Delta U\rangle$ by the incoming Poynting energy flux $S_z^{\rm inc}=(c/8\pi)nE_0E_0^*\cos\theta_i$, we obtain the desired delay

[5] G. Beck and H. M. Nussenzveig, Nuovo Cimento **16**, 416 (1960).
[6] H. M. Nussenzveig, Nuovo Cimento **20**, 694 (1961).
[7] N. Austern, in *Selected Topics in Nuclear Theory* (International Atomic Energy Agency, Vienna, 1963).
[8] M. Froissart, M. L. Goldberger, and K. M. Watson, Phys. Rev. **131**, 2820 (1963).
[9] T. Ohmura, Progr. Theoret. Phys. (Kyoto) Suppl. **29**, 108 (1964).

[10] Hereafter the solutions without mirror will be written with script characters.

time, which is

$$\Delta t = -\frac{2\epsilon\omega}{c^2 k_z}\frac{\partial\delta}{\partial k_z}. \qquad (10a)$$

From $\partial k_z/\partial\omega = \epsilon\omega/c^2 k_z$, and since we take $S = e^{-2i\delta}$, we can write Δt as

$$\Delta t = -i\frac{\partial S}{\partial\omega}S^*. \qquad (10b)$$

The last formula is identical with (1) when one sets $E = \hbar\omega$. From the phase shift (by differentiation), the delay time Δt can be written as

$$\Delta t = \frac{\lambda}{(c/n)\pi}\frac{\sin\theta_i \tan\theta_i}{(\sin^2\theta_i - 1/n^2)^{1/2}}, \qquad (10c)$$

where λ is the wavelength of the light in the dielectric and $\theta_i \geq \theta_{\rm crit}$. As given by (10c), Δt is probably beyond experimental detection: It becomes infinite for $\theta_i = \theta_{\rm crit}$, but when θ_i deviates as little as 10 min of arc from $\theta_{\rm crit}$, Δt is about 10^{-14} sec and then remains always of the order of $(\lambda/c) \approx 10^{-15}$ sec. However, a mechanism can be imagined by which this delay time, unobservable as a time process, might lead to measurable results in a stationary experiment. During all the time the wave propagates through the vacuum, it must travel with a certain group velocity $v_x^{(g)}$ in order that momentum may be conserved along the reflecting surface. Upon dividing the Poynting vector along the surface by the energy density, we obtain $v_x^{(g)} = (c/n)\sin\theta_i$, and thus the displacement Δx of the wave in the vacuum parallel to the surface is given by

$$\Delta x = (\Delta t)v_x^{(g)} = \frac{\lambda}{\pi}\frac{\tan\theta_i}{(\sin^2\theta_i - 1/n^2)^{1/2}}. \qquad (11)$$

Now, if the incident wave is collimated by a slit, the center of the reflected beam, on reappearing in the dielectric, will show a parallel displacement relative to the beam reflected at the mirror given by

$$\Delta x_{||} = \Delta x \cos\theta_i = \frac{\lambda}{\pi}\frac{\sin\theta_i}{(\sin^2\theta_i - 1/n^2)^{1/2}}. \qquad (12)$$

This shift is also very small, but may be amplified

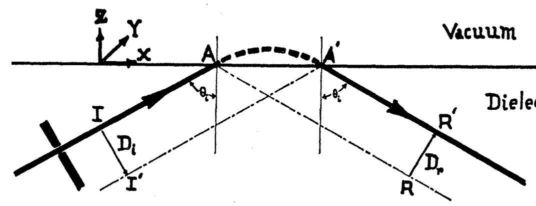

FIG. 1. The experiment of Goos and Hänchen.

by successive reflections to produce an observable effect as in the Goos and Hänchen experiment[11] (see Fig. 1). The optical beam IA, striking the interface under an angle $\theta_i \geq \theta_{\rm crit}$, is reflected not directly as a beam AR but as a beam $A'R'$, displaced parallel by a distance D_r, which they found to be given by

$$D_r = 0.52\lambda\frac{1}{(\sin^2\theta_i - 1/n^2)^{1/2}}, \qquad (13)$$

in fairly good agreement with Eq. (12), since[12]

$$1 \geq \sin\theta_i \geq (1/n).$$

Now, following Ref. 9, we can say that the displacement of the "incident orbit" is $\bar{D}_i$, normal to IA, and it is seen that $D_i = D_r$. Setting $E = \hbar\omega$, since the momentum transfer $\Delta\bar{p}$ is normal to the surface and equal to $2\hbar k_z$, its projection on a direction perpendicular to IA will be $\Delta p_{||} = 2\hbar k_z \sin\theta_i$, and thus according to (10), (11), and (12) we write

$$D_i = \Delta x_{||} = -2\frac{\partial\delta}{\partial k_z}\frac{1}{\sin\theta_i} = -2i\hbar S^*\frac{\partial S}{\partial\Delta p_{||}}, \qquad (14)$$

which shows that the displacement of the incident orbit in a direction normal to it, is given by the derivative of the S matrix with respect to the momentum transfer in the same direction. This is the result given by Froissart, Goldberger, and Watson.[8]

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[11] F. Goos and H. Hänchen, Ann. Physik 6, 333 (1947).

[12] The presence of $\sin\theta_i$ in the numerator of (12) was also obtained in the earliest stationary treatments of the Goos and Hänchen experiment given by K. Artmann [Ann. Physik 6, 87 (1948)] and C. von Fragstein [Ann. Physik 6, 271 (1949)].