for tachyons up to some energy are already filled. If this energy is above the energy of the photons used in this experiment, no tachyons would have been produced. The limit that is presented assumes that these two effects are not important.

The limit that is presented does not consider the process of a photon decaying into a photon plus tachyons.[2] If such a process has a high probability of occurring, the limit set by this experiment would be lower than the quoted result.

It is important to note that faster-than-light particles of any "rest mass" could have been created in the experiment.[9] It is possible that the production cross section is pathologically small below energies corresponding to the "rest mass" μ of the tachyons. This possibility, although not based on any theoretical arguments, lends some support to efforts to extend the measurements to higher photon energies. Also, there is some interest in placing a limit on the existence of completely neutral faster-than-light particles. Experiments along both of these lines are now being planned, as well as plans to extend the sensitivity of the present experiment. It is felt that it is possible to increase the sensitivity of the present experiment by several orders of magnitude with only minor changes in the detector system.

[9] It should be noted that very low mass tachyons might not have been seen in this experiment. This case has been considered in another experiment [T. Alväger, P. Erman, and A. Kerek (unpublished)] which also gave a negative result.

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Gravitationally Induced Electric Field near a Conductor, and Its Relation to the Surface-Stress Concept

CONYERS HERRING
Bell Telephone Laboratories, Murray Hill, New Jersey 07971
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The contact potential difference between points just outside different parts of the surface of a conducting body is altered when the body is acted on by a gravitational field, because of the effect of gravity on the work function of the surface. A reciprocity relation due to Schiff and Barnhill relates this effect of gravity to the shift in mass moment of the conductor produced by shifting the position of a test charge near it, and shows how the effect can be expressed as the sum of a purely electronic term and a nuclear term due to distortion of the crystal lattice. It is shown here how the nuclear term, which describes the effect on the work function of the distortion of the body by its own weight, can also be described, via the reciprocity relation, in terms of the local alteration of the surface stress of the body by the electric field due to the test charge. The contribution of the nuclear term to the electric field in a vertical metal tube is expected to be of the order of 10^{-9} to 10^{-8} V/cm, much larger than the Schiff-Barnhill electronic contribution of -5.6×10^{-13} V/cm.

1. THE PROBLEM

CONFLICTING views have recently been expressed[1,2] regarding the theoretical effect of gravity on the electrostatic potential just outside the surface of a conductor. This question arises in connection with any experiment aimed at measuring the force of gravity on charged elementary particles: To avoid having gravity swamped by stray electrostatic fields, one must introduce metallic shields; one must then worry about the electrostatic fields inside these shields. These fields can conceivably arise from random causes such as fluctuations in surface contamination or crystal grain texture, from space charge in the evacuated region, and from a systematic effect of gravity in modifying the charge distribution that would exist in the bulk and surface of the metal in the absence of gravity. The present paper, like those cited,[1,2] will discuss only this gravitational effect; it will show how to reconcile the large field estimated by Dessler *et al.*[2] with the reciprocity-theorem approach of Schiff and Barnhill,[1] which led them to predict a much smaller field.

The most obvious way to attack the problem in question is to view it as one of computing (or at least estimating) the effect of stress on the work function. When the different parts of a conducting body are in equilibrium with one another in a gravitational field, the electrochemical potential $\bar{\mu}$ of the electrons must be the same in all these different parts. In a gravitational field the role played by the electrostatic potential

[1] L. I. Schiff and M. V. Barnhill, Phys. Rev. **151**, 1067 (1966).
[2] A. J. Dessler, F. C. Michel, H. E. Rorschach, and G. T. Trammel, Phys. Rev. **168**, 737 (1968).

energy $-e\Phi$ in the usual definition[3] of the work function φ of any region of the surface of a conductor is now played by the total potential energy $-e\Phi+m\Psi$, where Ψ is the gravitational potential. Thus

$$e\varphi=-e\Phi-\bar{\mu}+m\Psi, \tag{1}$$

and so it follows that the electrostatic potential difference $\Delta\Phi$ between points just outside two different regions of the conductor must be

$$\Delta\Phi=-\Delta\varphi+(m/e)\Delta\Psi, \tag{2}$$

where $\Delta\varphi$ is the difference in the work functions of these two regions, $\Delta\Psi$ the difference of their gravitational potentials. This is essentially the well-known expression for contact potential difference. Now imposition of a gravitational field on the conductor will cause stresses and strains to be developed in it, which will in general depend on its geometry and on the way in which it is supported; if the stress or strain is different at the two points being compared in Eq. (2), there will be, in addition to the second term of (2), a gravity-induced term $-\Delta_g\varphi$, representing the difference in the effect of strain on the work functions at the two points.

The second term of Eq. (2) gives a downward vertical contribution to the electric field $-\nabla\Phi$ just outside the conductor, of magnitude $gm/e=5.6\times10^{-13}$ V/cm, where g is the acceleration of gravity. The first term of Eq. (2) can be expected to give a much larger contribution. From the known relationships between the work function and quantities entering the theory of cohesion of metals,[4] one expects that a compressive strain u of the order of unity will alter the bulk contribution to the work function of a typical metal by an amount $d\varphi/du$ of the order of a volt; the surface-dipole contribution, which adds to the bulk contribution to give the total work function, should have a comparable dependence on strain. Now one expects the zz component of the stress tensor in a body of density ρ to have a variation with height z something like $-\rho gz$ plus a constant; this is in fact exactly correct for a vertical rod or tube of uniform cross section. From this and the elastic constants one can calculate the variation of compressive strain with height, and hence, with the estimate $d\varphi/du\sim1$ V, the field $E_z=-d\Phi/dz$. The result, as shown by Dessler *et al.*,[2] is $|E_z|\sim10^{-9}-10^{-8}$ V/cm, depending on the amount of accidental cancellation of the bulk and surface-dipole terms.

An alternative approach to the problem, due to Schiff and Barnhill,[1] makes use of a reciprocity relation connecting $\Delta_g\Phi$, the first-order gravity-induced contribution to the difference of potential between two points just outside two spots on the surface of the conductor, with Δ_qM_z, the first-order change in the vertical component of the mass moment of the conductor when a test charge q is moved from the one point to the other. Treating the magnitude q of the test charge and the acceleration g of gravity as infinitesimal perturbations on the thermodynamic state of the conductor, we have, to first order in g and q,

$$\Delta_g\Phi=g\frac{\partial\Delta\Phi}{\partial g}=g\left[\frac{\partial}{\partial g}\frac{\partial\Delta\mathfrak{F}}{\partial q}\right]_{g=q=0}, \tag{3}$$

$$\Delta_qM_z=q\frac{\partial\Delta M_z}{\partial q}=q\left[\frac{\partial}{\partial q}\frac{\partial\Delta\mathfrak{F}}{\partial g}\right]_{g=q=0}, \tag{4}$$

where $\Delta\mathfrak{F}$ is the difference in free energy (or, at $T=0$, in ground-state energy) between situations with the test charge at the two points, respectively. Thus $\Delta g\Phi$ is just g/q times Δ_qM_z.

Now Δ_qM_z is made up of shifts in the electronic and nuclear mass distributions; it is convenient to write

$$\Delta_qM_z=\Delta_qM_z^{(e)}+\Delta_qM_z^{(n)}, \tag{5}$$

where $\Delta_qM_z^{(e)}$ is the contribution of the shift in the electronic mass distribution when all the nuclei are held fixed, and $\Delta_qM_z^{(n)}$ is the remaining shift in M_z (mostly, but not entirely, nuclear) when the nuclei are allowed to relax. For a conducting body, the first of these terms can easily be evaluated by noting that the perturbation of the electronic density in the conductor, due to a charge q near some point of its surface, consists in the accumulation of an extra charge $-q$ near this point (the image charge) and of a charge $+q$ in remote regions of the surface, with a distribution determined by the condition that the surface of the conductor be an equipotential. If the conductor is a closed hollow body, the distribution of the $+q$ on the outer surface will be independent of the position of q in the interior space, and Schiff and Barnhill gave a simple proof that the centroid of the image charge is exactly at the position of q. We thus have the simple result that

$$\Delta_qM_z^{(e)}=(m/e)q\Delta z, \tag{6}$$

where m is the mass of the electron, e the magnitude of its charge, and Δz is the difference in the vertical coordinates of the two points being compared. According to the reciprocity relation, this corresponds to a contribution

$$E_z^{(e)}=-gm/e=-5.6\times10^{-13}\text{ V/cm} \tag{7}$$

to the electric field in the space internal to the conductor. Clearly, Eq. (7) will also apply as a good approximation in the interior of a long hollow tube, etc. Note that this is exactly the same downward field which we computed above from the second term of Eq. (2): *Neglect of $\Delta_qM_z^{(n)}$ in Eq. (5) is equivalent to neglecting the effect of gravity on the work function.* This could have been anticipated, since holding the nuclei fixed prevents gravity from affecting the electron density significantly.

Schiff and Barnhill made a rough estimate of the other contribution $E_z^{(n)}$ to the field inside a hollow

[3] C. Herring and M. H. Nichols, Rev. Mod. Phys. **21**, 185 (1949), Sec. I.2.

[4] E. Wigner and J. Bardeen, Phys. Rev. **48**, 84 (1935); Ref. 3, Sec. IV.1.

conductor, arising via the reciprocity theorem from the second (mainly nuclear) term of Eq. (5). They concluded that for conductors of macroscopic size this term would be much smaller than Eq. (7). They reached this conclusion, despite the fact that nuclear masses are much larger than electronic masses, on the basis of reasoning which seemed to indicate that nuclear displacements of an atomic scale were to be compared with electronic displacements of a macroscopic scale. That the latter are macroscopic is obvious, since for situations of the type considered in the preceding paragraph the image charge can be shifted over macroscopic distances by moving q, while the compensating external surface charge shifts little or not at all. Whether the effect of q on the nuclear coordinates extends to a macroscopic distance or not is a little less obvious; it depends both on the nature of the forces that q exerts on the metal, and on the solution of a problem in elasticity. We show, in Secs. 2 and 3, that the nuclear displacements due to q do indeed extend to macroscopic distances, and that the term $\Delta_q M_z^{(n)}$ in Eq. (5) will in general be larger than $\Delta_q M_z^{(e)}$ by something of the order of the ratio of the nuclear to the electronic mass: the reciprocity-theorem approach agrees with the approach via strain dependence of the work function in yielding an expected gravity contribution to the vertical electric field of the order of 10^{-9}–10^{-8} V/cm. In Sec. 4, finally, we shall discuss a few further aspects of the problem. A few words are added in Appendix A about the case of a nonconducting body.

2. "SURFACE PINCH"

As has just been indicated, the first step in the calculation of the effect of a test charge q on the nuclear mass moment of a metal specimen must be an evaluation of the forces which the introduction of q exerts on the nuclei. More precisely, our procedure will be first to introduce q holding all nuclei fixed, to estimate the forces that must be applied to the nuclei to do this, and then to calculate, by elasticity theory, the displacements of the nuclei when these forces are removed. Our concern in this section will be with the estimation of the forces; Sec. 3 deals with the elastic response.

We are interested only in forces of the first order in q, since our basic equations [(3) and (4)] involve derivatives with respect to q at $q=0$. To this order the *net* force is just the attraction of q for its image, and is of order q^2. Moreover, even if we separate the metal into electrons and nuclei or ions, the net force on the latter alone, due to q and to the modification q induces in the electron distribution, can easily be shown to be of order q^2 also. Thus the $\Delta_q M_z^{(n)}$ of Eq. (5) can arise only from higher moments of the force distribution, i.e., from the fact that forces are felt by nuclei in a region of diameter comparable with d, the distance of q from the surface (though perhaps only an atom layer or so deep), equal and opposite forces being felt in two regions a distance

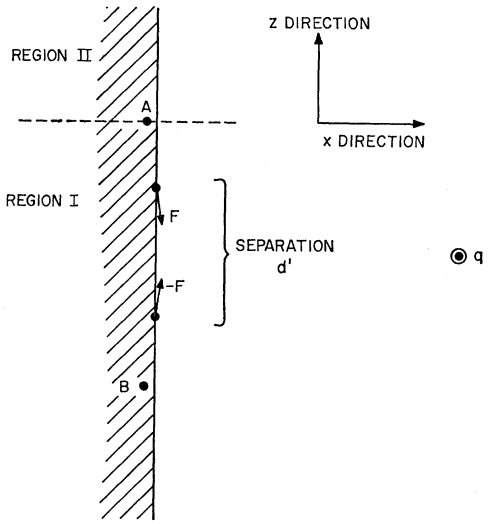

FIG. 1. Schematic diagram of a metal surface showing (1) the formation of a "surface pinch" by two equal and opposite forces **F** acting in the plane at the surface at different points; (2) the contribution to a surface pinch from the forces on two nuclei A,B due to a test charge q and its image; and (3) the definition of surface stress in terms of force and momentum transmitted between two regions I,II.

of order d apart. As will now be shown in detail, the effect of q in shifting the vertical component of the nuclear-mass moment of the metal is equivalent to that of applying a pair of equal and opposite tangential forces **F** to the surface, as shown in Fig. 1, so related to the separation d' of their points of application as to make the product Fd' proportional to q and independent of the distance d of q from the surface, if d is $\ll$ radii of curvature, etc. Such a pair of forces we shall call a "surface pinch of strength $-Fd'$."

It is obvious, from the analogy of pinching a rubber wall with one's fingers, that when the metal is allowed to respond elastically to such a pinch there will be a net transfer of mass from remote regions into the pinched region. We shall calculate this effect quantitatively in Sec. 3.

A. Microscopic Approach

Let us first consider the nuclei one at a time. When the test charge q is introduced near the surface, holding the nuclei fixed in position, the only nuclei that will experience appreciable forces will be those within a few atom layers of the surface, the deeper ones being screened from all influence by the metallic electrons. Let us choose a coordinate system with the z direction parallel to the surface and the x direction normal to it, as shown in Fig. 1. Consider the variation of E_z, the z component of the electric field due to q and its image charge, as we vary x along some line like the dashed line in Fig. 1, chosen to pass through the nucleus of some atom A of a surface layer. This variation might look something like the curve of Fig. 2; it may oscillate in sign a little inside the metal, as most electronic

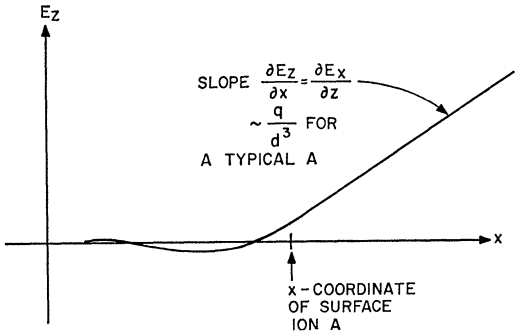

FIG. 2. Schematic variation of the electric field parallel to the surface of Fig. 1 with position along a line normal to this surface. The ordinate represents only that part of the field which is due to the test charge q and its image, d being the distance of q from the surface.

responses do, but its zero will not normally occur at the exact position of A. Thus we expect the force $\mathbf{F}^{(A)}$ required to hold A fixed to have a z component

$$F_z^{(A)}=Zeb_A\left(\frac{\partial E_z}{\partial x}\right)_{\text{out}}=qZe\frac{b_A(z_A-z_q)d}{|\mathbf{r}_A-\mathbf{r}_q|^5}, \tag{8}$$

where Z is the nuclear charge, b_A is a length of atomic magnitude, dependent on the lattice constant and the screening length, $(\partial E_z/\partial x)_{\text{out}}$ is the slope of the right-hand part of Fig. 2, d is the distance of q from the surface, and $\mathbf{r}_A$, $\mathbf{r}_q$ are the coordinates of A and q, respectively. Clearly an atom B, located as far below q as A is above it (see Fig. 1), will require a force $\mathbf{F}^{(B)}$ with $F_z^{(B)}=-F_z^{(A)}$. Any such pair A,B thus gives a surface pinch of the sort described in the preceding paragraph. Summing over all atoms in the surface layers gives a total pinch strength

$$P_{zz}=\sum_A F_z^{(A)}z_A\approx\frac{1}{a^2}\int\int dy_A dz_A \times\sum_{\text{line}}[\text{right of (8)}]_{z_A=\frac{1}{3}\pi}=\frac{qZeb}{a^2}, \tag{9}$$

where $\sum_{\text{line}}$ means a summation over a line of atoms running inward from the surface beneath atom A, a is the lattice constant, and b, derived from the b_A's in Eq. (8), is also a length of atomic dimensions. Note that Eq. (9) is independent of the distance d of q from the surface.

There will be, of course, a yy pinch $P_{yy}\equiv\sum_A F_y^{(A)}y_A$ of the same order as Eq. (9), and for anisotropic surfaces perhaps a P_{yz} and P_{zy}. It is obvious from the symmetry of Fig. 1 with regard to reversal of the z or y direction that there can be no P_{xz} or P_{xy} of this order; we shall show in Sec. 2 B that in fact the sum of $F_x^{(A)}$ over atoms A underneath any significant element of area of the surface is smaller by a factor of order a/d than the corresponding sum of $F_z^{(A)}$.

B. Macroscopic Approach

A less detailed but neater and in some ways more satisfactory approach to the surface-pinching phenomenon can be given in terms of purely macroscopic concepts. The key concept is that of surface stress,[5] defined as the surface excess, in the sense of Gibbs, of force transmitted across a plane. Specifically, consider a plane normal to the z direction in Fig. 1, dividing the metal into two regions, as shown by the dashed line. The force exerted by region I (above) on region II (below) is defined, as usual, as the sum of all forces exerted by particles of I on particles of II, plus the rate of kinetic transfer of momentum across the plane. Let K_z be the value of this force, out to a distance Δx to the left of the surface, and p_{zz} the zz component of the stress tensor well inside the surface. Then the surface stress component G_{zz} is defined by

$$G_{zz}=(K_z/\Delta y)-p_{zz}\Delta x, \tag{10}$$

where Δy is the width, in the direction of the line of intersection of the dividing plane with the surface, of the portion of dividing plane being considered; if p_{zz} is small in atomic units, ambiguities in the location of the surface or in the inner limit Δx will have a negligible effect on G_{zz}. Obviously other components of a two-dimensional tensor $G_{\mu\nu}$ can be defined similarly as the surface excess of force in the μ direction transmitted across a plane normal to a direction ν in the surface.

The important thing for us is that the surface stress can be modified by the presence of an electric field at the surface. However, for a conducting body a (normal) electric field at the surface implies charging of the surface, and for a uniformly charged infinite plane surface the force K_z as we have defined it is infinite, the electrostatic repulsions of the surface charges diverging at infinity. The obvious way to modify our definition of $G_{\mu\nu}$ for such a case is to replace the K_z in Eq. (10) by K_z', the total force less the interaction of the macroscopic distribution of surface charges, the latter being described by some two-dimensional density $\sigma(\mathbf{r})$ localized on the two-dimensional "surface" of the body, and varying appreciably only over macroscopic distances. Consideration of the equilibrium of a pill box of matter just beneath an element of the surface then gives, as the boundary condition on the macroscopic stress field $p_{\mu\nu}$ inside the body,

$$\sum_\nu p_{\mu\nu}n_\nu=\sum_{\alpha\text{ in surf.}}\frac{\partial G_{\mu\alpha}}{\partial x_\alpha}+\sigma E_\mu \tag{11}$$

on the surface, where $\mathbf{n}$ is the unit outward normal to the surface, $\mathbf{E}$ is the macroscopic electric field, and the

[5] For a brief review of this concept, see C. Herring, in *Structure and Properties of Solid Surfaces*, edited by R. Gomer and C. S. Smith (University of Chicago Press, Chicago, 1953), p. 5, especially Sec. II. Note that while for a liquid the surface stress can be shown to equal the ordinary surface tension, defined in terms of free energy, this equality does not hold for crystals.

summation on the right is just the surface divergence of $G_{\mu\alpha}$.

Note that rotational equilibrium of elementary regions requires that $\sum_\mu n_\mu G_{\mu\alpha}=0$ in the absence of a macroscopic surface density of externally applied torque. Thus the effective surface force density (11) has no normal component in the present macroscopic approximation; this justifies the statement made at the end of Sec. 2 A.

The problem of interest to us is the elastic distortion of a conducting body when an infinitesimal charge q is introduced at a distance d from a portion of its surface, d being small compared with the dimensions of the body but still of macroscopic size. The σE_μ term in Eq. (11) is of the second order in q, so we can neglect it. Whatever stress field may originally have been present in the body will now have added to it an amount $p_{\mu\nu}(r)$, determined by the equations of elasticity and a boundary condition

$$\sum_\nu p_{\mu\nu}n_\nu=\sum_{\alpha \text{ in surf.}}\frac{\partial G_{\mu\alpha}}{\partial\sigma}\frac{\partial\sigma}{\partial x_\alpha} \tag{12}$$

on the surface near q.[6] With the coordinate directions of Fig. 1, $\partial\sigma/\partial z$ is q times a positive quantity at positions above q, a negative quantity below q. The right of Eq. (12) is thus equivalent to a distribution of surface forces whose zz pinch is

$$P_{zz}=\frac{\partial G_{zz}}{\partial\sigma}\int\int dzdy\, z\frac{\partial\sigma}{\partial z}=-\frac{\partial G_{zz}}{\partial\sigma}\int\int\sigma dzdy$$
$$=q\frac{\partial G_{zz}}{\partial\sigma};$$

this can obviously be generalized to

$$P_{\mu\nu}=q\frac{\partial G_{\mu\nu}}{\partial\sigma}. \tag{13}$$

We can expect charging of the surface to alter G_{zz} by a large percentage when the excess charge in the surface cells becomes of the order of the valence-electron charge initially in them, i.e., when the surface charge density σ becomes of order Xe/a^2, where X is the valence and a the lattice constant. Since G_{zz} is of order $e^2/a^2\delta$, where δ is a length of atomic dimensions (usually of the order of $10a$), we get for Eq. (13)

$$P_{zz}\approx qXe/\delta, \tag{14}$$

i.e., a value in essential agreement with our earlier estimate [Eq. (9)].

3. ELASTIC RESPONSE TO THE SURFACE PINCH

As we have mentioned above, it is intuitively obvious that a surface pinch will move matter elastically into or out of the pinched region, and it is not hard to convince oneself that it will produce significant displacements at large distances, so that the effective transport of matter is from remote regions of the specimen to the pinched region. We shall illustrate this explicitly for a particular geometry in the next paragraph. Let us first, however, make the almost trivial observation that one can use a reciprocity argument like that of Sec. 1 to relate the effect of a surface pinch on the mass moment to the effect of gravity on the state of strain at the surface. For if, as in Sec. 1, we let $\mathfrak{F}$ be the free energy of the body, $\mathbf{M}$ its mass moment, g the acceleration of gravity (acting in the negative z direction), we have for the response to a surface pinch $P_{\mu\nu}$ applied to a point $\boldsymbol{r}$ of the surface

$$\frac{\partial M_z}{\partial P_{\mu\nu}(\mathbf{r})}=\frac{\partial^2\mathfrak{F}}{\partial P_{\mu\nu}(\mathbf{r})\partial g}=\frac{\partial}{\partial g}\frac{\partial\mathfrak{F}}{\partial P_{\mu\nu}(\mathbf{r})}=-\frac{\partial}{\partial g}u_{\mu\nu}(\mathbf{r}), \tag{15}$$

where $u_{\mu\nu}(\mathbf{r})$ is the strain tensor at the point $\mathbf{r}$.

Since this strain in general depends on the complete geometry of the body and the location of its supports, it is obvious that a direct calculation of the left of Eq. (15), such as we shall in fact perform in the next paragraph, must involve significant displacements in regions of the body remote from $\mathbf{r}$, and can be correctly performed only by solving an elasticity problem that takes correct account of the exact boundary conditions at such remote points. If one compares the mass moments for two positions of the test charge q that are infinitesimally close together, one can compute the $\Delta M_z^{(n)}$ to be used in Eq. (5) from the elastic response to a pair of equal and opposite surface pinches applied at points an infinitesimal distance apart. In a semi-infinite medium the displacement in such a response would die off as the second derivative of that in response to an applied surface force, but as this is still only as the inverse cube of the distance,[7] the integral over the specimen cannot be computed unambiguously for the semi-infinite model, and account must be taken of the shape-dependent boundary conditions at remote points.

For an explicit illustration of (15), let us take a body in the form of a long cylinder with a cross section of arbitrary shape; this is the geometry assumed in the celebrated Saint-Venant problem of elasticity theory. Let the long direction of the body be the z direction, and let it be constrained at its lower end, which we take at $z=0$. Application of a vertical force F_z at a point at height z_1 will produce a strain u_{zz} equal to F_z/SY at

[6] To readers versed in the theory of electrocapillarity, it may seem surprising that after throwing away the second term of Eq. (11) as being of the second order in q, we have retained the first term in the form of Eq. (12), treating it as of the first order: For a fluid surface, $\partial G_{\mu\nu}/\partial\sigma$ is itself of order σ, hence of q in a case like ours. The surface of a solid behaves differently however, and its $\partial G_{\mu\nu}/\partial\sigma$ is, in general, finite as $\sigma\to 0$. This difference in the behavior of solids and fluids is elucidated in Appendix B; it arises ultimately from the long-range lattice periodicity, the same cause, in fact, as is responsible for making the surface stress $G_{\mu\nu}$ of a solid surface different from $\delta_{\mu\nu}$ times the quantity usually called "surface tension."

[7] L. Landau and E. M. Lifshitz, *Theory of Elasticity* (Pergamon Press, Ltd., London, 1959), Sec. 8.

all points with z between 0 and z_1, except near the ends of this range, S being the cross-sectional area and Y being Young's modulus. It will shift the centroid of the matter between z_1 and the upper end of the body, at $z=L$, by the displacement of z_1 and shift the centroid of the matter below z_1 by half this (plus a small end correction in each case). Thus the force will alter the mass moment by

$$\delta_{F_z} M_z = (F_z\rho/Y)(Lz_1-\tfrac{1}{2}z_1^2)+\text{end correction}, \quad (16)$$

where ρ is the density. A second force $-F_z$ applied at $z_2=z_1-d'$ will affect M_z correspondingly, and since by Saint-Venant's principle the end correction will be practically independent of z_1 or z_2, the surface pinch P_{zz} formed by the two forces will affect the mass moment by

$$\begin{aligned}\delta_{P_{zz}} M_z &= (F_z d')(L-z_1)\rho/Y+O(d'^2)\\ &= P_{zz}(L-z_1)\rho/Y+O(d'^2), \end{aligned} \quad (17)$$

a relation clearly identical with the zz component of Eq. (15). The meaning of (17) is that the pinch effectively removes a mass $P_{zz}\rho/Y$ from the region in which it is applied, and transports it to the unconstrained end of the specimen.

A pinch P_{yy} in the horizontal direction of the surface will clearly also alter M_z by an amount of the same order as (17), though in this case it is easier to compute the alteration from (15) than to compute it directly: for an elastically isotropic body the result is, of course,

$$\delta_{P_{yy}} M_z = -P_{yy}(L-z_1)\rho\eta/Y, \quad (18)$$

where η is Poisson's ratio.

4. CONCLUSION

We have shown in Sec. 2 how a surface pinch is produced by bringing a test charge q close to some point of the surface of a conductor. The elastic shift in the mass moment of the conductor, produced by this pinch as computed in Sec. 3, is responsible for the $\Delta_q M_z^{(n)}$ term in Eq. (5), and adds to the value (6) already computed for the $\Delta_q M_z^{(e)}$ term. Thus, using Δ as in Sec. 1 to denote a difference between two locations for q and choosing the z direction upward, we have, from Eqs. (13) and (15),

$$\Delta_q M_z^{(n)} = -q\sum_{\mu,\nu}\frac{\partial G_{\mu\nu}}{\partial\sigma}\frac{\partial\Delta u_{\mu\nu}}{\partial g}. \quad (19)$$

Use of this in the reciprocity relation [Eqs. (3) and (4)] gives the contribution to the electric field near a vertical surface of a conductor:

$$E_z^{(n)} = -\frac{g}{q}\frac{\Delta_q M_z^{(n)}}{\Delta z} = \sum_{\mu,\nu}\frac{\partial G_{\mu\nu}}{\partial\sigma}\frac{\partial u_{\mu\nu}^{(g)}}{\partial z}, \quad (20)$$

where $u_{\mu\nu}^{(g)}$ is the strain produced in the conductor by gravity, at the point of its surface near which $\mathbf{E}$ is measured. The total contribution of gravity to the field is the sum of Eqs. (7) and (20). In general, for a body of unsymmetrical shape, there will also be other components of $\mathbf{E}^{(n)}$ besides the z component, which can be computed analogously; this contrasts with the field $\mathbf{E}^{(e)}$ of Eq. (7), which, since it corresponds to the second term of (2), is always exactly in the $-z$ direction.

Let us specialize (20) to the field in a long, hollow, vertical cylinder, or in general near a long vertical rod, assuming the surface stress tensor $G_{\mu\nu}$ to be isotropic. We then have, from Eq. (20), or from Eqs. (3), (4), (17), and (18),

$$E_z^{(n)} = \frac{\partial G_{zz}}{\partial\sigma}\frac{\rho g(1-\eta)}{Y}. \quad (21)$$

For copper take density $\rho=9.0$ g/cm^3, Poisson's ratio $\eta=0.37$, and Young's modulus $Y=1.2\times10^{12}$ erg/cm^3:

$$E_z^{(n)}(\text{Cu}) = 2.0\times10^{-6}\frac{\partial G \text{ erg/cm}^2}{\partial\sigma \text{ esu/cm}^2}\ V/\text{cm}. \quad (22)$$

According to the estimate leading to (14), this may be expected to be of the order of a few $\times10^{-9}$ V/cm, in agreement with the estimate made at the start of Sec. 1, or with that of Dessler *et al.*[2]

The agreement between the $E_z^{(n)}$ calculated in this way from the Schiff-Barnhill reciprocity relation and that calculated from the effect of strain on the work function is a consequence of the reciprocity relation between the charge dependence of surface stress and the strain dependence of the work function[8]:

$$\partial G_{\mu\nu}/\partial\sigma = \partial\varphi/\partial u_{\mu\nu}. \quad (23)$$

Although one may calculate $E_z^{(n)}$ by estimating either the right or the left of Eq. (23), it seems difficult to get an accurate estimate either way. The surface stress clearly depends on conditions in the difficult transition region between the interior and the vacuum; so does the work function, since an important part of it is the surface double layer contribution.[9] However, it may be possible to determine the strain dependence of the work function experimentally in a contact-potential measurement, or to measure the dependence of surface stress on normal field by using this dependence to excite acoustic oscillations.

Careful experiments[10] on the motion of charged particles in vertical metal tubes have been interpreted as indicating a total electric field much less than that expected from estimates of the strain derivative of the

[8] This relation is based, as is our preceding analysis, on the assumption that there are no mobile-adsorbed species, other than electrons. If such species are present, there will be a contribution to the $\Delta_q M_z^{(n)}$ of Eq. (5) from the migration of adsorbed atoms induced by the presence of q; this migration will also alter the surface stress. Such effects are taken into account in the general thermodynamic treatment of Appendix B.

[9] Reference 3, Secs. IV.1, IV.2; C. Herring, in *Metal Interfaces* (American Society for Metals, Cleveland, Ohio, 1952), p. 1.

[10] F. C. Witteborn, Ph.D. thesis, Stanford University, 1965 (unpublished); F. C. Witteborn and W. M. Fairbank, Phys. Rev. Letters **19**, 1049 (1967).

work function, and in fact of the order of Eq. (7). According to the arguments presented here and in Ref. 2, it seems inconceivable that the field induced purely by gravity can be this small. The proper interpretation of this large body of experimental data is thus a serious challenge for future work.

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APPENDIX A: GRAVITATIONALLY INDUCED FIELD NEAR AN INSULATOR

All the discussion in this paper so far has been about a conducting body, i.e., a body with enough free electrons to make the screening length much smaller than its macroscopic dimensions. Though the opposite extreme of a perfectly insulating body is of little practical importance, it may provide a helfpul perspective to add, as a postscript, a few words about the extent to which the various effects we have been discussing have analogs in this case. The reciprocity relation described by Eqs. (3) and (4) still holds, and in fact holds for any one of the multitude of metastable states that can be formed by distributing trapped charges arbitrarily on the surface of the insulator or in its volume. Considering for simplicity just the ideal neutral state, let us turn attention first to the Schiff-Barnhill effect which for a hollow metal body is described by Eqs. (6) and (7), i.e., the potential induced by gravity if the elastic deformation of the body is inhibited by holding all the nuclei fixed. According to the reciprocity relation this potential is determined by the shift in the mass moment of the electronic system induced by the presence of a test charge q. This shift can be computed either from the integral of $\mathbf{P}$ over the body, where $\mathbf{P}$ is the polarization vector in the dielectric, or alternatively from the moment of $-\nabla\cdot\mathbf{P}$ in the volume and $\mathbf{n}\cdot\mathbf{P}$ on the surface. (These two expressions are easily shown to be equal.)

To find the electric field produced at a given point by the action of gravity, we thus compute the total moment induced in the specimen by a dipole placed at this point. In general, this moment is of the same order as that of the perturbing dipole (though of course it goes to zero with the susceptibility of the medium), and so the gravity-induced field is of the same order as Eq. (7). However, for points near the surface of a vertical elongated specimen of uniform cross section, the moment can be much less. For this case it is simplest to use reciprocity again and compute the field that would be produced at the point in question if the body were placed in a uniform external field. (This is in fact just the gravity problem we started with.) For a point near the equator of an ellipsoidal body, and approximately for other elongated shapes, the modification of the field at the point in question, due to the dielectric, is just $(1+N\chi)^{-1}N\chi$, times the applied field, where N ($\ll 1$ for this case) is the depolarization factor and χ the susceptibility. When $N\chi\ll 1$, therefore, the gravity-induced field is $\ll$ Eq. (7), and when $N\chi\gg 1$ it approaches Eq. (7), as it must if a dielectric of infinite susceptibility is to behave like a metal.

Just as in the case of a conductor, introduction of a test charge q will produce a shift of the nuclear mass moment if we cease constraining the nuclear positions, which will be related by the reciprocity theorem to a contribution to the gravitationally induced field. If the medium is not piezoelectric, this contribution can be related as before to the effect of the electric field of the test charge on the surface stress; despite the penetration of the electric field, there will be no body forces of the first order in q if the dielectric contains no macroscopic volume charges. In such case the gravitationally induced field may be expected to be of the same order as that for a metal [Eqs. (20)–(22)] though involving the geometry in a different way. In a piezoelectric medium, on the other hand, there will be piezoelectric deformations which in general will give rise to vastly larger mass shifts; these correspond to the large piezoelectric fields that would be produced by gravitational distortion.

APPENDIX B: THERMODYNAMICS OF SURFACE STRESS AND THE DIFFERENCE BETWEEN SOLIDS AND FLUIDS

The Lippmann equation of electrocapillarity theory[11] is usually interpreted to mean that the surface tension of a fluid interface varies quadratically with the charge on it, at least under "ideal" conditions. Actually, this equation and the experiments supporting it refer to an electrolytic interface (e.g., mercury in contact with an electrolyte) that is neutral as a whole, but which contains a double layer made up of electronic charge on the mercury and ionic charge in the electrolyte. In this Appendix we shall consider only the much simpler system of a solid or fluid conductor bordered by empty space, with an electronic surface charge induced by an externally imposed field. We shall show that for a liquid body of this type it is indeed true that the surface stress $G_{\mu\nu}$ (here equal to the unit tensor times the ordinary surface tension γ) varies quadratically with the surface charge density σ when σ is small. But for a solid surface we shall find that there are additional terms in $G_{\mu\nu}$, not present for liquids, and that $(\partial G_{\mu\nu}/\partial\sigma)_{\sigma=0}$ does not normally vanish.

Before plunging into the abstract formalism of thermodynamics it will be instructive for us to consider

[11] D. C. Grahame, Chem. Rev. 41, 441 (1947).

a simple model of a solid surface for which $(\partial G_{\mu\nu}/\partial\sigma)_{\sigma=0}$ is nonvanishing. The fact that such a model can be thought up already shows that there can be no purely thermodynamic requirement for this quantity to vanish. The model consists of two layers of idealized atoms (not the same as those of the substrate). The first layer is attached epitaxially to the crystal lattice. The second layer contains an equal number of atoms, each one lying directly over the midpoint between three (for hexagonal packing) or four (for square packing) of the first-layer atoms; the vertical separation between the layers is to be of the order of the interatomic spacing within the layers. The atoms of the second layer are joined by springs to their nearest neighbors in the first layer. Unit + charges are attached to all atoms of the one layer, unit − charges to those of the other. Such a surface has no net charge, and application of a normal electric field (whose lines of force may be supposed to end on charges in the substrate below the first layer) will clearly stretch the springs in such way as to make a first-order alteration in the surface stress. Note that the possibility of such a model depends on there being a crystal lattice to anchor to.

Turning now to thermodynamics, let us first consider the scalar quantity γ usually called "surface tension" (the stress implication of the name is unfortunate and should be disregarded). For any solid or liquid body γ is defined as the specific surface excess, in the sense of Gibbs,[12] of the thermodynamic potential

$$\Psi = U - TS - \sum_i \bar{\mu}_i N_i, \tag{B1}$$

where U is the energy, T the absolute temperature, S the entropy, N_i the number of moles (or atoms) of the ith component, and $\bar{\mu}_i$ the electrochemical potential of that component. The γ so defined has the property of being practically independent of the precise location of the dividing surface bounding the body, if the radii of curvature of the surface are always much larger than atomic dimensions. It also has the property of being independent of the choice of zero for the electrostatic potential, since a change of this zero shifts U and $-\sum_i \bar{\mu}_i N_i$ by cancelling amounts. Since in a general change

$$d\Psi = DW - S\,dt - \sum_i N_i d\bar{\mu}_i, \tag{B2}$$

where DW is the work done on the body, we have the familiar relation

$$d(\gamma A) = DW^s - As^s dt - A\sum_i \Gamma_i d\bar{\mu}_i, \tag{B3}$$

where A is the area of the surface (assumed uniform for simplicity), s^s is the surface entropy per unit area, Γ_i is the surface excess of component i per unit area, and DW^s is the "surface work," i.e., the difference between DW and the volume work, the latter being defined for each bulk phase as the work done on unit volume times the volume on the appropriate side of the dividing surface.

All this is standard surface thermodynamics. However, for our present application to charged surfaces of conductors one must use these concepts with caution. For Gibbs's definition of surface-excess quantities is based on the assumption that things like energy can be localized in space, and thus assigned volume densities. The interaction energy of macroscopic distributions of charge cannot be so localized without giving up the requirement, necessary for the type of thermodynamic treatment we wish to use, that the local energy density is a function only of the local state variables. The obvious thing to do is the analog of what we have already done for surface stress in Sec. 2 B of the text: Define a "local energy" $\tilde{U}$ as the difference between the total energy U of the system and the electrostatic energy U_{es} of any macroscopic distributions of charge; then define γ as the surface excess of a $\tilde{\Psi}$ defined by an equation of the form of Eq. (B1), but with $\tilde{U}$ replacing U and a corresponding $\tilde{\mu}_i$ replacing $\bar{\mu}_i$. Such a procedure is not in general unique, since one can define U_{es} in various ways, e.g., with inclusion or exclusion of the energy of polarization of any dielectric material present. However, one can draw correct conclusions from any definition that leaves $\tilde{U}$ a function only of local state. Moreover, with any definition of the sort considered, the value of γ remains independent of the positioning of the dividing surface.

For a conducting body bordered by vacuum the obvious definition to adopt is

$$U_{es} = \int \sigma(\psi_0 + \tfrac{1}{2}\psi_\sigma)\,dA\,, \tag{B4}$$

where the integral is over the surface of the body, σ is the surface charge density, ψ_0 is the potential due to external charges, and ψ_σ is that due to the distribution σ itself. Correspondingly, we have

$$\tilde{\mu}_i \equiv \left[\frac{\partial}{\partial N_i}(U - TS)\right]_{T,u} = \bar{\mu}_i - e_i\psi^*, \tag{B5}$$

where u stands for all strain variables, e_i is the specific charge of the ith component, and ψ^* is the value of $(\psi_0+\psi_\sigma)$ on or inside the conductor. Note that ψ^* is not normally the same as the local space average $\bar{\psi}$ of the electrostatic potential inside the body, since $\bar{\psi}$ can be influenced by such things as surface double layers, strain fields, etc., which do not show up in the macroscopic charge density σ. Equation (B2) is now modified to

$$d\tilde{\Psi} = (DW - dU_{es} + \psi^* \sum_i e_i dN_i) - SdT - \sum_i N_i d\tilde{\mu}_i, \tag{B6}$$

[12] J. W. Gibbs, *Collected Works* (Longmans, Green, and Company, Inc., New York, 1928), Vol. 1, pp. 55–353, especially pp. 219 ff.

i.e., in addition to the replacement of Ψ by $\tilde{\Psi}$, μ_i by $\tilde{\mu}_i$, one must replace the work DW by its "nonelectrostatic" part: DW was the work done by forces applied directly to the body, e.g., at boundary surfaces; the quantity in parentheses in Eq. (B6) is this work less that part of it which represents work done by these forces against long-range electrostatic interactions. Similarly, (B3) is modified to

$$d(\gamma A)=D\tilde{W}^s-As^s dT-A\sum_i \Gamma_i d\tilde{\mu}_i, \qquad \text{(B7)}$$

where $D\tilde{W}^s$ is the surface excess of the quantity in parentheses in (B6).

Consider now a change in which the conductor is charged by addition or subtraction of electrons, keeping constant the temperature, volume, state of strain, and *internal* composition. Under these conditions the internal state of the body will not change, except for a possible shift in the mean electrostatic potential $\bar{\psi}$. In particular, the chemical potentials μ_i, related to $\tilde{\mu}_i$ by $\tilde{\mu}_i=\mu_i+e_i(\psi-\psi^*)$, will not change. Thus, from (B7) and with $\sum_i \Gamma_i e_i=\sigma$,

$$\left(\frac{\partial\gamma}{\partial\sigma}\right)_{\text{is}}=-\sigma\left(\frac{\partial(\bar{\psi}-\psi^*)}{\partial\sigma}\right)_{\text{is}}, \qquad \text{(B8)}$$

where the subscripts "is" mean that internal state is to be kept constant. The derivative on the right of (B8) measures the effect of charging on the magnitude of the surface double layer; it is much smaller than the capacitative quantity $\partial\psi^*/\partial\sigma$. Our treatment differs from that of the usual theory of electrocapillarity in that we have excluded the long-range classical electrostatic energy from γ, while in electrocapillarity theory the electric fields are confined to a Debye layer so thin that it is convenient to include their energy in γ; such a γ leads to an equation (the Lippmann equation) like (B8) but with ψ^* missing. For our present purpose, however, it is sufficient to note that (B8) has σ as a factor, hence vanishes at $\sigma=0$.

The relation of the surface-stress tensor $G_{\mu\nu}$ to γ can be seen by considering the effect of an infinitesimal strain applied so as to stretch or shear the surface; let the temperature, total charge, and internal mole numbers be kept constant. With $du_{\mu\nu}$ for the applied strain tensor, and noting that our removal of long-range electrostatic forces from the definition of $G_{\mu\nu}$ in Sec. 2 B corresponds to the removal of these forces from the $D\tilde{W}^s$ of (B7), we have

$$\sum_{\mu,\nu} G_{\mu\nu}du_{\mu\nu}=A^{-1}D\tilde{W}^s=\sum_{\mu,\nu}\left(\gamma\frac{\partial\ln A}{\partial u_{\mu\nu}}+\frac{\partial\gamma}{\partial u_{\mu\nu}}\right)du_{\mu\nu}+\sum_{\mu,\nu}\sum_i \Gamma_i\frac{\partial\tilde{\mu}_i}{\partial u_{\mu\nu}}du_{\mu\nu},$$

whence finally

$$G_{\mu\nu}=\gamma\delta_{\mu\nu}+\left(\frac{\partial\gamma}{\partial u_{\mu\nu}}\right)_{\text{int }N}+\sum_i \Gamma_i\left(\frac{\partial\tilde{\mu}_i}{\partial u_{\mu\nu}}\right)_{\text{int }N}, \qquad \text{(B9)}$$

where the subscript "int N" implies constancy of charge and internal mole numbers, as well as temperature. For a liquid both strain derivative terms vanish: Displacing lateral boundaries of the body does not change the nature of its surface, but merely alters the area or shape. For a solid the strain alters lattice spacings at the surface, hence in general changes γ and the surface double layer moment. The first two terms of (B9) agree with the form previously given[5,13] for $G_{\mu\nu}$ except that the latter has $(\partial\gamma/\partial u_{\mu\nu})_{\mu's}$; if all $e_i=0$, the last terms of (B9) can be combined into this form.

Let us now consider the derivative of $G_{\mu\nu}$ with respect to σ at $\sigma=0$, for the case of a solid. Equation (B8) ensures the vanishing of the contributions of the first two terms of (B9), so only that from the last term remains:

$$\left(\frac{\partial G_{\mu\nu}}{\partial\sigma}\right)_{\text{is};\,\sigma=0}=\sum_i\left(\frac{\partial\Gamma_i}{\partial\sigma}\right)_{\text{is};\,\sigma=0}\left(\frac{\partial\tilde{\mu}_i}{\partial u_{\mu\nu}}\right)_{\text{int }N;\,\sigma=0}+\sum_i\left(\Gamma_i\frac{\partial^2\tilde{\mu}_i}{\partial u_{\mu\nu}\partial\sigma}\right)_{\sigma=0}. \qquad \text{(B10)}$$

For a clean surface we may position the dividing surface so as to make $\Gamma_i=0$ for the nuclear component, and $=\Gamma\sigma/e$ for the electrons. The second term on the right of Eq. (B10) then vanishes and the first reduces to $(\partial\varphi/\partial u_{\mu\nu})_{\text{int}N;\,\sigma=0}$, where φ is the work function. We thus recover Eq. (23) of the text. When there are mobile absorbed components present other than electrons, the situation is more complicated: Charging the surface can affect both the Γ_i of these and the surface-stress forces to which they give rise.

[13] R. Shuttleworth, Proc. Phys. Soc. (London) **A63**, 444 (1950); C. Herring, in *The Physics of Powder Metallurgy*, edited by W. E. Kingston (McGraw-Hill Book Co., New York, 1951), p. 143.